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Equilibrium Unemployment Duration in an Urn-Ball Model of the Labour Market

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Abstract

We derive the equilibrium unemployment duration in response to a change in unemployment when matches are formed by an urn-ball process. We show that the duration is more sensitive to unemployment than conventionally thought. This result comes from solving the equilibrium of the model instead of just varying the unemployment-vacancy ratio.

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1   Introduction

Search theory is the main modelling framework used in the theory of labour markets. Finding a partner is time consuming and uncertain, and consequently unemployed and vacancies meet in some kind of random manner. Often the exact meeting process is side stepped by postulating a matching function that tells the number of employment relationships produced given the number of unemployed and vacancies. Denoting the number of unemployed by U and the number of vacancies by V the number of matches is then given by $M = m(U, V)$ where m is the matching function. Usually, m is assumed to be increasing in both arguments and homogenous of degree one. On average, an unemployed worker finds a job during a period with probability $m(U, V)/U$, and a vacant job is filled with probability $m(U, V)/V$. In a steady state equilibrium the inverses of these probabilities are the mean durations of unemployment and vacancies, respectively.

In a survey of matching functions Petrongolo and Pissarides (2001, p. 402) claim that a simple urn-ball model fails to produce unemployment durations that are observed empirically: "...It is, however, too naïve to be empirically a good approximation to matching in real labor markets. For example, it implies an implausible combination of levels and durations of unemployment. If the level of unemployment and vacancies is the same, the mean duration of unemployment is 1.58 periods, and if the level of unemployment is three times as high as that of vacancies, mean duration is 3.16. In actual labor markets duration would rise by more than the function $\left[M = V \left(1 - e^{-U/V} \right) \right]$ implies when the level of unemployment is higher."

There are basically two methods for assessing whether the simple urn-ball model produces empirically observed unemployment durations. As the duration depends only on ratio U/V, one can vary the ratio and compare the theoretical durations to the observed ones. This requires reliable data on both unemployed and vacancies. If data on vacancies, for example, is missing or unreliable but there is data on unemployment and job separation probability then one can evaluate the urn-ball model by using equilibrium analysis. We choose this approach.

We study how the duration of unemployment is affected in an urn-ball model when the

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absolute number of unemployed changes. We specify an equilibrium model where changes in one variable cannot take place without changes in other variables. In particular, an increase in the number of unemployed requires a decrease in the number of firms, or an increase in the labour force or in the job separation rate. Keeping track of the effects of these changes requires that one solves the new equilibrium values for the stocks of active firms and workers as well as the stocks of unemployed and vacancies. It turns out that when the number of unemployed is, say, tripled this necessitates a decrease in the number of vacancies such that the equilibrium ratio U/V more than triples. Thus, the duration is more sensitive to unemployment than one would expect by just tripling the unemployment-vacancy ratio.

We derive the urn-ball model in Section 2. In Section 3 we present the equilibrium analysis when unemployment increases because the number of firms decreases. In Section 4 we consider a model where change in unemployment is driven by a change in the labour force. Section 5 concludes.

2   The Urn-Ball Model

In the urn-ball model the vacancies present the urns, and the unemployed play the role of the balls which are randomly placed in the urns. The number of balls ending up in an urn is thus binomially distributed. Some urns end up with one ball, some urns end up with several balls, and some urns remain empty. The coordination failure is due to the lack of information about the other applicants' actions. The model can be enriched but here we focus just on the equilibrium meetings that are assumed symmetric and random. We do not say anything about how the meetings come about but for instance, a model with posted wages exhibits the matching function (1) in a symmetric equilibrium.

The matching function is

$$M = V \left[1 - (1 - 1/V)^U \right], \tag{1}$$

where $(1 - 1/V)^U$ denotes the probability that a vacancy receives no applications. The probability that a vacancy receives at least one application multiplied by V gives us (1),

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the expected number of matches. For large U and V the magnitude $(1 - 1/V)^U$ can be approximated with $e^{-U/V}$, i.e. the binomial distribution can be approximated with a Poisson distribution with parameter U/V, giving the following matching function:

$$M = V \left(1 - e^{-U/V}\right). \tag{2}$$

Denoting the mean duration of unemployment by D we have

$$D = (M/U)^{-1} = \left[(V/U)\left(1 - e^{-U/V}\right)\right]^{-1}. \tag{3}$$

We can easily verify the durations mentioned by Petrongolo and Pissarides. Let $U = V$. Then $D = (1 - e^{-1})^{-1} \approx 1.58$. If $U = 3V$, we have $D = [(1/3)(1 - e^{-3})]^{-1} \approx 3.16$.

3 The Duration of Unemployment when the Number of Employers Changes

Varying the number of employers, or production possibilities, is a natural way of modelling changes in unemployment. Employers enter or exit the economy because of shocks to productivity, demand, discount factor, recruiting cost, bargaining power etc. Analysing how these "ultimate" factors affect unemployment and its duration requires a job creation condition (see e.g. Pissarides 2000, ch. 1), but in order to test the matching function we take the change in the number of employers as given.

The labour force consists of W workers, of whom U are unemployed, and the rest are employed. There are E employers who either have hired one worker or are vacant. The number of vacancies is denoted V. There are equally many employed workers and filled jobs:

$$W - U = E - V. \tag{4}$$

We assume that these numbers are large and that the agents meet according to the urn-ball matching function (2). Jobs terminate at an exogenous separation probability $s \in [0, 1]$, and the worker and the employer start looking for partners again. The flow into unemployment is equal to $s(W - U)$, and it is equal to the flow $s(E - V)$ into vacancies.

We consider a steady-state equilibrium where the flows out of and into unemployment are equal:

$$V \left(1 - e^{-U/V}\right) = s(W - U). \tag{5}$$

Equation (5) is the Beveridge curve. For given W and s it depicts the steady state values for U and V. It is straightforward to show that it is downward sloping in (U, V) plane and convex to the origin.

The model consists of equations (4) and (5) where E, W, and s are exogenous variables, and U and V are determined in an equilibrium. It is trivial to show that (4) and (5) have a unique solution for U and V for given values of E, W and s even though we cannot solve for U and V explicitly. In the analysis of this section we let E change and determine the equilibrium responses of U and V. Also a variation in s affects the steady state, but there is strong empirical evidence that s is practically constant ("My findings that the job-finding rate is strongly procyclical and the separation rate is nearly acyclical oppose the conventional wisdom that recessions are primarily characterized by a high separation rate." (Shimer 2005. p. 493)).

We solve the unemployment duration by tracking the Beveridge curve numerically when E changes. To avoid computational problems we, instead of decreasing E directly, let U increase. The algorithm is the following: (i) Give initial values to E, W, and s. The values of E and W must be large in order to justify the use of matching function (2), and they must be close to each other to facilitate computation. (ii) Use equation (4) and the Beveridge curve (5) to calculate the initial values of U and V. (iii) Give increasing values to U, and use the Beveridge curve to compute the corresponding V. (iv) Use equation (3) to calculate the duration. Finally, one can use equation (4) to find the value of E which gives the U, V, D calculated.

A Numerical Example

Let $E = W$ initially, in other words, there is no structural unemployment. Then $U = V$ in equilibrium. We postulate that W and the initial value of E are one hundred million. The shape of the Beveridge curve and thereby unemployment duration depends on the value of s, and therefore we use two alternative period lengths: a week and a

month. We pick the monthly separation probability from U.S. data[1]: $s_m = 0.033$. The weekly separation probability s_w is then determined by $s_m = 1 - (1 - s_w)^{30/7}$. For $s_m = 0.033$ this gives $s_w = 0.008$. Initially, $U = V = 1249000$ using $s_w = 0.008$, and $U = V = 4961500$ using $s_m = 0.033$. We compute the unemployment duration when unemployment changes from the initial value to eightfold.
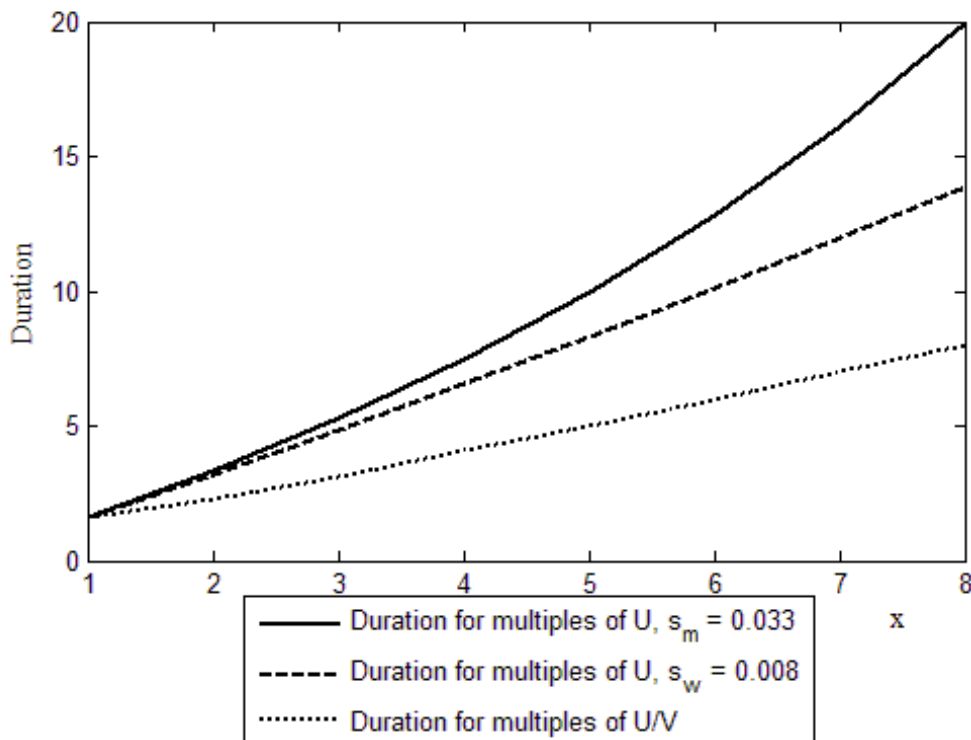
Figure 1: Unemployment duration for multiples of U and multiples of U/V.

Figure 1 depicts unemployment duration in periods when U/V changes from one to eight and when U changes from initial value (1249000 or 4961500, depending on the value of s) to eightfold. The duration responds more sharply to an increase in absolute unemployment than to an increase in U/V. If we are interested in how the duration

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[1] Total separations, nonfarm, seasonally adjusted, 2001 - 2007. Source: Department of Labor, Bureau of Labor Statistic. sana

changes when something happens to the absolute number of unemployed people, then this exercise is quite satisfying in the sense that the too small duration generated by the too straightforward use of the urn-ball model, namely 3.16 turns out not to be the true duration. When the separation rate becomes larger, the duration responds more strongly to a change in unemployment. Table 1 shows the computed durations and the associated value of U/V when U increases from the initial value to eightfold. In tables 1 and 2 we denote $U_n = nU_1$.

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Table 1

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Unemployment duration and unemployment/vacancy ratio when the number of firms changes.

	U_1	U_2	U_3	U_4	U_5	U_6	U_7	U_8
D when $s_w = 0.008$	1.58	3.20	4.87	6.58	8.33	10.13	11.98	13.89
U/V when $s_w = 0.008$	1	3.05	4.83	6.57	8.33	10.13	11.98	13.89
D when $s_m = 0.033$	1.58	3.34	5.30	7.50	10.00	12.85	16.13	19.95
U/V when $s_m = 0.033$	1	3.20	5.27	7.50	10.00	12.86	16.13	19.95

4 The Duration of Unemployment when the Labour Force Changes

Assume next that changes in unemployment are driven by changes in the labour force (or, equivalent in this context, participation) while keeping the number of employers fixed. The labour force changes because of exogenous shocks like a change in the value of leisure, but for our purposes we take the change in W as given. As before, each firm can hire one worker, which implies $W - U = E - V$. In a steady state

$$V \left(1 - e^{-U/V}\right) = s \left(E - V\right) \tag{6}$$

where $s \left(E - V\right)$ is the flow from the filled jobs to vacancies. Using (6) instead of (5) allows fixing the value of E. The algorithm for computing the equilibrium unemployment

duration is the same as above. For convenient computation we do not increase W directly but we let U increase.

A Numerical Example

Let W and E to be one hundred million initially, and we use the same separation probabilities as above. Table 2 depicts unemployment duration and the corresponding U/v when unemployment changes from the initial value (1249000 or 4961500, depending on the value of s) to eightfold.

Table 2

Unemployment duration and unemployment/vacancy ratio when the size of the labour force changes.

	U_1	U_2	U_3	U_4	U_5	U_6	U_7	U_8
D when $s_w = 0.008$	1.58	3.15	4.72	6.30	7.87	9.45	11.02	12.60
U/V when $s_w = 0.008$	1	3.00	4.68	6.29	7.87	9.45	11.02	12.60
D when $s_m = 0.033$	1.58	3.11	4.66	6.21	7.77	9.32	10.87	12.43
U/V when $s_m = 0.033$	1	2.95	4.61	6.20	7.77	9.33	10.87	12.43

Comparing the durations in tables 1 and 2 we see that the duration is more sensitive to a decrease in E than to a an increase in W, given the increase in the absolute unemployment. This is because Beveridge curve (6) is flatter than Beveridge curve (5) for all values of U (when U is on the horizontal axis and V on the vertical axis): Denote $U/V \equiv x$. On (5) we have $dV/dU = -\left(e^{-x} + s\right)/\left(1 - e^{-x} - xe^{-x}\right)$, and on (6) we have $dV/dU = -\left(e^{-x}\right)/\left(1 - e^{-x} - xe^{-x} + s\right)$. The difference of the absolute slopes is equal to $s\left(1 - xe^{-x} + s\right)/\left[\left(1 - e^{-x} - xe^{-x}\right)\left(1 - e^{-x} - xe^{-x} + s\right)\right] > 0 \ \forall x > 0$.

5   Conclusion

We have shown how to compute the equilibrium duration of unemployment when unemployment increases, in a model where unemployed and vacancies meet according to

a simple urn-ball matching function. The increase in unemployment is assumed to be driven by a decrease in the number of employers or an increase in the labour force. This method can be used to evaluate the empirical validity of the urn-ball matching function if one has data on unemployment and job separation probabilities but not on vacancies. We show that the duration is more sensitive to unemployment than one would expect by just changing the unemployment-vacancy ratio.

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