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Klaus Kultti
University of Helsinki and HECER

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HECER – Helsinki Center of Economic Research, P.O. Box 17 (Arkadiankatu 7), FI-00014 University of Helsinki, FINLAND, Tel +358-9-191-28780, Fax +358-9-191-28781,
E-mail info-hecer@helsinki.fi, Internet www.hecer.fi

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Abstract

I study an economy with sellers and buyers. The sellers are capacity constrained and face stochastic demand. They have a choice of locating geographically close to each other, i.e., clustering or locating separately. In the former case the buyers can visit any of them while in the latter case the buyers can visit only one of them. The sellers post prices which are observed by the buyers who base their decision to contact sellers on the prices. I explicitly derive the equilibrium prices or price strategies in the clustered and in the non-clustered market for an arbitrary distribution of demand. I show that the clustered market often yields higher profits than the non-clustered. Finally, I derive the equilibrium market structure.

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Klaus Kultti

Department of Economics
University of Helsinki
P.O. Box 17 (Arkadiankatu 7)
FI-00014 University of Helsinki
FINLAND

e-mail: klaus.kultti@helsinki.fi

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1 Introduction

It is well known that often competitors locate close to each other. Examples abound from petrol stations and market places where sellers of similar goods are side by side to shopping centres where firms specialising in very close substitutes, say outdoor equipment, gather in the same part of the centre. This is somewhat strange as it seems to foster competition.[1] Indeed, with symmetric firms and constant marginal costs Bertrand competition leads to zero profits.

To explain the phenomenon the standard setting of Bertrand competition must be altered. I introduce two extra features. The first one concerns capacity. While in the standard setting the firms are assumed to possess capacity up to the competitive level I assume that the firms are capacity constrained. This makes the firms' pricing and location choices non-trivial. The other crucial feature concerns demand. I assume that the demand is stochastic, and that firms learn about it only after they have made the pricing decisions.

Of particular interest is the effect of the firms' location choices and demand uncertainty on the intensity of competition. With limited capacity and stochastic demand the firms face a trade-off between clustering close to each other and choosing separate locations. In the former case they compete fiercely when demand is less than supply but when demand is greater than supply the firms can price like monopolists. When the firms are located separately they have to compete for buyers no matter how large the demand; when buyers use symmetric strategies they contact the firms randomly, in an un-coordinated manner, and it is always possible that a firm remains without any buyers. As the firms do not know the level of demand their pricing strategies reflect their expectations about the degree of competition.

Uncertain demand is crucial here since if it were known that there are more buyers than firms, the firms would certainly cluster; they could charge the monopoly price, and all of them would succeed in trading. Conversely, if it were known that there are fewer buyers than firms, the firms certainly would not cluster; locating in separate locations would save them from Bertrand-competition and zero profits.

The aim of this article is to demonstrate exactly how the trade-off above affects location, pricing and profits. To make the point as clear as possible I assume that the firms provide homogeneous goods, possess unit-capacity, and compete in prices before the magnitude of the demand is known. I explicitly model the price formation process as a non-cooperative activity, and I derive the equilibrium pricing strategies both when the firms are clustered and when they are non-clustered. Further, I assume that the buyers are perfectly informed about the prices offered by the firms.

Apriori it is not clear that the firms should either all cluster together or locate separately from each other, and I allow for any mixture of these choices.

[1] An obvious guess is that locating close to each other helps collusion. Addressing collusion requires a dynamic framework, while here I focus on a phenomenon that comes up also in a static setting.

The main result of the article concerns the equilibrium market structure. It tells how the firms locate and price in the market taking into account the buyers' reactions to the prices and to the firms' location choices. This is non-trivial since a good outcome from the firms' perspective tends to be a bad outcome from the buyers' perspective. Understanding market structure is important since there is no immediate reason why the market as such should be efficient.

The literature closest to this work consist of Stahl (1983) and Wolinsky (1983) as well as Dudey (1990). Stahl and Wolinsky study how the sellers' location choices affect the consumers' search decision when the consumers do not know the prices. They find that clustering may be a profitable strategy to the sellers; because of search costs the consumers find clusters more desirable than geographically separated sellers.

Dudey retains the feature that consumers do not know prices but he models the price formation stage carefully. The consumers expect higher degree of competition and lower prices in clusters, and thus clusters attract more consumers than sellers who are separated. In equilibrium the consumers' expectations about prices turn out true.

Dana (1993) recognises the importance of demand uncertainty but his interest is in the associated price dispersion. In his model capacity is costly, and the sellers set prices before the demand is known. The sellers then choose a menu of prices and the quantity to be sold at each price in the menu.

Deneckere and Peck (1995) study a situation with a finite number of non-clustered firms and a continuum of buyers. Demand is uncertain like in my present model, and the firms choose capacity. The authors derive the equilibrium prices and capacities in a symmetric equilibrium. Given the level of demand the assumption about a finite number of firms gets rid of uncertainty about the number of buyers while in the present model with a continuum of firms this uncertainty still remains in the non-clustered market. Deneckere and Peck do not address the location choices of the firms at all.

Burdett, Shi and Wright (2001) study the equilibrium price posting in a finite agent deterministic world that corresponds to the non-clustered market in this article. They note that when some firms have one unit and the rest of the firms have two units for sale the equilibrium prices of the higher capacity firms are higher. This means that more capacity per location is regarded as attractive by the buyers. But it is not obvious what happens when the capacity is offered by competing firms rather than one firm. A complete solution to this problem when prices are determined by auction, rather than price posting, and when there are fewer buyers than sellers is given in Kultti (2003a).

In the older literature already Chamberlin (1933) realised the trade-off between increased competition from locating close to each other and the positive effect this has on attracting consumers.

Methodologically the work here belongs to the directed search literature where firms are dispersed. By posting prices firms attract buyers. The driving force is the buyers' uncoordinated decisions about which firm to visit, which then may result locally in under or over demand. The seminal work is by Peters (1984, 1991) where he shows the existence of the equilibrium, and provides the foundation for the urn-ball matching technology with an infinite number of agents.

The article is organised as follows. In section 2 I present an example that makes the trade-off between clustering and not clustering clear. In section 3 I generalise the example to an infinite agent model where the demand follows an arbitrary distribution. In section 4 I take also the buyers' reactions into account and determine the equilibrium market structure. In section 5 I take up some caveats and in section 6 I summarise.

2 A motivating example

Even though this is intended just as an illustrating example I think that it is of independent interest and provides a rather neat result. Assume that there are two firms 1 and 2. The firms are capacity constrained with zero marginal costs. Both firms possess one unit of a good for sale both goods being identical. There are potential buyers each of whom demands one unit of a good from which he obtains one unit of utility. The demand is uncertain: With probability p_1 there is only one buyer, with probability p_2 there are two buyers and with probability $1 - p_1 - p_2$ there are three buyers.

I consider two scenarios or different market structures. In the first one the firms are physically separated so that the buyers can visit only one of them. The firms post prices to attract buyers, and I determine the equilibrium prices. This is called the non-clustered market

In the second scenario the firms are located in the same place so that buyers can visit both of them. The firms again post prices to attract buyers. This is called the clustered market.

There is a difference in the firms' pricing strategies in the two cases. In the non-clustered market there is a unique symmetric equilibrium in pure strategies, i.e., both firms post the same price. In the clustered market the unique symmetric strategy is a mixed strategy that is continuous on a closed interval.

2.1 Non-clustered market

First I derive the equilibrium price when the firms are not clustered. Denote the price of firm 1 by q_1 and that of firm 2 by q_2. Denote the probability that any buyer goes to firm 1 by π_1. To determine how the pricing decisions of the firms affect the buyers' probability of contacting them note that in equilibrium any buyer must be indifferent between visiting firm 1 and firm 2. When the buyers' actions are unco-ordinated they face a trade-off between low prices and the probability of acquiring a good. A buyer must assess the probability that the demand is one, two and three given the information that he (the buyer) exists. Assuming that each buyer is alike one gets the following probabilities by symmetry

$$r_1 = \frac{\frac{1}{3}p_1}{\frac{1}{3}p_1 + \frac{2}{3}p_2 + (1 - p_1 - p_2)} = \frac{p_1}{3 - 2p_1 - p_2} \tag{1}$$

$$r_2 = \frac{2p_2}{3 - 2p_1 - p_2} \tag{2}$$

$$r_3 = \frac{3\left(1 - p_1 - p_2\right)}{3 - 2p_1 - p_2} \tag{3}$$

where r_i is the updated probability that the demand is i.

A buyer's expected utility of visiting firm 1 is given by

$$\begin{aligned} & r_1\left(1 - q_1\right) + r_2 \frac{\pi_1}{2}\left(1 - q_1\right) + r_2\left(1 - \pi_1\right)\left(1 - q_1\right) + \\ & r_3 \frac{\pi_1^2}{3}\left(1 - q_1\right) + r_3 \frac{2\pi_1\left(1 - \pi_1\right)}{2}\left(1 - q_1\right) + \\ & r_3\left(1 - \pi_1\right)^2\left(1 - q_1\right) \end{aligned} \tag{4}$$

Analogously, the expected utility of contacting firm 2 is given by

$$\begin{aligned} & r_1\left(1 - q_2\right) + r_2\pi_1\left(1 - q_2\right) + r_2 \frac{1 - \pi_1}{2}\left(1 - q_2\right) + \\ & r_3\pi_1^2\left(1 - q_2\right) + r_3 \frac{2\pi_1\left(1 - \pi_1\right)}{2}\left(1 - q_2\right) + \\ & r_3 \frac{\left(1 - \pi_1\right)^2}{3}\left(1 - q_2\right) \end{aligned} \tag{5}$$

Equating (4) and (5) determines the buyers' behaviour, i.e., determines the value of π_1 given the posted prices q_1 and q_2. In the symmetric equilibrium both firms post price $q_1 = q_2 = q$, and then the buyers visit each firm with equal probability. Totally differentiating the equality and inserting the equilibrium conditions yields the following expression

$$\frac{\partial \pi_1}{\partial q_1} = -\frac{12r_1 + 9r_2 + r_3}{\left(1 - q\right)\left[12r_2 + 16r_3\right]} \tag{6}$$

firm 1's problem is the following

$$\max_{q_1} p_1\pi_1 q_1 + p_2\left(1 - \left(1 - \pi_1\right)^2\right) q_1 + \left(1 - p_1 - p_2\right)\left(1 - \left(1 - \pi_1\right)^3\right) q_1 \tag{7}$$

In the first term, with probability p_1 there is only one buyer and with probability π_1 he comes to firm 1. In the second term, with probability p_2 there are two buyers and at least one of them comes to firm 1 if it is not the case that both of them go to the firm 2. This happens with probability with $\left(1 - \left(1 - \pi_1\right)^2\right)$. The third term is interpreted analogously. Evaluating the first order condition of this problem at the symmetric equilibrium where $q_1 = q_2 = q$ and $\pi_1 = \frac{1}{2}$ yields the following quite nasty looking expression for the equilibrium price

$$q = \frac{\left(12r_2 + 16r_3\right)\left(4p_1 + 6p_2 + 7p_3\right)}{\left(12r_2 + 16r_3\right)\left(4p_1 + 6p_2 + 7p_3\right) + \left(12r_1 + 9r_2 + 7r_3\right)\left(8p_1 + 8p_2 + 6p_3\right)} \tag{8}$$

It is readily seen that this is always between zero and unity.

The expected utility of a firm is the probability that it makes a sale times the price, i.e.,

$$q\frac{4p_1 + 6p_2 + 7p_3}{8} \tag{9}$$

2.2 Clustered market

Assume that the firms are located so close to each other that the consumers are able to visit both of them. As long as $1 > p_1 > 0$ the firms' pricing in a symmetric equilibrium is in mixed strategies. Denote this strategy by F and assume that its support is a closed interval $[l, L]$. It is immediately clear that $L = 1$ since the firm that chooses L only makes a sale if at least two buyers appear and then it is not optimal to leave any surplus to the buyers. The firm quoting price $L = 1$ makes a sale with probability $1 - p_1$ and this is also its expected utility. The firm quoting price l makes a sale for certain (when the other firm follows the equilibrium strategy). To get the same expected utility as a firm with price unity it must be the case that $l = 1 - p_1$. If a firm posts price $\rho \in (l, L)$ its expected utility is given by

$$p_1 (1 - F(\rho)) \rho + (1 - p_1) \rho \tag{10}$$

This choice, too, must yield utility $1 - p_1$ from which the equilibrium strategy can be solved

$$F(\rho) = \frac{\rho - 1 + p_1}{\rho p_1} \tag{11}$$

Proposition 1 *Whenever $1 > p_1 > 0$ the expected utility of the firms is higher in the clustered market than in the non-clustered market.*
Proof. *Straightforward calculation.* ■

The heuristics of the result emanate from the trade-off between being able to price like a monopolist and being forced to engage in Bertrand-competition. When the firms are not clustered the ex-ante probability of ending up with no buyer when pricing symmetrically is $\frac{1}{2}p_1 + \frac{1}{4}p_2 + \frac{1}{8}(1 - p_1 - p_2)$, while when they are clustered it is $\frac{1}{2}p_1$. The higher probability in the non-clustered market induces the firms to engage in more severe price competition to attract buyers.[2]

3 The general case

3.1 The set up

[2] As the buyers take into account the probability of getting the good this is not a complete explanation.

What we can infer from the example in the previous section is that if the highest realisations of the (stochastic) demand are sufficiently larger than the supply then the firms find it optimal to cluster. The exact magnitude, however, remains unclear because demand of two or three is 100% or 150% of the supply (which is two in the example). To derive sharper results I consider a limit economy where the measure of firms is unity and the measure of buyers, θ, is distributed according to a distribution function H on an interval $[0, m]$, $m > 1$. For simplicity, but with no loss in generality as to the results, I assume that there are no atoms.

I conduct similar analysis as in the example, and to that end I need a buyer's expectation that there are exactly $a \in [0, m]$ buyers. By symmetry the updated probability is given by

$$g(a) = \frac{\frac{a}{m}h(a)}{\int_0^m \frac{x}{m}h(x)dx} = \frac{ah(a)}{E(\theta)} \tag{12}$$

Notice that this cannot be accomplished by iid random variables but this causes no problems as the agents are anonymous.

The timing of the model is such that first firms simultaneously quote prices, then buyers observe the prices and based on these they simultaneously approach the firms. To place the comparison of the markets on an equal footing I focus on symmetric equilibrium in both markets; in the clustered market this means pricing in mixed strategies, while in the non-clustered market pricing is in pure strategies.

Definition 2 *A symmetric equilibrium in a particular market consists of symmetric pricing strategies of the firms, and symmetric contact strategies of the buyers such that any firm's strategy is the best response to the other firms' and buyers' strategies, and any buyer's strategy is the best response to the firms strategies and other buyers' strategies.*

3.1.1 Non-clustered market

The firms quote prices and the buyers contact the firms using a symmetric mixed strategy. Thus, if all firms quote the same price the number of buyers a firm expects is Poisson-distributed with parameter $\frac{\#buyers}{\#firms} = \frac{\theta}{1} = \theta$. The probability that a firm meets exactly k buyers is $e^{-\theta}\frac{\theta^k}{k!}$.[3]

Denote the equilibrium price by q. Any firm is of measure zero and the criterion for the Nash-equilibrium is very weak. To determine q I assume that proportion ε of the firms deviate and quote price $\widetilde{q}$. Then I impose the equilibrium condition that $\widetilde{q} = q$. When ε approaches zero this technique yields the equilibrium price that is the limit of the equilibrium price in the finite agent model (eg. Kultti 2003b, Burdett, Shi and Wright 2001).

[3] The Poisson-distribution is the limit of the binomial distribution when the numbers of buyers and sellers go to infinity so that the ratio remains constant.

Now the buyers' equilibrium strategy is to mix between going to a deviating firm and a non-deviating firm. With probability z a buyer goes to a deviating firm and with probability $1-z$ to a non-deviating firm. Then the Poisson-rates that govern the meetings of the buyers and the non-deviating firms is

$$\alpha = \frac{1-z}{1-\varepsilon}\theta \tag{13}$$

and the corresponding rate for the buyers and deviating firms is

$$\beta = \frac{z}{\varepsilon}\theta \tag{14}$$

Notice that they are ex-post rates in the sense that they are true once the magnitude of demand has realised. The agents, however, do not know the real demand and they behave on expectations.

Given the demand θ and α the probability that a buyer gets a good is $\sum_{i=0}^{\infty} e^{-\alpha}\frac{\alpha^i}{i!}\frac{1}{i+1} = \frac{1-e^{-\alpha}}{\alpha}$. Thus, the utility of going to a non-deviating firm is

$$\int_0^m (1-q)\frac{1-e^{-\alpha}}{\alpha}g(\theta)d\theta \tag{15}$$

and the utility of going to a deviator is

$$\int_0^m (1-\widetilde{q})\frac{1-e^{-\beta}}{\beta}g(\theta)d\theta \tag{16}$$

In equilibrium these have to be equal. This equality determines z but for our purposes it is sufficient to totally differentiate it to get

$$\frac{dz}{d\widetilde{q}} = -\frac{\int_0^m \frac{1-e^{-\beta}}{\beta}g(\theta)d\theta}{\int_0^m (1-q)\frac{1-e^{-\alpha}-\alpha e^{-\alpha}}{\alpha^2}\frac{\theta}{1-\varepsilon}g(\theta)d\theta + \int_0^m (1-\widetilde{q})\frac{1-e^{-\beta}-\beta e^{-\beta}}{\beta^2}\frac{\theta}{\varepsilon}g(\theta)d\theta} \tag{17}$$

The objective of a deviating firm can be expressed as

$$\max_{\widetilde{q}} \int_0^m \widetilde{q}\left(1-e^{-\beta}\right)h(\theta)d\theta \tag{18}$$

The first order condition for a maximum is

$$\int_0^m \left(1-e^{-\beta}+\widetilde{q}e^{-\beta}\frac{\theta}{\varepsilon}\frac{dz}{d\widetilde{q}}\right)h(\theta)d\theta = 0 \tag{19}$$

In a symmetric Nash equilibrium $\widetilde{q} = q$ and $\alpha = \beta = \theta$. Inserting these data into (17) and (19) and letting ε approach zero the equilibrium price turns out

$$q = \frac{\int_0^m \left(1-e^{-\theta}-\theta e^{-\theta}\right)h(\theta)d\theta}{\int_0^m \left(1-e^{-\theta}\right)h(\theta)d\theta} \tag{20}$$

3.1.2 Clustered market

Just like in the example above it is clear that the firms' equilibrium pricing strategy is mixing. It is also immediate that if the mixed strategy is given by distribution function F on $[l, L]$ then $L = 1$, F has no atoms and no gaps, and $l = 1 - H(1)$.

3.2 Expected profits

A firm in the clustered market earns $1 - H(1)$ in expected profits. In the non-clustered market a firm earns $\int_0^m \left(1 - e^{-\theta}\right) qh(\theta)d\theta$ where q is the equilibrium price given in (20). The former is greater than the latter if

$$e^{-m}(1+m) + \int_0^m \theta e^{-\theta} H(\theta)d\theta > H(1) \tag{21}$$

where I have partially integrated the expected profit in the non-clustered market. Based on this, one can state the following result.

Proposition 3 *If the probability that there are at most as many buyers as sellers, here unity, is sufficiently small, i.e.,* $e^{-m}(1+m) + \int_0^m \theta e^{-\theta} H(\theta)d\theta > H(1)$ *then the firms fare better in the clustered market than in the non-clustered market.*

To get some idea about the required magnitudes note that if H is, for instance, uniform on $[0, m]$ the condition becomes

$$e^{-m}(2+m) < 1 \tag{22}$$

and this is satisfied for all $m > 1.11$.

4 Equilibrium degree of clustering

Above I have shown that far from being an unreasonable outcome it seems quite possible that firms rather cluster together than remain in separate locations when there is demand uncertainty. The analysis is, however, incomplete and one-sided. It remains silent about the buyers' reactions to the firms' location choices as well as about the firms' incentives; if all the firms are clustered together could it be the case that an individual firm would have an incentive to depart to a separate location. Or if all the firms are in separate locations does an individual firm have an incentive to move together with another firm? Both the buyers' and firms' interests have to be taken into account since a market that is very favourable to, say, the firms is likely to be unfavourable to the buyers.

Proper analysis requires that the whole market structure must be determined in equilibrium, and I assume that a non-clustered and a clustered market may coexist. The timing of the model with two markets is such that first the firms choose which market to go to, then the buyers choose which market to go to anticipating the firms' pricing strategies in each market, then the firms post prices, and finally the buyers choose the firms they contact based on the observed prices.[4] If there is an equilibrium with two markets the buyers must be indifferent between going to either market. Analogously, the firms must be indifferent between the markets as well.

Definition 4 *An equilibrium consists of firms choice of market, the firms symmetric pricing strategies in each market, the buyers choice of the market, and the symmetric contact strategies of the buyers in each market such that any firm's strategy is the best response to the other firms and buyers strategies, and any buyer's strategy is the best response to the firms strategies and other buyers strategies.*

I fix the proportion of firms that form the non-clustered market at σ. The rest of the firms form the clustered market. Of the buyers proportion z goes to the former market where the Poisson-rate governing the meetings is $\gamma = \frac{z\theta}{\sigma}$. The rest of the buyers go to the latter market. From the previous section it is known that the equilibrium price in the non-clustered market is given by

$$q = \frac{\int_0^m \left(1 - e^{-\gamma} - \gamma e^{-\gamma}\right) h(\theta) d\theta}{\int_0^m \left(1 - e^{-\gamma}\right) h(\theta) d\theta} \tag{23}$$

and the expected utility of a buyer by

$$\begin{aligned} & \int_0^m \frac{1 - e^{-\gamma}}{\gamma} g(\theta) d\theta (1 - q) \\ = & \int_0^m \frac{1 - e^{-\gamma}}{\gamma} g(\theta) d\theta \frac{\int_0^m \gamma e^{-\gamma} h(\theta) d\theta}{\int_0^m \left(1 - e^{-\gamma}\right) h(\theta) d\theta} \\ = & \frac{1}{E(\theta)} \int_0^m \theta e^{-\gamma} h(\theta) d\theta \end{aligned} \tag{24}$$

In the clustered market the highest value in the support of the firms' mixed strategy is unity. Let F denote the mixed strategy; $F(q)$ denotes the measure of firms that post a price at most q. Since the measure of firms in the clustered market is $1 - \sigma$ we have $F(1) = 1 - \sigma$. The support of F is $\left[1 - H(\frac{1-\sigma}{1-z}), 1\right]$. The lower bound is got as before; proportion $1 - z$ of buyers go to the clustered market and if the demand is exactly $\frac{1-\sigma}{1-z}$ then the measure of buyers exactly matches the measure of firms. The mixed strategy $F(q)$ is determined by

$$\left(1 - H\left(\frac{F(q)}{1 - z}\right)\right) q = 1 - H\left(\frac{1 - \sigma}{1 - z}\right) \tag{25}$$

On the left hand side there is the probability that at a firm quoting price q makes a sale and on the right hand side there is the probability that more than $1 - \sigma$

[4] Since there is an infinite number of agents we could as well assume that the buyers choose the market after they have observed all the prices quoted by all the sellers in both markets.

buyers appear which also equals the expected utility from posting the highest price, namely unity. From this an explicit expression for F can be determined

$$F(q) = (1-z)H^{-1}\left(1 - \frac{1 - H\left(\frac{1-\sigma}{1-z}\right)}{q}\right) \tag{26}$$

A buyer's utility depends on how many other buyers appear. If there are fewer buyers than firms, i.e.if $(1-z)\theta < 1-\sigma$ each buyer gets a good and the firms that price highest do not sell. If there are more buyers than firms, i.e.if $(1-z)\theta \geq 1-\sigma$ then the buyers are rationed so that each of them gets a good with the same probability. Let ω be the highest price at which trading takes place. It is given by $F(\omega) = \min\{1-\sigma, (1-z)\theta\}$. I can now express the utility of a buyer as

$$\begin{aligned} &\int_0^{(1-\sigma)/(1-z)} g(\theta) \int_{1-H\left(\frac{1-\sigma}{1-z}\right)}^{F^{-1}((1-z)\theta)} (1-q)\frac{f(q)}{1-\sigma} dq d\theta \\ &+ \int_{(1-\sigma)/(1-z)}^{m} g(\theta)\frac{1-\sigma}{\theta(1-z)} \int_{1-H\left(\frac{1-\sigma}{1-z}\right)}^{1} (1-q)\frac{f(q)}{1-\sigma} dq d\theta \end{aligned} \tag{27}$$

where the first term depicts low demand such that all the buyers get a good, and the second term high demand where the buyers are rationed.

This is a complicated expression, and a more convenient approach can be used when, as it is the case, the agents' utilities are linear. Assume that each firm charges price p. A firm's expected utility is then given by

$$\int_0^{(1-\sigma)/(1-z)} \frac{\theta(1-z)}{1-\sigma} p h(\theta) d\theta + \int_{(1-\sigma)/(1-z)}^{m} p h(\theta) d\theta \tag{28}$$

Forcing this to equal the firms' expected utility $1 - H\left(\frac{1-\sigma}{1-z}\right)$ in the clustered market I can solve for the price p that yields the same utility to the firms as the mixed strategy F. This turns out

$$p = \frac{1 - H\left(\frac{1-\sigma}{1-z}\right)}{1 - \int_0^{(1-\sigma)/(1-z)} \frac{1-z}{1-\sigma} H(\theta) d\theta} \tag{29}$$

Since the total number of trades is the same under mixed strategy F and under the scenario where the firms charge p the buyers' expected utility has to be the same, too, under the two scenarios. Price p in (29) yields a buyer the following utility

$$\int_0^{(1-\sigma)/(1-z)} (1-p)g(\theta) d\theta + \int_{(1-\sigma)/(1-z)}^{m} \frac{1-\sigma}{\theta(1-z)}(1-p)g(\theta) d\theta \tag{30}$$

$$= \frac{1}{E(\theta)}\left[\frac{1-\sigma}{1-z} H\left(\frac{1-\sigma}{1-z}\right) - \int_0^{(1-\sigma)/(1-z)} H(\theta) d\theta\right] \tag{31}$$

If there is an equilibrium where some firms are in the clustered and some in the non-clustered market the buyers must fare equally well in both markets. This means that

$$\left[\frac{1-\sigma}{1-z}H\left(\frac{1-\sigma}{1-z}\right)-\int_0^{(1-\sigma)/(1-z)}H(\theta)d\theta\right]=\int_0^m \theta e^{-\gamma}h(\theta)d\theta \tag{32}$$

This is equivalent to

$$\int_0^{(1-\sigma)/(1-z)}\theta h(\theta)d\theta=\int_0^m \theta e^{-\gamma}h(\theta)d\theta \tag{33}$$

Lemma 5 *When the buyers are indifferent between the markets, the firms in the clustered market fare better than the firms in the non-clustered market or*

$$\int_0^m (e^{-\gamma}+\gamma e^{-\gamma})h(\theta)d\theta > H\left(\frac{1-\sigma}{1-z}\right) \tag{34}$$

Proof. Using (33), expression (34) is equivalent to

$$\int_0^m e^{-\gamma}h(\theta)d\theta+\int_0^{(1-\sigma)/(1-z)}\frac{z}{\sigma}\theta h(\theta)d\theta > H\left(\frac{1-\sigma}{1-z}\right)=\int_0^{(1-\sigma)/(1-z)}h(\theta)d\theta \tag{35}$$

This is equivalent to

$$\int_0^{(1-\sigma)/(1-z)}\left(\frac{z}{\sigma}\theta-1+e^{-\frac{z}{\sigma}\theta}\right)h(\theta)d\theta+\int_{(1-\sigma)/(1-z)}^m e^{-\gamma}h(\theta)d\theta>0 \tag{36}$$

which certainly holds as the first integrand is of the form $x-1+e^{-x}\geq 0$. ■

The above Lemma implies the first of the two main results of this article.

Proposition 6 *There are two equilibria in the model, namely one where all the firms are clustered, and one where all the firms are non-clustered. In particular, in equilibrium the two markets do not co-exist.*

This result does not depend very much on the demand being stochastic. To see this assume that there are more firms than buyers in the market, and that half the firms are in each market. Then the firms in the clustered market make zero profits because of Bertrand competition, while in the non-clustered market they make positive profits. The buyers strictly prefer the clustered market to the non-clustered market. To make buyers indifferent between the markets requires that the ratio of buyers to firms goes up in the clustered market. But then the firms would prefer the clustered market where they can charge the monopoly price. Of course, with deterministic demand the firms would not price as with stochastic demand, and the above heuristics just show that the buyers' indifference between the markets makes the clustered market, in an informal sense, more attractive to the firms; the stochastic demand allows to me to formalise this idea.

Proposition (6) says that there are two equilibria: Either the firms cluster together or all the firms are non-clustered. But it is clear that the non-clustered equilibrium does not survive standard refinements, say, trembling-hand perfectness. If the firms make mistakes with a small probability when they try to locate as in a non-clustered equilibrium a non-zero measure of them end up in a clustered market. Then the buyers' optimal contact decisions are such that they are indifferent between the markets, and by Proposition (6) the sellers in the clustered market fare better. This reasoning shows the second main result

Proposition 7 *The clustered market is the unique perfect equilibrium.*

5 Caveats

5.1 Capacity

Perhaps the weakest point in the modelling so far is the assumption about exogenous capacity. One would expect that the nature of competition and profitability in a market affect the capacity choice of the firms. It turns out to be difficult to determine the Nash-equilibrium level of capacity even with constant unit costs; given the capacity the equilibrium prices can be determined in the same vein as above, though. Some insights to the problem can be attained by assuming free entry of the firms. Typically it is assumed that entry drives profits to zero but that approach is not compatible with the adjustment process of the previous section. From proposition 3 it is known that when both markets co-exist, and the buyers choose which market to go to optimally, the clustered market is more profitable to the firms than the non-clustered market. Thus, the profits in both markets cannot equal zero simultaneously.

Instead of applying the free entry to the adjustment process I determine the welfare under both market structures with free entry, and I also determine the socially optimal number of firms. Consider first the clustered market, and denote the number of firms there by n_C. Now the expected profit of a firm entering the market is $1 - H(n_C) - \xi$ where ξ is the entry cost to the market. The expected number of transactions is

$$\int_0^{n_C} \theta h(\theta) d\theta + n_C \int_{n_C}^{m} h(\theta) d\theta = n_C - \int_0^{n_C} H(\theta) d\theta \tag{37}$$

where I have partially integrated. The total welfare is then

$$n_C - \int_0^{n_C} H(\theta) d\theta - n_C \xi \tag{38}$$

The socially optimal number of firms is got from the first order condition

$$1 - \xi - H(n_C) = 0 \tag{39}$$

But this is the same as the free entry condition above. Thus, free entry guarantees the socially optimal number of firms in the clustered market.

In the non-clustered market the number of firms is denoted by n_N, and the Poisson parameter governing the meetings is denoted by $\phi \equiv \frac{\theta}{n_N}$. An entering firm's zero-profit condition is

$$\int_0^m \left(1 - e^{-\phi} - \phi e^{-\phi}\right) h(\theta) d\theta - \xi \tag{40}$$

The number of transactions in the non-clustered market is given by

$$\int_0^m n_N \left(1 - e^{-\phi}\right) h(\theta) d\theta \tag{41}$$

and total welfare is given by

$$\int_0^m n_N \left(1 - e^{-\phi}\right) h(\theta) d\theta - n_N \xi \tag{42}$$

The first order condition for the optimal number of firms, after some manipulation, simplifies to

$$\int_0^m \left(1 - e^{-\phi} - \phi e^{-\phi}\right) h(\theta) d\theta - \xi \tag{43}$$

which is exactly the same as the free-entry condition. This means that in both markets the meetings and terms of trade are determined in an efficient manner given the market structure.[5]

Inserting the free-entry conditions to the expressions for total welfare yields in the clustered market

$$n_C H\left(n_C\right) - \int_0^{n_C} H(\theta) d\theta$$

and in the non-clustered market

$$n_N \int_0^m \phi e^{-\phi} h(\theta) d\theta$$

These expressions are just the buyers' aggregate utilities in the two markets (firms make zero profits).

I can now utilise the following trick to determine which magnitude is greater. Assume that the buyers do not adapt but half of them go to the non-clustered market and half of them go to the clustered market. Assume also that the

[5] It is known that the non-clustered market with deterministic demand and price posting is utilitywise equivalent to a non-clustered market where the buyers randomly contact the sellers, and the terms of trade are determined in an auction. The auction is such that in a one-seller-one-buyer meeting the initiator of the contact, here the buyer, makes a take-it-or-leave-it offer, and in a one-seller-many-buyer meeting the buyers bid for the object. This, also known as the Mortensen-rule, is the correct way to distribute bargaining power to attain efficienct entry (see Julian, Kennes and King, 2006). In the clustered market all possible trades take place, and efficient entry is not surprising at all.

number of buyers is doubled, i.e., the density of bueyrs is $k(\theta) \equiv h(\theta/2)$ where $\theta \in [0, 2m]$. Then the clustered and non-clustered markets are as above, and firms fare equally well. But from proposition (5) it is known that if the buyers fare equally well then the firms in the clustered market fare better than the sellers in the non-clustered market. Thus, if the firms fare equally well there must be more buyers in the non-clustered market than in the case where the buyers are indifferent between the markets. This means that the buyers' utility in the non-clustered market is lower than in the clustered market. As I assumed that there are equal numbers of buyers in both markets this means that the welfare in the non-clustered market is lower than in the clustered market when there is free entry of firms.

5.2 Degree of clustering

Another feature that may raise doubts is that the firms are assumed to be either completely clustered or completely separated so that there is just one firm per location. One may expect that in some circumstances configurations where there are many locations occupied by $r \geq 2$ firms would feature the optimal degree of clustering. This possibility would complicate the analysis very much but it would most likely not contribute much to our understanding. The reason is that all configurations where there are a positive number of firms in a location constitute an inefficienct market as it is possible that no buyers contact some locations while some other locations are contacted by more buyers than there are sellers in the location. If the clustered market and a market with several firms in a location coexisted the former one, which is efficient, would still yield higher profits to the firms given that the buyers are indifferent between the markets.

6 Conclusion

I study two market structures under capacity constrained firms and demand uncertainty. In one market the firms are geographically separated so that a buyer can visit only one firm. The firms attract buyers by posting prices, and I derive the unique equilibrium price. In the other market the firms are geographically close to each other so that a buyer can choose the most attractive firm. In this market the firms attract buyers by posting prices, too. The unique symmetric equilibrium pricing strategy is a mixed strategy. I derive this explicitly.

Comparing the firms' expected utilities in the two markets shows that they prefer the clustered market if the demand variability is large. It is not, however, enough to take into account only one side of the market. The buyers choose which firms to visit, and to determine the relative attractiveness of the two markets it is necessary to allow both kind of markets to co-exist. The behaviour of the buyers is then such that some of them go to one market and the others to the other market, and in equilibrium they have to be indifferent. But it turns out that then the case for the clustered market is even stronger; whenever the buyers are indifferent between the markets the firms necessarily prefer the clustered market. This means that if there is a dynamic adjustement process in the economy there is a tendency towards the clustered market.

The reason for the firms to prefer the clustered market is that in the non-clustered market they compete for the buyers whatever the demand; indeed, it is possible that some firms do not meet any buyers even though there are more buyers than firms. In the clustered market the firms make a transaction with probability one in such cases, and they only compete for the buyers when there are fewer buyers than firms. Thus, the competition is less fierce in the clustered market.

If there are two markets and the buyers are indifferent between them, then the firms fare better in the clustered market because it is efficient. There all the possible trades are always executed. This fact is behind the stability of the clustered market when replicator dynamics is used to select between the two equilibria.

For the results of this work it is essential that there is demand variability and that there is a positive probability for excess demand, i.e, more buyers than firms. If it is certain that the number of buyers is less than the number of firms then the firms do not want to cluster since in equilibrium they would earn zero profits as a result of Bertrand-competition. I have determined the equilibrium market structure in this case in Kultti (2003a).

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