Helsinki Center of Economic Research

Discussion Papers

Backstop Technology Adoption

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Discussion Paper No. 94
January 2006

ISSN 1795-0562

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HECER
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Abstract

We consider how efficient markets adopt technologies that reduce dependence on volatile factors such as oil. We find a relationship between volatility and technology overlap: new technology entry rate exceeds old technology exit rate under sufficient uncertainty. From this follows that efficient adoption is characterized by prolonged coexistence of alternative technologies and that uncertainty increasingly propagates from input to output market despite the declining use of the volatile factor in production. The properties depend on (i) the option to remain idle rather than exit, (ii) heterogeneity in factor supply, and (iii) factor market volatility.

JEL Classification: D1; D9; O30; Q40

Keywords: technology adoption; factor markets; uncertainty; irreversible investment; energy

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* We thank Pierre Lasserre, Pierre-Olivier Pineau, and seminar participants at CAS-Oslo, HECER-Helsinki, MIT IO workshop, and CREE-Montreal for many useful comments. Funding from the Academy of Finland, Nordic Energy Research Program, and Yrjö Jahnsson Foundation is gratefully acknowledged.

"The concept that is relevant to this problem is the *backstop technology*, a set of processes that (1) is capable of meeting the demand requirements and (2) has a virtually infinite resource base".

(William D. Nordhaus, 1973, pp. 547-548)

1 Introduction

More than 30 years have passed since the first oil price shock but the dependency on oil is still at the forefront of public concern. It is perhaps no longer the finiteness of long-term factor supply but the risk of economic disruption due to volatility of prices that is concerning.[1] Will the alternative technologies – backstop technologies – that reduce the dependence on the volatile fossil fuel markets ever enter the market in large scale? Although much has been said about the potential market inefficiencies delaying the entry of new technologies, the more basic question of efficient market solution to the factor-dependency problem is yet to be answered. In this paper, we approach the question by considering how competitive equilibrium coordinates the irreversible entry of factor-free and exit of factor-dependent technologies when the factor supply is uncertain and declining over time. We find a relationship between factor market volatility and technology overlap: efficient new technology entry rate exceeds old technology exit rate under sufficient volatility. In this sense, factor market uncertainty provides an efficiency justification for prolonged coexistence of alternative technologies – it is socially optimal to adopt new technologies to coexists with the old factor demand infrastructure until the uncertainty about the future factor supply sufficiently resolves. Thus, no market failures are needed for the phenomenon that old technologies do not seem to give way to the new ones.

William Nordhaus (1973) introduced the concept of backstop technology and analyzed the timing of entry of such technologies in markets for factors that are finite in supply, a feature of most energy commodities. Following his reasoning, it is usual to think that the backstop technology entry depends on the overall factor supply that is exhausted before it is profitable for the new technology to enter. While scarcity rents may ultimately become important, it seems far less obvious today than in the 1970s that scarcity rents alone could be important for technology choices.[2] In contrast, most economists agree that

[1]Average year-to-year fluctuation of oil price was within 1% of the price level during the years 1949-1970, whereas this number jumbed to 30% from 1970 to 2002 (Smith, 2002).

[2]See Krautkramer (1999) for a survey of the empirical success of the Hotelling model.

factor markets, oil market in particular, are characterized by supply-side shocks. Yet, factor price volatility has no role in the existing elaborations of the backstop technology adoption. This seems potentially a serious omission since the volatility clearly affects the profitability of production using the volatile factor while backstop technologies are, by definition, free from factor market volatility. This asymmetry with which uncertainty enters together with the fact that the decisions to reduce the dependence on the factor market are irreversible suggest that the transition to backstop technologies may not be well understood without the factor market uncertainty.

While the energy sector is our prime motivation, we make general conclusions for the factor-market induced technology adoption under the following preconditions. First, factor demand infrastructure is long-lived and costly to maintain. When factor market conditions turn unfavorable, utilization of the technology can be adjusted or technology units can exit irreversibly. We thus consider situations where idleness, while costly, is an alternative to exit, which seems a particularly relevant case in the energy sector. Second, there is heterogeneity among factor supply sources, implying an upward sloping supply curve and ensuring that those who reduce the usage of the factor-dependent technology, all else equal, relax the factor market conditions for those who still use the technology. Third, the factor market is subject to supply-side shocks. In the oil market such shocks are related to wars and political instability, uncertain reservoir levels, accidents in refineries, sporadic success in market power, hurricanes hitting oil fields, etc. These occur around a deterministic trend reflecting the presence of scarcity rents if the overall factor supply is finite.[3] Finally, there is an alternative technology which can serve the same output market without using the volatile factor. We thus consider relatively mature technologies that can be irreversibly adopted by incurring a costly up-front investment. In the energy sector such technologies are nuclear, solar, wind, geothermal, biofuel and biomass, proving a backstop for fossil fuels. We may also include energy saving technologies in our definition of backstop technologies - investments in those technologies also reduce the demand for volatile factors in supplying some output market.[4]

Given these characteristics of the factor demand and supply, we describe the qualitative phases of the adoption path as a function of the declining factor supply. In the first

[3]However, we do not explicitly model the factor as an exhaustible resource.

[4]In general, adoption of energy saving technologies shows up in cross-section data across countries: energy use or investments in capital goods with different energy intensities are responsive to permanent differences in energy prices (Berndt and Wood, 1975; Atkenson and Kehoe, 1999).

phase the factor supply is still abundant, implying low factor prices and full utilization of the factor-dependent technology. Since the technology is in full use, it absorbs factor market shocks into its profits and, therefore, uncertainty is not transmitted to the output price. The alternative technology faces then no uncertainty and, if entry is profitable, it replaces old technology units one-to-one as factor market conditions gradually worsen.

In the second phase the factor supply declines to a level that forces fraction of the old technology units to idleness. In this phase, the technology usage is adjusted when factor market conditions change and, therefore, uncertainty is transmitted to the output price, i.e., to the profits of the entrant technology. This propagation of uncertainty makes the expected payoff to both technologies uncertain but the effect is asymmetric: whenever factor supply declines to a record level and new firms enter, they do not replace old firms one-to-one because a fraction of old firms chooses idleness instead. It is this buffer of idle firms between active and exiting firms that leads to the technology overlap; the overall availability of technology units increases as the factor supply continues to decline.

An important feature of the second phase is that aggregate output becomes less and less factor intensive – the market share for the new technology increases – and yet the factor-induced output price volatility increases during the transition. Factor market shocks are transmitted to the output market to a greater extent, the larger is the fleet of remaining but idle old technology units that can respond to factor market conditions. It is a general property of backstop adoption paths that large scale idleness precedes the final decline of the old technology, leading to the necessary existence of this volatile capacity.[5]

The third and final phase is about the old technology decline. All old technology firms are in the buffer of idle firms and each entering firm replaces more than one old firm whenever new factor market records are reached. The technology overlap thus declines and, for sufficiently small factor supply, entrants have replaced all old technology firms. The output uncertainty finally vanishes as the dependence on the factor market is completely eliminated.

The main difference between our work and the earlier literature on technology adoption is that we do not consider one-to-one replacement of technologies by assumption.[6]

[5]Macroeconomists find it puzzling that the oil prices have an aggregate effect despite the low cost share of oil in GDP (See, e.g., Barsky and Kilian (2004) and Hamilton (2005)). One potential explanation is that factor price changes are propagated through movements in other factor prices they induced. We do not consider macroeconomic effects but do intend the price volatility result to be suggestive of a propagation channel.

[6]See Reinganum (1989) and Hoppe (2002) for surveys of the technology adoption literature.

Instead, we model the equilibrium exit and entry of technologies, implying that the aggregate availability of technology units can change along the equilibrium path. However, we obtain the one-to-one replacement paths in equilibrium, for example, if volatility is sufficiently small or if the old technology does not have the option to remain idle but can only exit. Also, most of the adoption literature considers adoption in environments where strategic issues and externalities are important whereas we consider a competitive equilibrium without distortions; our backstop adoption paths maximize the social surplus.

There is a large but somewhat dated literature on backstop technologies (for example, Nordhaus 1973, Dasgupta and Heal 1974, Heal 1976). Without exceptions known to us, this research casts the adoption problem in an exhaustible-resource framework without uncertainty. The models from the 70s typically feature a switch to the backstop as soon as the resource is physically or economically depleted. While such models are helpful in gauging the limits to resource prices using the backstop cost data (see Nordhaus), the predictions for the backstop technology entry are not entirely plausible if one accepts the characteristics of factor markets and energy demand infrastructure that we have outlined above. A more realistic backstop technology entry is obtained in Charkravorty et al. (1997) where the demand for exhaustible factors is heterogenous and backstop technologies such as solar energy have a declining trend in adoption costs. We provide an alternative approach to gradual backstop technology transition where factor market price trends and volatility are distinct determinants of the expected long-run market shares for the technologies.

Methodologically our model is closely related to the real options approach on irreversible investment. As, e.g., Dixit (1989), Pindyck (1993), Leahy (1993), and Caballero and Pindyck (1996), we consider equilibrium behavior of a large number of rational agents in such a context. A distinct feature of our model in comparison to those papers is that we have a two-dimensional state space due to the capital stocks associated with the two technologies. In particular, our equilibrium concept and the technique for solving it can be seen as generalizations of Leahy (1993) to multiple dimensions. Other papers that consider costly adjustment in multiple dimensions include Dixit (1997) and Eberly and Van Mieghem (1997), but in a context that is in many ways quite different from ours.

The paper is organized as follows. In Section 2, we introduce the agents, technologies, markets, and define the equilibrium. We also state the main Theorem of existence which is proved in Appendix. Section 3 then progresses as a sequence of propositions characterizing the equilibrium. We explain how volatility determines the nature of the transition (Section 3.1), characterize the output price volatility (Section 3.2.), and discuss the determinants of the long-run market shares for technologies (Section 3.3.). In Section 4, we conclude by discussing the robustness of the qualitative features, and the lessons for energy policies.

2 Model

2.1 Production technologies

There are two technologies, the old and new, for producing the same homogenous output. The old technology is a fixed proportions technology using one unit of a factor (say, oil) for one unit of output. The old technology is embodied in old capacity units that constitute the demand infrastructure for the factor. The demand infrastructure is given by history and it can respond to output and factor market conditions by adjusting utilization and scrapping capacity units.

The new technology is embodied in backstop capacity units that do not use the factor – one installed backstop unit produces one unit of output for free but the installation of such a unit requires a costly up-front investment.

We assume that there is a continuum of infinitesimal firms, and each active firm has one unit of capital of either type. If we let k_t^f and k_t^b denote the respective total factor-dependent and backstop capacities at time t, then k_t^f and k_t^b denote also the numbers of firms at t. By k_0^f and k_0^b we refer to exogenously given initial capacity levels. Each factor-dependent firm that is still in the industry at some given t must choose one of the following options: produce, remain idle, or exit. To make the choice between idleness and exit interesting, we assume that staying in the industry implies an unavoidable cost per period. Let $c > 0$ denote this fixed flow cost. A producing unit in period t thus incurs cost $c + p_t^f$, where p_t^f is the factor price. An idle unit pays just c. An exiting unit pays a one-time cost $I_f > 0$ and, of course, avoids any future costs. Note that, in equilibrium, firms (discrete) choices between production and idleness determine the overall utilization of the old capacity. Let q_t^f denote the total output from the factor dependent capacity. Then, q_t^f is also the number of producing firms which satisfies $q_t^f = k^f$ if all remaining firms produce, and $0 \leq q_t^f < k^f$ if utilization is adjusted.

Consider then the build-up of the backstop technology. We assume infinitely many potential entrants which can adopt the technology by paying the up-front investment cost $I_b > 0$. Once installed, a backstop unit produces output without using the factor or other variable inputs. Without loss of generality, we also normalize the unavoidable cost flow from running a backstop plant to zero.[7] The assumption that the technology uses no variable inputs makes the idleness an irrevelant option for these units.[8] Thus, throughout this paper we have $q_t^b = k_t^b$, where q_t^b denotes the total output from backstop capacity units in period t.

All agents are risk neutral, have infinite time horizons, and discount with rate r (time is continuous). The following restriction holds throughout the paper:

$$I_f < \frac{c}{r} < I_f + I_b.$$

The first inequality implies that exit saves on unavoidable costs for an old capacity unit. The second inequality implies that replacing an old unit by a new unit is costly. Without the former restriction, old plants would never exit. Without the latter, the factor-dependent capacity would be scrapped and new capacity built immediately.

2.2 Output and factor markets

Ignore the firms entry and exit decisions for a while and suppose that the numbers of technology units, $(k_t^f, k_t^b) = (k^f, k^b)$, are fixed over time. In period t, the output price P_t that clears the market is given by inverse demand $P_t = D(q_t)$ that is continuously differentiable and decreasing in $q_t = q_t^f + q_t^b$. The inverse supply curve for the factor is

$$p_t^f = x_t + C(q_t^f) \geq 0, \tag{1}$$

where intercept $x_t \geq 0$ is a stochastic variable capturing the factor market volatility, and $C : \mathbb{R}_+ \to \mathbb{R}_+$ is a continuous and strictly increasing function for which $C(0) = 0$. Variable x_t follows Geometric Brownian Motion with drift $\alpha > 0$ and standard deviation σ,

$$dx_t = \alpha x_t dt + \sigma x_t dz_t. \tag{2}$$

Note that the trend in the intercept x_t captures the idea that the equilibrium supply is expected to decline over time.[9] We will use notation $\{x_t\}$ to denote to the stochastic

[7]Because the entry is irreversible, one may calculate the present value of such costs and include them in the investment cost.

[8]In fact, to make plant utilization of the backstop technology a relevant issue in our model, variable production cost should be made very high relative to the installation cost.

[9]The solution of the model does not require the positive trend but some qualitative results depend on the assumption; see Section 3.3.

process defined by (2), while x_t refers to the value of this process at time t.[10] Let x_0 denote a given initial value for this process.

The formulas (1) and (2) for the factor supply and shocks are somewhat restrictive. However, our main theorem (Theorem 1) would hold under a more general formulation, where factor supply is given by a function $p_t^f = C(q_t^f, x_t)$ with appropriate restrictions on its derivatives (e.g. admitting x_t to enter multiplicatively), and with (2) replaced by a more general form $dx_t = \alpha(x_t)\,dt + \sigma(x_t)\,dz_t$ (with some appropriate restrictions on functions $\alpha(\cdot)$ and $\sigma(\cdot)$). The reason for choosing to work with formulas (1) and (2) is that this allows a clean characterization and straight-forward interpretation of the effect of volatility, but at the same time it is good to keep in mind that the main message of our model is not dependent on those specific formulas.

Let us now consider the equilibrium quantities supplied to the output market. Remember that the new technology always supplies $q^b = k^b$,[11] and denote by $q^f\left(x_t; k^f, k^b\right)$ the equilibrium quantity supplied by the old technology units at the current shock value x_t. The output price can then be written as:

$$P_t = P(x_t; k^f, k^b) = D\left(q^f\left(x_t; k^f, k^b\right) + k^b\right). \tag{3}$$

The factor market uncertainty is transmitted to the output price if $q^f\left(x; k^f, k^b\right)$ is responsive to shocks, which in turn depends on the following critical values for x:

$$\begin{aligned} \underline{x}(k^f, k^b) &\equiv D(k^f + k^b) - C(k^f), \\ \overline{x}(k^f, k^b) &\equiv D(k^b). \end{aligned}$$

If $x < \underline{x}(k^f, k^b)$, factor supply is so high that it is optimal for all old technology units to produce. Then the overall capacity constraint is binding, $q^f\left(x_t; k^f, k^b\right) = k^f$, which drives a wedge between the equilibrium output and factor prices, $P_t > p_t^f$. See also Fig. 1. In that case we say that the factor market conditions are favorable to old firms. The factor market conditions are unfavorable to old firms if $x \geq \underline{x}(k^f, k^b)$, because then some firms must remain idle, and $q^f\left(x_t; k^f, k^b\right) < k^f$. Then also $P_t = p_t^f$ which implies no flow surplus covering the unavoidable cost c. If $\underline{x}(k^f, k^b) < x_t < \overline{x}(k^f, k^b)$, the equilibrium

[10] Formally, $\{x_t\}$ is a sequence of random variables indexed by $t > 0$ defined on a complete probability space $(\Omega, \mathcal{F}, P)$. We denote by $\{\mathcal{F}_t\}$ the filtration generated by $\{x_t\}$, i.e. $\mathcal{F}_t$ contains the information generated by $\{x_t\}$ on the interval $[0, t]$.

[11] In equilibium where (k^f, k^b) are endogenous, the market can always absorb this quantity, $D(k^b) > 0$.

output $q^f\left(x_t;k^f,k^b\right)$ is positive, and chosen to equate factor and output price, i.e. it is implicitly given by the condition:

$$x_t + C\left(q^f\left(x_t;k^f,k^b\right)\right) = D\left(q^f\left(x_t;k^f,k^b\right)+k^b\right).$$

If $x_t > \overline{x}(k^f,k^b)$, all capacity units k^f must remain idle, that is, $q^f\left(x_t;k^f,k^b\right) = 0$. Since $q^f\left(x;k^f,k^b\right)$ is responsive to shocks within the interval $\left(\underline{x}(k^f,k^b),\overline{x}(k^f,k^b)\right)$, this means that the factor market uncertainty is transmitted to the output market when the shock value lies within that interval, otherwise factor market and output market are disconnected.

Since an active backstop capacity has no production costs, the cash-inflow of such a unit is simply equal to the output price (3). Note that this captures the idea that the new technology's payoff is uncertain because of the factor market condition determining the competitiveness of the old technology. We can also see at this point that when k^f capacity goes to zero so does the output price uncertainty.

INSERT FIGURE 1 HERE OR BELOW

The factor-dependent capacity must buy the factor in order to produce, and hence it generates the following cash-flow:

$$\pi_f(x_t;k^f,k^b) = \begin{cases} \underline{x}(k^f,k^b) - x_t - c, \text{ when } x_t < \underline{x}(k^f,k^b) \\ -c, \text{ when } x_t \geq \underline{x}(k^f,k^b) \end{cases}. \tag{4}$$

Let us now pull together the basic assumptions as follows.

ASSUMPTIONS : *We consider a competitive industry where the following hold:*

1. *All agents are risk neutral and discount with rate $r > 0$.*
2. *There is a continuum of factor-dependent firms, each choosing one of the following options per period: produce a unit of output, remain idle, or exit. Production cost is the factor price, $p_t^f \geq 0$. Staying in the industry costs $c > 0$ per period for both producing and idle firms, and irreversible exiting costs $I_f > 0$.*
3. *There is a continuum of potential entrants to the industry. Entry is irreversible and costs $I_b > 0$. Each entrant produces a unit of output for free.*
4. *Exit saves on unavoidable costs but replacing technologies is costly:*

$$I_f < \frac{c}{r} < I_f + I_b.$$

5. *Inverse demand for output, $D(q)$, is continuously differentiable and decreasing in q.*

6. *Inverse supply for the factor is $x + C(q^f)$, where x follows Geometric Brownian Motion with a positive drift and $C(q^f)$ is continuous and strictly increasing in q^f.*

2.3 Equilibrium capacity paths

Let us now allow the capacities k_t^f and k_t^b to change over time as new plants are built and old ones are scrapped. The information on which the firms base their behavior at period t consists of the historical development of x_t, k_t^f, and k_t^b up to time t. This means that the resulting capacity paths are stochastic processes $\{k_t^f\}$and $\{k_t^b\}$ such that their values at time t depend on the history of $\{x_t\}$ up to that moment[12]. Since factor dependent capacity (backstop capacity) can only be decreased (increased), we must impose a restriction on the set of admissible capacity paths according to which $\{k_t^f\}$ ($\{k_t^b\}$) must be non-increasing (non-decreasing).

Even with this restriction the capacity levels at time t could in principle depend in complicated ways on the entire history of $\{x_t\}$ up to t. However, $\{x_t\}$ being a Markov process, it is not the entire history but the current value that matters to the firms' behavior. The higher the value of x_t, it becomes not only more attractive to exit or remain idle but also invest in backstop technology because entrants face less competition from active old technology firms. In any sensible description of the firms' behavior, it will always be the case that the capacities only change when x_t reaches new historical maximum values, and thereby, the capacities at time t will depend on the history of x_t only through the historical record value, which we denote by

$$\hat{x}_t \equiv \sup_{\tau \leq t}\{x_\tau\}.$$

In this paper we only need to consider capacity paths that describe the evolution of the capacities as functions of $\hat{x}_t$. In describing the equilibrium capacity paths, we treat the initial state consisting of the tuple $\{x_0, k_0^f, k_0^b\}$ as an exogenously given model parameter, but our characterization applies to any possible value combination for those parameters. Using boldface notation to denote capacities as such functions, we define admissible capacity paths as follows:

[12]That is, they are stochastic processes adapted to the filtration $\{\mathcal{F}_t\}$.

Definition 1 *An admissible capacity path is a pair* $\mathbf{k} = \left(\mathbf{k}^f, \mathbf{k}^b\right)$ *consisting of two mappings: a non-increasing, right-continuous function* $\mathbf{k}^f : [x_0, \infty) \rightarrow \mathbb{R}_+$ *and a non-decreasing, right-continuous function* $\mathbf{k}^b : [x_0, \infty) \rightarrow \mathbb{R}_+$*, where* $\mathbf{k}^f(\hat{x}_t)$ *gives the level of factor dependent capacity and* $\mathbf{k}^b(\hat{x}_t)$ *gives the level of backstop capacity at time* t *as functions of the historical maximum for* x_t*. We say that* $\mathbf{k}^f$ *(*$\mathbf{k}^b$*) adjusts at* $\hat{x} > x_0$ *if* $\mathbf{k}^f(\hat{x}) < \mathbf{k}^f(\hat{x} - \epsilon)$ *(*$\mathbf{k}^b(\hat{x}) > \mathbf{k}^b(\hat{x} - \epsilon)$*) for an arbitrarily small* $\epsilon > 0$*. We say that* $\mathbf{k}^f$ *(*$\mathbf{k}^b$*) adjusts at* x_0 *if* $\mathbf{k}^f(x_0) < k_0^f$ *(*$\mathbf{k}^b(x_0) > k_0^b$*).*

INSERT FIGURE 2 HERE

Note that an admissible capacity path admits one to describe the evolution of the capacities as stochastic processes $\{k_t^f\} \equiv \left\{\mathbf{k}^f(\hat{x}_t)\right\}$ and $\left\{k_t^b\right\} \equiv \left\{\mathbf{k}^b(\hat{x}_t)\right\}$. As we progress, we will illustrate the results using Fig. 2. At this point, ignore all else but the admissible capacity paths, $\mathbf{k}^f$ and $\mathbf{k}^b$. Let us now consider individual firms' optimal investment and scrapping decisions. Consider first a firm, which owns a unit of factor dependent capacity. Assume that this firm anticipates correctly the capacity path $\mathbf{k} = \left(\mathbf{k}^f, \mathbf{k}^b\right)$ induced by the behavior of all other firms, and chooses the optimal time to scrap its own capacity unit at cost I_f. The value of this firm at t is a function of the current value x_t and the historical maximum value $\hat{x}_t$:

$$V_f(x_t, \hat{x}_t; \mathbf{k}) = \sup_{\tau^* \geq t} E\left[\int_t^{\tau^*} \pi_f(x_\tau; \mathbf{k}^f(\hat{x}_\tau), \mathbf{k}^b(\hat{x}_\tau)) e^{-r(\tau - t)} d\tau - I_f e^{-r(\tau^* - t)}\right], \qquad (5)$$

where τ^* is an optimally chosen scrapping time[13]. Note that all active units are alike and therefore solve the same exit problem, but as will be formalized shortly, in equilibrium there is rationing of exit such that the firms staying and leaving make the same ex-ante profit.[14]

On the other hand, the owner of backstop capacity has no decisions to make, and hence the value of an infinitesimal unit of such capacity is given by:

$$V_b(x_t, \hat{x}_t; \mathbf{k}) = E\int_t^{\infty} P(x_\tau; \mathbf{k}^f(\hat{x}_\tau), \mathbf{k}^b(\hat{x}_\tau)) e^{-r(\tau - t)} d\tau. \qquad (6)$$

[13] τ^* is a stopping time adapted to the filtration $\{\mathcal{F}_t\}$.

[14] Without affecting the equilibrium we can also assume that factor-dependent firms are heterogenous and produce the factor "in house" rather than buy it from the market. Then, heterogeneity is equivalent to assuming an upward sloping supply curve for the factor. In this interpretation, x_t is a productivity shock common to all firms. Yet another interpretation is that firms buy the factor with price x_t and differ in their efficiency in using the factor. Also, we could let x_t affect firms asymmetrically by introducing it multiplicatively into the model. This would not affect the main Theorem of the paper.

One unit of the backstop technology can be adopted by paying cost $I_b > 0$. All potential entrants to the backstop sector are effectively holding an option to install one unit, so they solve the following stopping problem:

$$F_b(x_t, \hat{x}_t; \mathbf{k}) = \sup_{\tau^* \geq t} E\left[\int_{\tau^*}^{\infty} P(x_\tau; \mathbf{k}^f(\hat{x}_\tau), \mathbf{k}^b(\hat{x}_\tau))e^{-r(\tau-t)}d\tau - I_b e^{-r(\tau^*-t)}\right], \tag{7}$$

where $F_b(\cdot)$ is the value of the option to enter. Again, all the potential entrants are alike and solve the same entry problem, but in equilibrium with unrestricted entry there is rationing that makes each entrant indifferent between entering and staying out. Of course, this means that $F_b(\cdot) = 0$ in equilibrium.

Let us now define formally the competitive equilibrium as a rational expectations Nash equilibrium in entry and exit strategies such that, given the entry and exit points of all firms, no firm can find any strictly more profitable entry and exit points (including the possibility of not entering or exiting at all). More precisely, we want to find capacity path $\mathbf{k}$ such that when firms take it as given, entering firms are indifferent between investing and remaining inactive, and exiting firms are indifferent between staying and leaving.

Consider first entering firms for which we must in equilibrium have for all $\hat{x}_t \geq x_0$, and $x_t \leq \hat{x}_t$:

$$F_b(x_t, \hat{x}_t; \mathbf{k}) = 0\text{, and} \tag{8}$$

$$V_b(x_t, \hat{x}_t; \mathbf{k}) - I_b = 0 \text{ if } x_t = \hat{x}_t \text{ and } \mathbf{k}^b(\cdot) \text{ adjusts at } x_t\text{, and} \tag{9}$$

Equation (8) means that no entrant can make a positive ex-ante profit (free entry condition), and equation (9) means that entrants do not make loss upon entry, i.e., every entrant makes a zero ex-ante profit.

To develop the equilibrium conditions for the old technology firms, let

$$\tau_{x^*} \equiv \inf\{t \geq 0 \,| x_t \geq x^*\}$$

be the stochastic time it takes for the process to reach some given level x^*. We want to think of x^* as any such future factor market condition at which some firms exit, and to ensure that we have an equilibrium, we must require that those firms can not do better by choosing some alternative exit strategy. Formally, we require that for all $\hat{x}_t \geq x_0$, $x_t \leq \hat{x}_t$, and for all $x^* \geq \hat{x}_t$ such that $\mathbf{k}^f(x^*)$ adjusts (i.e. some firms exit):

$$E\left[\int_t^{\tau_{x^*}} \pi_f(x_\tau; \mathbf{k}^f(\hat{x}_\tau), \mathbf{k}^b(\hat{x}_\tau))e^{-r(\tau-t)}d\tau - I_f e^{-r(\tau_{x^*}-t)}\right] = V_f(x_t, \hat{x}_t; \mathbf{k}). \tag{10}$$

To understand this condition, recall that $V_f(x_t, \hat{x}_t; \mathbf{k})$ is the value generated by the optimal exit time τ^* (see (5)). The condition (10) thus says that the firm who will exit at x^* cannot achieve more by choosing some other exit time. Since (10) must hold for all x^* where some firms exit, it means that all firms who exit along $\mathbf{k}$ do so at an ex-ante optimal moment.

Finally, we must require that whenever some firms stay, there is some future exit time that gives them as high payoff as they would get by exiting. The purpose of this final requirement is to rule out the capacity path where some firms stay at infinitely high shock values. Formally, we require that for all $\hat{x}_t \geq x_0$, $x_t \leq \hat{x}_t$, and for all $x^* \geq \hat{x}_t$ such that $\mathbf{k}^f(x^*) > 0$:

$$\sup_{\tau^* \geq \tau_{x^*}} E\left[\int_t^{\tau^*} \pi_f(x_\tau; \mathbf{k}^f(\hat{x}_\tau), \mathbf{k}^b(\hat{x}_\tau)) e^{-r(\tau - t)} d\tau - I_f e^{-r(\tau_{x^*} - t)}\right] = V_f(x_t, \hat{x}_t; \mathbf{k}). \quad (11)$$

Thus, whenever some k_f-firm remains in the market (i.e. $\mathbf{k}^f(x^*) > 0$), this firm can not do better by exiting earlier (in the ex-ante sense).

Definition 2 *An admissible capacity path* $\mathbf{k} = (\mathbf{k}^f, \mathbf{k}^b)$ *is an equilibrium, if (8)-(11) hold.*

For intuition, we will now discuss equilibrium conditions for entering and exiting firms in more detail. Consider first entry and suppose that current period t is an entry point, i.e., factor market condition hits a new record, $x_t = \hat{x}_t$, and thus $V_b(\hat{x}_t, \hat{x}_t; \mathbf{k}) - I_b = 0$ by equation (8). Because entrants must be indifferent between entering now or at the "next" entry time, we have

$$E\{I_b(1 - e^{-r(\tau^{**} - \tau^*)})\} = E\int_{\tau=\tau^*}^{\tau^{**}} P_\tau e^{-r(\tau - \tau^*)} d\tau. \quad (12)$$

where $P_\tau = P(x_\tau; \mathbf{k}^f(\hat{x}_t), \mathbf{k}^b(\hat{x}_t))$ and τ^*, τ^{**} are two consecutive entry points.[15] Note that capacities are constants between the two time points since $\mathbf{k}$ changes only when x_t reaches a new record value, which occurs at the next entry time. The LHS is what, in expectations, the firm could save in costs by postponing entry to the next point at which factor market conditions favor entry. Because this reasoning must hold between any two consecutive entry points, we can write the indifference condition (12) as follows

$$I_b = E\int_{\tau^*}^{\infty} P_\tau e^{-r(\tau - \tau^*)} d\tau. \quad (13)$$

[15] Since the purpose is merely to give correct intuition here, we use somewhat loose argumentation. Think of (τ^*, τ^{**}) as a time interval during which x_t makes an "excursion" downwards from a historic maximum value and back to that same value.

There will be no exceptions to this rule: the discounted value of the equilibrium price process will be equal to the entry cost at any equilibrium entry point.

Consider now exit and suppose that current period t is an exit point, i.e., factor market condition hits a new record, $x_t = \hat{x}_t$, and thus $V_f(\hat{x}_t, \hat{x}_t; \mathbf{k}) = -I_f$. Following the logic from above, an exiting firm must be indifferent between two consecutive exit points:

$$E\{(\frac{c}{r} - I_f)(1 - e^{-r(\tau^{**}-\tau^{*})})\} = E\int_{t=\tau^*}^{\tau^{**}} \{P_\tau - p_\tau^f\} e^{-r(\tau-\tau^*)} d\tau, \tag{14}$$

where $P_\tau = P(x_\tau; \mathbf{k}^f(\hat{x}_t), \mathbf{k}^b(\hat{x}_t))$ and $p_\tau^f = x_\tau + C(q^f(\mathbf{k}^b(\hat{x}_t), x_\tau))$. Again, $\mathbf{k}$ is fixed between two consecutive entry and exit points. The LHS is the expected cost from delaying exit, recall that $\frac{c}{r} - I_f > 0$, and the RHS is the expected surplus from this delay (see Fig. 1 to see why the flow payoff takes this form). Note that the reason to stay in the industry is that in expectations the total capacity constraint will bind before the next entry point, implying rents for the old capacity units under favorable factor market conditions, i.e., when $x_\tau < \underline{x}(\mathbf{k}^f(\hat{x}_t), \mathbf{k}^b(\hat{x}_t))$ and thus $P_\tau - p_\tau^f > 0$. This same reasoning holds for any two consecutive equilibrium exit points: the cost from staying rather than exiting at an equilibrium exit point equals the expected present value of rents from being able to produce under favorable factor market conditions.

While the indifference conditions for marginal entering and exiting firms are intuitive, they are not yet helpful in characterizing the technology transition, i.e., the entire capacity path $\mathbf{k}$. The key to the characterization is the observation that a marginal firm which understands the stochastic process $\{x_t\}$ but disregards the other firms' entry and exit decisions will choose the same entry or exit time as a firm that optimizes against the equilibrium capacity path $\mathbf{k}$. For example, an exiting firm that thinks the current capacities $(\mathbf{k}^f(\hat{x}_t), \mathbf{k}^b(\hat{x}_t)) = (k^f, k^b)$ remain unchanged in the future solves the exit time from

$$V_f^m(x_t; k) = \sup_{\tau^* \geq t} E\left[\int_t^{\tau^*} \pi_f(x_\tau; k^f, k^b) e^{-r(\tau-t)} d\tau - I_f e^{-r\tau^*}\right]$$

and finds the same exit time as the sophisticated firm that solves (5) with the understanding of the aggregate capacity development. This myopia result is due to Leahy (1993).[16] It can be used to transform each firm's problem into a simple Markov decision problem determining entry and exit tresholds in terms of $\hat{x}$, for any given pair (k^f, k^b). Alternatively, we can take any $\hat{x} \in \mathbb{R}^+$ as given and look for capacity pairs (k^f, k^b) that make $\hat{x}$ the investment treshold for myopic firms. This way we can map from all conceivable myopic tresholds $\hat{x} \in \mathbb{R}^+$ to an equilibrium path $\mathbf{k} = \mathbf{k}^*$. In Appendix, we show

[16] However, our extension of the result to a two-dimensional model is non-trivial.

that the equilibrium is unique and that it can indeed be computed solving the myopic problems.

Theorem 1 *The model has a unique equilibrium* $\mathbf{k} = \left(\mathbf{k}^f, \mathbf{k}^b\right)$ *with the following properties:*

- $\mathbf{k}^f$ *is everywhere continuous, strictly decreasing on some interval* $(a_f, b) \subset \mathbb{R}^+$*, and constant on* $\mathbb{R}^+ \setminus (a_f, b)$.
- $\mathbf{k}^b$ *is everywhere continuous, strictly increasing on some interval* $(a_b, b) \subset \mathbb{R}^+$*, and constant on* $\mathbb{R}^+ \setminus (a_b, b)$.

Proof. See Appendix A. ■

Before turning to characterization, let us note two basic implications of the theorem. The exit of the old technology may start before or after the entry of the new one (i.e., $a_f \neq a_b$), but both transitions end at the same factor market condition, $\hat{x} = b$. The theorem also implies that as long as the transition is going on for both technologies, there is both exit and entry every time $\hat{x}$ reaches a new record value.

3 Characterization

3.1 Volatility and the transition

In this section, we describe technology transition $\mathbf{k}(\hat{x}) = \left(\mathbf{k}^f(\hat{x}), \mathbf{k}^b(\hat{x})\right)$ as the factor supply gradually declines, i.e., as the supply curve reaches new record levels captured by $\hat{x}$. In particular, we characterize the relationship between the degree of uncertainty σ and the nature of the technology transition. However, before progressing we want to limit attention to technology transitions that are relevant for technology adoption. For example, we are not particularly interested in situations where there is so much initial backstop capacity that the transition is merely about the exit. We are also not interested in transitions that more or less jump to the long-run equilibrium (small initial k^f). Interesting transitions are such that there is both entry and exit as $\hat{x}$ reaches new values. Along such a path it makes sense to talk about technology replacement.

Definition 3 *Adoption path is an equilibrium path* $\mathbf{k}(\hat{x})$ *with both entry and exit for all* $\hat{x} \in (0, b) \subset \mathbb{R}^+$.

Recall from Theorem 1 that b is the factor market condition at which the transition is over for both technologies. The definition of the adoption path confines attention to an equilibrium path $\mathbf{k}(\hat{x})$ with the property that $a_f = a_b = 0$ in Theorem 1, i.e., the transition in both technologies should start already when factor supply is abundant ($\hat{x}$ close to zero). The adoption path is a generic equilibrium path in the sense that any given equilibrium starting from arbitrary initial conditions (x_0, k_0^f, k_0^b) will ultimately coincide with the adoption path for all $\hat{x} \in (a, b) \subset \mathbb{R}^+$ where $a = \max\{a_f, a_b\}$. That is, as soon as the transition has started for both technologies, the equilibrium coincides from that point on with the adoption path. By confining attention to the path along which there is entry and exit for all $\hat{x} \in (0, b)$, we can find the equilibrium path that is not constrained by the starting point and this way characterize all cases at once. Note that $a_f = a_b = 0$ requires that the old technology units cannot meet the demand alone at $\hat{x} = 0$ so that some entry must take place already at very favorable factor market conditions. For ease of exposition, let us assume that demand is large enough for the adoption path, as defined above, to exist.[17]

Two observations are important for understanding how the technology transition depends on uncertainty. First, if there is little uncertainty about the future factor market development, the factor supply situation gets worse almost surely in the near future (recall trend $\alpha > 0$) and, therefore, an old technology unit has no reason to accept temporary losses from idleness in expectations of more favorable conditions. Second, if the old technology does not adjust utilization through idleness, an entrant is completely isolated from the factor market uncertainty which, together with free entry, implies an output price that does not change as new entry takes place (otherwise entrants at different times would not be indifferent). Because the output price depends only on the total capacity (in the absence of utilization adjustment), the total capacity should then not change as the transition progresses, i.e., the replacement ratio is one.

To formalize the above reasoning, recall that

$$\underline{x}(\mathbf{k}^f(\hat{x}), \mathbf{k}^b(\hat{x})) = D(\mathbf{k}^f(\hat{x}) + \mathbf{k}^b(\hat{x})) - C(\mathbf{k}^f(\hat{x}))$$

is the critical value for capacity adjustment. Note that $\underline{x}(\mathbf{k}^f(\hat{x}), \mathbf{k}^b(\hat{x})) - \hat{x}$, if positive, equals $P_t - p_t^f$ which is the rent from binding overall capacity for the old technology units.

[17] The assumption is not needed for Theorem 1. This assumption is without loss of generality because it can be relaxed by adding more initial backstop capital k_0^b, leading to lower residual demand to start with. Therefore, if the assumption on demand is not satisfied, the effect on the equilibrium is the same as that from a large k_0^b. Any equilibrium that is constrained by excessively large k_0^f or k_0^b will ultimately follow the path $\mathbf{k}$ identified by the adoption path.

Remark 1 *Along the adoption path, exit from activity (idleness) implies*

$$\underline{x}(\mathbf{k}^f(\hat{x}), \mathbf{k}^b(\hat{x})) - \hat{x} \geq 0 \ (< 0).$$

Proposition 1 *Assume $\sigma = 0$. Then, all factor-dependent units exit from activity. Along the adoption path, each entrant replaces one factor-dependent unit.*

Proof. For $\sigma = 0$, we can write the exit condition as

$$\begin{aligned} c - rI_f &= D(\mathbf{k}^f(\hat{x}) + \mathbf{k}^b(\hat{x})) - C(\mathbf{k}^f(\hat{x})) - \hat{x} \\ &= \underline{x}(\mathbf{k}^f(\hat{x}), \mathbf{k}^b(\hat{x})) - \hat{x} > 0, \end{aligned}$$

meaning that if there is exit, the old units exit from activity. We can write the entry condition (13) as

$$rI_b = D(\mathbf{k}^f(\hat{x}) + \mathbf{k}^b(\hat{x})),$$

implying that if there is entry, the total capacity must be a constant between two entry points; the entry-exit ratio is one. ■

Small uncertainty does not change the old technology units' willingness to exit before idleness becomes an option. This is best illustrated by considering the last old technology firm in the industry whose value V_f satisfies

$$\frac{1}{2}\sigma^2 x^2 V_f'' + \alpha x V_f' - rV_f + \pi_f = 0,$$

where arguments are omitted and primes denote derivatives with respect to x. Noting that the factor price for the last infinitesimal firm is just the intercept of the supply curve, $p^f = x + C(0) = x$, and the output price is a constant given by entrants' indifference condition $D(k_\infty) = rI_b$, where k_∞ denotes the final long-run capacity when all old firms are replaced by new ones, we can use standard procedures to find the exit treshold for a firm that exits from activity:

$$b(\sigma) = \frac{\beta_1(\sigma)}{\beta_1(\sigma) - 1}(r - \alpha)(I_b + I_f - \frac{c}{r})$$

where $\beta_1(\sigma) > 1$ and is given by

$$\beta_{1,2}(\sigma) = \frac{1}{2} - \frac{\alpha}{\sigma^2} \pm \sqrt{\left[\frac{\alpha}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}}. \tag{15}$$

Note that as $\sigma \to 0$, the expression for $b(\sigma)$ approaches the exit condition in Proposition 1. Now, the last exiting firm indeed exits without accepting a period of idleness if

$D(k_\infty) - b(\sigma) = P - p^f > 0$. The greater is uncertainty, the better are the chances for improving factor market conditions, so that the critical compensation $D(k_\infty) - b(\sigma)$ diminishes in σ. Let $\sigma = \sigma^* > 0$ denote the unique solution to

$$D(k_\infty) - b(\sigma^*) = 0.$$

Proposition 2 *For any given $\sigma \leq \sigma^*$, Proposition 1 holds.*

Let us next consider larger factor market volatility, $\sigma > \sigma^*$. Now, the chances for improving factor market conditions are good enough to justity the postponing of the last-firm exit to a point where the last firm is no longer producing at the exit point. We now solve the same stopping problem as above but this time under the assumption that the last firm exits from idleness. Without reporting the routine details we note that the exit treshold for the last factor-dependent unit is[18]

$$b(\sigma) = \left[\frac{(r-\alpha)(rI_f)^{\beta_2(\sigma)-1}(c - rI_f)}{\frac{r}{\beta_1(\sigma)} - \alpha} \right]^{\frac{1}{\beta_2(\sigma)}} \quad \text{for } \sigma > \sigma^*.$$

It is straightforward to verify that $b(\sigma)$ increases in σ and that the above two expressions for $b(\sigma)$ coincide when $\sigma = \sigma^*$.

Having now demonstrated that the last exiting firm exits from idleness, we continue working "backwards" from this last exiting firm. Consider how the industry reached the situation where there is only one remaining idle firm? This situation must have been preceded by more favorable market conditions with lower $\hat{x}$ and, thereby, more room for old technology firms. But still, if $\hat{x}$ is only slightly below $b(\sigma)$, the group of remaining firms must be idle at exit points because $\hat{x} > D(\mathbf{k}^b(\hat{x}))$. In this phase, the remaining firms are thus rationed between a buffer of idle units and exiting firms each time the factor supply declines to a record level, i.e., as $\hat{x}$ reaches new values.

Ask next, how did the industry reach the phase where a fraction of firms remain in the idle buffer and another fraction exits as the supply declines? This situation must have been preceded by a phase where at least some firms produce at exit points since production is profitable for sufficiently abundant supply ($\hat{x}$ low). In particular, when

$$D(\mathbf{k}^b(\hat{x})) > \hat{x} > D(\mathbf{k}^f(\hat{x}) + \mathbf{k}^b(\hat{x})) - C(\mathbf{k}^f(\hat{x}))$$

[18]The solution procedure is standard if the firm exits from activity. If the firm exits from idless, the value matching and smooth pasting conditions change and there are also boundary conditions for values of x at which the firm switches from production to idleness. But the procedure is still standard and not reported here.

some firms must remain active producers while others move to the idle buffer at the exit points. One might envision the idle buffer as a waiting-room for exit: as $\hat{x}$ increases, producing firms move to the idle buffer, and some firms from the idle buffer exit. However, starting from very low $\hat{x}$, there cannot be an idle buffer but all remaining firms produce, because $D(\mathbf{k}^f(\hat{x}) + \mathbf{k}^b(\hat{x})) - C(\mathbf{k}^f(\hat{x})) > \hat{x}$ for $\hat{x}$ sufficiently low. In this phase, if firms exit, then they exit directly from activity.[19]

In Appendix we formalize this reasoning. For this purpose we call the above described phases of the technology transition the active, volatile, and idle capacity phases because of the following facts: in the first phase ($\hat{x}$ low) all remaining firms are active producers; in the second phase ($\hat{x}$ higher) some firms produce while others are idle so that the overall capacity in use is responding to shocks; and in the final phase ($\hat{x}$ even higher) all firms are idle at exit points.

Proposition 3 *Assume $\sigma > \sigma^*$. Then, the adoption path entry-exit points pass through three phases:*

1. *active capacity phase,* $0 < \hat{x} \leq \underline{X}$;
2. *volatile capacity phase,* $\underline{X} < \hat{x} \leq \overline{X}$;
3. *idle capacity phase,* $\overline{X} < \hat{x}$,
   where the tresholds $\underline{X}$ *and* $\overline{X}$ *are unique in* $(0, b) \subset \mathbb{R}^+$.

Proof. See Appendix B. ■

To grasp the precise picture, see Fig. 2 again where we assume that the admissible capacity paths are the equilibrium paths. In the active capacity phase, the old technology is fully used at the entry-exit points. Utilization is depicted by the shaded area under path $\mathbf{k}^f(\hat{x})$. In the volatile capacity phase, utilization is always less than 100 per cent at the entry-exit points. Note that full utilization can be reached at other than entry-exit points. Finally, in the idle capacity phase, utilization drops to zero at the entry-exit points. In this sample path, utilization never becomes positive once the equilibrium enters the last phase, but positive utilization must always occur at a positive probability as long as some k^f units remain in the market.

We can now describe the technology overlap: new technology units are adopted to coexist with old units so that the overall availability of technology units increases.

[19] Recall that adoption path is defined by the requirement of exit (and entry) already at low levels of $\hat{x}$.

Proposition 4 *Assume $\sigma > \sigma^*$. Then, the adoption path exhibits technology overlap:*

1. $\mathbf{k}^f(\hat{x}) + \mathbf{k}^b(\hat{x}){=}k_\infty$ *is a constant in active capacity phase;*
2. $\mathbf{k}^f(\hat{x}) + \mathbf{k}^b(\hat{x})$ *increases and stays above k_∞ in volatile capacity phase;*
3. $\mathbf{k}^f(\hat{x}) + \mathbf{k}^b(\hat{x})$ *declines back to k_∞ at the end of the idle capacity phase.*

See again Fig. 2 to visualize the result. It follows from the following reasoning. In the first phase, no firm is in the idle buffer so that there is no capacity adjustment between any two consecutive exit points and, therefore, the output price and thus the payoff for the entrants is deterministic. By the entrants' indifference between entry points, the price and thus the overall capacity must remain constant across entry-exit points during the active capacity phase. In the second phase, old technology production is in expectations volatile and thus P is expected to visit below and also above rI_b during during the volatile capacity phase (see the indifference condition 13). But if $P < rI_b$, then it must be the case that $\mathbf{k}^f(\hat{x}) + \mathbf{k}^b(\hat{x}) > k_\infty$. In the third phase, the overall capacity must decline back to k_∞ because the last old technology firm exits when $P = rI_b$. By Theorem 1, the capacity functions are continuous which completes the proof.[20]

To better understand the result, let us now discuss what destroys it.

Factor market volatility. One-to-one replacement is obtained with low factor market volatility, $\sigma \leq \sigma^*$, as already explained. It is thus the sufficient uncertainty about the factor market development that makes the old technology units to move to the idle buffer rather than exit. This implies that the exit rate falls short of the entry rate.

Option to remain idle. If idleness is ruled out by assumption, leaving exit as the only response option to the declining factor supply, then replacement is again one-to-one. Clearly, if the old technology units cannot adjust utilization, the output price cannot not change between consecutive entry points and, by the equilibrium entry condition, price must be a constant along adoption paths. Hence, the entry-exit ratio is one and there is no technology overlap.

Heterogeneity. Heterogeneity in the factor supply is necessary for the technology overlap. To see this, suppose temporarily that the cost of supplying a marginal unit of

[20] Proposition 4 means that the total capacity peaks somewhere along the way, but it does not specify whether this happens in the volatile or idle capacity phase. With linear demand and supply we are able to show that the capacity always peaks in the idle capacity phase, but we can not rule out the possibility of capacity peaking in the volatile capacity phase with some highly non-linear demand and supply curves (although we do not think this would be a typical case).

the factor is zero, $C(\cdot) = 0$, so that the inverse factor supply curve is horizontal, $p^f = x$. Consider then the first replacement of an old technology unit by a new one. The cost of this irreversible replacement is $I_f + I_b - \frac{c}{r} > 0$, and the expected benefit is $\frac{\hat{x}}{r-\alpha}$ where $\hat{x}$ is the factor market condition at which the replacement is chosen to occur. The solution to this standard stopping problem satisfies

$$\hat{x} = \frac{\beta_1}{\beta_1 - 1}(r - \alpha)(I_f + I_b - \frac{c}{r}) \tag{16}$$

where β_1 is given by (15).[21] However, since the factor market price is just $p^f = \hat{x}$, it is independent of exit, meaning that all active old capacity producers can be profitably and instantaneously replaced as well. Just before the replacement these units produced q^f satisfying $D(q^f) = \hat{x}$, where $\hat{x}$ is given by (16), so the optimal amount of entry is given by

$$\mathbf{k}^b = D^{-1}(\frac{\beta_1}{\beta_1 - 1}(r - \alpha)(I_f + I_b - \frac{c}{r})).$$

After this large scale one-to-one replacement of technologies, the overall capacity declines for all $\hat{x} > D(\mathbf{k}^b)$. Therefore, heterogeneity in factor supply is necessary for the increase in total capacity.

We now turn to elaborate additional properties of the technology overlap.

3.2 Output price volatility

In this section we describe how the factor market volatility is transmitted into the output price along the adoption path that exhibits technology overlap. In particular, we want to demonstrate that it is a feature of the equilibrium transition that the factor price volatility translates into a larger output price volatility as the equilibrium progresses through the volatile capacity phase although production becomes less intensive in the factor.

We are first interested in describing the set of output prices that are achievable for each $\hat{x}$. Let $\overline{P}(\hat{x})$ and $\underline{P}(\hat{x})$ denote the maximum and minimum output prices that can be observed at current capacities, i.e., without strictly exceeding the current factor market record $\hat{x}$. Clearly,

$$\begin{aligned} \underline{P}(\hat{x}) &= D(\mathbf{k}^f(\hat{x}) + \mathbf{k}^b(\hat{x})), \\ \overline{P}(\hat{x}) &= D(q^f(\hat{x}; \mathbf{k}^f(\hat{x}), \mathbf{k}^b(\hat{x})) + \mathbf{k}^b(\hat{x})). \end{aligned}$$

[21] Note that this is the same treshold as for the last exiting firm when $\sigma \leq \sigma^*$. This is because the equilibrium factor supply curve is horizontal for the last exiting firm.

That is, the lowest price is achieved when the current technologies are in full use, and the highest when the factor market condition is so bad that the technology utilization cannot be adjusted further. In equilibrium, any price from the set $[\underline{P}(\hat{x}), \overline{P}(\hat{x})]$ can be reached depending on the realization $x \leq \hat{x}$.

To describe how $[\underline{P}(\hat{x}), \overline{P}(\hat{x})]$ develops as the factor supply declines, i.e., as $\hat{x}$ increases, we must impose more structure on the model. Rather than imposing tedious curvature restrictions on demand and supply relations, we choose to assume linearity:

$$D(q) = A - Bq \tag{17}$$

$$C(q^f) = x + Cq^f, \tag{18}$$

where A, B, C are strictly positive constants.

Proposition 5 *Assume $\sigma > \sigma^*$ and $D(q)$ and $C(q^f)$ satisfying (17)-(18). Then,*

1. *$\overline{P}(\hat{x}) = \underline{P}(\hat{x}) = rI_b$ in the active capacity phase;*

2. *$\overline{P}(\hat{x})$ increases and $\underline{P}(\hat{x})$ decreases throughout the volatile capacity phase;*

3. *$\overline{P}(\hat{x})$ decreases throughout the idle capacity phase, and $\underline{P}(\hat{x})$ increases in the end of the idle capacity phase and $\underline{P}(b) = \overline{P}(b) = rI_b$.*

Proof. See Appendix C. ■

The price set is depicted in Fig. 3. The set of volatile prices thus expands during the volatile capacity phase. We will next demonstrate that also the price volatility increases: for any given $\hat{x} \in (\underline{X}, \overline{X}]$ and price P from the set $(\underline{P}(\hat{x}), \overline{P}(\hat{x}))$, a change in $x < \hat{x}$ translates into a greater change in price P as the equilibrium progresses through the volatile capacity phase. Using Ito's Lemma, we can write the price process under the linear structure as follows:

$$dP = \begin{cases} 0 \text{ for } P = \underline{P}(\hat{x}) \\ (P - Q(\mathbf{k}^b(\hat{x}))(\alpha dt + \sigma dz) \text{ for } \underline{P}(\hat{x}) < P < \overline{P}(\hat{x}) \end{cases},$$

where

$$Q\left(\mathbf{k}^b(\hat{x})\right) = \frac{C\left(A - B\mathbf{k}^b(\hat{x})\right)}{B + C}.$$

Note that, given $x < \hat{x}$, $Q(\cdot)$ is a constant, and the volatility of P is

$$\frac{P - Q(\mathbf{k}^b(\hat{x}))}{P}\sigma \text{ for } \underline{P}(\hat{x}) < P < \overline{P}(\hat{x}).$$

Now, when a new entry point is reached, $Q(\cdot)$ drops to a lower level and therefore the expression for volatility, which holds until the next entry point, is larger. We can thus conclude that the output prices become more volatile during the volatile capacity although production becomes less intensive in the factor.

Consider now the output price in Fig 2. During the active capacity phase, the output market is isolated from the factor market volatility. This follows because the old technology is fully used and therefore absorbing the volatility. When the old capacity turns volatile, we see a considerable transmission of uncertainty to the output sector. Finally, during the idle capacity phase the volatility gradually levels off.

INSERT FIGURE 3 HERE

3.3 Probability of the transition

How likely is it that the technology transition is completed in the sense the backstop technology takes over the market and eliminates the dependence on the volatile factor entirely in the long run? To get an idea of this, consider the probability of reaching the factor market condition b at which the process is completed within T periods. Let Φ denote the cumulative distribution function for the standard normal distribution. Then, starting from $x_0 < b$, the probability of $\hat{x} \geq b$ at T is

$$\begin{aligned} \text{Prop}(\hat{x}_T \geq b) = & \\ & \Phi\left(\frac{\ln(x_0/b) + (\alpha - \sigma^2/2)T}{\sigma\sqrt{T}}\right) + \\ & \left(\frac{b}{x_0}\right)^{\frac{2\alpha}{\sigma^2}-1}\Phi\left(\frac{\ln(x_0/b) - (\alpha - \sigma^2/2)T}{\sigma\sqrt{T}}\right). \end{aligned}$$

Remark 2 *The backstop technology takes over the market with probability one if $\alpha \geq \sigma^2/2$ and with probability $(\frac{b}{x_0})^{\frac{2\alpha}{\sigma^2}-1}$ if $0 < \alpha < \sigma^2/2$, where $x_0 < b$.*

Proof. As $T \to \infty$,

$$\begin{aligned} \text{Prop}(\hat{x}_T &\geq b) \to 1, \text{ if } \alpha \geq \sigma^2/2 \\ \text{Prop}(\hat{x}_T &\geq b) \to \left(\frac{b}{x_0}\right)^{\frac{2\alpha}{\sigma^2}-1}, \text{ if } 0 < \alpha < \sigma^2/2. \end{aligned}$$

■

While the result follows directly from the properties of the stochastic process $\{x_t\}$, it gives some idea of the distinct roles of scarcity rents and volatility in the technology transition. Although we do not explicitly link the overall availability of the factor and the factor price trend α, the above result suggests that the trend must be sufficiently large relative to the factor price volatility for the backstop to fully take over the market. In other words, volatility satisfying $\sigma^2/2 > \alpha$ tends to protect the old technology in the sense that, in expectations, the old technology has a market share.

4 Concluding Remarks

We considered socially efficient adoption of technologies that reduce dependence on volatile factors of production. Three assumptions are essential for the nature of the technology transition. First, factor-dependent technology units have the option to remain idle rather than exit when factor markets develop unfavorably. Thus, the aggregate technology utilization can be adjusted. Second, the factor supply sources are heterogenous so that the supply curve is upward sloping. Third, factor market is subject to sufficient supply-side uncertainty.

Under these circumstances, we found that the technology adoption process has three qualitatively distinct phases, depending on the historical performance of the factor market. In the active capacity phase, the factor market is still relatively favorable to the factor-using production but continually worsening. Then, the adoption process has the traditional form where the existing technology units are replaced one-to-one by the new units.

In the volatile capacity phase, the factor use becomes so expensive that the old capacity cannot be fully utilized. However, because the factor use may still become cheaper in the near future, it is a profitable option to leave some capacity idle rather than scrap the old units. A general property of this phase is that the new technology units are built to coexist with the old ones so that the total availability of production units increases. It is important to emphasize that it is socially efficient to expand the portfolio of production forms in this way since both scrapping and adoption are irreversible decisions which are made under uncertainty about the future profitability of both production forms. As a result of this capacity expansion, the factor market uncertainty is increasingly transmitted to the output market.

If the volatile capacity phase is about the old technology's fight against its decline, the final phase, the idle capacity phase, is about the decline. The old technology exit rate exceeds new technology entry rate, and the output market volatility gradually diminishes. Yet, the factor-dependent technology may have a positive long-run market share because there may be a persistent possibility of improving factor market conditions.

At the theoretical level, there are some obvious sources of criticism. For tractability, we could not allow expansion of the factor-dependent capacity.[22] We do not believe that this restriction is central to the results. This holds in particular if the factor price process has a trend large enough to imply no long-run market share for the old technology. Moreover, the explicit inclusion of the option to remain idle serves as a partial substitute for the option to expand: under improving factor market development new production capacity comes from the idle reserve before any new investment should take place. Another shortcoming is the fact that the factor price trend is exogenous. Ideally, the trend should reflect the Hotelling-type rents due to the finiteness of the overall factor supply. Making this link explicit would allow addressing the roles of scarcity rents and volatility in the backstop technology adoption in detail.

The most recent revival of interests in reducing dependence on some key factors such as energy commodities is due to various externalities caused by the use of these factors. Reducing dependence on oil may contribute to road safety through the reduced size of the vehicles. In general, fossil fuels cause local and global externality problems. We deliberately excluded any externalities from the analysis to provide insights regarding the determinants of the prolonged transition to the factor-free environment in a well-functioning market economy. However, these insights remain intact under an alternative interpretation of the model that incorporates the externality pricing. Without affecting the equilibrium we can think that factor users face a horizontal supply curve, $p^f = x$, but are heterogenous in their efficiency of using the factor. The efficiency in factor use may relate to emission rates and thereby to externality payments, making firms exit the industry in the order given by their emission rates.

This alternative interpretation of the model can provide important policy implications. Penalizing the use of factors causing pollution or other externalities may not cause a decline of the factor demand infrastructure but only its utilization decline. If externalities are correctly priced, the persistence of the polluting technology together with the new clean technology is socially optimal for the reasons that we have underscored in this paper.

[22]The myopia result of Leahy would not extend to a two-dimensional model, where both expansion and scrapping of one technology were allowed.

References

[1] A. Atkeson; P. J. Kehoe, , 1999. Models of Energy Use: Putty-Putty versus Putty-Clay, The American Economic Review, Vol. 89, No. 4, pp. 1028-1043.

[2] Baldurson, F., and I. Karatzas, 1997, Irreversible Investment and Industry Equilibrium. Finance and Stochastics 1, 69-89.

[3] R. B. Barsky, and L. Kilian, 2004. Oil and the Macroeconomy Since the 1970s, Journal of Economic Perspectives, Vol. 18, No. 4, pp. 115-134.

[4] E.R. Berndt, and D.O. Wood, 1975. Technology, Prices, and the Derived Demand for Energy, Review of Economics and Statistics, Vol. 57, No. 3, pp. 259-68.

[5] Caballero, R., and R. Pindyck, 1996. Uncertainty, Investment, and Industry Evolution, International Economic Review, Vol. 37, No. 3., 641-662.

[6] Chakravorty, U., Roumasset, J., Tse, K., 1997. Endogenous substitution among energy resources and global warming. Journal of Political Economy 195, 1201-1234.

[7] Dasgupta, P., Heal, G., 1974. The optimal depletion of exhaustible resources. Review of Economic Studies (Symposium), 3-28.

[8] Dixit, A.,1989. Hysteresis, Import Penetration, and Exchange Rate Pass-Through, The Quarterly Journal of Economics, Vol. 104, No. 2., 205-228.

[9] Dixit, A., 1997. Investment and Employment Dynamics in the Short Run and the Long Run, Oxford Economic Papers, New Series, Vol. 49, No. 1., 1-20.

[10] Dixit, A.K, and R.S. Pindyck, 1994. Investment under Uncertainty, Princeton University Press, Princeton, New Jersey.

[11] Eberly, J.C. and Van Mieghem, J.A., 1997. Multi-factor Dynamic Investment under Uncertainty, Journal of Economic Theory, Vol. 75, No. 2, 345-387.

[12] J. D., Hamilton, 2005. Oil and the Macroeconomy, prepared for the New Palgrave Dictionary of Economics.

[13] Heal, G., 1976. The relationship between price and extraction cost for a resource with a backstop technology. Bell Journal of Economics 7, pp. 371-378.

[14] H. C.Hoppe, 2002. The Timing of New Technology Adoption: Theoretical Models and Empirical Evidence, The Manchester School, Vol. 70, pp. 56-76.

[15] Krautkraemer, J., 1999. Nonrenewable resource scarcity. Journal of Economic Literature XXXVI, 2065-2107.

[16] Leahy, J.V., 1993. Investment in Competitive Equilibrium: The Optimality of Myopic Behaviour, The Quarterly Journal of Economics, Vol. 108, No. 4, 1105-1133.

[17] Nordhaus, W., 1973. The Allocation of Energy Reserves. Brookings Papers 3, 529-570.

[18] Pindyck, R.S., 1993. A Note on Competitive Investment under Uncertainty, The American Economic Review, Vol. 83, No. 1., 273-277.

[19] Reinganum, J.F., 1989. The Timing of Innovation: Research, Development, and Diffusion. In: R. Schmalensee and R.D. Willig (Eds.), the Handbook of Industrial Organization, Vol. I., 850-908.

[20] Smith, J.L., 2002. Oil and the Economy: Introduction. Quarterly Review of Economics and Finance 42, 163-168.

APPENDIX A: Proof of Theorem 1.

To prove Theorem 1, we build on Leahy (1993), and utilize the correspondence between equilibrium and what we call a myopic adjustment path (see Definition 4 below). The logic of the proof is the following. First, in Proposition 6 we show that an equilibrium is always a myopic adjustment path. Second, in Proposition 7 we prove the converse: a myopic adjustment path is always an equilibrium. This means that equilibrium and myopic adjustment path are equivalent, and to prove Theorem 1 it sufficies to establish the required properties for the latter, which we do in Proposition 8. Along the way, we will make use of a number of lemmas.

For a start, let $\Pi_b(x,k)$ ($\Pi_f(x,k)$) be the marginal payoff of an entrant (exiting firm) from adjusting entry (exit) threshold upwards from x:

$$\Pi_b(x,k) \equiv \lim_{\epsilon\downarrow 0}\frac{E\int_{\tau=\tau_x}^{\tau_{x+\epsilon}}\left(rI_b - P\left(x_\tau;k^f,k^b\right)\right)e^{-r(\tau-\tau_x)}d\tau}{\varepsilon}, \tag{19}$$

$$\Pi_f(x,k) \equiv \lim_{\epsilon\downarrow 0}\frac{E\int_{\tau=\tau_x}^{\tau_{x+\epsilon}}\left(rI_f + \pi_f\left(x_\tau;k^f,k^b\right)\right)e^{-r(\tau-\tau_x)}d\tau}{\varepsilon}. \tag{20}$$

These marginal payoffs of delay are linked to the optimality of stopping. In particular, in equilibrium all agents stop optimally, and the following must hold:

Lemma 1 *Let* $\mathbf{k} = \left(\mathbf{k}^f, \mathbf{k}^b\right)$ *be an equilibrium. Then,*

i) for any $x \geq x_0$*:*

$$\Pi_b(x,\mathbf{k}(x)) \geq 0 \ (=0 \textit{ if } \mathbf{k}^b(\cdot) \textit{ adjusts at } x)\textit{, and} \tag{21}$$

ii) for any $x \geq x_0$ *s.t.* $\mathbf{k}^f(\cdot) > 0$*:*

$$\Pi_f(x,\mathbf{k}(x)) \geq 0 \ (=0 \textit{ if } \mathbf{k}^f(\cdot) \textit{ adjusts at } x). \tag{22}$$

Proof. Let $\mathbf{k} = \left(\mathbf{k}^f, \mathbf{k}^b\right)$ be an equilibrium. Assume that for some $x \geq x_0$ we have $\Pi_b(x,\mathbf{k}(x)) < 0$. Fix $k = \left(k^f, k^b\right) = \mathbf{k}(x)$. Then, $E\int_{\tau=\tau_x}^{\tau_{x+\varepsilon}}\left(rI_b - P\left(x_\tau;k^f,k^b\right)\right)e^{-r(\tau-\tau_x)}d\tau < 0$ for a sufficiently small ε, and in fact, since $P(\cdot)$ is increasing in x, we must have $E\int_{\tau=\tau_x}^{\tau_{x+h}}\left(rI_b - P\left(x_\tau;k^f,k^b\right)\right)e^{-r(\tau-\tau_x)}d\tau < 0$ for any $h > 0$. By the fact that $\mathbf{k}$ is continuous from right, we must also have

$$E\int_{\tau=\tau_x}^{\tau_{x+\varepsilon}}\left(rI_b - P\left(x_\tau;\mathbf{k}^f(x_\tau),\mathbf{k}^b(x_\tau)\right)\right)e^{-r(\tau-\tau_x)}d\tau < 0$$

for ε small enough. Fix $\varepsilon > 0$ so small that this is the case. Let

$$x' = \inf\left\{\widetilde{x} \geq x+\varepsilon \,\middle|\, \mathbf{k}^b(\cdot) \text{ adjusts at } \widetilde{x}\right\}.$$

If $\mathbf{k}^b(\cdot)$ never adjusts above $x+\varepsilon$, then we have $x'=\infty$ and $\tau_{x'}=\infty$. Since P is decreasing in k^f (so that possible decreases in k^f during the excursion between τ_x and $\tau_{x'}$ only increase P), we clearly have

$$E\int_{\tau=\tau_x}^{\tau_{x'}}\left(rI_b-P\left(x_\tau;\mathbf{k}^f(x_\tau),\mathbf{k}^b(x_\tau)\right)\right)e^{-r(\tau-\tau_x)}d\tau<0. \tag{23}$$

Since $\mathbf{k}^b(\cdot)$ adjusts at x' (or else $x'=\infty$), we have by (9) $V_b(x',x';\mathbf{k})-I_b=0$, or equivalently

$$E\int_{\tau=\tau_{x'}}^{\infty}\left(rI_b-P\left(x_\tau;\mathbf{k}^f(x_\tau),\mathbf{k}^b(x_\tau)\right)\right)e^{-r(\tau-\tau_x)}d\tau=0. \tag{24}$$

Summing (23) and (24), we have

$$E\int_{\tau=\tau_x}^{\infty}\left(rI_b-P\left(x_\tau;\mathbf{k}^f(x_\tau),\mathbf{k}^b(x_\tau)\right)\right)e^{-r(\tau-\tau_x)}d\tau<0,$$

which is equivalent to $F_b(x,x;\mathbf{k})>0$, thus contradicting (8). Therefore, it must be that $\Pi_b(x,\mathbf{k}(x))\geq 0$ for any $x\geq x_0$.

Assume then that $\mathbf{k}^b(\cdot)$ adjusts at x. If then $\Pi_b(x,\mathbf{k}(x))>0$, an agent would strictly benefit from at least slightly delaying investment, meaning that $F_b(x,x;\mathbf{k})>V_b(x,x;\mathbf{k})-I_b$. However, since $\mathbf{k}^b(\cdot)$ adjusts at x, we have $V_b(x,x;\mathbf{k})-I_b=0$ by (9), and hence $F_b(x,x;\mathbf{k})>0$. This contradicts (8), so we must have $\Pi_b(x,\mathbf{k}(x))=0$, and (21) holds.

The proof of (22) is quite similar. Assume that for some $x\geq x_0$ we have $\mathbf{k}^f(x)>0$ and $\Pi_f(x,\mathbf{k}(x))<0$. Fixing $k=\left(k^f,k^b\right)=\mathbf{k}(x)$, and noting that $\pi(\cdot)$ is decreasing in x, we must have

$$E\int_{\tau=\tau_x}^{\tau_{x+h}}\left(rI_f+\pi_f\left(x_\tau;k^f,k^b\right)\right)e^{-r(\tau-\tau_x)}d\tau<0$$

for any $h>0$. By the fact that $\mathbf{k}$ is continuous from right, we must also have

$$\mathbf{k}^f(x+\varepsilon) > 0, \text{ and} \tag{25}$$

$$E\int_{\tau=\tau_x}^{\tau_{x+\varepsilon}}\left(rI_f+\pi_f\left(x_\tau;\mathbf{k}^f(x_\tau),\mathbf{k}^b(x_\tau)\right)\right)e^{-r(\tau-\tau_x)}d\tau < 0 \tag{26}$$

for ε small enough. Fix $\varepsilon>0$ so small that (25) and (26) hold and let

$$x'=\inf\left\{\widetilde{x}\geq x+\varepsilon\,\middle|\mathbf{k}^f(\cdot)\text{ adjusts at }\widetilde{x}\right\}.$$

Again, if $\mathbf{k}^f(\cdot)$ never adjusts above $x+\varepsilon$, we have $x'=\infty$ and (25) implies that $\mathbf{k}^f(x)>0$ for all $x>0$. Since π is decreasing in k^b (so that possible increases in k^b during the excursion between τ_x and $\tau_{x'}$ decrease π), we clearly have

$$E\int_{\tau=\tau_x}^{\tau_{x'}}\left(rI_f+\pi_f\left(x_\tau;\mathbf{k}^f(x_\tau),\mathbf{k}^b(x_\tau)\right)\right)e^{-r(\tau-\tau_x)}d\tau<0. \tag{27}$$

On the other hand, if $x' < \infty$, $\mathbf{k}^f(\cdot)$ adjusts at x', and we have by a rearrangement of (10):

$$E\int_{\tau=\tau_x}^{\tau_{x'}} \left(rI_f + \pi_f\left(x_\tau; \mathbf{k}^f(x_\tau), \mathbf{k}^b(x_\tau)\right)\right) e^{-r(\tau-\tau_x)}d\tau - I_f = V_f(x, x; \mathbf{k}). \qquad (28)$$

If $x' = \infty$, then $\mathbf{k}^f(\cdot) > 0$ for all $x > 0$, and (11) implies that (28) holds also in that case.

By the definition of $V_f(\cdot)$ given in (5), we have $V_f(x, x; \mathbf{k}) \geq -I_f$, which means that (27) and (28) contradict each other. Therefore, it must be that $\Pi_f(x, \mathbf{k}(x)) \geq 0$ for any $x \geq x_0$.

Finally, assume that $\mathbf{k}^f(\cdot)$ adjusts at x. Then, inserting $x_t = \widehat{x}_t = x^* = x$ in (10) gives $V_f(x, x; \mathbf{k}) = -I_f$. But if $\Pi_f(x, \mathbf{k}(x)) > 0$, we would have for small enough ε:

$$E\int_{\tau=\tau_x}^{\tau_{x+\varepsilon}} \left(rI_f + \pi_f\left(x_\tau; \mathbf{k}^f(x_\tau), \mathbf{k}^b(x_\tau)\right)\right) e^{-r(\tau-\tau_x)}d\tau > 0,$$

or equivalently $E\int_{\tau=\tau_x}^{\tau_{x+\varepsilon}} \pi_f\left(x_\tau; \mathbf{k}^f(x_\tau), \mathbf{k}^b(x_\tau)\right) e^{-r(\tau-\tau_x)}d\tau - I_f e^{-r(\tau_{x+\varepsilon}-\tau_x)} > -I_f$, which by (5) implies that $V_f(x, x; \mathbf{k}) > -I_f$. We have a contradiction, so it must be that $\Pi_f(x, \mathbf{k}(x)) = 0$. ■

Following Leahy (1993) and Baldursson and Karatzas (1997), we will utilize the correspondence between equilibrium and optimal stopping of myopic agents that do not take into account any further stopping decisions of other firms. The key is to note that the marginal payoffs of additional delay defined in (19) and (20) determine the optimality of stopping for myopic agents just as they do for fully rational agents in equilibrium. Let us denote by $V_b^m(x_t; k)$ the value of a unit of backstop technology, by $F_b^m(x_t; k)$ the value of an option to build such unit, and by $V_f^m(x_t; k)$ the value of a unit of factor dependent capital, all calculated by a myopic firm that assumes that the industywide capacity levels will be fixed at $k = \left(k^f, k^b\right)$ for ever:

$$\begin{aligned}
V_b^m(x_t; k) &= E\int_t^\infty P(x_\tau; k^f, k^b)e^{-r(\tau-t)}d\tau, \\
F_b^m(x_t; k) &= \sup_{\tau^*\geq t} E\left[\int_{\tau^*}^\infty P(x_\tau; k^f, k^b)e^{-r(\tau-t)}d\tau - I_b e^{-r(\tau^*-t)}\right], \\
V_f^m(x_t; k) &= \sup_{\tau^*\geq t} E\left[\int_t^{\tau^*} \pi_f(x_\tau; k^f, k^b)e^{-r(\tau-t)}d\tau - I_f e^{-r(\tau^*-t)}\right].
\end{aligned}$$

We now proceed to describe the capacity processes that are implied by myopic behavior. We will work in (k^f, k^b)-plane, and define various subsets of $\mathbb{R}_+ \times \mathbb{R}_+$. First, let us define the *strict inaction regions* $\mathbf{K}^b(x)$ and $\mathbf{K}^f(x)$ as the sets of values of k^f and k^b such that myopic agents are strictly better off waiting than stopping, given that $x_t = \hat{x}_t = x$:

$$\begin{aligned}
\mathbf{K}^b(x) &\equiv \left\{\left(k^f, k^b\right) \in \mathbb{R}_+ \times \mathbb{R}_+ \left| F_b^m\left(x; k^f, k^b\right) > V_b^m\left(x; k^f, k^b\right) - I_b \right.\right\}, \\
\mathbf{K}^f(x) &\equiv \left\{\left(k^f, k^b\right) \in \mathbb{R}_+ \times \mathbb{R}_+ \left| V_f^m\left(x; k^f, k^b\right) > -I_f \right.\right\}.
\end{aligned}$$

At the boundaries of those regions the myopic agents are just indifferent between stopping and continuing. We call those boundaries the *indifference regions*, and denote them by $\partial\mathbf{K}^b(x)$ and $\partial\mathbf{K}^b(x)$:

$$\begin{aligned}
\partial\mathbf{K}^b(x) &\equiv \left\{Cl\left(\mathbf{K}^b(x)\right) \cap Cl\left(\left[\mathbf{K}^b(x)\right]^C\right)\right\}, \\
\partial\mathbf{K}^f(x) &\equiv \left\{Cl\left(\mathbf{K}^f(x)\right) \cap Cl\left(\left[\mathbf{K}^f(x)\right]^C\right)\right\},
\end{aligned}$$

where $Cl(A)$ and A^C denote the closure and complement of A, respectively. The following lemma links the signs of the marginal net payoffs of delay with those regions:

Lemma 2 *The signs of* $\Pi_b(x,k)$ *and* $\Pi_f(x,k)$ *are:*

$$\Pi_b(x,k) \begin{cases} > 0 \text{ iff } k \in \mathbf{K}^b(x) \\ = 0 \text{ iff } k \in \partial\mathbf{K}^b(x) \\ < 0 \text{ iff } k \in \left(\mathbf{K}^b(x) \cup \partial\mathbf{K}^b(x)\right)^C \end{cases}$$

$$\Pi_f(x,k) \begin{cases} > 0 \text{ iff } k \in \mathbf{K}^f(x) \\ = 0 \text{ iff } k \in \partial\mathbf{K}^f(x) \\ < 0 \text{ iff } k \in \left(\mathbf{K}^f(x) \cup \partial\mathbf{K}^f(x)\right)^C \end{cases}$$

Proof. $\Pi_b(x,k) > 0$ implies that for small enough ε, we have

$$\begin{aligned}
& E\left[\int_{\tau_{x+\varepsilon}}^{\infty} P(x_\tau; \mathbf{k}^f(\hat{x}_\tau), \mathbf{k}^b(\hat{x}_\tau)) e^{-r(\tau-\tau_x)} d\tau - I_b e^{-r(\tau_{x+\varepsilon}-\tau_x)}\right] \\
> \; & E\left[\int_{\tau_x}^{\infty} P(x_\tau; \mathbf{k}^f(\hat{x}_\tau), \mathbf{k}^b(\hat{x}_\tau)) e^{-r(\tau-\tau_x)} d\tau - I_b\right],
\end{aligned}$$

which, using (6) and (7), means that $F_b^m\left(x; k^f, k^b\right) > V_b^m\left(x; k^f, k^b\right) - I_b$, i.e. $k \in \mathbf{K}^b(x)$. Conversely, if $F_b^m\left(x; k^f, k^b\right) > V_b^m\left(x; k^f, k^b\right) - I_b$, there must be some x^* such that

$$\begin{aligned}
& E\left[\int_{\tau_{x^*}}^{\infty} P(x_\tau; \mathbf{k}^f(\hat{x}_\tau), \mathbf{k}^b(\hat{x}_\tau)) e^{-r(\tau-\tau_x)} d\tau - I_b e^{-r(\tau_{x^*}-\tau_x)}\right] \\
> \; & E\left[\int_{\tau_x}^{\infty} P(x_\tau; \mathbf{k}^f(\hat{x}_\tau), \mathbf{k}^b(\hat{x}_\tau)) e^{-r(\tau-\tau_x)} d\tau - I_b\right],
\end{aligned}$$

which means that

$$E\int_{\tau=\tau_x}^{\tau_{x^*}}\left(rI_b - P\left(x_\tau; k^f, k^b\right)\right)e^{-r(\tau-\tau_x)}d\tau > 0.$$

Since $P\left(x; k^f, k^b\right)$ is increasing in x, this implies that $\Pi_b\left(x,k\right) > 0$. So, $\Pi_b\left(x,k\right) > 0$ if and only if $k \in \mathbf{K}^b\left(x\right)$. Noting that $\Pi_b\left(x,k\right)$ is continuous and strictly increasing in k^b, we find that $\Pi_b\left(x,k\right) = 0$ if and only if $k \in \partial\mathbf{K}^b\left(x\right)$. It then also follows that $\Pi_b\left(x,k\right) < 0$ if and only if $k \in \left(\mathbf{K}^b\left(x\right) \cup \partial\mathbf{K}^b\left(x\right)\right)^C$. The proof is analogous for $\Pi_f\left(x,k\right)$. ■

We will next define the capacity path induced by a mass of agents behaving myopically. For a given value of x, we define the *admissible region* in $\left(k^f, k^b\right)$-space as a region where no myopic agents induces adjustments in k by investing or exiting. For k^b, the admissible region is simply the closure of the strict inaction region:

$$\overline{\mathbf{K}}^b\left(x\right) \equiv \mathbf{K}^b\left(x\right) \cup \partial\mathbf{K}^b\left(x\right).$$

For k^f, the admissible region also contains the k^b-axis $\left\{k \in \mathbb{R}_+ \times \mathbb{R}_+ \left| k^f = 0\right.\right\}$, because it is no longer possible to adjust k^f when $k^f = 0$:

$$\overline{\mathbf{K}}^f\left(x\right) \equiv \mathbf{K}^f\left(x\right) \cup \partial\mathbf{K}^f\left(x\right) \cup \left\{k \in \mathbb{R}_+ \times \mathbb{R}_+ \left| k^f = 0\right.\right\}.$$

We will call the boundaries of the admissible regions the *adjustment curves*, because once myopic agents adjust k from outside the admissible region, the adjustment stops as soon as k hits the boundary, and as will be shown further below, the adjustment will thereafter move along that boundary. For k^b, the adjustment curve is simply the same as the indifference region,

$$\overline{\partial\mathbf{K}}^b\left(x\right) \equiv \overline{\mathbf{K}}^b\left(x\right) \backslash \mathbf{K}^b\left(x\right) = \partial\mathbf{K}^b\left(x\right),$$

but for k^f the adjustment region also contains the k^b axis outside the strict inaction region:

$$\overline{\partial\mathbf{K}}^f\left(x\right) \equiv \overline{\mathbf{K}}^f\left(x\right) \backslash \mathbf{K}^f\left(x\right) = \partial\mathbf{K}^f\left(x\right) \cup \left[\left\{k \in \mathbb{R}_+ \times \mathbb{R}_+ \left| k^f = 0\right.\right\} \backslash \mathbf{K}^f\left(x\right)\right].$$

We may now define a *myopic adjustment path* as follows:

Definition 4 *An admissible capacity path* $\mathbf{k} = \left(\mathbf{k}^f, \mathbf{k}^b\right)$ *is a myopic adjustment path, if for all* $x \geq x_0$

$$\mathbf{k}\left(x\right) \in \overline{\mathbf{K}}^b\left(x\right) \cap \overline{\mathbf{K}}^f\left(x\right), \tag{29}$$

and further,

$$\mathbf{k}(x) \in \overline{\partial\mathbf{K}}^b(x) \text{ if } \mathbf{k}^b \text{ adjusts at } x, \text{ and} \quad (30)$$

$$\mathbf{k}(x) \in \overline{\partial\mathbf{K}}^f(x) \text{ if } \mathbf{k}^f \text{ adjusts at } x. \quad (31)$$

We are next going to show that myopic adjustment path and equilibrium are equivalent concepts in our model. We start with:

Proposition 6 *Let* $\mathbf{k}$ *be an equilibrium. Then* $\mathbf{k}$ *is a myopic adjustment path.*

Proof. This follows from Lemmas 1 and 2. ■

To prove the converse, it is helpful to first introduce two more lemmas. Lemma 3 says that the admissible region of capacity pairs shrinks as x grows:

Lemma 3 *Let* $x_0 \leq x' < x''$. *Then* $\overline{\mathbf{K}}^b(x'') \subseteq \overline{\mathbf{K}}^b(x')$ *and* $\overline{\mathbf{K}}^f(x'') \subseteq \overline{\mathbf{K}}^f(x')$.

Proof. Take $k \in \overline{\mathbf{K}}^b(x'')$. Then $\Pi_b(x'', k) \geq 0$ by Lemma 2. Since P is increasing in x, it must then also be that $\Pi_b(x', k) \geq 0$, which implies that $k \in \overline{\mathbf{K}}^b(x')$.

Similarly, take $k \in \overline{\mathbf{K}}^f(x'') = \mathbf{K}^f(x) \cup \partial\mathbf{K}^f(x) \cup \{k \in \mathbb{R}_+ \times \mathbb{R}_+ \,|k^f = 0\}$. If $k \in \{k \in \mathbb{R}_+ \times \mathbb{R}_+ \,|k^f = 0\}$, Lemma 3 holds trivially. If $k \in \mathbf{K}^f(x) \cup \partial\mathbf{K}^f(x)$, then $\Pi_f(x', k) \geq 0$ (by Lemma 2). Since $\pi_f(\cdot)$ is decreasing in x, it must also be that $\Pi_f(x', k) \geq 0$, which implies that $k \in \overline{\mathbf{K}}^f(x')$. ■

From this follows Lemma 4, which implies that a myopic adjustment path will move along the adjustment curves:

Lemma 4 *Let* $\mathbf{k}$ *be a myopic adjustment path. If* $\mathbf{k}(x) \in \overline{\partial\mathbf{K}}^f(x)$ *for some* $x \geq x_0$, *then* $\mathbf{k}(x'') \in \overline{\partial\mathbf{K}}^f(x'')$ *for all* $x'' > x$. *Similarly, if* $\mathbf{k}(x) \in \overline{\partial\mathbf{K}}^b(x)$ *for some* $x \geq x_0$, *then* $\mathbf{k}(x'') \in \overline{\partial\mathbf{K}}^b(x'')$ *for all* $x'' > x$.

Proof. Assume that $\mathbf{k}(x) \in \overline{\partial\mathbf{K}}^f(x)$ for some $x \geq x_0$, but $\mathbf{k}(x'') \notin \overline{\partial\mathbf{K}}^f(x'')$ for some $x'' > x$. It then follows from (29) that $\mathbf{k}(x'') \in \mathbf{K}^f(x'')$, and on the other hand, it follows from (31) that there must be some $x' \in [x, x'')$ such that $\mathbf{k}(x'') \in \overline{\partial\mathbf{K}}^f(x')$ (if $\mathbf{k}(x'') = \mathbf{k}(x)$, then $x' = x$ will do). But this means that $\overline{\mathbf{K}}^f(x'') \nsubseteq \overline{\mathbf{K}}^f(x')$, which contradicts Lemma 3. The proof is analogous for $\overline{\partial\mathbf{K}}^b(x)$. ■

Now we are ready to prove:

Proposition 7 *Let* $\mathbf{k}$ *be a myopic adjustment path. Then* $\mathbf{k}$ *is an equilibrium.*

Proof. We must show that (8) - (11) hold for a given myopic adjustment path $\mathbf{k}$. Start with (9). Take any x at which $\mathbf{k}^b$ adjusts. Let τ' and τ'', $\tau_x \leq \tau' < \tau''$, be any two consecutive stopping times at which k^b increases after x_t has hit x: $\tau' = \inf\{\tau < t \,|\widehat{x}_\tau = \widehat{x}_t\}$ and $\tau'' = \sup\{\tau > t \,|\widehat{x}_\tau = \widehat{x}_t\}$ for some $t \in (\tau', \tau'')$. By Lemma 4, $\mathbf{k}(x') \in \overline{\partial\mathbf{K}}^f(x')$ for all $x' > x$, and hence by Lemma 2, $\Pi_b(x', \mathbf{k}(x')) = 0$ for all $x' > x$. This implies (note that $\mathbf{k}^f(x_\tau)$ and $\mathbf{k}^b(x_\tau)$ are fixed through the period (τ', τ'')):

$$E\int_{\tau=\tau'}^{\tau''} \left(rI_b - P\left(x_\tau; \mathbf{k}^f(x_\tau), \mathbf{k}^b(x_\tau)\right)\right) e^{-r(\tau-\tau_x)} d\tau = 0. \qquad (32)$$

Because τ' and τ'' are arbitrary time points at which x_t reaches new record values, and since the time that x_t spends at its historic record values is measure zero almost surely (property of Brownian motion), (32) implies that

$$E\int_{\tau=\tau_x}^{\infty} \left(rI_b - P\left(x_\tau; \mathbf{k}^f(x_\tau), \mathbf{k}^b(x_\tau)\right)\right) e^{-r(\tau-\tau_x)} d\tau = 0,$$

which is the same as $V_b(x, x; \mathbf{k}) - I_b = 0$. Since x was an arbitrary point at which $\mathbf{k}^b$ adjusts, this means that (9) holds for $\mathbf{k}$.

Consider then (8). By the definition of myopic adjustment path, $\mathbf{k}(x) \in \overline{\mathbf{K}}^b(x)$ for all $x \geq x_0$, implying that $\Pi_b(x, \mathbf{k}(x)) \geq 0$ for all $x \geq x_0$. By the same argumentation as above, this implies $E\int_{\tau=\tau_x}^{\infty} \left(rI_b - P\left(x_\tau; \mathbf{k}^f(x_\tau), \mathbf{k}^b(x_\tau)\right)\right) e^{-r(\tau-\tau_x)} d\tau \geq 0$, which is the same as $V_b(x, x; \mathbf{k}) - I_b \leq 0$. Since this holds for any x, this means that there are no strictly profitable investment opportunities available, and we must have $F_b(x_t, \widehat{x}_t; \mathbf{k}) \leq 0$ for any $x_t \leq \widehat{x}_t$. On the other hand, it follows directly from the definition (7) that $F_b(\cdot) \geq 0$ always (simply choose $\tau^* = \infty$), so (8) must hold.

Consider (10). Given x_t and $\widehat{x}_t$, $V_f(x_t, \widehat{x}_t; \mathbf{k})$ is by definition the maximal payoff attainable. The question is, what kind of stopping rule gives this maximal payoff. Denote the payoff of stopping at a given $x \geq x_t$ by

$$\widetilde{V}_f(x, x_t, \widehat{x}_t; \mathbf{k}) \equiv E\left[\int_t^{\tau_x} \pi_f(x_\tau; \mathbf{k}^f(\hat{x}_\tau), \mathbf{k}^b(\hat{x}_\tau)) e^{-r(\tau-t)} d\tau - I_f e^{-r(\tau_x - t)}\right].$$

Denote $x' = \inf\left\{x \geq \widehat{x}_t \left|\mathbf{k}^f(x) \text{ adjusts}\right.\right\}$ and compare the payoffs of stopping at various thresholds. First, consider stopping at some τ_{x^-}, where $x_t \leq x^- < x'$. This gives payoff

$$\widetilde{V}_f\left(x^-, x_t, \widehat{x}_t; \mathbf{k}\right) = \widetilde{V}_f(x', x_t, \widehat{x}_t; \mathbf{k}) - E\left[\int_{\tau_{x^-}}^{\tau_{x'}} \pi_f(x_\tau; \mathbf{k}^f(\hat{x}_\tau), \mathbf{k}^b(\hat{x}_\tau)) e^{-r(\tau-t)} d\tau - I_f e^{-r(\tau_{x^*}-t)}\right].$$

Since $\mathbf{k}(x) \in \overline{\mathbf{K}}^f(x)$ for all $x \geq x_0$, it must hold that by the same line of reasoning as before that $E\left[\int_{t_{x^-}}^{\tau_{x'}} \pi_f(x_\tau; \mathbf{k}^f(\hat{x}_\tau), \mathbf{k}^b(\hat{x}_\tau)) e^{-r(\tau-t)} d\tau - I_f e^{-r(\tau_{x^*}-t)}\right] \geq 0$, which means that

$$\widetilde{V}_f\left(x^-, x_t, \widehat{x}_t; \mathbf{k}\right) \leq \widetilde{V}_f\left(x', x_t, \widehat{x}_t; \mathbf{k}\right).$$

On the other hand, by Lemmas 4 and 2, $\Pi_f\left(x, \mathbf{k}\left(x\right)\right) \geq 0$ for all $x^+ \geq x'$, and hence

$$\widetilde{V}_f\left(x^+, x_t, \widehat{x}_t; \mathbf{k}\right) \leq \widetilde{V}_f\left(x', x_t, \widehat{x}_t; \mathbf{k}\right)$$

for any $x^+ \geq x'$, meaning that the payoff is maximized by stopping at $\tau_{x'}$. However, what we want to show is that there are also other stopping points that give equal payoff. In particular, note that Lemma 2 says that $\Pi_f\left(x, \mathbf{k}\left(x\right)\right) = 0$ whenever $\mathbf{k}\left(x\right) \in \partial\mathbf{K}^f\left(x\right)$, which is the case for all $x'' \geq x'$ as long as $\mathbf{k}^f\left(x''\right) > 0$ (by Lemma 4 and the fact that $\overline{\partial\mathbf{K}}^f\left(x\right) \cap \left\{k \in \mathbb{R}_+ \times \mathbb{R}_+ \left| k^f > 0\right.\right\} = \partial\mathbf{K}^f\left(x\right)$). This means that for any $x'' \in [x', x_\infty)$, where $x_\infty = \sup\left\{x > x' \left| \mathbf{k}^f\left(x\right) > 0\right.\right\}$, we have by the same reasoning as before:

$$E\left[\int_{\tau_{x'}}^{\tau_{x''}} \pi_f(x_\tau; \mathbf{k}^f\left(\hat{x}_\tau\right), \mathbf{k}^b\left(\hat{x}_\tau\right))e^{-r(\tau-t)}d\tau - I_f e^{-r(\tau_{x^*}-t)}\right] = 0.$$

Hence:

$$\begin{aligned}
& \widetilde{V}_f\left(x'', x_t, \widehat{x}_t; \mathbf{k}\right) \\
= \ & \widetilde{V}_f\left(x', x_t, \widehat{x}_t; \mathbf{k}\right) + E\left[\int_{\tau_{x'}}^{\tau_{x''}} \pi_f(x_\tau; \mathbf{k}^f\left(\hat{x}_\tau\right), \mathbf{k}^b\left(\hat{x}_\tau\right))e^{-r(\tau-t)}d\tau - I_f e^{-r(\tau_{x^*}-t)}\right] \\
= \ & \widetilde{V}_f\left(x', x_t, \widehat{x}_t; \mathbf{k}\right).
\end{aligned}$$

Summing all this up, the payoff is maximized by stopping at $\tau_{x'}$ or alternatively at any $\tau_{x''}$, $x'' \in [x', x_\infty)$. Now, take any $x^* \geq \widehat{x}_t$ such that $\mathbf{k}^f\left(\cdot\right)$ adjusts at x^*. Clearly $x^* \in [x', x_\infty)$, and hence payoff is maximized by stopping at τ_{x^*}. So,

$$\widetilde{V}_f\left(x^*, x_t, \widehat{x}_t; \mathbf{k}\right) = E\left[\int_t^{\tau_{x^*}} \pi_f(x_\tau; \mathbf{k}^f\left(\hat{x}_\tau\right), \mathbf{k}^b\left(\hat{x}_\tau\right))e^{-r(\tau-t)}d\tau - I_f e^{-r(\tau_{x^*}-t)}\right] = V_f\left(x_t, \widehat{x}_t; \mathbf{k}\right),$$

that is, (10) holds.

Finally, consider (11). Assume $\mathbf{k}^f\left(x^*\right) > 0$. Then, there must be some $x' > x^*$ such that $\left(\mathbf{k}^f\left(x^*\right), \mathbf{k}^b\left(x'\right)\right) \notin \overline{\mathbf{K}}^b\left(x'\right) \cap \overline{\mathbf{K}}^f\left(x'\right)$ (in words, it must eventually become optimal for a myopic holder of factor dependent unit to exit when x climbs high enough). But by the definition of myopic adjustment path $\left(\mathbf{k}^f\left(x'\right), \mathbf{k}^b\left(x'\right)\right) \in \overline{\mathbf{K}}^b\left(x'\right) \cap \overline{\mathbf{K}}^f\left(x'\right)$ meaning that $\mathbf{k}^f\left(x'\right) < \mathbf{k}^f\left(x^*\right)$, so $\mathbf{k}^f\left(\cdot\right)$ must adjust for some $x^{**} \in (x^*, x')$. Hence, (10) holds for $\tau_{x^{**}}$, and since $\tau_{x^{**}} > \tau_{x^*}$, also (11) holds. ■

Now that we have established the equivalence between equilibrium and myopic adjustment path, we can safely work with the latter. To complete the proof of Theorem 1, it suffices to show that there is a unique myopic adjustment path with the properties stated in the Theorem. We will do that in Proposition 8 with the help of a number of Lemmas. First, in Lemmas (5) - (7) we will describe some geometrics of the admissible regions, which is useful for establishing the existence and uniqueness of the myopic adjustment path.

Lemma 5 *Let* $\left(k^f, k^b\right) \in \overline{\mathbf{K}}^b(x) \cap \overline{\mathbf{K}}^f(x)$. *Then* $\left(k'^f, k'^b\right) \in \overline{\mathbf{K}}^b(x) \cap \overline{\mathbf{K}}^f(x)$ *whenever* $0 \le k'^f < k^f$, $k'^b > k^b$, *and* $k'^f + k'^b = k^f + k^b$.

Lemma 6 $\overline{\partial\mathbf{K}}^b(x) \cap \overline{\partial\mathbf{K}}^b(x)$ *is a singleton that we denote by* $\widehat{k}(x) = \left(\widehat{k}^f(x), \widehat{k}^b(x)\right)$. *If* $k^f > \widehat{k}^f(x)$ *and* $k^b < \widehat{k}^b(x)$, *then* $\left(k^f, k^b\right) \notin \overline{\mathbf{K}}^b(x) \cap \overline{\mathbf{K}}^f(x)$.

Lemma 7 *Take some* $k = \left(k^f, k^b\right)$, $k_- = \left(k_-^f, k_-^b\right)$, *and* $k_+ = \left(k_+^f, k_+^b\right)$ *such that* $0 \le k_-^f \le k^f \le k_+^f$ *and* $0 \le k_-^f \le k^f \le k_+^f$. *Then the following hold:*

- *If* $k \in \overline{\mathbf{K}}^f(x)$, *then* $k_- \in \overline{\mathbf{K}}^f(x)$.
- *If* $k \notin \overline{\mathbf{K}}^f(x)$, *then* $k_+ \notin \overline{\mathbf{K}}^f(x)$.
- *If* $k \in \overline{\mathbf{K}}^b(x)$, *then* $k_+ \in \overline{\mathbf{K}}^b(x)$.
- *If* $k \notin \overline{\mathbf{K}}^b(x)$, *then* $k_{\_} \notin \overline{\mathbf{K}}^b(x)$.

Lemmas (5) - (7) can be proved using Lemma 2 and the monotonicity properties of $P(\cdot)$ and $\pi_f(\cdot)$ with respect to k^f and k^b. Next, Lemma 8 will be used to show that myopic adjustments are continuous after the initial adjustment:

Lemma 8 *Let* $k = \left(k^f, k^b\right) \in \overline{\mathbf{K}}^b(x) \cap \overline{\mathbf{K}}^f(x)$. *Then, for any* $\varepsilon > 0$, *there is some* $\delta > 0$ *such that* $\left[\overline{\mathbf{K}}^b(x') \cap \overline{\mathbf{K}}^f(x')\right] \cap \left[\left[k^f - \varepsilon, k^f\right] \times \left[k^b, k^b + \varepsilon\right]\right]$ *is non-empty whenever* $x' \le x + \delta$.

Proof. After noting that $\Pi_b(x, k)$ and $\Pi_f(x, k)$ are continuous in k^f and k^b, this result follows from Lemma 2, Lemma 5, and Lemma 6. ■

Lemma 9 will imply that once an adjustment begins, there will be adjustment every time x hits a new record value, until the whole adjustment process is over:

Lemma 9 *Let* $x' \neq x''$. *Then*

$$\partial\mathbf{K}^f(x') \cap \partial\mathbf{K}^f(x'') = \emptyset$$

and

$$\partial\mathbf{K}^b(x') \cap \partial\mathbf{K}^b(x'') \cap \left\{k \in \mathbb{R}_+ \times \mathbb{R}_+ \left| k^f + k^b > k_\infty \right.\right\} = \emptyset.$$

Proof. This follows from Lemma 2 and the monotonicity properties of $P(\cdot)$ and $\pi_f(\cdot)$. ■

We are now ready to complete the proof of Theorem 1 by posing:

Proposition 8 *There is a unique myopic adjustment path* $\mathbf{k} = \left(\mathbf{k}^f, \mathbf{k}^b\right)$ *with the following properties:*

- $\mathbf{k}^f$ *is everywhere continuous, strictly decreasing on some interval* $(a_f, b) \subset \mathbb{R}^+$*, and constant on* $\mathbb{R}^+ \setminus (a_f, b)$*.*
- $\mathbf{k}^b$ *is everywhere continuous, strictly increasing on some interval* $(a_b, b) \subset \mathbb{R}^+$*, and constant on* $\mathbb{R}^+ \setminus (a_b, b)$*.*

Proof. This is a constructive proof. Take $\left\{x_0, k_0^f, k_0^b\right\}$, and consider the initial adjustment implied by Definition 4. If $\left(k_0^f, k_0^b\right) \in \overline{\mathbf{K}}^b(x) \cap \overline{\mathbf{K}}^f(x)$, no initial adjustment is taken, and Definition 4 requires that $\mathbf{k}(x_0) = \left(k_0^f, k_0^b\right)$. If $k_0^f > \widehat{k}^f(x_0)$ and $k_0^b < \widehat{k}^b(x_0)$, then there must be initial adjustment for both k^f and k^b, and Lemmas 6 and 7 ensure that $\mathbf{k}(x_0) = \left(\widehat{k}^f(x_0), \widehat{k}^b(x_0)\right)$ is a unique starting point for the myopic adjustment path. If, $k_0^f \leq \widehat{k}^f(x_0)$ and $k_0^b \notin \overline{\mathbf{K}}^b(x_0)$, then Lemmas 5 and 7 imply that there is a unique $k'^b > k_0^b$ such that $\left(k_0^f, k'^b\right) \in \overline{\partial\mathbf{K}}^b \cap \overline{\mathbf{K}}^f(x)$, and hence $\mathbf{k}(x_0) = \left(k_0^f, k'^b\right)$ is the unique point satisfying Definition 4. Similarly, If, $k_0^b \leq \widehat{k}^b(x_0)$ and $k_0^f \notin \overline{\mathbf{K}}^f(x_0)$, then Lemmas 5 and 7 imply that there is a unique $k'^f < k_0^f$ such that $\left(k'^f, k_0^b\right) \in \overline{\mathbf{K}}^b \cap \overline{\partial\mathbf{K}}^f(x)$, and hence $\mathbf{k}(x_0) = \left(k'^f, k_0^b\right)$. These cases cover all possible initial value combinations, and hence we may conclude that there is a unique initial adjustment that satisfies Definition 4. Let us then increase $\widehat{x}$ continuously from x_0 up to infinity. In the same way as with the initial adjustments, Lemmas 5 - 7 ensure that there is always a unique $\mathbf{k}(\widehat{x})$ satisfying Definition 4 as x reaches new record values. Lemma 8 ensures that Definition 4 forces $\mathbf{k}$ to adjust continuously, and Lemma 9 ensures that once adjustment has started for k^f or k^b (excluding the possible initial adjustment for k^b), there will be adjustment every time x hits a new record value until $\mathbf{k}^f$ falls to zero at which point all adjustments stop (let the value of x at which this happens be denoted b). This means that $\mathbf{k}^f$ is strictly decreasing on some interval $(a_f, b) \subset \mathbb{R}^+$, and constant on $\mathbb{R}^+ \setminus (a_f, b)$, and $\mathbf{k}^b$ is strictly increasing on some interval $(a_b, b) \subset \mathbb{R}^+$, and constant on $\mathbb{R}^+ \setminus (a_b, b)$. ■

APPENDIX B: Proof of Proposition 4.

Lemma 10 *Assume $\sigma > \sigma^*$. Then, the adoption path defines unique tresholds $\underline{X}$ and $\overline{X}$ in $(0, b) \subset \mathbb{R}^+$such that*

$$\begin{aligned} \hat{x} &= \underline{x}(\mathbf{k}^f(\hat{x}), \mathbf{k}^b(\hat{x})) \textit{ for } \hat{x} = \underline{X} \\ \hat{x} &= \overline{x}(\mathbf{k}^f(\hat{x}), \mathbf{k}^b(\hat{x})) \textit{ for } \hat{x} = \overline{X} > \underline{X}. \end{aligned}$$

Proof. We will show that

$$0 < \frac{\partial}{\partial \hat{x}} \underline{x}(\mathbf{k}^f(\hat{x}), \mathbf{k}^b(\hat{x})) < 1 \tag{33}$$

holds along the adoption path, which implies that $\hat{x} = \underline{x}(\mathbf{k}^f(\hat{x}), \mathbf{k}^b(\hat{x}))$ holds exactly once in $(0, b)$. Because

$$\begin{aligned} \frac{\partial}{\partial \hat{x}} \overline{x}(\mathbf{k}^f(\hat{x}), \mathbf{k}^b(\hat{x})) &= \frac{\partial}{\partial \hat{x}} D(\mathbf{k}^b(\hat{x})) < 0 \text{ and} \\ \overline{x}(\mathbf{k}^f(\hat{x}), \mathbf{k}^b(\hat{x})) &> \underline{x}(\mathbf{k}^f(\hat{x}), \mathbf{k}^b(\hat{x})) \text{ in } (0, b)\,, \end{aligned}$$

it follows that the equation $\hat{x} = \overline{x}(\mathbf{k}^f(\hat{x}), \mathbf{k}^b(\hat{x}))$ holds exactly once in $(0, b)$. Thus, after proving (33), the proof is complete. To this end, note that by the exiting firm's indifference condition (14), the expected payoff must be the same at different exit points along the adoption path:

$$\begin{aligned} & E_{t=\tau^*} \int_{\tau^*}^{\tau^{*\prime}} \pi_f(x_\tau; \mathbf{k}^f(x_{\tau^*}), \mathbf{k}^b\,(x_{\tau^*})) e^{-r(\tau - \tau^*)} d\tau \qquad (34) \\ = \; & E_{t=\tau^{**}} \int_{\tau^{**}}^{\tau^{**\prime}} \pi_f(x_\tau; \mathbf{k}^f(x_{\tau^{**}}), \mathbf{k}^b\,(x_{\tau^{**}})) e^{-r(\tau - \tau^{**})} d\tau, \end{aligned}$$

where $\pi_f(\cdot)$ is given by (4), τ^* and $\tau^{**} > \tau^*$ are two equilibrium exit points, and $\tau^{*\prime}$ and $\tau^{**\prime}$ are the corresponding "next" exit points (we are not entirely rigorous here in defining the "next" exit points, but the argument we are making is so simple that we believe this will do here; see Proof of Theorem 1 for more formalism in this issue). The payoff function evaluated at τ^* and τ^{**} is piecewise linear (see (4)) with the intercept $\underline{x}$ changing as the capacities change. It is straightforward to verify that the intercept must increase with $\hat{x}$, $0 < \frac{\partial}{\partial \hat{x}} \underline{x}(\mathbf{k}^f(\hat{x}), \mathbf{k}^b(\hat{x}))$, otherwise the payoff function would be strictly worse for larger $\hat{x}$ values, and the equality of expected payoffs in (34) could not hold (formally, this can be seen by using the normalization explained in Appendix C). However, if $1 < \frac{\partial}{\partial \hat{x}} \underline{x}(\mathbf{k}^f(\hat{x}), \mathbf{k}^b(\hat{x}))$, then again the equality of expected payoffs cannot hold (the second line in (34) would be strictly larger, which can be seen directly from the form of the payoff function). Thus, (33) must hold. ■

The Proposition follows readily from this Lemma. For $\hat{x}$ close to zero, $\underline{x}(\mathbf{k}^f(\hat{x}), \mathbf{k}^b(\hat{x})) > \hat{x}$, so the adoption path must start in the active capacity phase. By the Lemma, the equilibrium leaves this phase only once. Also by the Lemma, the equilibrium enters and leaves the volatile capacity phase only once. Finally, it follows that the equilibrium cannot leave the idle capacity phase once reached. This completes the proof.

APPENDIX C: Proof of Proposition 5.

Proof. We prove the result by studying the entrants' indifference condition (13) along the adoption path. Let τ^* and $\tau^{**} > \tau^*$ denote two equilibrium entry points in the volatile capacity phase, $(\underline{X}, \overline{X}]$. By the indifference condition, the expected present-value revenues between consecutive entry points must be the same at the two entry points. Therefore, we can write

$$E_{t=\tau^*} \int_{\tau^*}^{\tau^{*\prime}} P(x_\tau; \mathbf{k}^f(x_{\tau^*}), \mathbf{k}^b(x_{\tau^*})) e^{-r(\tau-\tau^*)} d\tau \tag{35}$$

$$= E_{t=\tau^{**}} \int_{\tau^{**}}^{\tau^{**\prime}} P(x_\tau; \mathbf{k}^f(x_{\tau^{**}}), \mathbf{k}^b(x_{\tau^{**}})) e^{-r(\tau-\tau^{**})} d\tau \tag{36}$$

$$= E_{t=\tau^{**}} \int_{\tau^{**}}^{\tau^{**\prime}} P(\frac{1}{\gamma} \cdot y_\tau; \mathbf{k}^f(x_{\tau^{**}}), \mathbf{k}^b(x_{\tau^{**}})) e^{-r(\tau-\tau^{**})} d\tau \tag{37}$$

where τ^* and $\tau^{**} > \tau^*$ are two equilibrium entry points, $\tau^{*\prime}$ and $\tau^{**\prime}$ are the corresponding next entry points (as in Proof of Proposition 4, we are somewhat vague in defining the next entry points; see Proof of Theorem 1 for more formalism). The first and second lines follow from the entrants' indifference condition, and the third is simply the second rewritten after defining $y_\tau \equiv \gamma x_\tau$, where $\gamma \equiv \frac{x_{\tau^*}}{x_{\tau^{**}}} < 1$. We make use of the fact that a variable following a Geometric Brownian Motion can be scaled without changing the process. That is, when $x \sim GBM(\alpha, \sigma)$, also $y \equiv \gamma x \sim GBM(\alpha, \sigma)$. The idea is to normalize process at the entry point τ^{**} such that the starting value of the process is the same as at τ^*, that is $y_{\tau^{**}} = x_{\tau^*}$. In this way we are replicating time τ^* entry problem at time τ^{**} but with two changes in the price function: 1) the argument has been scaled by term $\frac{1}{\gamma}$, and 2), the capacities have changed by time τ^{**}. Now we can consider how the equilibrium price function must change to retain equality between (35) and (37). By the linearity, the price function in the volatile capacity phase is

$$P(x_\tau; \mathbf{k}^f(\hat{x}), \mathbf{k}^b(\hat{x})) = \begin{cases} A - B(\mathbf{k}^f(\hat{x}) + \mathbf{k}^b(\hat{x})), \text{ when } x_\tau < \underline{x}(\mathbf{k}^f(\hat{x}), \mathbf{k}^b(\hat{x})) \\ Q(\mathbf{k}^b(\hat{x})) + R x_\tau, \text{ when } \underline{x}(\mathbf{k}^f(\hat{x}), \mathbf{k}^b(\hat{x})) \leq x_\tau \leq \hat{x} \end{cases}$$

where

$$Q(\mathbf{k}^b(\hat{x})) = \frac{C(A - B\mathbf{k}^b(\hat{x}))}{B + C}$$
$$R = \frac{B}{B + C}.$$

Now consider the lines (35) and (37) above. By the fact that $\mathbf{k}^f(\hat{x}) + \mathbf{k}^b(\hat{x})$ increases in the volatile capacity phase (Proposition 4), we must have $\underline{P}(x_{\tau^{**}}) < \underline{P}(x_{\tau^*})$, which is equivalent to $\underline{P}(\frac{1}{\gamma} \cdot y_{\tau^{**}}) < \underline{P}(x_{\tau^*})$. To retain the equality of (35) and (37) it must then be that

$$\overline{P}(\frac{1}{\gamma} \cdot y_{\tau^{**}}) > \overline{P}(x_{\tau^*})$$

otherwise the scaled price function associated with τ^{**} would be strictly worse than the original for the excursion until the next entry point (this argument rests on the linearity of the price function, and the fact that scaling the argument by the term $\frac{1}{\gamma}$ increases its slope). We have now shown that $\overline{P}(x_{\tau^{**}}) = \overline{P}(\frac{1}{\gamma} \cdot y_{\tau^{**}}) > \overline{P}(x_{\tau^*})$ for arbitrary entrypoints $x_{\tau^{**}} > x_{\tau^*}$, meaning that $\overline{P}(\hat{x})$ must be increasing for all $\hat{x} \in (\underline{X}, \overline{X}]$. The remaining cases follow trivially from Proposition 4. ■

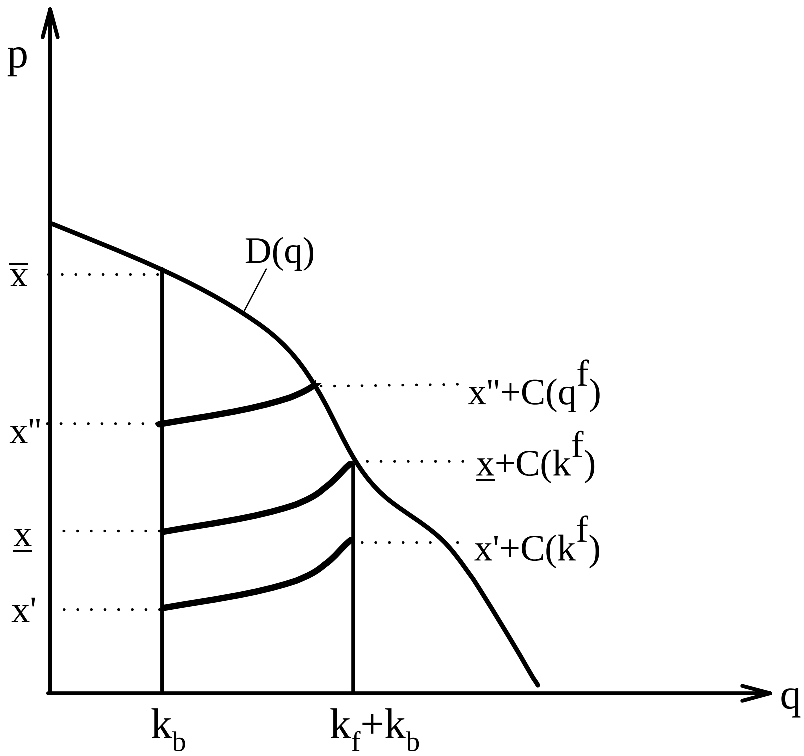


FIGURE 1

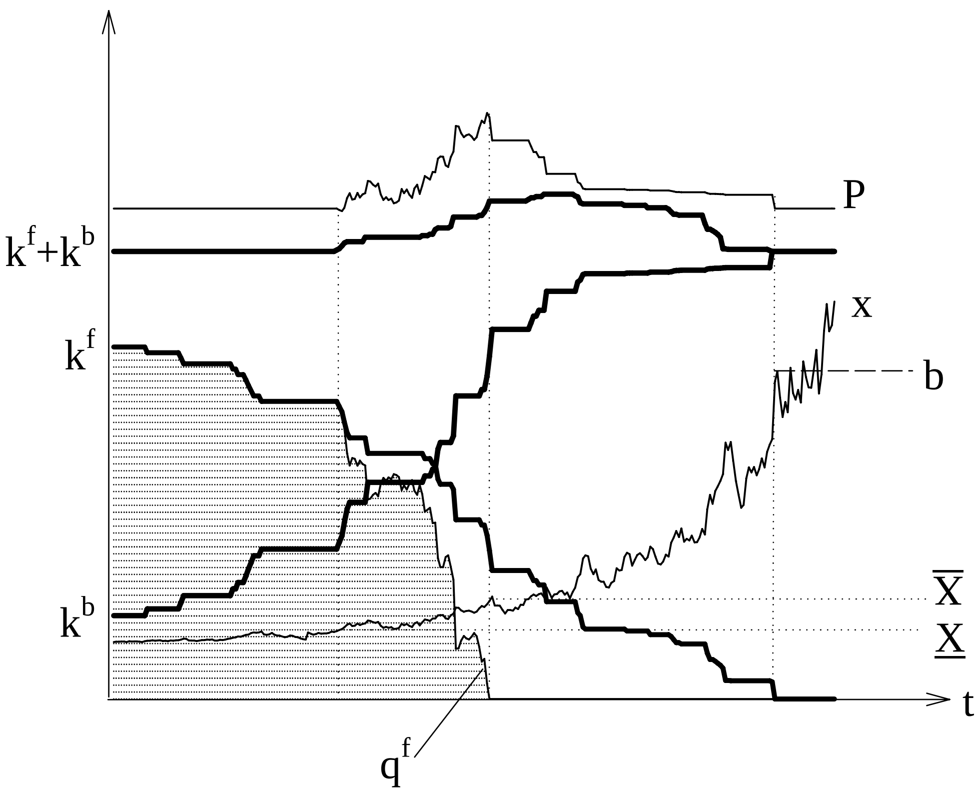
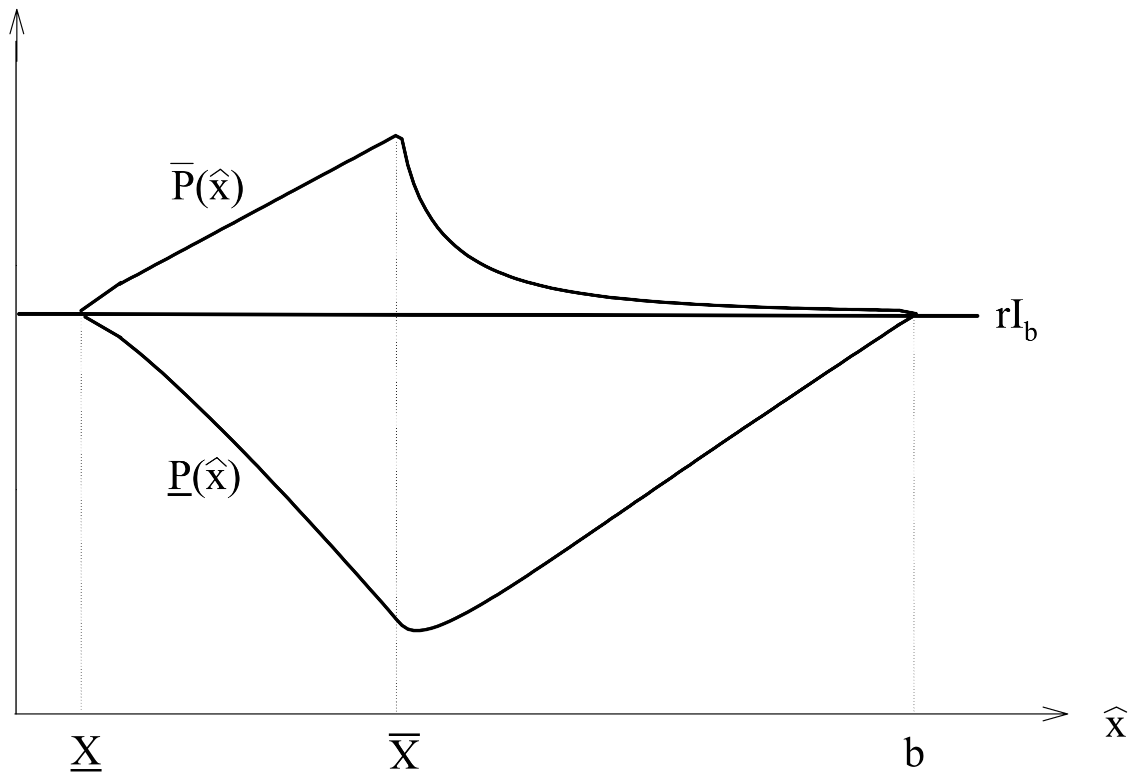


FIGURE 2



$\overline{P}(\hat{x})$
$\underline{P}(\hat{x})$
rI_b
$\underline{X}$
$\overline{X}$
b
$\hat{x}$

FIGURE 3