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Liquidity

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Abstract

This paper explores liquidity, deposit insurance and bank runs in a dynamic overlapping generation economy, when only a fraction of the agents can participate in capital markets. Under maturity matching, banks transform the constant liquidity of their assets to deposits with varying liquidity: liquid demand deposits and time deposits, which are uninterruptible but marketable. Everyone obtains a liquid saving asset, since the agents who can not participate in capital markets save with demand deposits, whereas the others prefer marketable time deposits. Once maturity mismatch is eliminated, bank panics are prevented.

JEL Classification: G21, G22, G28

Keywords: Banking, Bank Runs, Deposit Insurance, Financial Crises, Bank Regulation

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1. Introduction

Frequent and widespread bank panics have been ruining banking sectors most commonly in emerging economies: 1995-1997 in Bulgaria, 1997 in Hungary, 1994 in Estonia, 1995 in Latvia and Lithuania, 1998 in Russia, etc (Tang et al., 2000). Given the disastrous consequences of bank panics, alternative options to mitigate them are urgently needed.[1]

Theoretical research on panics is inspired by the influential article by Diamond & Dybvig (1983), who analyzes intertemporal risk sharing, liquidity insurance (consumption smoothing) and panics in static intragenerational model.[2] The model was advanced to the dynamic overlapping generation economies by Qi (1994), Bhattacharya & Padilla (1996), Fulghieri & Rovelli (1998) and Bhattacharya & Fulghieri & Rovelli (1998). The articles contribute to the theoretical debate on the relative merits of banks and capital markets in promoting economic development. In view of the broad agreement of the relevance of the contributions, it is a bit surprising that the overlapping generation model has not been utilized in investigating panics, even through it supplies a rich framework for the investigation.[3] Several aspects that cannot be fully

[1] Deposit insurance offers the standard option to prevent panics. Yet, according to theoretical research, e.g. Merton (1977), deposit insurance encourages banks to excessive risk taking. The theoretical research is supported by empirical evidence. Demirguc-Kunt & Detragiache (2002), for example, find that deposit insurance tends to increase the likelihood of banking crises. These findings have led to calls for the eliminating deposit insurance.

[2] Models, which utilize the Diamond & Dybvig framework, are still extensively used: Von Thadden (1997, 1999), Allen & Gale (1997, 1998), Freixas & Parigi (1998), Freixas & Parigi & Rochet (2000), Green & Lin (2000), Qi (2003), Ennis & Keister (2003), Rochet & Vives (2004), Goldstein & Pauzner (2005) and Chen & Hasan (2006).

[3] Qi's (1994) article is an exception. He demonstrates that suspension of convertibility may not prevent panics in the OLG-framework, since a newborn generation may refuse to save their endowments in the bank. This gives an interesting example on the result that is unachievable in the intragenerational framework.

investigated in the intragenarational model, can be scrutinized in the overlapping generations model.[4] This gap in the literature is in part fulfilled by this paper.

Niinimäki (2003) shows in the intragenerational model how panics can be prevented if a bank not only attracts liquid demand deposits but also time deposits which have low interruption value. When depositors have a differing risk of encountering a preference shock, high-risk agents will hold their savings as demand deposits, whereas low-risk agents will prefer time deposits. A low-risk agent appreciates the substantial long-term return of a time deposit and is willing to bear the low-probability risk of encountering a preference shock and interrupting his time deposits. The analysis has two main shortcomings. The bank is a monopoly and the interruption value of a time deposit is lower than the initial deposit. This makes time deposits risky. An agent is willing to resell an ongoing time deposit before maturity instead of interrupting it, but reselling is not possible in the model. These shortcomings are corrected in this paper. The banking sector is perfectly competitive and time deposits can be resold in secondary markets.

In this paper it is assumed that some agents are able to participate in capital markets whereas others are not. A panic-free bank system is constructed utilizing maturity matching. A bank attracts both liquid demand deposits and time deposits (alternatively stocks) so that the total liquidation value of deposits is equal to the liquidation value of the bank assets. Given the equality, it may appear that the bank does not boost the liquidity of the economy. The appearance is defective. The bank provides a fundamental service to an economy by transforming the constant liquidity of its assets to deposits with varying liquidity. Demand deposits are more liquid than bank assets. Time deposits are nominally less liquid than bank assets, since they cannot be interrupted before maturity. They can, however, be resold in secondary markets and thus are effectively liquid for the

[4] In the static intragenerational setting, it is extremely important that the agents cannot contact each others. If agents can contact each other, no bank can be established (see Jacklin, 1987; Wallace, 1988; Diamond, 1997). These problems can be eliminated in the OLG-framework (see Qi, 1994).

agents who can participate in the markets. Time deposits (or bank stocks) are optimally chosen by the agents who can participate in capital markets whereas others prefer demand deposits. Given the marketability of time deposits, both demand deposits and time deposits are more liquid than the assets of the bank, and thus the bank boosts the liquidity of the economy even under maturity matching.

The paper displays that time deposits and stock capital stabilize banking system efficiently and almost in an identical manner. Stabilization may, however, be expensive, since the agents who can participate in capital markets would require the same return on their time deposits (and bank stocks) to what they can obtain by investing directly in firms. If the costs of stabilization are high, they may make banking unprofitable. To avoid this, a bank regulator may decide to offer deposit insurance. Panics can be prevented without the excessive burden of time deposits (or stock capital) and thus banks can pay moderate return on demand deposits.

The assumption that only a fraction of agents can participate in the capital markets is borrowed from Diamond (1997). However, he uses the intragenerational model, whereas this paper adopts the framework of overlapping generations, which generates different results. Furthermore, Diamond concentrates on liquidity insurance, whereas this paper also examines panics.

2. Economy

Consider an infinite horizon economy with an infinite sequence of overlapping generations of agents. A new generation is born at every time $t \in \{0,1,2,...\}$ and consists of a continuum of agents of measure 1. Newborn agents are endowed with 1 unit of a homogeneous good, which can be spent for either consumption or production. An agent born at time t lives with certainty at $t+1$ and possibly also at $t+2$. The agents of the first type thus live for one period only, consume at time $t+1$, and are labelled *early consumers*. Those of the second type live for two periods, consume at $t+2$ and are identified as *late consumers*. An agent born at t learns his type (early or later consumer) privately at time $t+1$. Agents become early or late consumers with constant probabilites ε and $1-\varepsilon$ respectively. The population is assumed to be so large that there is no uncertainty regarding the aggregate distribution between early and late consumers. A constant share ε of agents born at t consumes at $t+1$, whereas the rest consume at $t+2$. Therefore, a newborn has an expected utility

$$W = \varepsilon U(c_1) + (1-\varepsilon)U(c_2) \ . \tag{1}$$

Here c_1 denotes the level of consumption of an early consumer and c_2 represents the consumption of a later consumer. The utility function is strictly increasing and concave: $U'(.), U''(.) < 0$, $U'(0) = \infty, U'(\infty) = 0$. The assumption $U'(0) = \infty$ implies $U(0) = -\infty$.

Thus, at each time t , $t \geq 2$, there are four groups of agents. One, newborns of generation t (these agents are born at t). Their total measure is 1. Two, early consumers of generation $t-1$ (measure ε). Three, late consumers of generation $t-1$ (measure $1-\varepsilon$). Four, later consumers of generation $t-2$ (measure $1-\varepsilon$). At each time point there are $3-\varepsilon$ agents and groups two and four consume. Since there is no uncertainty regarding the aggregate distribution between early and late consumers, the demographic structure of the population is constant through time.

Two technologies are available. The first is risk-free long-term *production* with constant returns of scale. An investment of one unit at t produces $R>1$ units at $t+2$. The liquidation value of production is $0<l<1$ units. The second technology is *storage*. Consumption can be transferred from one period to the next without any loss or depreciation. To emphasize the importance of banks and capital markets the following assumption is made

Assumption 1. $\varepsilon l + (1-\varepsilon)R<1$.

This assumption states that in autarky (each agent produces his own consumption) an agent prefers storage to the long-term production. Given the risk to liquidate early and the low liquidation value, the expected returns from production are lower than the returns from storage. Therefore, there is no production in autarky. The following assumption guarantees that the liquidation value of long-term production is low and makes a bank vulnerable to bank panics

Assumption 2. $l < 1-\varepsilon$.

New production can be started at each time point. This generates a sequence of overlapping technologies which are at different stages of the production process. At each time point, different stages exist: production at the start up stage, production at the intermediate stage and production that materializes.

The following extension is based on Diamond (1997). It is assumed that a fraction $0<\alpha<1$ of agents are *active*. They can contact each other and thus participate in a capital markets. The rest of the agents, $1-\alpha$, are *passive*, live in isolation (see Wallace, 1988), cannot contact each other and thereby cannot participate in capital markets. They can, however, contact a bank. The fractions of early consumers and late consumers are assumed to be independent of the agent's ability to participate in capital markets.

As is standard for the OLG models, the analysis focuses on steady-state allocations that yield identical ex ante return for all current and future generations. For the dynamic transition to the steady state, see Bhattacharya & Fulghieri & Rovelli (1998) and Qi (1994).

3. Capital markets

This section, which borrows from Bhattacharya & Padilla (1996) and Fulghieri & Rovelli (1998), characterizes the optimal steady-state consumption and investment allocations that can be achieved by active agents, who are able to contact each other and trade stocks among themselves.[5]

The stock markets operate as follows. At each time point t there are newborn agents who are endowed with a unit of consumption good. Each newborn sets up a firm, which invests a fraction of the endowment, $I \leq 1$, in production. A part of the firm's stocks is retained by the newborn, who sells the remaining stock to other agents. The selling revenue and the rest of the endowment, $1 - I$, are invested by him in the stocks of firms set up by the other agents. There are three age groups of firms at each time point: new firms which have started their production at t, intermediate firms that started their production at $t - 1$ and old firms that started their production at $t - 2$. The production of old firms materializes and is paid out as dividends.

The economy has three markets: (1) A market for goods, with their unit price normalized to one. (2) A market for the stocks of new firms; (3) a market for the stocks of intermediate firms. The following labels are used.

$P_n =$ The unit price for the stock of a new firm, n

[5] For initial research on stocks in the Diamond & Dybvig model, see Jacklin (1987) and Jacklin & Bhattacharya (1988).

$P_i =$ The unit price for the stock of an intermediate firm, i

$\theta_n^0 =$ The agent's, whose age is 0 (a newborn), ownership in new firms, n

$\theta_i^0 =$ The agent's, whose age is 0, ownership in intermediate firms, i

$\theta_n^1 =$ The agent's, whose age is 1 (an intermediate agent), ownership in new firms

$\theta_i^1 =$ The agent's, whose age is 1, ownership in intermediate firms

To maximize his expected utility, an agent optimizes his consumption and investment decisions. A newborn's budget constraint is

$$1 + P_n I = I + \theta_n^0 P_n + \theta_i^0 P_i . \tag{2}$$

The L.H.S constitutes the initial income: an endowment and the selling revenues from the stocks of his firm, $P_n I$. The R.H.S shows that the income is invested in long-term production I (the set up cost of the firm), in the stocks of new firms, $\theta_n^0 P_n$ (including his own firm) and in the stocks of the intermediate firms, $\theta_i^0 P_i$.

At time $t+1$ a newborn learns whether he is an early or late consumer. If he turns out to be an early consumer, he avoids the interruption of long-term production by selling his stocks and by consuming the selling revenue as well as his dividend income

$$C_1 = \theta_n^0 P_i + \theta_i^0 R . \tag{3}$$

At time t, the agent invested $\theta_n^0 P_n$ in new firms. The updated selling price of these stocks is $\theta_n^0 P_i$. Furthermore, at t the agent invested $\theta_i^0 P_i$ in the intermediate firms. The value of these stocks is now $\theta_i^0 R$, since the production materializes.

At $t+1$ the newborn alternatively can learn that he is a late consumer. He owns, of course, the same stocks as if he were an early consumer, $\theta_n^0 P_i + \theta_i^0 R$. This wealth can be reallocated by him. The late consumer's budget constraint at $t+1$ is

$$\theta_n^0 P_i + \theta_i^0 R = \theta_n^1 P_n + \theta_i^1 P_i . \tag{4}$$

The L.H.S indicates the inherited portfolio from time t , whereas the reallocated portfolio is on the R.H.S. The reallocated portfolio is consumed by the late consumer at $t+2$

$$C_2 = \theta_n^1 P_i + \theta_i^1 R \quad . \tag{5}$$

The prices of stocks must be such that arbitrage in the stock markets is eliminated

$$\frac{P_1}{P_0} = \frac{R}{P_1} = \sqrt{\frac{R}{P_0}} . \tag{6}$$

The term P_1/P_0 expresses the returns from new stocks and R/P_1 indicates the returns from the stocks of intermediate firms. The term $\sqrt{R/P_0}$ can be constructed using P_1/P_0 and R/P_1.

No arbitrage is allowed between stock markets and production. A unit of production yields R units after two periods. This must be as productive as the investment in new stocks. Hence, it requires that $P_0 = 1$. This, together with (6) implies $P_1 = \sqrt{R}$.[6]

Both production markets and stock markets must clear (for simplicity, the size of the stock markets here is 1 although only active agents can participate in the markets)

$$
\begin{aligned}
&i.)\ \ \varepsilon C_1 + (1-\varepsilon) C_2 = 1 - I + IR \\
&ii.)\ \ \theta_n^0 + (1-\varepsilon)\theta_n^1 = I \\
&iii.)\ \ \theta_i^0 + (1-\varepsilon)\theta_i^1 = I\ .
\end{aligned}
\tag{7}
$$

Here $i.)$ states that consumption is equal to production. At every time point t, both the early consumers of generation $t-1$ and the late consumers of generation $t-2$ consume. Production, IR, is based on the investment, I, by generation $t-2$. Note that $IR < R$, if not all of the endowment is invested in new production, $I < 1$. Additionally, $ii.)$ determines that the volume of new stocks is

[6] It is necessary to investigate arbitrage in more detail. The following analysis is borrowed from Bhattacharya & Padilla (1996). Suppose first that $P_0 > 1$ and consider the following arbitrage strategy. A newborn short sells $1/P_1$ units of intermediate stocks. He invests the selling revenue, one unit, in long-term production, sells the stocks of production and earns $P_0 > 1$. The selling revenue is invested in P_0/P_1 intermediate stocks. These stocks represent the agent's long position, whereas his short position consists of $1/P_1$ units of intermediate stocks. Since $P_0 > 1$, the newborn makes an arbitrage profit, $R(P_0 - 1)/P_1$. Suppose now that $P_0 < 1$, but $R/P_1 = P_1/P_0$. Then, a newborn will not set up a firm of his own. Instead, he invests the endowment in the stocks of the other new firms and obtains either a two-period return $R/P_0 > R$ or one-period return $P_1/P_0 > P_1$. In sum, it must be $P_0 = 1$. It is easy to see that when $P_0 = 1$, it must be $P_1 = \sqrt{R}$ (recall (6)).

equal to the amount of production, while *iii.)* expresses the same requirement for intermediate stocks. The optimal consumption and investment allocations can now be reviewed.[7]

Proposition 1 *(Fulghieri & Rovelli, 1998). In the stock markets, the steady state competitive equilibrium satisfies*

$$P_0 = 1\,,\; P_1 = \sqrt{R}\,,\; C_1 = \sqrt{R}\,,\; C_2 = R,\; 0 < I^* = 1 - \frac{\varepsilon(R-\sqrt{R})}{R-1} < 1\,.$$

Here $I < 1$ since not all of the endowment is invested in new production, but $1-I$ is invested in intermediate stocks. Recall that in autarky an agent optimally stores the endowment and obtains utility $u(1)$. Hence, the formation of stock markets boosts the expected utility. It is possible to invest in production and obtain liquidity by trading the stocks of production so that the interruption of long-term production is avoided.

[7] Above, it was shown that $P_0 = \sqrt{R}$, $P_1 = R$. Substituting these into (2)-(5) provides after some manipulation $C_1 = \sqrt{R}$, $C_2 = R$. Inserting this into 7i.) gives $I^* = 1 - \varepsilon(R-\sqrt{R})/(R-1)$.

4. Bank

This section highlights liquidity provision through a bank system. Since a fraction $1-\alpha$ of agents are passive and thus unable to participate in capital markets, a bank system offers them the only option for investment and liquidity. The section is based on the analysis of Qi (1994), Bhattacharya & Padilla (1996) and Fulghieri & Rovelli (1998). To begin, the occurrence of panics is clarified.

Definition 1*. The agents predict that a panic may occur. The predictions come true and a panic occurs, if a late consumer obtains a higher expected return by joining a predicted panic than by waiting for the next time point, or if a newborn will not save his endowment in a bank.*

Here a late consumer is an agent, whose true consumption time is in the next time point.

Definition 2 . *A bank operates under maturity mismatch, when the total liquidation value of its deposits exceeds the liquidation value of its assets.*

Given Definition 1, maturity mismatch triggers a panic with certainty, since only the very first withdrawers can regain possession of their deposits. Thereafter, once bank assets are exhausted, it fails and the last withdrawers obtain nothing. Therefore, it is optimal to panic.

A bank is established by passive agents who save their endowment in it. The bank attracts only demand deposits and promises to pay D_1 for withdrawals made after one period and D_2 for withdrawals made after two periods. The payments are chosen so that the expected utility of a depositor is maximized[8]

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[8] Here the bank is modelled as if its size were 1 instead of $1-\alpha$. That is, the multiplications by $1-\alpha$ are dropped.

$$\varepsilon u(D_1) + (1-\varepsilon)u(D_2) \tag{8a}$$

$$s.t. \quad \varepsilon D_1 + (1-\varepsilon)D_2 = 1 - I + L + R(I-L) - (2-\varepsilon)B \tag{8b}$$

$$(D_1)^2 \leq D_2 \tag{8c}$$

$$0 \leq I \leq 1 \tag{8d}$$

$$L \leq lI \tag{8e}$$

$$\varepsilon(R-\sqrt{R}) < B < (R-1)/(2-\varepsilon) \tag{8f}$$

The expected utility (8a) is maximized subject to the resource constraint (8b). The L.H.S of (8b) represents consumption; at t, both the early consumers of generation $t-1$ and the late consumers of generation $t-2$ consume. The R.H.S shows resources. The first term is the endowment, 1, from which the production investment, I, is subtracted. The resources also include the returns from the liquidated production, L, production output, $R(I-L)$, and the costs of banking per a size unit, B. (8c) is needed to eliminate arbitrage by depositors; a later consumer will not mimic an early consumer by withdrawing D_1 at $t+1$ and redepositing it in another bank for a period. This strategy would yield $(D_1)^2$ and it cannot be more profitable than retaining the savings in the original bank (see Qi, 1994). Since the agents are risk averse, (8c) is binding. As regards to (8d), $I \leq 1$ states that the production investment cannot exceed the amount of the endowment. Additionally, in (8e), the returns from liquidation, L, cannot exceed the liquidation value of the whole production, lI. It is easy to see that the R.H.S of (8b) is maximized when $L = 0$; no production is liquidated. Constraint (8f) determines the operating costs of banking. Since $B < (R-1)/(2-\varepsilon)$, the operating costs are so low that banking is profitable. The second part, $B > \varepsilon(R-\sqrt{R})$, sets a lower limit for the operating costs.[9] Given $L = 0$ and $D_2 = (D_1)^2$, the maximization problem simplifies to

[9] The positive costs of operation have been adopted for several reasons. First, they are realistic. Mishkin (2007, p. 221) documents: "In recent years, interest paid on deposits (checkable and time) has accounted for around 25% of total bank

$$\varepsilon\, u(\sqrt{D_2}\,) + (1-\varepsilon) u(D_2\ ) \tag{9a}$$

$$s.t. \quad \varepsilon \sqrt{D_2} + (1-\varepsilon) D_2 = 1 + I(R-1) - (2-\varepsilon) B \tag{9b}$$
$$0 \le I \le 1 \tag{9c}$$
$$\varepsilon(R-\sqrt{R}) < B < (R-1)/(2-\varepsilon) \tag{9d}$$

It is easy to see from (9a) that the expected utility is maximized when D_2 is maximal. The maximal value of D_2 is determined by the resource constraint, (9b). Since the resources are largest when $I = 1$, (9b) can be rewritten as $\varepsilon \sqrt{D_2} + (1-\varepsilon) D_2 = R - (2-\varepsilon) B$. Given this and (9d), it is known that $D_2 > 1$; interest on deposits is positive. Furthermore, the rewritten resource constraint satisfies $D_2 - \varepsilon (D_2 - \sqrt{D_2}) = R - (2-\varepsilon) B$. If $D_2 = R$, the consumption (the R.H.S) exceeds the resources (the L.H.S) due to (9d). Thus, it is known that $D_2 < R$. A conclusion follows.

Proposition 2 . *The bank's optimal allocation satisfies* $1 < D_1^* < D_2^* < R$, $D_1{*} = \sqrt{D_2^*}$ and $I = 1$.

Therefore, the bank can offer positive return and liquidity to agents. However, they receive lower return than what could be achieved in the capital markets (Proposition 1) due to the costs of banking.

Unfortunately, the bank is threaten by panics. The late consumers of generation $t - 2$ withdraw $(1-\varepsilon) D_2$ and the early consumers of the next generation withdraw εD_1 even without a

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operating expenses, while the costs involved in serving accounts (employee salaries, building rent, and so on) have been approximately 50% of operating expenses." Second, due to the operating costs, demand deposits are less productive than stocks. Without the operating costs, demand deposits would be as productive as stocks, which would be unrealistic. Third, it is seen later that the operating costs motivate the bank to minimize the amount of bank capital. Fourth, the lower limit of the operating costs is so high that the payments on demand deposits satisfy $D_2 \le R$. Thus, it is not necessary to set an extra restriction $D_2 \le R$ in order to eliminate interbank deposits as in Bhattacharya & Padilla (1996).

panic, since their true consumption time point is present. During a panic, the late consumers of generation $t-1$ mimic early consumers and withdraw $(1-\varepsilon)D_1$. If the newborns panic and do not save their endowments in the bank, the resources of the bank consist of materializing production, R, and from the liquidated intermediate production, $lI = l$. The resources cover the withdrawals if

$$R - B(2-\varepsilon) - (1-\varepsilon)D_2 - \varepsilon D_1 - (1-\varepsilon)D_1 + l\,. \tag{10}$$

The sum of the first 4 terms is zero owing to the resource constraint. Given Assumption 2, it is known that $1-\varepsilon > l$. The liquidation value of the intermediate production is so low that it does not cover payments on deposits and the bank fails due to maturity mismatch, $-(1-\varepsilon)D_1 + l < 0$. Therefore, if an agent predicts a bank panic, he rationally withdraws his deposits and the panic actually occurs. Given Definition 1, a panic occurs with certainty. Furthermore, since the last withdrawers do not get anything, their utility is $u(0) = -\infty$. As a result, a rational agent will not deposit initially his endowment in the bank, but instead he stores it. No bank is established although a panic-free bank system could raise the expected returns of the agents and provide the desired liquidity. Consequently, it is important to investigate different options for preventing panics. [10]

[10] For somewhat sceptical studies regarding the optimality of demand deposits in the Diamond & Dybvig setting, see Jacklin (1987), Postlewaite & Vives (1987), Jacklin & Bhattacharya (1988),Wallace (1988) and Green & Lin (2000).

5. Bank with subordinated time deposits

This section examines how panics can be prevented if the bank attracts not only liquid demand deposits, but also time deposits, which cannot be interrupted before maturity. Passive agents favour demand deposits, whereas active agents save with marketable time deposits.

5.1 Market of time deposits

The characteristics of time deposits and the operations of the market for time deposits are approached first. It is assumed that there are sufficiently active agents to save in time deposits. The assumption is later dropped.

A time deposit lasts for two periods. It cannot be interrupted after the first period, but can then be resold. The bank attracts $\frac{1}{2}A$ new time deposits at each time point and thus the volume of outstanding time deposits is equal to A. How much does the bank pay on time deposits? The bank is established by passive agents and it maximizes their expected utility, thereby minimizing payments on time deposits. Time deposits need to be at least as productive as the stocks of firms, which yield an allocation $\sqrt{R}$, R to an active agent (an early consumer, a late consumer). Suppose that the bank pays interest R on a time deposit at maturity, that is, after two periods. Let P_1^{TD} denote the resale price of an intermediate time deposit. Arbitrage is eliminated if

$$\frac{P_1}{P_0} = \frac{R}{P_1} = \sqrt{\frac{R}{P_0}} = \frac{R}{P_1^{TD}}. \tag{11}$$

The first 3 terms are the same as before (recall (6)), while the fourth term indicates the return of a time deposit from the intermediate stage to maturity. It has been shown above that

$P_0 = 1$, $P_1 = \sqrt{R}$. It must be $P_1^{TD} = \sqrt{R}$. Thus, at maturity the bank pays interest R on time deposits, which cannot be interrupted before maturity, but which can be resold at the market price $\sqrt{R}$ at the intermediate stage.

After the introduction of time deposits, the budget constraints and consumption allocations of an active agent are[11]

$$1 = \beta_n^0 + \sqrt{R}\,\beta_i^0 + \theta_n^0 + \sqrt{R}\,\theta_i^0 \quad . \tag{12}$$

$$C_1 = \beta_n^o \sqrt{R} + R\beta_i^0 + \sqrt{R}\,\theta_n^0 + R\theta_i^0 = \sqrt{R}\,.$$

$$\sqrt{R} = \beta_n^1 + \sqrt{R}\,\beta_i^1 + \theta_n^1 + \sqrt{R}\,\theta_i^1.$$

$$C_2 = \beta_n^1 \sqrt{R} + \beta_i^1 R + \theta_n^1 \sqrt{R} + \theta_i^1 R = R\,.$$

Here β_n^0 (β_n^1) denotes the newborn's (late consumer's) savings in new time deposits and β_i^0 (β_i^1) represents his savings in intermediate time deposits. In (12), the first line expresses the newborn's budget constraint; he splits the endowment between new time deposits, intermediate time deposits, the stocks of new firms and the stocks of intermediate firms. The price of a new stock or a new time deposit is one unit and the price of an intermediate stock or time deposit is $\sqrt{R}$ units. The second line indicates the consumption of an early consumer. With each asset portfolio, the realized consumption is $\sqrt{R}$. If the agent becomes a later consumer, he can reallocate his wealth, $\sqrt{R}$, at the intermediate stage. This is showed in the third line, whereas the fourth line indicates the realized consumption of a late consumer. Whatever his portfolio, he can consume R units.

Both stock markets and the markets of time deposits must clear

[11] It is assumed that each active newborn chooses the same allocation and each active late consumer chooses the same allocation.

$$i.)\, \alpha\beta_n^0 + \alpha(1-\varepsilon)\beta_n^1 = \tfrac{1}{2}A \qquad ii.)\, \alpha\beta_i^0 + \alpha(1-\varepsilon)\beta_i^1 = \tfrac{1}{2}A, \tag{13}$$

$$iii.)\, \alpha\theta_n^0 + \alpha(1-\varepsilon)\theta_n^1 = \alpha I^* - \tfrac{1}{2}A \qquad iv.)\, \alpha\theta_i^0 + \alpha(1-\varepsilon)\theta_i^1 = \alpha I^* - \tfrac{1}{2}A\ .$$

Here $i.$ $(ii.)$ states that total savings in new (intermediate) time deposits are equal to the supply of new (intermediate) time deposits. Additionally, $iii.$ $(iv.)$ indicates that total savings in new (intermediate) stocks are equal to the supply of new (intermediate) stocks. The supply of stocks is flexible; the larger the supply of time deposits, the less can be invested directly in the stocks of the firms, $\alpha I^* - \tfrac{1}{2}A$. The term is positive when there are sufficiently active agents in the economy to fulfil the supply of time deposits. Recall that the amount of direct investment in long-term production, αI^*, is smaller than the volume of active agents, α, since the agents optimally invest a part of their endowment in intermediate assets.

One market clearing solution is the following: $\beta_n^0 = \tfrac{1}{2\alpha}A$, $\beta_n^1 = 0$, $\theta_n^0 = I^* - \tfrac{1}{2\alpha}A$, $\theta_n^1 = 0$, $\theta_i^0 + \beta_i^0 = 1 - I^*$ and $\beta_i^1 + \theta_i^1 = 1$. Late consumers do not invest in new assets (time deposits or stocks), which are channelled to newborns, who invest the rest of their endowments in intermediate assets. If a newborn later becomes an early consumer, he keeps the returns from maturing assets and sells the intermediate assets ($\beta_n^0 = \tfrac{1}{2\alpha}A$, $\theta_n^0 = I^* - \tfrac{1}{2\alpha}A$) to the late consumers of his generation (β_i^1, θ_i^1) and to the newborns of the next generation (β_i^0, θ_i^0). The market for intermediate time deposits clears since their supply, $\varepsilon\beta_n^0$, is equal to demand, $\beta_i^0 + (1-\varepsilon)(\beta_i^1 - \beta_n^0)$, that is, $\beta_n^0 = \beta_i^0 + (1-\varepsilon)\beta_i^1$, or

$$\tfrac{1}{2\alpha}A = \beta_i^0 + (1-\varepsilon)\beta_i^1\ . \tag{14}$$

This is equal to (13ii) and thus it is always true. Equally, the market for stocks clears because their supply, $\varepsilon\,\theta_n^0$ is equal to demand, $\theta_i^0 + (1-\varepsilon)(\theta_i^1 - \theta_n^0)$, that is, $\theta_n^0 = \theta_i^0 + (1-\varepsilon)\,\theta_i^1$, or

$$I^* - \tfrac{1}{2\alpha} A = \theta_i^0 + (1-\varepsilon)\,\theta_i^1 \quad , \tag{15}$$

which is equal to (13iv). Therefore, the markets of intermediate time deposits and stocks clear. It is easy to check that under these allocations an early consumer can consume $\sqrt{R}$ units and a late consumer R units (recall (12)).

5.2 The optimal amount of time deposits

This subsection shows that the optimal amount of time deposits is the minimum amount that is needed to satisfy the maturity matching constraint. With time deposits, the resource constraint of the bank is

$$R\left[(1-\alpha) + \tfrac{1}{2}\mathrm{A}\right] - B\left[(1-\alpha)(2-\varepsilon) + \mathrm{A}\right] \;=\; \tfrac{1}{2} R\,\mathrm{A} + (1-\alpha)\left[\varepsilon\sqrt{D_2} + (1-\varepsilon) D_2\right]. \tag{16}$$

The first term on the L.H.S represents production output. At each time point the bank attracts $1-\alpha$ new demand deposits and $\frac{1}{2}\mathrm{A}$ new time deposits and invests the funds in production. The second term states the costs of banking that are dependent on the amount of deposits. Passive newborns, $1-\alpha$, save with demand deposits. In addition, the passive late consumers of the previous generation have demand deposits, $(1-\alpha)(1-\varepsilon)$ in all. Given time deposits, A , the total amount of deposits adds up to $(1-\alpha)(2-\varepsilon)+\mathrm{A}$. The first term on the R.H.S indicates payments on maturing time deposits and the second term displays payments on demand deposits. Some manipulation gives

$$R - B\left[(2-\varepsilon) + \frac{\mathrm{A}}{1-\alpha}\right] = \varepsilon\sqrt{D_2} + (1-\varepsilon)D_2.$$

This reveals $d\,D_2/d\,A < 0$; time deposits reduce payments on demand deposits. It is optimal to minimize the amount of time deposits. Intuitively, each bank account, including both demand and time deposits, incurs an equal cost B to the bank. Yet, active agents require the very same return on their time deposits to what they could achieve by investing directly in firm stocks. Thus, the costs of time deposits, B, are borne entirely by the depositors who save in demand deposits.

The minimum amount of time deposits is determined by maturity matching. The late consumers of generation $t-2$ as well as the early consumers of generation $t-1$ withdraw their deposits with certainty. During a panic, newborns do not save their endowment in the bank. Additionally, the late consumers of generation $t-1$ panic and mimic early consumers by intending to withdraw their deposits. Only intermediate time deposits cannot be withdrawn. The maturity matching constraint is satisfied if

$$R\left[(1-\alpha)+\tfrac{1}{2}\mathrm{A}\right] - B\left[(1-\alpha)(2-\varepsilon)+\mathrm{A}\right] - \tfrac{1}{2}R\,\mathrm{A} - (1-\alpha)\left[\varepsilon\sqrt{D_2}+(1-\varepsilon)D_2\right] - (1-\alpha)(1-\varepsilon)\sqrt{D_2} + l\left[(1-\alpha)+\tfrac{1}{2}\mathrm{A}\right] \geq 0. \tag{17}$$

The first 4 terms together form a resource constraint and their sum is equal to zero. The bank can pull through the panic if $(1-\alpha)(1-\varepsilon)\sqrt{D_2} = l\left[(1-\alpha)+\tfrac{1}{2}A^*\right]$. The minimum amount of time deposits that prevents panics, $A*$ (which is also the optimal amount of time deposits), satisfies

$$A^* = \frac{2(1-\alpha)\left[(1-\varepsilon)\sqrt{D_2} - l\right]}{l}. \tag{18}$$

Intuitively, the stabilizing effect of time deposits is simple. Since time deposits increase the liquidation value of intermediate production but cannot be interrupted, they help create maturity matching. More precisely, without time deposits the value of demand deposits exceeds the liquidation value of intermediate production, $(1-\alpha)(1-\varepsilon)\sqrt{D_2} > \; l(1-\alpha)$, which makes the bank vulnerable to panics. However, an amount of time deposits exists $\frac{1}{2}A^*$ so that the value of demand deposits is equal to the increased liquidation value of intermediate production, $(1-\alpha)(1-\varepsilon)\sqrt{D_2} = \; l(1-\alpha) + \; l\,\frac{1}{2}A^*$. Maturity matching is satisfied and the bank can pay back deposits if a panic occurs.

5.3 Special rules

This subsection confirms that panic-free banking system can be achieved only if the maturity matching constraint is supported with special rules.

First, it is shown that time deposits need to be subordinated to demand deposits. In the next time point after a panic, the bank cannot settle the promised payments on deposits in full, because some production has been liquidated during the panic, and the materializing value of the production is low. Suppose that in this case the bank adopts "*a fear sharing rule*". Each depositor receives an equal share, f , of the promised payment when the bank cannot settle the payments in full. That is, the bank pays $f * D_2$ units on long-term demand deposits and $f * R$ units on long-term time deposits, $f < 1$. Without a panic, the bank can pay the promised payments in all, $f = 1$.

Suppose now that the maturity matching requirement is satisfied, but a panic occurs. Only intermediate time deposits, which cannot be interrupted, are retained in the bank. Given maturity matching, the value of bank assets, intermediate production, erodes to zero. Thus, at the next time point, the bank has no materializing production, it cannot pay anything on maturing time deposits and it fails.

Is a panic withdrawal of demand deposits rational? Suppose that one late consumer opts to sit out the panic and does not withdraw D_1. The bank has then D_1 units of intermediate production. In the next time point, the production materializes yielding $D_1 R$ units. Under the fair sharing rule, $D_1 R$ units are shared equally among the agents; that is, between the late consumer with demand deposits and the agents who have maturing time deposits. As a result, the agents with time deposits can share $D_1 R$, whereas the late consumer with demand deposits does not obtain anything (the measure of his demand deposits is zero and the total measure of the time deposits is positive. Hence, the whole return is paid on time deposits). By joining the panic and withdrawing his demand deposits the late consumer can obtain D_1. Thus, the panic withdrawal of demand deposits is rational.

Consequently, maturity matching is a necessary condition to prevent panics, but not a sufficient condition. It guarantees that a late consumer, who saves in demand deposits, can obtain the allocation of the early consumer by withdrawing immediately. Unfortunately, it does not guarantee that the late consumer obtains a higher return in waiting for his true consumption time than in withdrawing the allocation of the early consumer at once.

Consider an identical case, but in which time deposits are now subordinated to demand deposits. One late consumer does not join the panic and withdraws D_1. The bank has D_1 units of intermediate production. In the next time point, the production materializes yielding $D_1 R$ units. Since $D_1 R > D_2$ and since demand deposits are senior to time deposits, the late consumer receives the promised return, D_2. Thus, he rationally opts to sit out the panic. Since each late consumer acts in the same manner, each waits for the next time point and thus demand deposits are retained in the bank. No intermediate production is interrupted and it materializes in the next time point yielding $R\left[(1-\alpha)+\frac{1}{2}\mathrm{A}\right]$, which covers the operating costs as well as the payments on long-term demand deposits, D_2 , and time deposits, R (recall the resource constraint (16)). Something is even left over for the early consumers of the next generation, $(1-\alpha)\varepsilon D_1$. A conclusion follows.

Lemma 1. *When time deposits are subordinated to demand deposits, both demand deposits and time deposits are retained in the bank and no intermediate production is liquidated.*

Consider time point t. Under Lemma 1, the agents of generation $t-1$ do not panic and withdraw their deposits. The agents of the new generation, generation t, need to be explored. These newborns, who ought to save their endowment in the bank, can panic by keeping their endowments out of the banking system. The following specification is made.

Assumption 3. *To avoid a maturity mismatch, the bank promises to pay back the deposits of newborns at once if it cannot attract the sufficient shares of both deposit types.*

The promise protects newborns. If the bank cannot remain maturity matching, the newborns receive their endowments back. Thus, they cannot lose anything at time point t.

The agents of generation t need to be certain that their savings are also protected in the next time points. Consider time point $t+1$. The case of time deposits is easy to analyze, since they cannot be interrupted. In addition, under Lemma 1 the panic withdrawal of intermediate demand deposits is prevented at $t+1$. Thus, both deposit types are retained in the bank and no intermediate production is interrupted. Furthermore, the value of intermediate demand and time deposits at $t+1$ is independent of whether or not the next generation, generation $t+1$, saves their endowment in the bank.

The agents of generation t need to be assured that their savings are also protected at time point $t+2$. Then, the decision of the newest generation (generation $t+2$), whether or not to save in the bank, has no effect on the payments on maturing time deposits and long-term demand deposits. Neither has the decision of the previous generation (generation $t+1$) any effect on payments. To see this, two cases need to be examined. First, if generation $t+1$ saves their endowment in the bank at $t+1$, the bank keeps on operating normally and it can pay R on time deposits and D_2 on long-term demand deposits at $t+2$. Second, if generation $t+1$ panics and does not save its endowments in the bank at $t+1$, the returns of generation t do not change. The production output is R at $t+2$ and is sufficient to pay R on time deposits and D_2 on long-term demand deposits. Since a part of the output, $(1-\alpha)\varepsilon D_1$, is reserved for the early consumers of generation $t+1$, but as this generation did not even save in a bank, the bank has extra returns, $(1-\alpha)\varepsilon D_1$. Thus, the agents of generation t can be sure that their deposits are safe at $t+2$.

Therefore, under Lemma 1 the newborns of generation t can be sure that their deposits will be safe regardless of whether the newborns of the next generations panic or save their endowments in the bank. Consequently, the newborns of each generation will save their endowments in the bank and no panics occur.

Since time deposits are now subordinated to demand deposits, the panic-preventing amount of time deposits needs to be updated

$$A^* = \frac{2(1-\alpha)\left[(1-\varepsilon)\sqrt{D_2}-l\right]}{R+l}. \tag{18'}$$

The previous analysis can be summarized as follows.

Proposition 3. *Panics can be prevented using maturity matching. The bank attracts not only demand deposits but also time deposits, which are uninterruptible, but which can be resold in secondary markets. Passive agents obtain liquidity by saving in demand deposits and active agents by saving in marketable time deposits. Since time deposits incur excessive costs for the bank, it* *minimizes the volume of time deposits so that the maturity matching constraint is only barely satisfied. Due to the operating costs of banking, the bank can offer a lower return to agents than what these could obtain by investing directly in firms.*

5.4 Numeric example I

Suppose the following economy: $R = 1.15$, $B = 0.025$, $\varepsilon = 0.3$, $\alpha = 0.35$, $l = 0.64$. The agents who can participate in capital markets obtain $R = 1.15$ (a long-term return). Without time deposits, the bank can offer $D_2 = 1.127$ to its depositors, but the bank is vulnerable to runs since $1-\varepsilon > l$. With time deposits, the bank can offer $D_2 = 1.124$. The amount of time deposits is 0.074, whereas the amount of demand deposits is $2-\varepsilon = 1.7$. Hence, the ratio of time deposits to total deposits is 4.2%.

It is easy to see that capital markets can supply higher returns than a bank, since banking incurs costs. In addition, the bank can pay more on demand deposits without stabilizing time deposits. Thus, the bank regulator can raise the expected utility of passive agents if it can offer deposit insurance at no cost. Then, the bank can avoid panics without the burden of time deposits and pay more on demand deposits. In this example the differences in payments on demand deposits are small, since the necessary amount of time deposits is modest and the operating costs are low.[12]

[12] If the operating costs of banking are zero, $B = 0$, the bank can offer the same optimal allocation $(\sqrt{R}, R)$ to passive agents to what the active agents can obtain by investing directly in firms.

6. Bank with stock capital

This section extends the analysis by exploring whether stock capital can be used to prevent panics. How does stock capital differ from time deposits in this context? Is there any difference at all?

6.1 Firm-E

The operation of capital markets was reviewed in section 4. Each agent set up a firm of his own and sold a part of its stock to other agents. The sales revenue was invested by him in the stocks of the other firms. Suppose now that the agents coordinate their actions and put together numerous small firms of unit size. The size of the big firm, Firm-E, is E . It has $\frac{1}{2}E$ units of production started in $t-2$ and $\frac{1}{2}E$ units of production started at $t-1$. The process continues in the following time points. At each time point, old production materializes yielding $\frac{1}{2}E\,R$. The firm reinvests a part of it, $\frac{1}{2}E$, in new production and pays out the remainder, $\frac{1}{2}E(R-1)$, as dividends.

6.2 Bank with stock capital

The bank makes the following suggestion to agents. Instead of setting up Firm-E, the agents should invest the same amount, E, as stock capital in the bank, which commits to pay a fixed dividend $\frac{1}{2}E(R-1)$ at each time point. Since stock capital is naturally subordinated on demand deposits, the order of moves is the following at each time point

1. The bank pays back demand deposits.
2. The bank pays out dividends.
3. Stockholders can trade stocks.

At each time point, after paying out dividends, the market price of a stock unit is

$$P_S = \frac{\delta \frac{1}{2}(R-1)}{1-\delta} = \tfrac{1}{2}(1+\sqrt{R}), \quad \delta = 1/\sqrt{R}. \tag{19}$$

For simplicity, it is assumed that some fraction, e , of active newborns invest only in bank stocks, whereas other active agents, $\alpha - e$, invest only in firm stocks. Consider first the active newborns who invest in bank stocks. Given the endowment, each of them can initially purchase $1/P_S$ bank stocks. After the first period, the wealth of the agent is

$$1 + \frac{\frac{1}{2}(R-1)}{P_S} = \sqrt{R}, \tag{20}$$

where the first term shows the value of $1/P_S$ bank stocks and the second term represents dividend payments on the stocks. Thus, an early consumer can consume $\sqrt{R}$. If the agent becomes a late consumer, he invests the dividend income in additional bank stocks. After the additional acquisition, he has $\sqrt{R}/P_S$ bank stocks. After the second period the wealth of the agent amounts to

$$P_S \frac{\sqrt{R}}{P_S} + \frac{\sqrt{R}\,\frac{1}{2}(R-1)}{P_S}, \tag{21}$$

which is equal to R, which can be consumed by the late consumer.

The market of bank stocks clears. To see this, note that at each time point the supply of bank stocks consists of the stocks sold by the early consumers of generation-t and of the sales by the late consumers of generation- t-1

$$\varepsilon \frac{1}{P_S} e + (1-\varepsilon) \frac{\sqrt{R}}{P_S} e \,, \tag{22}$$

or

$$\frac{e}{P_S} + (1-\varepsilon) e \, (\sqrt{R}-1) \frac{1}{P_S} \,. \tag{23}$$

The stocks are demanded by newborns, e/P_S , and by late consumers, who invest their dividend income in additional stocks, $(1-\varepsilon)e \; \tfrac{1}{2}(R-1)/P_S \; * 1/P_S$. After some manipulation, the total demand can be expressed as in (23).

The rest of the active agents, $\alpha - e$, invest their endowment directly in firm stocks. The consumption allocations of the agents are identical to the above, $\sqrt{R}\,, R$. The market clearing constraints for the stocks of the firms are

$$(\alpha - E)\, \theta_n^0 + (\alpha - E)(1-\varepsilon)\theta_n^1 = (\alpha - E) I^* \,, \qquad (\alpha - E)\, \theta_i^0 + (\alpha - E)(1-\varepsilon)\theta_i^1 = (\alpha - E) I^* \,. \tag{24}$$

The first equality states that the investments in new stocks are equal to the supply of stocks. The second equality gives the same information for intermediate stocks. One solution is, again, the following. Late consumers do not invest in new stocks ($\theta_n^1 = 0$), which are channelled to newborns, who invest the rest of their endowment in intermediate stocks. If a newborn later becomes an early consumer, he keeps the income from maturing stocks and sells the intermediate stocks, $\theta_n^0 = I^*$ units, to the late consumers of his generation (θ_i^1) and to the newborns of the next generation (θ_i^0). The market for intermediate firm stocks clears, because their supply, $\varepsilon\, \theta_n^0$ is equal to demand, $\theta_i^0 + (1-\varepsilon)(\theta_i^1 - \theta_n^0)$, that is, $\theta_n^0 = \theta_i^0 + (1-\varepsilon)\theta_i^1$. This is always true (see (24) when $\theta_n^1 = 0$). Thus, the market of firm stocks clears.

The fraction of agents who need to invest in bank stocks, e, can be solved by noting that at each time point bank stocks are held by newborns, e/P_S , and by late consumers, $(1-\varepsilon)\, e\sqrt{R}/P_S$. The total holdings need to equal to the amount of bank stocks, E. It is easy to solve

$$e = \frac{\frac{1}{2}E}{I^*}. \tag{25}$$

6.3 The optimal amount of stock capital, E*

The resource constraint of the bank is

$$R\left[(1-\alpha)+\tfrac{1}{2}E\right] - B\left[(1-\alpha)(2-\varepsilon)+E\right] = \tfrac{1}{2}E\,R + (1-\alpha)\left[\varepsilon\sqrt{D_2} + (1-\varepsilon)D_2\right] . \tag{26}$$

The first term represents production output. At each time point, 50% of stock capital is tied to intermediate production and the remainder of the capital is invested in new production. The second term gives the operating costs, which depend on the bank size. The first term on the R.H.S expresses the costs of stock capital. The costs total $\frac{1}{2}RE$, consisting of dividends, $\frac{1}{2}(R-1)E$, and investments in new production, $\frac{1}{2}E$. The last term reveals the payments on demand deposits. The resource constraint can be rewritten as

$$R - B\left[(2-\varepsilon)+\frac{E}{1-\alpha}\right] = \varepsilon\sqrt{D_2} + (1-\varepsilon)D_2. \tag{27}$$

From this it is easy to see that $dD_2/dE < 0$; the bigger the amount of stock capital, the lower the payments on demand deposits. Hence, the amount of stock capital is minimized. The intuition is the same as in the context of time deposits. Each size unit incurs operating costs to the bank. Yet, active agents require the very same return on bank stocks to what they could achieve by investing directly in firms. Therefore, the operating costs are borne entirely by those depositors who save with demand deposits.

The minimum amount of stock capital is determined by the maturity matching constraint. The liquidation value of bank assets covers the liquidation value of bank deposits if

$$\begin{aligned} &R\left[(1-\alpha)+\tfrac{1}{2}E\right]-B\left[(1-\alpha)(2-\varepsilon)+E\right]-(1-\alpha)\left[\varepsilon\sqrt{D_2}+(1-\varepsilon)D_2\right]-(1-\alpha)(1-\varepsilon)\sqrt{D_2}- \\ &\quad +l\left[(1-\alpha)+\tfrac{1}{2}E\right] \geq 0. \end{aligned} \tag{28}$$

Dividend payments are excluded from (28) since the bank pays dividends only after the withdrawal of deposits. Using the resource constraint, the panic-preventing amount of stock capital can calculated

$$E^* = \frac{2(1-\alpha)\left[(1-\varepsilon)\sqrt{D_2}-l\right]}{R+l}. \tag{29}$$

This is equal to the panic-preventing amount of time deposits, (18’). Consider a bank with the amount E^* of stock capital, which is naturally subordinated to demand deposits. Thanks to maturity matching and subordination, passive agents, who have demand deposits, can rely on the bank being able to pay the promised interest on their deposits. Thus, the agents have no reason to panic. Furthermore, when maturity matching was created with time deposits, at each time point 50% of time deposits matured. This caused a small problem. There was some confusion whether or not a new generation was ready to save in new time deposits or not. This problem is eliminated when maturity matching is created with stock capital, which does not mature similarly to time deposits. Therefore, it may be easier to satisfy the maturity matching constraint with stocks than with time deposits. A conclusion follows.

Proposition 4. *Stock capital helps prevent bank panics efficiently. The needed amount of stock capital is equal to the panic-preventing amount of time deposits.*

Up to now, it has been implicitly assumed that enough active agents exist who are ready to save in bank stocks, $\alpha > e$. The opposite case is explored next.

7. Too few active agents

7.1 Model

Until now some agents have been passive and others have been active. The ratio of active agents has been sufficient to fulfil the supply of time deposits or bank stocks. This assumption is now abandoned. Initially there are no active agents at all, because the agents live in isolation as in Wallace (1988) and cannot contact each other and capital markets. The concept of isolation, however, is not taken as seriously as in Wallace (1988). It is somewhat realistic to assume that isolation is imperfect. Everyone can contact other agents and capital markets, but this incurs costs. To keep the analysis simple, it is assumed that the cost of market participation or "the cost of breaking through the isolation" erodes $(1-m)$ percent of the asset return in each period. As before, suppose that the initial value of a firm stock is one unit and that its value is $\sqrt{R}$ after a period. Yet, after deducing of the costs of market participation, the value of agent's wealth is $\sqrt{R}\,m$, $0<m<1$. If he invests this in production for the second period, the value is $\sqrt{R}\,m * \sqrt{R}\,m$ or $R\,m^2$. The erosion is assumed to be so severe that $\sqrt{R}\,m < 1$. Therefore, the erosion makes direct investment in firm stocks unprofitable and the agents live de facto in isolation. No agent will participate in the markets and thus there is no capital markets at all. Alternatively, a bank could supply a liquid investment opportunity. Yet, as has been seen above, the bank is subject to panics without time deposits or stocks. Unfortunately, no-one is willing to invest in time deposits or stocks without the existence of capital markets. Thus, no banks can be established. It seems that there is no way to invest in production.

Fortunately, the case is more optimistic. Suppose that a bank attracts time deposits, but does not issue stocks. The initial size of a time deposit is a unit and it pays D_{TD} units at maturity. The bank chooses D_{TD} so that

$$\varepsilon\, u\,(m\sqrt{D_{TD}}\,) + (1-\varepsilon)u(m^2\, D_{TD}) \;=\; \varepsilon\, u\,(\sqrt{D_2}\,) - (1-\varepsilon)u(D_2)\;,\; D_2 > 1. \tag{30}$$

An agent obtains the same expected utility by becoming an active agent and saving in marketable time deposits as by remaining passive and saving in demand deposits. Since the bank maximizes the total utility of its depositors, the interest on time deposits meets $D_{TD} \;=\; D_2 / m^2 \;>\; R$. Thus, the interest on time deposits exceeds the output of long-term production. However, some time deposits are needed to prevent panics.

Above, the market value of an intermediate time deposit, P_1^{TD}, was solved from the non-arbitrage constraint

$$\frac{m\,P_1^{TD}}{1} \;=\frac{m^2\, D_{TD}}{m\,P_1^{TD}} \;= \sqrt{D_2}\;. \tag{31}$$

It is easy to see $P_1^{TD} = \sqrt{D_{TD}}$. The updated resource constraint of the bank is

$$R \;\;-\;\; (2-\varepsilon)\,B \;\;=\;\; \tfrac{1}{2}\,A\,D_{TD} \;\;+\;\; (1-\alpha^*)\left\lfloor \varepsilon\sqrt{D_2} + (1-\varepsilon)D_2 \right\rfloor. \tag{32}$$

The whole production of the economy takes place through the bank. The volume of production is R at each time point, because the size of the generation (= the investment input) is 1. The operating costs amount to $(2-\varepsilon)B$, since the size of the bank is $2-\varepsilon$, newborns and the late consumers of the previous generation. The R.H.S includes payments on time deposits and demand deposits. The optimal fraction of active agents satisfies

$$\alpha^* = \frac{\frac{1}{2}A^*}{I^{**}} \quad , \quad where \quad I^{**} = 1 - \frac{\varepsilon\sqrt{D_2}}{1+\sqrt{D_2}}. \tag{33}$$

Since it is expensive to be active, the optimal fraction of active newborns, α^*, only barely fulfils the supply of time deposits. Here ½ stems from the fact that at each time point 50% of time deposits mature. Additionally, a part I^{**} of the endowment is saved in new time deposits, while the rest are allocated to intermediate time deposits. The optimal supply of time deposits, A^*, must be sufficient to ensure maturity matching when time deposits are subordinated to demand deposits

$$R \;-\; (2-\varepsilon)B \;-\; (1-\alpha^*)\left[\varepsilon\sqrt{D_2} + (1-\varepsilon)D_2\,\right] + \;l\; - \;(1-\alpha^*)(1-\varepsilon)\sqrt{D_2} \;\geq\; 0\;. \tag{34}$$

Given the budget constraint, the sum of the first 3 terms adds up to $\frac{1}{2}AD_2/m^2$. It is easy to solve

$$A^* \;=\; \frac{2\left[(1-\varepsilon)\sqrt{D_2} - l\right]}{\dfrac{D_2}{m^2} + \dfrac{\sqrt{D_2}(1-\varepsilon)}{I^{**}}}. \tag{35}$$

The budget constraints and consumption allocations of an active agent are

$$\begin{aligned} &i.)\; 1 = \beta_n^0 + m\sqrt{D_{TD}}\,\beta_i^0 \;, && ii.)\; C_1 = m\sqrt{D_{TD}}\,\beta_n^0 + m^2 D_{TD}\,\beta_i^0 = \sqrt{D_2}\;. \\ &iii.)\; \sqrt{D_2} = \beta_n^1 + m\sqrt{D_{TD}}\,\beta_i^1, && iv.)\; C_2 = m\sqrt{D_{TD}}\,\beta_n^1 + m^2 D_{TD}\,\beta_i^1 = D_2\,. \end{aligned} \tag{36}$$

Recall that β_n^0 (β_n^1) denotes the newborn's (late consumer's) savings in new time deposits and β_i^0 (β_i^1) symbolizes his savings in intermediate time deposits. In (36), i.) expresses the newborn's budget constraint and ii.) the consumption of an early consumer. If the agent becomes a late consumer, he can reallocate his wealth at the intermediate stage. This is shown by iii.), whereas iv.) reveals the realized consumption of a late consumer.

Additionally, the markets for time deposits need to clear

$$i.)\alpha^{*}\beta_n^0 + \alpha^{*}(1-\varepsilon)\beta_n^1 = \tfrac{1}{2}A^{*} \qquad ii.)\,\alpha^{*}\beta_i^0 + \alpha^{*}(1-\varepsilon)\beta_i^1 = \tfrac{1}{2}A^{*}. \tag{37}$$

Here $i.$ $(ii.)$ states that the total savings in new (intermediate) time deposits are equal to the supply of new (intermediate) time deposits. One market clearing solution is, again, the following: recall $\alpha^{*} = \tfrac{1}{2}A^{*}\big/I^{*}$ and suppose $\beta_n^0 = I^{**}$, $\beta_n^1 = 0$, $\beta_i^0 = (1-I^{**})\big/\sqrt{D_2}$, $\beta_i^1 = 1$. Late consumers do not save in new time deposits, which are channelled to newborns. They save the rest of their endowment in intermediate time deposits. If a newborn later becomes an early consumer, he keeps the returns from maturing time deposits and sells the intermediate time deposits to the late consumers of his generation and to the newborns of the next generation. The market for intermediate time deposits clears when their supply, $\varepsilon\beta_n^0$ is equal to demand, $\beta_i^0 + (1-\varepsilon)(\beta_i^1 - \beta_n^0)$. This means that $\beta_n^0 = \beta_i^0 + (1-\varepsilon)\,\beta_i^1$, or

$$I^{**} = 1 - \frac{\varepsilon\sqrt{D_2}}{1+\sqrt{D_2}}. \tag{38}$$

This is equal to (33) and thus is always true.

In sum, the agents are initially passive. Some active agents are, however, needed to save in time deposits so that a panic-free banking system can be constructed. Thus, the bank pays very high interest on time deposits in order to motivate a few agents to break through the isolation and become active. The bank determines interest on demand deposits and interest on time deposits so that each agent obtains the same expected consumption allocation ($\sqrt{D_2}$, D_2) and a few agents are motivated to become active. Each agent then obtains a productive saving asset. The secondary market for time deposits clears. Due to maturity matching, the bank system is panic-free.[13]

Consequently, under some parameter values (this is shown later by using a numeric example) there is no production without a bank and a bank can be established only if it pays very high interest on time deposits. The establishment of the bank generates capital markets as a by-product. This case is most likely to arise in the context of a small, unknown bank that operates in a periphery, probably in an emerging economy, where there are no natural secondary markets for time deposits. Time deposits can be made lucrative and their secondary markets can be created by paying very high interest on them. But even then, the secondary markets of time deposits are likely to be modest. It may take plenty of time and effort to find a willing trade partner. The required high payments on time deposits reduce interest on demand deposits. Therefore, although it may be possible to establish a panic-free bank system through maturity matching, the costs may be substantial. The bank is able to pay more interest on demand deposits if the bank regulator would offer deposit insurance and the bank could abandon stabilization through time deposits.

[13] Wallace (1996) explores maturity matching from the other point of view: narrow banking. Maturity mismatch is avoided although the bank attracts only demand deposits, since it invests funds in safe, short-term assets.

7.2 Numeric example II

Consider an economy: $R = 1.069$, $B = 0.025$, $\varepsilon = 0.3$, $l = 0.4$. First, suppose a case where the erosion effect is $m = 0.9$. It is easy to note that $D_2 \approx 1.02$, $A^* \approx 0.29$ and $\alpha^* \approx 0.17$. Therefore, it is possible to motivate some agents to "break through the insolvency" by paying high interest on time deposits. Interest on demand deposits is positive. However, it is possible to solve from $R - (2-\varepsilon)B = \varepsilon\sqrt{D_2} + (1-\varepsilon)D_2$ that without time deposits the bank could pay higher interest on demand deposits, $D_2 \approx 1.031$. Thus, the regulator can boost the agents' expected utility by offering deposit insurance. Then, panics can be avoided without the burden of time deposits.

Consider an identical case but where $m = 0.5$. Now banking is unprofitable. The bank cannot pay positive interest on demand deposits. Therefore, it is not possible to prevent panics by using time deposits. Yet, a panic-free bank system can be established without time deposits, if the regulator can offer deposit insurance.[14]

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[14] In some contexts the results of this paper differ from Diamond (1997). Diamond finds that banks may provide more liquidity than markets (cross subsidization), whereas in this paper capital markets provide more liquidity than banks. Additionally, according to Diamond (1997), increasing participation in capital markets weakly reduces the liquidity that banks can create. In this paper, increasing participation in capital markets weakly raises the liquidity that banks can create. When participation in capital markets is large – that is, the fraction of active agents is high – these agents can fulfil the supply of marketable time deposits. The bank needs to pay interest R on time deposits. In contrast, when there is no market participation and active agents, the bank needs to pay very high interest on time deposits in order to motivate some agents to become active. Interest on time deposits exceeds R and thereby strongly reduces payments on demand deposits. Therefore, the bank can offer more liquidity (higher interest on demand deposits) when participation in capital markets is large than in the opposite case.

8. Moral hazard is eliminated

It is often argued that deposits need to be liquid in order to eliminate moral hazard (e.g. Calomiris & Kahn, 1991). This section briefly indicates that moral hazard can be eliminated in the panic-free bank system, in which the bank also attracts time deposits.

Let us enrich the model by assuming a risky long-term project. It requires a unit of investment and yields $\hat{R}$, $\hat{R} > R$ after two periods if it succeeds. The risky project succeeds with probability s in every period. If it fails, the failure is irreversible and the value of the project is zero. Thus, the final output, $\hat{R}$, materializes with probability s^2. The NPV of the risky project is assumed to be negative, $s^2\hat{R} < 1$. If a risky project is successful, but liquidated after the first period, its liquidation value is $\hat{l} < l$.

Suppose that a banker establishes a bank in a perfectly competitive economy. If he operates as above and invests in safe long-term projects, which yield R after two periods, he earns zero returns due to competition. This tempts him to risk taking. If he promises the same interest on deposits as above, but invests the funds in risky projects and the risk taking is successful, he earns profits $\hat{R} - R$. If the risk taking fails, the costs are suffered by depositors.

Suppose that the bank changes its investment strategy at time point t and begins risk taking. The novel strategy is observed by depositors during the same period. How do they react at $t+1$? The depositors who have intermediate time deposits cannot interrupt their deposits. The depositors with demand deposits know that the maturity matching constraint is no longer satisfied since the liquidation value of long-term risky production, $\hat{l}$, is less than the liquidation value of long-term safe production, l. Given this together with Definitions 1 and 2, the agents with demand deposits panic immediately. Knowing this, the banker will never switch to risk taking. He knows that he cannot win. If he switches to risk taking, a disciplinary panic immediately occurs and the bank fails. Therefore, moral hazard is eliminated even through the bank also attracts time deposits.

The result follows, of course, from the assumption that the liquidation value of a risky project is lower than the liquidation value of a safe project. In our opinion, this assumption is rather realistic.

9. Conclusion

The novel Basel II Accord stipulates capital requirements for banks. Two key types of capital exist: tier 1 (e.g. common stock) and tier 2 (e.g. subordinated time deposits). The capital requirements are determined in order to price bank risk correctly. Their existence, however, poses the question of whether the same kind of capital requirements could also be used to prevent bank panics. Is stock capital in this use more effective than time deposits? These questions are examined in this paper.

The paper confirms that both stock capital and time deposits offer an effective option preventing panics. In reality, large-denomination time deposits (CDs), which can be resold in secondary markets, are an important source of funds for banks. CDs represent approximately 16% of the liabilities of commercial banks in the U.S and they are typically hold by corporations, money market mutual funds and other financial institutions (Mishkin, 2007, p.221). Consequently, since marketable time deposits represent a natural source of funds for banks, they might also offer a natural option for preventing panics.

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