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Principles of Defining Index Numbers and Constructing Index Series

Yrjö Vartia
University of Helsinki and HECER

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HECER – Helsinki Center of Economic Research, P.O. Box 17 (Arkadiankatu 7), FI-00014 University of Helsinki, FINLAND, Tel +358-9-191-28780, Fax +358-9-191-28781,
E-mail info-hecer@helsinki.fi, Internet www.hecer.fi

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Abstract

Conceptional background of the index number problem is still unclear in its literature. Consensus of the general definition of the Index Number Formula as a function of prices and quantities is still lacking. Likewise, Index Number Formula and the Construction Strategy for an index series have not been properly separated. This unfortunate situation present also in the PPI Manual of IMF is discussed and tentatively corrected in the paper.

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Yrjö Vartia

Department of Economics
University of Helsinki
P.O. Box 17 (Arkadiankatu 7)
FI-00014 University of Helsinki
FINLAND

email: yrjo.vartia@helsinki.fi

Yrjö Vartia[1]: Principles of Defining Index Numbers and Constructing Index Series

1 Introduction

Systematic gradual progress has been going on in the *theoretical basis* of index number problem during the last 30 years. Knowledge of its importance for practical applications has increased considerably. Here the efforts of EuroStat, the Ottawa Group, IMF and other economic organizations have been very important. The production of the PPI Manual organized by IMF is an important undertaking from this point of view, see IMF (2004). Good atmosphere in the co-work of academic experts and official appliers is an important explanatory variable in success of the projects (or of lack of it). The PPI Manual can be considered humorously as an updated version of the "Fisher's Index Number Bible" from 1922.

There are not too many experts on index numbers (*IN*) who understand both its axiomatic and economic approaches. It is essential that the experts behind the Manual clearly master the both main views. More commonly, only one of these valid partial views is applied and the other ignored or even opposed. This has been seen clearly in the index number development of EuroStat. The most common level of knowledge among academic economics and statistics in this special subject is that neither the axiomatic nor the economic approaches are well understood. Most economists and statisticians regard index numbers only as simple mechanical calculations needed to produce official statistics.

This is a common elementary but faulty view, which would win by high margin in a sampling survey or democratic voting even among professional economists.

I concentrate in the paper on some conceptual viewpoints. These are:

1. What does an index number *IN* and
2. a construction strategy for an index series mean?
3. How consistency in aggregation *CA* and the proportionality tests *PT* are interrelated?

I try to evaluate shortly, what kind of views are found of these issues.

My general conviction is that the index number problem in its whole complexity becomes much simpler in a suitable general mathematical framework, where we can concentrate on fundamental issues. The whole issue culminates in the question, what its problems actually are. In this respect, index numbers and especially its extension, the aggregation theory are - despite of intensive high-level research efforts allocated to them for decades - still partly underdeveloped areas. The reason for this is in my opinion that so many different disciplines meet and inference in the area.

I try *to see the forest from the trees* in this forest of statistics, mathematics, economics, sampling surveys, data collection, quality corrections, hedonic regressions, computation systems, experts of the official statistics, consultants and numerous tests, requirements and desiderata. It is evident, that quite many trees, bushes, flowers

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[1] *This paper is based on my comments on "Axiomatic and Economic Approaches to Index Numbers" by Bert Balk (Friday 27, August 2004) presented in the International Conference on Producer Price Index Manual - Helsinki August 2004*. Comments of Heikki Pursiainen are gratefully acknowledged.

and even imaginary objects grow in this abstract forest, whose actual answers culminate in figures as 2,7% annually from the preceding month.

The two chapters 16 and 17 of the IMF Manual are

A. The Axiomatic and Stochastic Approaches to Index Numbers

E. The Economic Approach to Index Numbers.

Clearly A is part of mathematics and E part of economics. My Finnish colleagues, especially Markus Halonen, Eugen Koev, Heikki Pursiainen, Antti Suoperä and myself have developed new aggregation methods during some years mainly as an extension of A. Instead of aggregation – the term that only economists understand – I have called this research area as *analysis and synthesis*. For our research in this area, see Koev (2003), Pursiainen (2005), Lintunen et al (2009) and Vartia (2008a-b, 2009).

The basic problem of index numbers can be summarized in one row. It is the connection between sum, product and changes, both in micro and macro levels. It is astonishing, that sums of products are so easy to calculate and understand but hard to decompose.

2 Axiomatic approach

We have here "the problem of too many tests" illustrated already by Fisher (1922). They can be shown to be contradictory taken all together and also in many subgroups, see Eichorn (1976), Balk (1995). (These are like Arrow's requirements for social choice, which are contradictory, unless we choose *some dictator*. Arrow strangely got the Nobel Price in Economics from this.) The role of axiomatics is explained in the next chapters. We propose that four axioms *CRT*, *UMT*, *MUT* and *PT2* are used to define the concept of the index number. Our second point is that (a) the index number formula (*IN*) and (b) the construction strategy for index series (say base and chain strategies) are two completely independent questions. Our ten questions tie all the different problems of index series construction neatly together. These topics were discussed already in my dissertation in 1976 and more tightly in my lecture note Vartia (1992). Recent studies of *IN* and especially *CA* (consistency in aggregation) based on a new but natural technique of semigroups is Pursiainen (2005, 2008).

3 Consistency in Aggregation CA

Pursiainen (2008) describes *CA* as follows: *In the calculation of economic aggregates it is often necessary to compute the value of these aggregates in some relevant subgroups as well as for the whole data. A method of calculation is said to be consistent in aggregation if it gives the same result regardless of whether it is applied directly to the whole data or to sub-aggregates calculated using the same method.*

Values $v_i = p_i q_i$ and their differences $\Delta v_i = v_i^1 - v_i^0$ are aggregated consistently simply by adding. How is this transformed to the log-scale? This is the basic problem of my dissertation in 1976 and the solution is particularly simple. Define the *logarithmic mean* of positive numbers by

(1) $$\hat{v} = L(v^1, v^0) = \frac{\Delta v}{\Delta \log v} = \frac{v^1 - v^0}{\log v^1 - \log v^0}.$$

This is a homogenous function and lies always between the geometric and arithmetic's means (nearer the geometric mean), see Vartia (1976a-b, 9-25) or Törnqvist et al (1985). Identically for all positive variables

(2) $$\Delta v = \hat{v} \Delta \log v = L(v^1, v^0) \Delta \log v.$$

Thus the additive identity (which is considered as naïve)

(3) $$\Delta V = \sum \Delta v_i$$

for *total values* $V = \sum v_i$ transforms (term by term) to

(4) $$\hat{V} \Delta \log V = \sum \hat{v}_i \, \Delta \log v_i \ .$$

This is not naïve any more, although it is the log-scale equivalent of (3). Dividing by the logarithmic average of the total value $\hat{V} = L(V^1, V^0)$ gives an equivalent representation[2]

(5) $$\Delta \log V = \sum \tfrac{\hat{v}_i}{\hat{V}} \Delta \log v_i \ \approx \sum \tfrac{\sqrt{v_i^1 v_i^0}}{\sqrt{V^1 V^0}} \Delta \log v_i \ = \sum \sqrt{w_i^1 w_i^0} \Delta \log v_i \ .$$

This decomposes to price and quantity terms by inserting $\Delta \log v_i \ = \Delta \log p_i \ + \Delta \log q_i$. Here the weights $\frac{\hat{v}_i}{\hat{V}} = \frac{L(v_i^1, v_i^0)}{L(V^1, V^0)} = \hat{w}_i \ \approx \sqrt{w_i^1 w_i^0}$ are those of the *Montgomery-Vartia index*, which “almost trivially” satisfies both time and factor reversal tests *TRT & FRT* and is consistent in aggregation *CA*, see Vartia (1976a-b). Montgomery-Vartia is the only log-change index with these properties, because the value contributions $\hat{w}_i \, \Delta \log v_i \ \approx$ $\sqrt{w_i^1 w_i^0} \Delta \log v_i$ in (5) are objectively and uniquely determined[3] by values in (3). Its approximation here having weights $\sqrt{w_i^1 w_i^0}$ is better than the Törnqvist index with arithmetic average weights $\frac{1}{2}(w_i^1 + w_i^0)$. Its strong point (not a weakness) is the fact that that its weights $\hat{w}_i$ (like $\sqrt{w_i^1 w_i^0}$) sum generally to a number slightly smaller than one. This sum is unity only if all values change proportionally. Also this is a straight consequence of just looking at the value changes in the log-scale. If Laspeyres, Paasche and Fisher indices are written as log-change *IN*, also the sum of the weights is generally less than one as noted by Vartia (1976b, p. 123). Thus, we cannot demand that it must be one or an *IN* should be generally a weighted arithmetic average in the log-scale.

It is illuminating to write Laspeyres price index in logarithm form:

(6) $$\log P_0^1(La) = \log \frac{p^1 \cdot q^0}{p^0 \cdot q^0} = \frac{p^1 \cdot q^0 - p^0 \cdot q^0}{L(p^1 \cdot q^0, p^0 \cdot q^0)}$$

$$= \sum \frac{q_i^0 L(p_i^1, p_i^0)}{L(p^1 \cdot q^0, p^0 \cdot q^0)} \frac{(p_i^1 - p_i^0)}{L(p_i^1, p_i^0)} = \sum \overline{w}_i \ (La) \Delta \log p_i \ ,$$

where $\overline{w}_i \ (La) = \dfrac{L(p_i^1 q_i^0, p_i^0 q_i^0)}{L(p^1 \cdot q^0, p^0 \cdot q^0)}$. Similarly $\overline{w}_i \ (Pa) = \dfrac{L(p_i^1 q_i^1, p_i^0 q_i^1)}{L(p^1 \cdot q^1, p^0 \cdot q^1)}$ and the weights of the Fisher index in the log-scale are $\overline{w}_i \ (F) = \frac{1}{2}(\overline{w}_i \ (La) + \overline{w}_i \ (Pa))$. All these weights sum to one at most like the weights of the Montgomery-Vartia index. Note that the value contributions of the Fisher index $\overline{w}_i \ (F) \Delta \log v_i$ differ from the only objective ones in (5) though their sum equals (5).

From proportionality tests *PT Montgomery-Vartia* satisfies the *natural requirement for all index numbers PT2:*

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[2] Note that in the illuminating approximation the weights are the geometric means of the value shares and *their sum is <u>not</u> forced to unity!* This is natural and reflects how the weights in the log-scale behave. By conventions only, we are accustomed to weaken the approximations in the log-scale by forcing the index number formulae to be literally *weighted means*. I regard that as unnecessary and even as a slight mistake in this context.

[3] Stuvel index satisfies also *FRT* and is *CA*. Therefore, also its reproduces exactly the correct *objective contributions of the value changes* but on the arithmetic scale (instead of the log-scale). It decomposes prices and quantities slightly differently to Montgomery-Vartia, although they do the same right thing for values. Professor Törnqvist had probably something like this in mind, because he needed a numerical example to admit that these two indices are not identical. For a formal treatment of the contributions on the arithmetic and logarithmic scales see Pursiainen (2005).

(*PT2*) $$f(kp^0, p^0, mq^0, q^0) \equiv kf(p^0, p^0, mq^0, q^0) = k\,.$$

As Pursiainen (2005) has proved that from all *CA & FRT* indices, Stuvel is the only one satisfying a stronger *PT*. This is the *desiderata PT3:*

(*PT3*) $$f(kp^0, p^0, q^1, q^0) \equiv kf(p^0, p^0, q^1, q^0) = k\,.$$

No *CA*-index can satisfy *FRT* and any stronger *PT* such as *linear homogeneity in prices or PT4:*

(*PT4*) $$f(kp^1, p^0, q^1, q^0) \equiv kf(p^1, p^0, q^1, q^0) \text{ (regardless of the quantities).}$$

My personal view is that we should be more careful of disqualifying index numbers by such strong *PT-requirements.* We do not actually need them. They are not natural basic properties, but more like *wishes* connected with the "simplifying assumption" that real income has no effect on proportions or that preferences are homothetic, see Vartia (1976b, s. 123-4)

These kinds of "simplifications" make everything very complicated when the models are forced to be systematically unrealistic. Why not "simply assume *Cobb-Douglas"* with universally constant value shares or *CES,* if it is a virtue of modelling to have *little to do* with reality. The point forgotten is: *Our assumptions are not magic which changes reality.*

4 Proportionality tests PT and factor antithesis FA

FA of an index number formula f denoted by f^{FA} is often called *the implicit quantity index* corresponding to using f as a price index. They *always go in pairs,* so that either both in (f, f^{FA}) or neither should be qualified as an index number. This is a group theoretic property: the group of index number formulas should be *closed* under taking *FA* or in the *FA-operation.* This should be clear and self-evident, but it seems to be often forgotten in general discussions of "good index number formulae". The axiom system presented in Vartia (1976a) is refined later in my Karslruhe paper (1985) as follows. It takes an index number as *a sequence of functions* $f = (f_1, f_2, ..., f_n, ...)$ one for any number of commodities, just like in Pursiainen (2005). Index number formulae should be specified for all possible number of commodities especially to define consistency in aggregation, where several numbers of commodities are treated at the same time. Index number formula is *a binary comparison* between positive price and quantity vectors of a given number of commodities from two situations, and attains strictly positive values. There are only four axioms[4], namely

- *CRT* = Commodity Reversal Test
- *UMT* = Unit of Measurement Test
- *MUT* = Money Unit Test and
- *PT2* = a Weak Proportionality Test, namely $f_n(kp^0, p^0, mq^0, q^0) \equiv kf_n(p^0, p^0, mq^0, q^0) = k$.

Then by standard functional equation methods, different useful *representations* such as $f_n(p^1, p^0, q^1, q^0) \equiv g_n(\pi, v^1, v^0)$ and the φ*-representation* $f_n(p^1, p^0, q^1, q^0) \equiv \varphi_n(\pi, w^1, w^0, V^1/V^0)$ are derived for any *IN.* These representations are unique as functions, although they can be expressed in infinitely many ways (as any function can). It is essential that our group if index numbers is closed e.g. under *TA-* and *FA-operations.*

Consider as an example the log-Laspeyres (called also the geometric Laspeyres):

[4] Functions satisfying all four may be called symmetric index number (*SIN*) and last three generalized index number (*GIN*) as proposed in Vartia (1985).

(7) $$f_n(p^1, p^0, q^1, q^0) \equiv \varphi_n(\pi, w^1, w^0, V^1/V^0) = \prod_{i=1}^{n} \pi(i)^{w_i^0} \text{ or } P_0^1(l) = \exp(\sum w_i^0 \log \pi(i)).$$

For its relation to other 5 basic price indices see Vartia (1978), Vartia, Y. & Vartia, P. (1984). Formula (7) satisfies strong versions of *PT*, even linear homogeneity in prices *PT4*. But it's *FA*

(8) $$P_0^1(l^{FA}) = \exp(\log(V^1/V^0) - \sum w_i^0 \log \kappa(i)) = (V^1/V^0)/\exp(\sum w_i^0 \log \kappa(i))$$

does not. It satisfies "only" *PT2*. Thus if *PT3* or *PT4* are used as *requirements* (and not only as *desiderata* which is our suggestion), both the geometric Laspeyres l and its natural pair l^{FA} *must be disqualified as IN's.* The same reasoning extends immediately to Törnqvist. If *PT3* or *PT4* is a *requirement*, then the pair $(f, f^{FA}) = (Törnqvist, Törnqvist^{FA})$ and thus both its members must be disqualified!

Proponents of these *stronger proportionality tests* seem to have forgotten, that all indices have "another side", their factor antithesis, and *they should be evaluated as a pair* (f, f^{FA}). Possible weaknesses of either of them count in the evaluation. The weakest point matters here! Especially the log-change indices show remarkable asymmetry to their *FA´s*. It is hard to believe, that proponents of strong *PT's* are ready to disqualify e.g. Törnqvist or Walsh formulae *because of* the weaknesses of their *FA's or* consider them as good formulas *despite of* the weaknesses of their "implicit pairs". Both these views seem unwarranted to me.

The logical thing to do is to accept as *IN's* all the formulae satisfying axioms like *CRT, UMT, MUT and PT2*, which automatically hold for both members of the pair (f, f^{FA}) or for neither of them. All additional properties for these *index numbers* (which is now defined as an exact mathematical concept without the annoying problems above), are presented as *desiderata*[5]. In the case when *factor reversal test FRT* holds, the pair becomes $(f, f^{FA}) = (f, f)$ as $f = f^{FA}$. Thi~~s~~ holds together with $f = f^{TA}$ for Stuvel and Montgomery-Vartia indices, which are two excellent *CA*-indices. This is my somewhat delayed comment to Diewert's Econometrica article (1978) on consistent aggregation and especially its conclusion.

5 Ten questions, the binary comparison index number and the construction strategy for an index series

Previous chapters demonstrated, that even the most basic concept, an index number formula, has not yet been properly defined. Imagine a similar situation in general mathematics: mathematician work continuously with *functions* but have not agreed what they are. This was the situation roughly 150 years ago before Weierstrass and Cantor. Index numbers is a concept still resembling partly that of *equity (All men are created equal)*: it sounds good but does not have a precise meaning. My suggestion for the meaning of an *IN* was given in chapter 4.

In addition to some vagueness in *IN*, index numbers and construction strategies for index series are not clearly separated in the literature. As a concrete example, Törnqvist did not make that distinction in his publications. Logically, these two are completely different operations:

- *An index number* controls, how any two situations or periods are compared, while
- *the construction strategy for the index series* tells, how to choose these different twin periods.

Base and chain strategies are only two simple extreme cases and they do not restrict the choice of an index number formula in any way. It is strange, that this self-evident fact is not generally recognized. It would require, for instance, a more explicit treatment in the PPI Manual. These together with other crucial tasks of the index number construction can be summarized in less than one page shortened from Vartia (1976a, 92-95)

[5] This is the Latin equivalent for "wishes".

Ten critical questions when a price index is constructed

Intended use of the index

1. <u>Characterisation of commodities:</u> How to characterise the set A of commodities whose prices interest us here?

2. <u>Reference group of economic agents:</u> What will be the group of economic agents (e.g. consumers, producers) from whose point of view the prices are examined?

3. <u>Length of the time periods:</u> What are the lengths of the periods t_0, t_1, ... for which the index will be calculated?

Technical problems

4. <u>Index commodities:</u> How to classify the commodities in A into disjoint subsets or index commodities A_1, ... , A_k in such way that the quality of each index commodity A_k will stay reasonably stable and the necessary information about it can be estimated?

5. <u>Price information:</u> How to collect for every period t_m enough price information from the commodities in A, so that the proper price ratios for the A_k:s can be estimated?

6. <u>Proper weights:</u> How to collect for some period(s) enough information so that proper weights (e.g. means of value shares) for the index commodities can be estimated?

Method of calculation

7. <u>Index number formula:</u> How to choose an index number formula in such a way that the information at our disposal will be well utilised?

8. <u>Strategy for constructing an index series:</u> How to choose the general strategy for constructing the index series from available binary comparisons between various periods?

Special problems

9. <u>Quality changes:</u> How to take into account the quality changes in our index commodities A_k?

10. <u>New and disappearing commodities:</u> How to handle new or disappearing commodities?

The basic strategies of constructing index series are base and chain strategies. They can be applied for any index number formula, so the formula and the construction strategy are independent phases of constructing an index series. Finnish research concerning especially mathematics of quality corrections and hedonic regressions include Vartia, Y. and Vartia, P. (1995), Vartia and Koev (1996), Hyrkkö et al (1998), Vartia (1996,1998) and Koskimäki & Vartia (2001)

6 Economic approach

Economic theory is important as an abstract framework, which gives index constructors useful insight what can be done in some idealized special situations. I have considered how different index number formulae and nonparametric demand systems can be used to measure welfare change, see Vartia (1983). Revealed preference arguments based on Laspeyres-Paasche limits are considered in Vartia and Weymark (1981). Economic theory is useful both conceptually and theoretically, but practical applications in official statistics should not be founded solely on the constructs of the economic theory. It is not yet realistic and robust enough for that. Fortunately, classical index number theory based on sound axiomatics ends up and supports in important special cases the suggestions of the economic theory – or should we view this from the other angle.

There are numerous jokes about professional economists. But they do not find these necessarily amusing. A good one is the following. *Economists – like artists – tend to fall in love in their models.*

This shows, that we have in economics too many fighting schools of though. It is difficult to judge what makes sense and why. What about to incorporate general equilibrium, Cranger causality or co-integration in our hedonic regressions? More intellectual honesty (instead of political correctness) in revealing our subjective preferences – i.e. simply straight talk – could help to cure this particular disease.

7 Conclusions

I congratulate the expert writers of the PPI Manual of IMF from a comprehensible survey of the state of arts of the index numbers during the last century. It will provide a good basis for further development in the area, both theoretical and applied. Our main point of the paper is that the very concept of the index number formula is still vague in the literature and the strategy for constructing an index series is not included in its methodology.

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