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Modeling Commercial Piracy of Information Goods *

Abstract

I discuss the competition between a copyright owner and several commercial pirates who offer copies of an information good to the same consumers. I view the increased risk of a punishment that offering a pirate copy to a consumer causes as an advertising cost, whose value is chosen by the government. In its presence pirate copies have a positive price and there is price dispersion in the market for pirate copies when there are at least two commercial pirates. In addition, the government determines indirectly also the structure of the market for pirate copies by influencing the fixed costs of the pirates. I analyze the effects of these two policy variables on the markets for pirated and legitimate copies. An increase in the fixed costs of the commercial pirates has always a non-negative effect on the profit of the copyright owner. However, an increase in the advertising costs might increase the profits of the commercial pirates and decrease the profits of the monopolist.

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1. Introduction

The distribution of illegal *pirate copies* of information goods might have a variety of motives. Such copies are distributed on the one hand by the members of *peer-to-peer networks*, who deliver digital goods on the Internet without monetary compensation and who are motivated by e.g. a feeling of identification with the other network members, and on the other hand by commercial pirates who are, more conventionally from the perspective of the economist, motivated by the revenue that results from their activities. Somewhat less obviously, the consumers of an information good might also form *clubs* each of which buys a single copy of the information good, produces further copies of it, and distributes one of them to all club members (cf. Varian, 2000).

There is a relatively large economic literature on *end-user copying*,¹ i.e. of situations in which pirate copies are produced without monetary compensation for the own use of their producer. *Dyuti S. Banerjee* has recently put forward several closely related models of commercial software piracy in (Banerjee, 2003, 2006a, 2006b), but it nevertheless seems that until now economists have given much less attention to the *commercial piracy* than to end-user copying.²

Given that both commercial and non-commercial forms of piracy are illegal, there is no obvious way of estimating to which extent the pirate copies of information goods that are currently in use exemplify commercial piracy. Such estimates are nevertheless available in the case of physical piracy in the *music industry*. E.g., according to a report of International federation of the Phonographic Industry (IFPI), approximately 37% of all the [music] CDs that were purchased in 2005 globally were pirate copies.³ However, in the case of the *software industry*, estimates of the extent to which pirate copies get sold, rather than distributed for free, are more difficult to find.

The Business Software Alliance (BSA) publishes yearly a piracy study which contains estimates for the piracy rate (i.e. the ratio of the number of pirated software

¹ For a survey, see Peitz and Waelbroeck (2006a).

² See, however, Yao (2005), which discusses counterfeiting on a more general level, without restricting attention to counterfeited information goods.

³ International federation of the Phonographic Industry (IFPI), *The Recording Industry 2006 Piracy report*, p. 4, available at <http://www.ifpi.org/site-content/library/piracy-report2006.pdf>, accessed on October 29, 2006.

units to the total number of installed software units) for different countries of the world, and also for different regions of the world as a whole. For example, according to the BSA the worldwide piracy rate was 35% in 2005.⁴ However, such estimates do not make a distinction between commercial and non-commercial forms of piracy.⁵ Nevertheless, e.g. the other surveys of the BSA suggest that both the commercial and non-commercial forms of software piracy are of a considerable economic significance.⁶

The production costs of pirate copies are low – in the case of the pirate copies which are distributed in an electric form via the Internet, they are almost zero – and the fact that their prices are not driven to the level of their production costs via Bertrand competition seems at first glance puzzling to the economist. An obvious answer to this puzzle is that it is costly for pirates to inform potential consumers of their products, since this increases the risk of getting caught and receiving a punishment. For example, if an illegally operating Internet site which offers pirate copies of software products for sale informs its potential customers by sending e-mail messages to randomly chosen addresses, each e-mail message increases the risk of getting caught and receiving a punishment. In this case the expected cost from a punishment can be viewed as an *advertising cost* which is analogous with a *variable cost of production* in so far that it is an increasing function of the number of the pirate copies that are sold.

Information goods can be protected not only by copyright and other intellectual property rights but also by *digital rights management* (DRM) systems. Digital rights management tools can, broadly speaking, be divided into *cryptology* (i.e. the

⁴ *Third Annual BSA and IDC Global Software Piracy Study*, <http://www.bsa.org/globalstudy/upload/2005%20Piracy%20Study%20-%20Official%20Version.pdf>, accessed on October 29, 2006, p. 13.

⁵ This is because the estimates of the BSA have been calculated from an estimate of the total number of installed software units, which is based on the number of the sold hardware units and surveys concerning their average software load, and an estimate of the number of the sold software units, which is based on information concerning the market revenues of software vendors and software pricing. Cf. *ibid.*, p. 14.

⁶ In one of such surveys, the BSA has investigated the attitudes of the online consumers from six different countries towards *spam*, i.e. commercial emails that they have received without requesting or signing up for them (BSA, *Consumer Attitudes Toward Spam in Six Countries*, 2004, available at <http://www.bsa.org/usa/events/loader.cfm?url=/commonspot/security/getfile.cfm&pageid=20654>, accessed on October 29, 2006). In each country, more than 80% of the respondents stated that they had received spam which was concerned with computer software (*ibid.*, p. 6), and 27% reported that, in the product category “computer software”, they had “purchased an item or taken advantage of an offer” which was suggested to them in spam (*ibid.*, p. 13). Only 31% of the respondents stated that they agreed with the statement that they would “never buy commercial software using this method because it is most likely unlicensed and illegal” (*ibid.*, p. 16).

distribution of information goods in an enciphered format) and *watermarking* (i.e. embedding information into a digital product in such a way that each copy of the good becomes different).⁷ Watermarks can be used for tracking down the person who has originally bought the legitimate copy of an information good from which the pirate copies on the market have been produced, which makes it easier to penalize commercial pirates.⁸ Clearly, unlike the “advertising costs” of the pirates that were mentioned above, the costs caused by DRM systems constitute *fixed costs of production* for commercial pirates.

Below I shall put forward a model of commercial piracy, i.e. of the trade of pirate copies at a positive price. I shall refer to the agents who produce pirate copies for sale as *bootleggers*. The model allows one to analyze the effects of DRM systems and the policy instruments of the government on the profits of the bootleggers and the copyright owner. In the model the “advertising costs” which are caused by an increased risk of legal sanctions keep the price of pirate copies positive also when there are several bootleggers on the market. Accordingly, the model differs from previous work in the same field in so far that it provides tools for analyzing also *the market structure* of the market for pirate copies and its effects on the market for legitimate copies.⁹

A *welfare analysis* of the model is somewhat problematic, given that it is concerned with an illegal business model. Such an analysis would yield an answer to the question which choice of the policy instruments by the government is optimal in the sense of maximizing the value of a welfare function. However, illegal activities are not normally included in the measured GDP even if they are known,¹⁰ and it is questionable whether one should include the welfare which is obtained by illegal

⁷ For a survey, see Eskicioglu and Delp (2001).

⁸ Since a large-scale commercial pirate can be expected to be able to break down a cryptographic system, watermarking seems to be the more relevant DRM tool in the context of large-scale commercial piracy (cf. *ibid.*, pp. 683-684). See also Park and Scotchmer (2005), which contains an analysis of DRM and of the different effects of the use of *shared* and *independent* DRM systems on end-user piracy.

⁹ Cf. (Banerjee, 2003, 2006a, 2006b), which discuss models of the competition between the copyright owner and a single pirate. In them the pirate copies are of a lower quality than legitimately bought copies, and this stops the price of the good from sinking to zero through Bertrand competition. However, in these models the number of the pirates is by assumption 1, and they do not address the question why the price of the pirate copies does not sink to zero through a Bertrand competition *between the pirates* (rather than between them and the copyright owner).

¹⁰ Cf. Myles (1995, pp. 383-5).

means (like the profits from selling pirate copies, or the utility from using them) in the welfare function of a social planner.¹¹

Below I shall not present a systematic welfare analysis of the current model. Rather, I shall rest content with addressing the question how the choice of the fixed and variable costs of bootlegging affects the profits of the bootleggers and the copyright owner. It turns out that, whereas it is always in the interest in the copyright owner that the fixed costs of the bootleggers are increased, an increase in the variable costs of the bootleggers might also *decrease* the profits of the copyright owner. As the current model contains just a single information good without any network effects, this result is distinct from the familiar results that the producer of an information good might profit from piracy if this makes it easier to sell complementary goods or services to consumers,¹² or if this leads network effects which increase its popularity.¹³

2. The Main Features of the Model

The agents of the model that will be considered below are 1) a *monopolist* who sells copies of an information product legally, 2) k *bootleggers* who sell illegitimate pirate copies of the same good, and 3) a unit mass of consumers, which is indexed by $\theta \in [0,1]$. The bootleggers can inform the consumers of the availability of their products by sending them advertisements at random. As a paradigmatic example of a situation of this type, one might think of an illegally operating Internet site which sends advertisements of pirated software products to randomly chosen e-mail addresses. The sending of an advertisement is associated with a cost b which should be interpreted as the increase in expected cost of punishment that sending a single advertisement causes.

¹¹ Cf. Sandmo (1981), in which the analogous problem is considered in the context of *tax evasion*. It is problematic whether a welfare function of a model of tax evasion should include the utility of tax-evaders since one might argue that the Pareto principle should not be extended to cases in which the utility of an individual is increased by illegal means (ibid., p. 275).

¹² For example, the demand for concerts by an artist might be increased by the pirate copies of her recordings (Gayer and Shy, 2006). Somewhat less obviously, when the consumers do not know in advance which information products they prefer (e.g. which musical recordings they would enjoy listening), the possibility to sample pirate copies might make them willing to pay more for their preferred product. Cf. Peitz and Waelbroeck, 2006b.

¹³ See e.g. Conner and Rumelt (1991, Proposition 4 on p. 133).

More precisely, I shall assume that sending an advertisement causes an increase α in the risk of getting caught, that the bootlegger receives a punishment G if she gets caught, and that the bootleggers are risk neutral. In this case the advertising cost b is given by¹⁴

$$(1) \quad b = \alpha G$$

It is easy to see that the problem of choosing α and G so that the costs of the government are minimized, given the constraint (1), does not have a well-defined solution: if an increase in monitoring causes costs for the monitoring authorities, but an increase in punishments (like fines) does not, any increase of G and decrease in α which keeps (1) valid seems always beneficial from the perspective of the government.¹⁵ Hence, it seems that a meaningful discussion of the optimal choice of α and G would require a more general model, in which the welfare function of the social planner depends also on other considerations besides the utility that consumers receive from using information goods, the revenue of the monopolist, the revenue of the bootleggers, and the monitoring costs of the government. Below I shall not discuss the problem of choosing the optimal α and G for a given b , but I shall just analyze the effects of the choice of b on the markets for legitimate and pirate copies.

I shall assume that the bootleggers cannot keep track of the consumers to whom they have already sent an advertisement. Rather, each of the bootleggers sends each of the advertisements with the same probability to each consumer. This implies that a bootlegger might send to the same consumer several advertisements in which the product is offered for sale at different prices. This assumption is particularly plausible in the context of trade on the Internet, since the potential customers of bootleggers might have several e-mail addresses. In addition, if the consumers are divided into groups whose members inform each other of the advertisements that they have

¹⁴ For a discussion of the econometric problem of actually constructing an index which measures the strength of legal software protection in a given country, see Andrés (2006, pp. 34-37).

¹⁵ In note 11 above it was pointed out that the problems in choosing an appropriate welfare function for an illegal business model have an obvious analogy in the context of the models of tax evasion. The same applies to the problem of choosing α and G optimally. If the tax collector observes tax evasion with a probability p and taxes unreported income with a penalty tax rate π , a lowering of p will lower the monitoring costs of the tax collector, so that any combination of a decrease of p and an increase of π which keeps the tax yield constant seems to be in the interest of the tax collector. Accordingly, the most cost-efficient way to pursue tax evaders seems to be to “hang tax evaders with probability zero” (Kolm, 1973, p. 266). A standard answer to this implausible conclusion is that the aim of a tax collector is not just to maximize tax revenue. One possibility is to view the tax collector as aiming at maximizing the utility that the representative consumer receives from public goods and from their net income, but this leaves out the relevant ethical considerations (ibid., pp. 267-270).

received, a single advertisement might reach individuals with different e-mail addresses, and in this case a bootlegger cannot eliminate the possibility that the same group of consumers receives many advertisements from her. In this case one should interpret b as the average cost of reaching a single consumer with a single advertisement.

This means that the description of the competition between the bootleggers resembles the classical *model of advertising by Gerard R. Butters* which was put forward in Butters (1977). However, the current model differs from Butters's model in several essential respects. As it was already seen, the current is a model of an oligopoly of k bootleggers, where $k \geq 1$,¹⁶ and since the advertising costs have been meant to represent the expected costs from a punishment for copyright violation, it will be for simplicity assumed that the monopolist may advertise for free, and that all consumers have the option of buying the product from her. In addition, since I wish to model a situation in which only a part of the consumers prefer buying a pirate copy to buying a legitimate copy, unlike Butters I shall assume that the reservation prices of the consumers differ.

I shall assume that reservation price of the consumer θ (where $0 \leq \theta \leq 1$) for a legitimate copy of the good is θ and that for a pirate copy her reservation price is $q\theta$, where $q \in (0,1)$ is a constant. In Banerjee (2003), from which I have borrowed this notation, it was assumed that q corresponds to the probability with which a pirate copy is operational.¹⁷ However, here the parameter q has been meant to represent not only the fact that a pirated product might be technically of a worse quality than a legal one or not operational at all, but also features like that the consumers might prefer legally bought copies also for ethical reasons, because there might be legal sanctions against using (and not just against selling) pirate copies, or because buying a pirate copy requires giving credit card information to criminals.

Different values of q seem plausible in the different applications of the model. One might expect that e.g. music files downloaded from a peer-to-peer network are experienced by their users to be of an almost identical quality with legally bought ones, and in this case it seems plausible to assume that $q \approx 1$. However, the other

¹⁶ More precisely, Butters (1977) is for the most part concerned with a model with a large number of sellers with no market power. See, however, Appendix A (ibid., pp. 483- 488) for a short discussion of an oligopoly of N sellers.

¹⁷ Banerjee (2003, p. 100).

considerations besides the technical quality seem more relevant in the case of e.g. illegally bought software products, and this motivates the assumption that they correspond to essentially lower values of q . Finally, if one applied the current model to pirate copies of other branded articles instead of information goods, they could be associated with even lower values of q .

Given that the pricing of the monopolist is by assumption known to all consumers, whereas the bootleggers try to avoid being detected by the authorities, it is natural to assume that the bootleggers can react to the price chosen by the monopolist, but not *vice versa*. Accordingly, I shall below consider a *leader-follower game* in which the monopolist first sets the price p_M of legitimate copies, and the bootleggers choose the prices in their advertisements only after p_M has become known.¹⁸ It is clear that for the monopolist the considered game is preferable to a *Bertrand game* in which the price p_M and the prices of the pirate copies are chosen simultaneously.¹⁹

More precisely, in the model the agents play the following three-stage game:

- 1) The monopolist chooses the price p_M of a legally bought copy of the good.
- 2) The bootleggers decide the number of the advertisements that they send and their price distribution and send them to randomly chosen consumers. The bootleggers are not constrained to offering the product at the same price in different advertisements.
- 3) The consumers choose whether to buy the product. If a consumer θ has not received any advertisements, she will buy the product from the monopolist if $\theta - p_M \geq 0$, and she will buy nothing otherwise. If a consumer θ has received at least one advertisement, and if the lowest price suggested in the advertisements that she has received is p , she will buy the product from the monopolist if $\theta - p_M \geq \max\{0, q\theta - p\}$. If this is not the case, she will buy the product from a bootlegger at price p if $q\theta - p \geq 0$. If neither of these conditions is valid, she will not buy anything. If the consumer buys the product from a bootlegger at price p and if

¹⁸ Cf. *ibid.*, which discusses both a leader-follower game and a Bertrand game in the context of a model with a single pirate.

¹⁹ This is obvious, because in the equilibriums of both games the bootleggers choose the price distribution which is optimal relative to the actual p_M , and in the leader-follower game the monopolist chooses the value of p_M for which this procedure leads to the largest revenue for her. Cf. also Proposition 3 in *ibid.*, p. 106.

there are several bootleggers who have offered the product at price p to her, she will choose one of them at random.

In this game the aim of the monopolist is to maximize her profit, which is simply equal to her revenue

$$(2) \quad R_M(p_M) = p_M D_M(p_M)$$

where $D_M(p_M)$ is the demand function of the monopolist. Each bootlegger $i \in \{1, 2, \dots, k\}$ aims at maximizing the profit $P_i = R_i - bA_i$, where R_i is the revenue of the i th bootlegger and A_i is the number of the advertisements that she sends.

As it was explained in the introduction, we wish to allow for the possibility that, in addition to the advertising cost b , the bootleggers might have also “fixed costs of production” because of DRM systems or because the risk of receiving a punishment is partially independent of the number of the sent advertisements. This idea can easily be incorporated into the current model by assuming that the number of the bootleggers k is endogenously determined by a fixed cost F . More precisely, it can be assumed that there will be new bootleggers who enter the market until market entry becomes unprofitable, i.e. that

$$(3) \quad k = \max \{k' \mid R_i - bA_i \geq F \text{ when the number of the bootleggers is } k'\},$$

I shall below view b and k , rather than b and F as the *policy variables* whose values are chosen by the government, since the government can affect the number of the bootleggers k by changing the two kinds of costs b and F . I shall briefly return to a discussion of the connections between k and the two kinds of costs at the end of Section 4.

Part 3) of the above definition implies that if the cheapest price at which a consumer θ can buy the considered good from a bootlegger is p , she will buy it from the monopolist if and only if

$$(4) \quad \theta \geq \frac{P_M - P}{1 - q} \text{ and } \theta \geq p_M$$

In this case the consumer will buy the product from the bootlegger if and only if

$$(5) \quad \theta < \frac{P_M - P}{1 - q} \text{ and } q\theta \geq p.$$

Clearly, (4) implies that if it were the case that

$$\frac{p_M - p}{1 - q} \leq \frac{p}{q}$$

there would be no consumers who would buy the product at price p . Hence, in equilibrium all prices suggested in the advertisements must satisfy the condition

$$\frac{p_M - p}{1 - q} > \frac{p}{q}$$

which is equivalent with

$$(6) \quad qp_M > p.$$

If the condition (6) is valid for the cheapest price p which is suggested in the advertisements that the consumer θ has received, then it will be the case that:

If $\theta < p/q$, the consumer does not buy anything.

$$(7) \quad \text{If } p/q \leq \theta < (p_M - p)/(1 - q), \text{ the consumer buys a pirate copy for price } p.$$

If $\theta \geq (p_M - p)/(1 - q)$, the consumer buys a legitimate copy.

I shall denote the demand function of the pirate copies by $x(p)$. More precisely, I shall let $x(p)$ denote the proportion of the consumers who would buy a pirate copy at the price p if p was the cheapest price which is suggested to them in the advertisements that they have received. This definition implies that $x(p)$ does not depend on the prices that are suggested in the other advertisements although it does, of course, depend on the price of the legitimate copies p_M . Clearly, (7) implies that

$$(8) \quad x(p) = \min \left\{ (p_M - p)/(1 - q), 1 \right\} - p/q$$

Here the case in which $(p_M - p)/(1 - q) > 1$ represents the situation in which there are no consumers who would buy a legitimate copy if they are offered a pirate copy at price p . Since the condition $(p_M - p)/(1 - q) > 1$ is equivalent with $p < p_M + q - 1$,

$$(9) \quad x(p) = \begin{cases} x_A(p), & p < q + p_M - 1 \\ x_B(p), & p \geq q + p_M - 1 \end{cases} = \begin{cases} (q - p)/q, & p < q + p_M - 1 \\ (qp_M - p)/[q(1 - q)], & p \geq q + p_M - 1 \end{cases}$$

However, in Section 4 below it will be seen that in equilibrium the choice of p_M by the monopolist and the choices of the values of p by the bootleggers will be such that

$x(p) = x_B(p)$ for all the advertisements that the bootleggers send.

In the next section I deduce the equilibrium distribution of the prices of pirate copies for a given value of p_M . This will be utilized in the subsequent discussion of the optimizing problem of the monopolist who chooses p_M . However, as it was explained in the Introduction, reliable estimates of the size of the market for pirate copies and of the risk of getting caught which bootleggers face are not available, and accordingly it is also of some interest to study also the situation in which p_M has not been determined with an optimization procedure, but is exogenously given. This situation can be thought of as a model of a case in which the copyright owner has no information concerning the bootleggers, or bases her decisions on incorrect estimates of the parameters that characterize the market for pirate copies.

3. The Market for Pirate Copies

Analogously with notation used in Butters (1977),²⁰ I define $A_i(p)$ to be the measure of the advertisements sent by advertiser i at a price smaller than or equal with p , and the function $A(p)$ by

$$(10) \quad A(p) = \sum_{i=1}^k A_i(p)$$

Further, I define $a_i(p)$ and $a(p)$ to be the derivatives of $A_i(p)$ and $A(p)$, whenever these derivatives exist.

If there were altogether A advertisements per consumer and the set of the consumers was a finite set of size N , the probability with which a given consumer would receive an advertisements would equal

$$1 - \left(1 - \frac{1}{N}\right)^{AN}$$

In the limit in which $N \rightarrow \infty$, this probability approaches the limit

$$Q = 1 - \lim_{N \rightarrow \infty} \left(1 - \frac{1}{N}\right)^{AN} = 1 - e^{-A}$$

Since in the currently considered case the set of all consumers has by assumption the measure 1, this expresses also measure of the consumers who receive an

²⁰ Cf. Butters (1977, p. 469).

advertisement when there are A advertisements per consumer. In particular, the probability with which a given consumer receives at least one advertisement from advertiser i with a price not larger than p is

$$Q_i(p) = 1 - e^{-A_i(p)}$$

Suppose next that $k=1$, i.e. that there is just one bootlegger, and that the single bootlegger sends δ_j advertisements per consumer at the prices p_j ($j=1,2,\dots,J$), where $p_1 \leq p_2 \leq \dots \leq p_J$. In this case the probability with which a given consumer has not received an offer which is cheaper than p_j is $\exp\left(-\sum_{j'=1}^{j-1} \delta_{j'}\right)$, and the probability with which she receives an offer with the price p_j is $1 - \exp(-\delta_j)$, so that the profit of the bootlegger is given by

$$P = \sum_{j=1}^J \left[p_j x(p_j) e^{-\sum_{j'=1}^{j-1} \delta_{j'}} (1 - e^{-\delta_j}) \right] - b \sum_{j=1}^J A_{1,j}$$

Now it can be concluded, by considering each term of the sum separately, that for any given values $\delta_1, \delta_2, \dots, \delta_J$, it is optimal for the single bootlegger to choose each of the values p_j to be price which maximizes $px(p)$. Since the profits that result when the distribution function A_1 is continuous can be approximated with the profits that result when A_1 is a step function with a finite number of prices, the conclusion is valid also when A_1 is continuous. Hence, if $k=1$, it is optimal for the bootlegger to specify in all of her advertisements the price which maximizes $px(p)$. For reasons which will become soon obvious, I shall denote this price by p_{\max} in the following proposition.

PROPOSITION 1. Suppose that there is just one bootlegger on the market. The single bootlegger specifies the same price p_{\max} in all her advertisements, and the number of the advertisements that she sends is given by

$$(11) \quad A = \ln(r_{\max}/b).$$

where $r_{\max} = p_{\max} x(p_{\max})$.

The claim concerning the value of A follows from the fact that the profit of the single bootlegger equals $P_1 = p_{\max} x(p_{\max})(1 - e^{-A}) - bA$. Intuitively, r_{\max} is the expected revenue that the bootlegger would receive from a single advertisement with the price p_{\max} if there were no other advertisements on the market.

Turning to the task of solving p_{\max} explicitly, it is first observed that (9) implies that $px_A(p)$ obtain its largest value when $p = p_A = q/2$, and that $px_B(p)$ obtain its largest value when $p = p_B = qp_M/2$. Clearly, $p_A > p_B$. It is easy to see with elementary methods that if $p_A > p_B \geq p_M + q - 1$, the optimal price is p_B and that if $p_M + q - 1 \geq p_A > p_B$, the optimal price is p_A . It is also clear if neither of these conditions is valid, it must be the case that $p_A > p_M + q - 1 > p_B$, and in this case the optimal price must be the corner solution $p = p_M + q - 1$. In other words,

$$p_{\max} = \begin{cases} (qp_M)/2, & (qp_M)/2 \geq p_M + q - 1 \\ p_M + q - 1, & q/2 > p_M + q - 1 > (qp_M)/2 \\ q/2, & p_M + q - 1 \geq q/2 \end{cases}$$

This can be equivalently expressed as

$$(12) \quad p_{\max} = \begin{cases} (qp_M)/2, & p_M \leq 2((1-q)/(2-q)) \\ p_M + q - 1, & 2((1-q)/(2-q)) < p_M < 1 - q/2 \\ q/2, & p_M \geq 1 - q/2 \end{cases}$$

Hence,

$$(13) \quad x(p_{\max}) = \begin{cases} p_M/(2(1-q)), & p_M \leq 2((1-q)/(2-q)) \\ (1-p_M)/q, & 2((1-q)/(2-q)) < p_M < 1 - q/2 \\ 1/2, & p_M \geq 1 - q/2 \end{cases}$$

Together with Proposition 1, (12) and (13) imply that

$$(14) \quad A = \begin{cases} \ln[qp_M^2/(4b(1-q))], & p_M \leq 2((1-q)/(2-q)) \\ \ln[((p_M + q - 1)(1 - p_M))/(qb)], & 2((1-q)/(2-q)) < p_M < 1 - q/2 \\ \ln(q/(4b)), & p_M \geq 1 - q/2 \end{cases}$$

The following proposition describes a symmetric equilibrium with k bootleggers, where $k > 1$.

PROPOSITION 2. Suppose that p_M has some fixed, given value, that the number of the bootleggers k is larger than 1, and that there is a symmetric Nash equilibrium in the market for pirate copies. The largest price suggested in the advertisements has still the value p_{\max} , and the smallest price p_{\min} suggested in them is determined by the condition

$$(15) \quad r_{\min} = b^{1-1/k} r_{\max}^{1/k}$$

where $r_{\min} = p_{\min} x(p_{\min})$ and $r_{\max} = p_{\max} x(p_{\max})$. The amount of advertisements with a price not larger than $p \in (p_{\min}, p_{\max})$ sent by a bootlegger i is given by

$$(16) \quad A_i(p) = \frac{1}{k-1} \ln \frac{px(p)}{p_{\min} x(p_{\min})} = \frac{1}{k-1} \ln \frac{px(p)}{r_{\min}}$$

The total number of advertisements sent by each bootlegger is given by

$$(17) \quad A_i = \frac{A}{k} \frac{1}{k} \ln \left(\frac{r_{\max}}{b} \right) \ln \left(\frac{r_{\min}}{b} \right)$$

and the revenue of each bootlegger equals

$$(18) \quad R_i = r_{\min} (1 - e^{-A_i}) = r_{\min} - b$$

The profit of each bootlegger is

$$(19) \quad P_i = R_i - b A_i = r_{\min} - b - b \ln \frac{r_{\min}}{b}$$

Obviously, the quantity r_{\min} which determines p_{\min} in accordance with (15) is the expected revenue that a single advertisement with the price p_{\min} would yield if there were no other advertisements.

The expression of the revenue of the bootleggers, (18), has an obvious intuitive interpretation: it is the revenue that bootlegger i would earn if she changed all the prices in her advertisements to p_{\min} , the minimum price that occurs in them, provided that the other bootleggers did not change their price distributions.

Propositions 1 and 2 make it easy to study the effects of the policy variables – i.e., the advertising costs b which are caused by the monitoring by the government, and the fixed costs which implicitly determine k via (3), and which might also be caused by DRM systems rather than monitoring – on the market for pirate copies when the monopolist does *not* optimize p_M .

PROPOSITION 3. Suppose that p_M is fixed.

(a) Both the total number of the advertisements that are sent by the bootleggers and the largest price p_{\max} suggested in their advertisements are independent of the number of the bootleggers on the market. However, an increase in their number shifts the price distribution of the advertisements downwards and decreases the profit of each bootlegger.

(b) An increase in the advertising costs b reduces the number of the advertisements. If there is just one bootlegger, her profit is a decreasing function of b , but the price of the pirate copies is independent of b . When there are at least two bootleggers, an increase of b shifts the price distribution of the pirate copies upwards. In this case the profit P_i of each bootlegger receives its maximum value for single value b_E of b within the interval $b \in (0, r_{\max})$. The profit P_i is an increasing function of b when $b < b_E$ and a decreasing function of b when $b > b_E$, and

$$\lim_{b \rightarrow 0} P_i = \lim_{b \rightarrow r_{\max}} P_i = 0.$$

The part (b) of this result is concerned with the question how the profits of each bootlegger depend on the parameter b . This question is important for obvious reasons: when the number of the bootleggers on the market depends on their profits in accordance with (3), it is in the interest of the monopolist that their profits are low. It is clear that sending advertisements cannot be profitable if the cost parameter b is larger than the value r_{\max} , i.e. the revenue from a single advertisement which is sent at the optimal price when there are no other advertisements on the market. On the other hand, Proposition 3 (b) states that when $b \rightarrow 0+$ and $k \geq 2$, the revenue and the profit earned from the advertisements approaches zero because of the increased competition in the pirate copy market, although the number of the advertisements approaches infinity according to Proposition 2. A situation of this kind might arise when the bootleggers do not have fixed costs, and it can be viewed as the analogy of a peer-to-peer network in the context of the current model.

Figure 1 shows the profit of a bootlegger as a function of b , and it also illustrates an important implication of this proposition: it suggests that it is not necessarily in the interest of the monopolist that b is increased, because an increase in b might raise the

profits from bootlegging and give potential bootleggers an incentive to enter the market by paying the fixed cost F . However, a rigorous demonstration of the fact that a case of this kind is possible must be postponed after the discussion of the optimization problem of the monopolist.

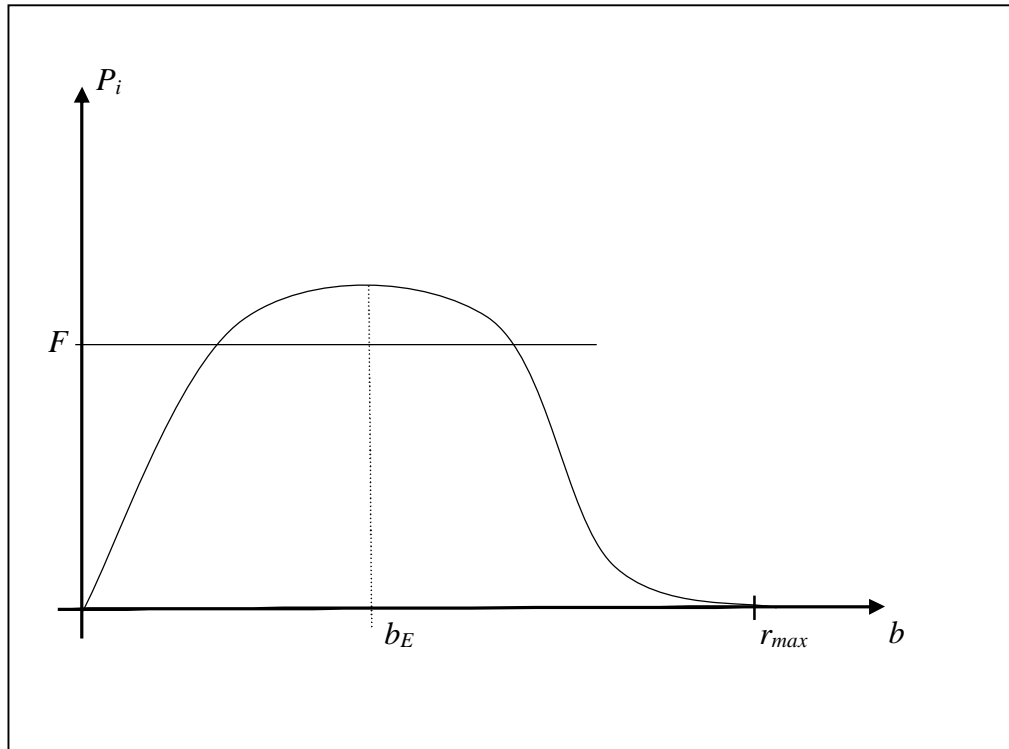


Figure 1.

4. The market for legitimate copies

In the current model the profit of the monopolist is equal with her revenue $R_M(p_M) = p_M D_M(p_M)$. The characterization of the demand $D_M(p_M)$ of the monopolist is easy when $p_M > 2((1-q)/(2-q))$. In this case p_{\max} is according to (12) such that none of the consumers who have the option of buying a pirate copy will choose a legitimate copy, and the demand of the monopolist is simply

$$D_M(p_M) = e^{-A}(1 - p_M)$$

Here the total number of the advertisements A is according to Propositions 1 and 2 given by

$$A = \ln(r_{\max}/b) = \ln((p_{\max} x(p_{\max}))/b)$$

so that one can conclude from (12) and (13) that

$$(20) \quad D_M(p_M) = \begin{cases} (qb)/(p_M + q - 1), & 2((1-q)/(2-q)) < p_M < 1 - q/2 \\ (4b(1 - p_M))/q, & p_M \geq 1 - q/2 \end{cases}$$

It is essentially more difficult to calculate $D_M(p_M)$ in the more interesting case in which $p_M \leq 2((1-q)/(2-q))$, however. In the following lemma $D_M(p_M)$ has been calculated by dividing the set of the potential customers – i.e., the consumers for whom $\theta \geq p_M$ – into three subsets, one of which might be empty. By the *low-valuation* consumers, I mean the potential customers who would not have bought a legitimate copy, if they had been offered a pirate copy for the price p_{\max} . Clearly, this is true of a consumer θ if and only if $\theta < (p_M - p_{\max})/(1-q)$. The *high-valuation consumers* (if any) are the potential customers who would have bought a legitimate copy even if they had been offered a pirate copy for the price p_{\min} . Clearly, this is true of a consumer θ if and only if $\theta \geq (p_M - p_{\min})/(1-q)$. Finally, the rest of the potential customers (if any) will be called *medium valuation consumers*.

LEMMA 1. Suppose that $p_M \leq 2((1-q)/(2-q))$, and put

$$p'_{\min} = \max\{p_{\min}, p_M + q - 1\}.$$

Now the demand of the monopolist is given by

$$(21) \quad D_M(p_M) = D_H(p_M) + D_{Med}(p_M) + D_L(p_M)$$

where

$$(22) \quad D_H(p_M) = 1 - (p_M - p'_{\min})/(1-q) \quad \max\{0, 1 - (p_M - p_{\min})/(1-q)\}$$

is the demand from the high-valuation consumers,

$$(23) \quad D_{Med}(p_M) = \frac{1}{1-q} \int_{p'_{\min}}^{p_{\max}} e^{-kA_1(p)} dp$$

is the demand from the medium-valuation consumers, and

$$(24) \quad D_L(p_M) = \frac{2b}{p_M}$$

is the demand from the low-valuation consumers.

Intuitively, p'_{\min} is the lowest price p which occurs in the advertisements and which is such that some consumers prefer buying a legitimate copy to buying a pirate copy for the price p . This means that when $p'_{\min} > p_{\min}$, some advertisements are such that none of the consumers who receive them will buy a legitimate copy. It should be observed that this lemma is formally valid also when $k=1$, but in this case $p'_{\min} = p_{\min} = p_{\max}$ so that $D_{Med}(p_M) = 0$.

Unfortunately, it seems that one cannot express the value of the integral which occurs in (23) in terms of elementary functions. The results in the rest of this section are based on a change of variables in this integral. Clearly, using (16) $D_{Med}(p_M)$ can be expressed in the form

$$(25) \quad D_{Med}(p_M) = \frac{1}{1-q} \int_{p'_{\min}}^{p_{\max}} e^{-kA_1(p)} dp = \frac{1}{1-q} \int_{p'_{\min}}^{p_{\max}} \left(\frac{p_{\min} x(p_{\min})}{px(p)} \right)^{k/(k-1)} dp$$

and (15), (12), and (13) together imply that

$$(26) \quad (p_{\min} x(p_{\min}))^{k/(k-1)} = b(p_{\max} x(p_{\max}))^{1/(k-1)} = b \left(\frac{qp_M^2}{4(1-q)} \right)^{1/(k-1)}$$

If one now puts

$$(27) \quad z = p/(qp_M)$$

it follows from (9) that

$$(28) \quad (px(p))^{k/(k-1)} = \left(\frac{p(qp_M - p)}{q(1-q)} \right)^{k/(k-1)} = \left(\frac{qp_M^2}{1-q} \right)^{k/(k-1)} (z(1-z))^{k/(k-1)}$$

Together (26) and (28) imply that

$$(29) \quad \left(\frac{p_{\min} x(p_{\min})}{px(p)} \right)^{k/(k-1)} = \frac{b(1-q)}{4^{1/(k-1)} qp_M^2} \frac{1}{(z(1-z))^{k/(k-1)}} = \frac{1-q}{qp_M^2} f(z)$$

where

$$(30) \quad f(z) = \frac{b}{4^{1/(k-1)} (z(1-z))^{k/(k-1)}}$$

Since $dz = dp/(qp_M)$, (25) now implies that

$$(31) \quad D_{Med}(p_M) = \frac{1}{p_M} \int_{p'_{\min}/qp_M}^{1/2} f(z) dz$$

In addition, (29) immediately implies that

$$(32) \quad f\left(\frac{p_{\min}}{qp_M}\right) = \frac{qp_M^2}{1-q}$$

These results make it easy to analyze the comparative statics of the profit of the monopolist relative to the policy variables b and k , when the price set by the monopolist is fixed. This analysis produces unsurprising results.

PROPOSITION 4. Suppose p_M is fixed. For a fixed number of bootleggers, the profit of the monopolist is an increasing function of b . When $p_M \geq 2((1-q)/(2-q))$, the profit of the monopolist is independent of the market structure in the market for pirate copies, but when $p_M < 2((1-q)/(2-q))$, the profit of the monopolist is a decreasing function of the number of the bootleggers.

Although it seems that the profit of the monopolist $R_M(p_M)$ cannot be expressed in terms of elementary functions, the result (31) nevertheless allows one to give an relatively elegant characterization for its derivative. This is presented in the next proposition.

PROPOSITION 5. The optimal price p_M of legitimate copies is never such that some pirate copies would be so cheap that there would be no consumers who prefer buying a legitimate copy to buying one of them. If p_M is such that this is not the case, and if there are pirate copies on the market,

$$(33) \quad \frac{dR_M(p_M)}{dp_M} = 1 - \frac{2p_M}{1-q} + \frac{2p_{\min}}{1-q}.$$

Since according to this proposition $p_{\min} \geq p_M + q - 1$ in equilibrium, (9) implies that in equilibrium all pirate copies have a price for which $x(p) = x_B(p)$.

PROPOSITION 6. The monopolist will choose a price for which pirate copies do not become available $b \geq q(1-q)/\left(4(2-q)^2\right)$. If this condition is valid, the price chosen by the monopolist is given by

$$(34) \quad p_M = \begin{cases} \sqrt{4(1-q)b/q}, & q(1-q)/(4(2-q)^2) \leq b \leq q(1-q)/(2-q)^2 \\ \min\{1/2, 1-q/2 - \sqrt{q^2/4 - bq}\}, & b > q(1-q)/(2-q)^2 \end{cases}$$

If $b < q(1-q)/(4(2-q)^2)$, pirate copies become available. In this case the equilibrium value of p_M is uniquely determined by the condition

$$(35) \quad 1 - \frac{2p_M}{1-q} + \frac{2p_{\min}}{1-q} = 0$$

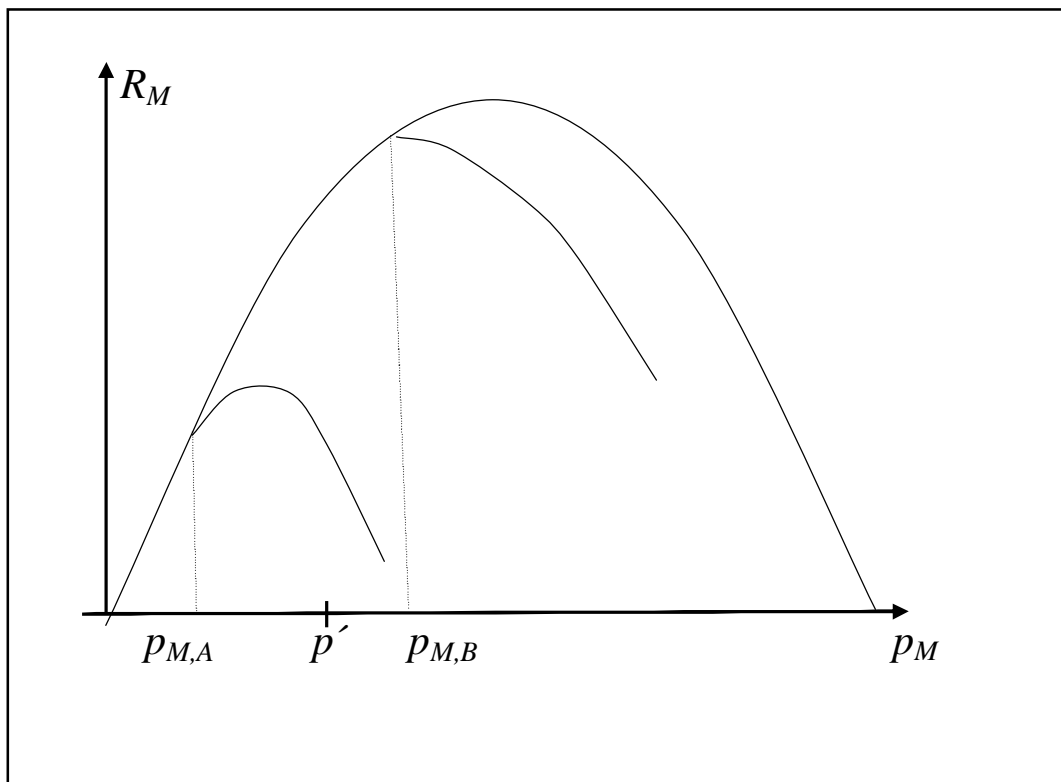


Figure 2.

Proposition 6 makes it possible to give a simple qualitative characterization of $R_M(p_M)$. The formula $R_M(p_M) = p_M(1-p_M)$ expresses the profit of a monopolist who does not compete with bootleggers. This formula corresponds to the parabola which has been drawn in Figure 2, and it is valid as long as the price p_M remains below the threshold $p_{M,0}$ at which the pirate copy market emerges. Obviously, this threshold is determined by the condition

$$b = p_{M,0}x(p_{M,0})$$

The figure shows also the profit of the monopolist as a function of p_M for two values of b , which correspond to the threshold values $p_{M,A}$ and $p_{M,B}$. Proposition 6 easily implies that the monopolist raises the price p_M above $p_{M,0}$ if $p_{M,0} < p'$, where

$$p' = (1-q)/(2-q)$$

The curve which begins at $p_M = p_{M,A}$ illustrates this case in the figure. On the other hand, the derivative $dR_M(p_M)/dp_M$ is negative when $p_M = p_{M,0}$ if $p_{M,0} > p'$. The curve which begins at $p_M = p_{M,B}$ corresponds to a case of this kind, and in this case it is optimal for the monopolist to choose $p_M = p_{M,0}$ so that the market for pirate copies does not emerge.

It is, of course, a trivial consequence of Proposition 4 that also when the monopolist chooses p_M optimally, the profit of the monopolist increases if b increases, and if k decreases and $p_M < 2((1-q)/(2-q))$. These results are based on the assumption that the two the policy variables, the advertising costs b and the number of the bootleggers k , are independent of each other. However, as we have already suggested, this is not necessarily the case: if the bootleggers have other costs beside the advertising costs, an increase in b might increase the number of the active bootleggers, indirectly decreasing the profits of the monopolist. I shall conclude this section by demonstrating that a case of this kind is, indeed, possible. For this reason, I consider the following game between the monopolist and a large number of potential bootleggers.

- 1) The monopolist sets the price p_M of legitimate copies.
- 2) Each potential bootlegger decides whether to enter the market and to pay a fixed cost F .
- 3) The bootleggers who have entered send their advertisements, just like in step 2) of the game that was defined in Section 2.
- 4) The consumers make their buying decisions, just like in step 3) of the game that was defined in Section 2.

Clearly, if the number of the potential bootleggers is sufficiently large, in equilibrium the number of the bootleggers who enter the market must satisfy (3).

PROPOSITION 7. Consider the game with a fixed cost F which was defined above.

- 1) An increase in F has always a non-negative effect on the equilibrium profit of the monopolist.
- 2) An increase of b will in some cases increase and in some cases decrease the profit of the monopolist.

5. Concluding Remarks

Above I discussed commercial piracy and the significance of government policy in preventing it. I started by observing that it seems at first glance puzzling that commercial piracy can be profitable, and that the prices of pirate copies of information goods do not always decrease to zero through Bertrand competition. Above this puzzle was answered by drawing a distinction between the different effects that the monitoring by the government has on the illegal business model of a bootlegger, i.e. of a commercial pirate: the expected cost of a punishment can partially be viewed as a *fixed cost of production* of the pirate, and partially as an *advertising cost* which depends on the number of the consumers to whom the bootlegger offers her products. The fact that commercial piracy can be profitable was explained by assuming that the fixed costs restrict the entry to the market for pirate copies, and that the advertising costs keep the prices of pirate copies above their production costs.

In the model the advertising costs and the number of the commercial pirates were *policy variables* which represented the extent to which and the ways in which the government fights piracy. However, I did not present a detailed *welfare analysis* from which one could have deduced the optimal values of these policy variables. As it was pointed out above, a welfare analysis of an illegal business model is somewhat problematic, since it is not clear whether a social planner should aim at maximizing welfare which is obtained by illegal means. It is easy to see that a welfare analysis would have yielded the result that consumer welfare is always increased by a decrease in the costs of the pirates. However, the effects of the policy variables on the profits of the bootleggers and the copyright owner are more difficult to analyze.

Comparative static analysis revealed that the effects of the two policy variables on the profits of the copyright owner and the commercial pirates were different: whereas it was always in the interest of the monopolist that the fixed costs were increased, this

was not true of the advertising costs, because without them pirate copies do not have a positive price. Hence, an increase in the advertising costs might *increase* the incentive to enter the market for pirate copies. Since fixed costs of production could also be caused by technical protection devices of information goods, whereas – at least in the case of the commercial piracy on the Internet – the “advertising costs” result almost completely from the increased risk of a punishment, one way to interpret the above results would be that if an information good is distributed in a form which makes it difficult to copy, e.g. if it is protected by a DRM system, an improvement of the technical protection is always in the interest of the copyright owner, but this is not necessarily true of an increase in the legal protection of information goods.

On the other hand, when there are no fixed costs, an increase of the advertising costs is always in the interest of the monopolist. As it was pointed out above, a *peer-to-peer network* can be viewed as the limiting case of the current model in which the both the advertising costs and the fixed costs have sunk to zero, and since a peer-to-peer network makes pirate copies easily available to anyone for free, in the current model it corresponds to the minimum of the profit of the copyright owner.

APPENDIX. PROOFS OF PROPOSITIONS.

PROOF OF PROPOSITION 2.

Below the smallest price which is suggested in the advertisements will be denoted by p_{\min} and the largest one will be denoted by p'_{\max} . Using these notations, the revenue of each bootlegger i can be expressed as

$$(A1) \quad R_i = \int_{p_{\min}}^{p'_{\max}} px(p) \exp\left(-\sum_{k=1}^K A_k(p)\right) a_i(p) dp$$

Next we shall show that p'_{\max} has the value p_{\max} , i.e. the price which would be optimal if there was only a single bootlegger on the market. It is clear that p'_{\max} cannot be larger than p_{\max} , since the quantity $px(p)$ obtains its largest value when $p = p_{\max}$, so that when $p > p_{\max}$,

$$px(p) \exp\left(-\sum_{k=1}^K A_k(p)\right) < p_{\max} x(p_{\max}) \exp\left(-\sum_{k=1}^K A_k(p_{\max})\right)$$

and if a bootlegger advertised at a price larger than p_{\max} , she could increase the value of the integral in (A1) by replacing all the prices which are above p_{\max} by the price p_{\max} .

Next we observe that in equilibrium there cannot be an interval $[p', p''] \subset [p_{\min}, p_{\max}]$ of a positive length such that no bootleggers advertised at any of the prices belonging to $[p', p'']$. This can be shown by supposing, on the contrary, that $[p', p'']$ is an interval with this property, and by letting $[p''', p''']$ be the largest interval which contains $[p', p'']$ and which has this property. Now the function

$$\exp\left(-\sum_{k=1}^K A_k(p)\right)$$

is a constant within the interval $[p''', p''']$, and for sufficiently small values of δ it will be the case that each bootlegger i can increase her profits by replacing in her advertisements all the prices which belong to $[p''' - \delta, p''']$ by the price p''' . Hence, the considered situation cannot be an equilibrium.

It can now be concluded that the problem of maximizing the integral in (A1) cannot have a corner solution in which the value of the function a_i is zero in a part of the interval $(p_{\min}, p'_{\max}) = (p_{\min}, p_{\max})$. Since this corner solution has now been ruled out, the problem of maximizing this integral can be solved with the standard tools of the calculus of variations.

When one puts

$$f(A_i, a_i, p) = px(p) \exp\left(-\sum_{j \neq i} A_j(p)\right) e^{-A_i} a_i(p)$$

the Euler equation which must be valid in the interval (p_{\min}, p_{\max}) for a function $A_i(p)$ for which the function R_i given by (A1) obtains its maximum value turns out to be

$$(A2) \quad \frac{\partial f}{\partial A_i} - \frac{d}{dp} \frac{\partial f}{\partial a_i} = 0$$

This Euler equation is equivalent with

$$-px(p) \exp\left(-\sum_{j \neq i} A_j(p)\right) e^{-A_i} a_i(p) - \frac{d}{dp} \left[px(p) \exp\left(-\sum_{j \neq i} A_j(p)\right) e^{-A_i} \right] = 0$$

with

$$\left[-px(p) a_i(p) - \frac{d}{dp} (px(p)) + px(p) \sum_{j=1}^k a_j(p) \right] \exp\left(-\sum_{j=1}^k A_j(p)\right) = 0$$

and with

$$x(p) + px'(p) = px(p) \sum_{j \neq i} a_j(p)$$

It can now be concluded that if the bootlegger i has chosen the price distribution optimally, then it must be the case that for all $p \in (p_{\min}, p_{\max})$

$$(A3) \quad \sum_{j \neq i} a_j(p) = \frac{1}{p} + \frac{x'(p)}{x(p)}$$

Since we are considering a symmetric equilibrium, it can be concluded from (A3) that

$$a_i(p) = \frac{1}{k-1} \left(\frac{1}{p} + \frac{x'(p)}{x(p)} \right)$$

so that

$$A_i(p) = \frac{1}{k-1} \int_{p_{\min}}^p \left(\frac{1}{u} + \frac{x'(u)}{x(u)} \right) du = \frac{1}{k-1} \ln \frac{px(p)}{p_{\min}x(p_{\min})}$$

This proves the validity of (16).

Now it can be concluded that the revenue of each bootlegger is given by

$$\begin{aligned} R_i &= \int_{p_{\min}}^{p_{\max}} px(p) \exp\left(-\sum_{j \neq i} A_j(p)\right) e^{-A_i(p)} a_i(p) dp \\ (A4) \quad &= \int_{p_{\min}}^{p_{\max}} px(p) \left(\frac{p_{\min}x(p_{\min})}{px(p)} \right) e^{-A_i(p)} a_i(p) dp \\ &= p_{\min}x(p_{\min})(1 - e^{-A_i}) \end{aligned}$$

This proves the validity of (18).

The profit of each bootlegger is given by

$$(A5) \quad P_i = R_i - bA_i = p_{\min}x(p_{\min})(1 - e^{-A_i}) - bA_i$$

This result establishes a connection between the values of A_i and p_{\min} in equilibrium, since A_i must have the value which maximizes profits for the given value of p_{\min} . In other words, in equilibrium it must be the case that

$$\frac{\partial P_i}{\partial A_i} = p_{\min}x(p_{\min})e^{-A_i} - b = 0$$

so that

$$(A6) \quad A_i = \ln \frac{p_{\min}x(p_{\min})}{b}$$

On the other hand, the values of A_i and p_{\min} are connected also by the fact that the total number of the advertisements that the advertiser i sends is $A_i(p_{\max})$. The result (16), which has already been proved, implies that

$$(A7) \quad A_i = A_i(p_{\max}) = \frac{1}{k-1} \ln \frac{p_{\max}x(p_{\max})}{p_{\min}x(p_{\min})}$$

Together the results (A6) and (A7) imply that

$$\ln \frac{p_{\min}x(p_{\min})}{b} = \frac{1}{k-1} \ln \frac{p_{\max}x(p_{\max})}{p_{\min}x(p_{\min})}$$

and this is equivalent with

$$p_{\min}x(p_{\min}) = b^{1-1/k} (p_{\max}x(p_{\max}))^{1/k}$$

This proves the validity of (15). Plugging this result into (A6) one gets

$$A_i = \ln \frac{(P_{\max} x(P_{\max}))^{1/k}}{b^{1/k}} = \frac{1}{k} \ln \frac{P_{\max} x(P_{\max})}{b}$$

Together with (A6), this proves the validity of (17) and completes the proof. Now (19) follows trivially from the results that have already been proved.

PROOF OF PROPOSITION 3.

All the statements in this proposition follow trivially from Proposition 1 and 2, except for the claim concerning the profit P_i in the case in which $k \geq 2$. In order to demonstrate it, we assume that $k \geq 2$ and observe that in this case (19) implies that

$$(A8) \quad \frac{dP_i}{db} = (1-1/k) \frac{b_{\max}^{1/k}}{b^{1/k}} - 1 - \frac{1}{k} \ln \frac{r_{\max}}{b} + \frac{1}{k}$$

and that

$$(A9) \quad \frac{d^2 P_i}{db^2} = -\frac{k-1}{k^2} \frac{r_{\max}^{1/k}}{b^{1+1/k}} + \frac{1}{kb}$$

This immediately implies that there is precisely one value of $b \in (0, r_{\max}]$ for which $\partial^2 P_i / \partial b^2 = 0$, and this further implies that there are at most two values of $b \in (0, r_{\max}]$ for which $\partial P_i / \partial b = 0$. It is now observed that (A8) implies that

$$(A10) \quad \left[\frac{dP_i}{db} \right]_{b=r_{\max}} = (1-1/k) - 1 + \frac{1}{k} = 0$$

Hence, there can be at most one value of b within the interval $(0, r_{\max})$ for which $\partial P_i / \partial b = 0$. On the other hand, since the function P_i is by construction positive in the interval $(0, r_{\max})$, and since it has the limit 0 when $b \rightarrow 0$ and when $b \rightarrow r_{\max}$, it must be the case that $\partial P_i / \partial b = 0$ for at least one value $b \in (0, r_{\max})$. Since it has now been seen that there is precisely one value $b_E \in (0, r_{\max})$ with this property, it can be concluded that P_i is an increasing function of b when $b < b_E$ and that P_i is a decreasing function of b when $b > b_E$.

PROOF OF LEMMA 1. When $p_M \leq 2((1-q)/(2-q))$, (12) implies that $p_{\max} = qp_M/2$.

In this case an agent θ will be a high-valuation consumer if and only if

$$\theta \geq \frac{p_M - p_{\min}}{1-q}$$

so that the demand from such consumers is 0 if $(p_M - p_{\min})/(1-q) \geq 1$, and

$$1 - (p_M - p_{\min})/(1-q)$$

otherwise. This proves (22).

On the other hand, a consumer θ is a low-valuation consumer if and only if

$$\theta < \frac{p_M - p_{\max}}{1-q}$$

A consumer θ for whom this condition is valid will nevertheless buy a legitimate copy if $\theta \geq p_M$ and if she does not receive any advertisements, which is the case with probability e^{-A} . Hence, the demand from such consumers is given by

$$(A11) \quad D_L(p_M) = \left(\frac{p_M - p_{\max}}{1-q} - p_M \right) e^{-A}$$

Since (14) implies that $A = \ln(qp_M^2/(4b(1-q)))$, (A11) simplifies to the form

$$D_L(p_M) = \frac{2b}{p_M}$$

This proves (24). Finally, the demand from the medium-valuation consumers (if any) is the demand from the consumers for whom

$$(A12) \quad \frac{p_M - p_{\max}}{1-q} \leq \theta < \frac{p_M - p_{\min}}{1-q}$$

Clearly, a medium valuation consumer θ will buy a legitimate copy if she has not received an advertisement which would specify a price below $p(\theta)$, where $p(\theta)$ is given by

$$\frac{p_M - p(\theta)}{1-q} = \theta$$

This is equivalent with

$$(A13) \quad p(\theta) = p_M - (1-q)\theta$$

The demand of the monopolist from the consumers for whom (A12) is valid is now seen to be given by

$$(A14) \quad D_{Med}(p_M) = \int_{(p_M - p_{\max})/(1-q)}^{\min\{(p_M - p_{\min})/(1-q), 1\}} e^{-kA_1(p(\theta))} d\theta$$

Putting $p'_{\min} = \max\{p_{\min}, p_M + q - 1\}$ and replacing the variable θ with the variable $p(\theta)$ which is given by (A13), the formula (A14) is seen to be equivalent with

$$D_{Med}(p_M) = \frac{1}{1-q} \int_{p'_{\min}}^{p_{\max}} e^{-kA_1(p)} dp$$

This proves (23) and completes the proof of this lemma.

PROOF OF PROPOSITION 4.

When $p_M \geq 2((1-q)/(2-q))$, the claims of this proposition follow trivially from (20) and the continuity of $D_M(p_M)$. When $p_M < 2((1-q)/(2-q))$, according to Lemma 1 and (31)

$$(A15) \quad D_M(p_M) = 1 - \frac{p_M - p'_{\min}}{1-q} + \frac{1}{p_M} \int_{p'_{\min}/qp_M}^{1/2} f(z) dz + \frac{2b}{p_M}.$$

For the purposes of a comparative static analysis, $f(z)$ will be viewed as given by the formula (30) for arbitrary real values of k although, of course, (A15) has a meaningful economic interpretation only when k is an integer. Clearly, (A15) is formally valid also when $k=1$, since in this case $p'_{\min}/(qp_M) = p_{\max}/(qp_M) = 1/2$, so that the integral on the right-hand side of (A15) vanishes.

The analysis of the partial derivatives of $\partial D_M(p_M)/\partial b$ and $\partial D_M(p_M)/\partial k$ is complicated by the fact that the values of the parameters b and k implicitly affect p'_{\min} when $p'_{\min} = p_{\min}$. However, (A15) and (32) imply that in this case

$$\frac{\partial D_M(p_M)}{\partial p'_{\min}} = \frac{1}{1-q} + \frac{-1}{p_M(qp_M)} f\left(\frac{p_{\min}}{qp_M}\right) = 0$$

On the other hand, p'_{\min} is independent of b and k when $p'_{\min} = p_M + q - 1$, so that in both cases

$$(A16) \quad \frac{\partial p'_{\min}}{\partial b} \frac{\partial D_M(p_M)}{\partial p'_{\min}} = \frac{\partial p'_{\min}}{\partial k} \frac{\partial D_M(p_M)}{\partial p'_{\min}} = 0$$

Together with (A15) and (30), this implies that

$$(A17) \quad \frac{\partial D_M(p_M)}{\partial b} = \frac{1}{p_M} \int_{p'_{\min}/qp_M}^{1/2} \frac{\partial f(z)}{\partial b} dz + \frac{2}{p_M} > 0$$

so that the demand of the monopolist is an increasing function of b . Similarly, since $4z(1-z) < 1$ when $z < 1/2$ it must be the case that

$$(A17) \quad \begin{aligned} \frac{\partial D_M(p_M)}{\partial k} &= \frac{1}{p_M} \int_{p'_{\min}/qp_M}^{1/2} \frac{\partial f(z)}{\partial k} dz \\ &= \frac{1}{p_M} \int_{p'_{\min}/qp_M}^{1/2} \frac{b}{z(1-z)} \frac{\partial}{\partial k} \left(\frac{1}{4z(1-z)} \right)^{1/(k-1)} dz < 0 \end{aligned}$$

This completes the proof.

PROOF OF PROPOSITION 5. First it is observed that if $p_M > 2((1-q)/(2-q))$ and there are bootleggers who send advertisements, none of the consumers who receives one of them will buy a legitimate copy, and (20) immediately implies that $R_M(p_M) = p_M D_M(p_M)$ is a decreasing function of p_M and that the monopolist can increase her profits by lowering the price p_M . Suppose now that $p_M \leq 2((1-q)/(2-q))$. In this case (15) is valid, so that the profit of the monopolist is given by

$$(A18) \quad R_M(p_M) = p_M \left(1 - \frac{p_M - p'_{\min}}{1-q} \right) + \int_{p'_{\min}/qp_M}^{1/2} f(z) dz + 2b$$

and its derivative is seen to be

$$(A19) \quad \frac{dR_M(p_M)}{dp_M} = \frac{d}{dp_M} \left[p_M \left(1 - \frac{p_M - p'_{\min}}{1-q} \right) \right] - \frac{d(p'_{\min}/qp_M)}{dp_M} f\left(\frac{p'_{\min}}{qp_M}\right)$$

If $p'_{\min} = p_M + q - 1$, this implies that

$$\frac{dR_M(p_M)}{dp_M} = -\frac{d}{dp_M} \left(\frac{1 - (1-q)/p_M}{q} \right) f\left(\frac{p'_{\min}}{qp_M}\right) < 0$$

and the monopolist has an incentive to lower the price. Hence, in equilibrium $p'_{\min} = p_{\min} > p_M + q - 1$, and some consumers would buy a legitimate copy even if they could get a pirate copy for p_{\min} .

Assume now that $p'_{\min} = p_{\min}$. Now (A19) and (32) imply that

$$\begin{aligned} \frac{dR_M(p_M)}{dp_M} &= \left(1 - \frac{2p_M - p_{\min}}{1-q} \right) + \frac{p_M}{1-q} \frac{dp_{\min}}{dp_M} - \left[\frac{1}{qp_M} \frac{dp_{\min}}{dp_M} - \frac{p_{\min}}{qp_M^2} \right] \frac{qp_M^2}{1-q} \\ &= 1 - \frac{2p_M - p_{\min}}{1-q} + \frac{p_{\min}}{1-q} = 1 + \frac{2p_{\min} - 2p_M}{1-q} \end{aligned}$$

This completes the proof.

PROOF OF PROPOSITION 6.

Proposition 5 and (12) imply that the monopolist will not choose a price p_M for which $p_M > 2((1-q)/(2-q))$ if pirate copies become available with this choice of p_M , because in this case none of the consumers who can buy a pirate copy would buy a legitimate one. The largest value of b for which the bootleggers advertise if the monopolist chooses $p_M = 2((1-q)/(2-q))$ is the value b'' for which $b'' = p_{\max}x(p_{\max})$ when $p_M = 2((1-q)/(2-q))$. Clearly,

$$(A20) \quad b'' = \frac{q(1-q)}{(2-q)^2}$$

If $b \geq b''$, the monopolist choose a price for which the bootleggers do not advertise. If the largest price p_M for which this is the case is not larger than $1/2$, it is according to (12) such that $p_{\max} = q + p_M - 1$, and it is determined by the condition

$$b = p_{\max}x(p_{\max}) = \frac{(q + p_M - 1)(1 - p_M)}{q}$$

In this case the equation $p_{\max}x(p_{\max}) = b$ is valid if and only if

$$(A21) \quad p_M = 1 - q/2 - \sqrt{q^2/4 - bq}$$

This proves the latter part of formula (34).

Suppose now that $b < b''$, and let $p_{M,0}$ be the smallest value of p_M for which the bootleggers still advertise. Clearly, $p_{M,0} < 2((1-q)/(2-q))$ so that when $p_M = p_{M,0}$, it must be the case that $p_{\max} = qp_M/2$ and the condition $b = p_{\max}x(p_{\max})$ receives the form

$$b = \frac{qp_{M,0}}{2}x\left(\frac{qp_{M,0}}{2}\right) = \frac{qp_{M,0}^2}{4(1-q)}$$

Hence,

$$(A22) \quad p_{M,0} = \sqrt{\frac{4(1-q)b}{q}}$$

When $p_M < 2((1-q)/(2-q))$, solving p_{\min} from (15), (12), and (13) yields

$$p_{\min} = \frac{qp_M}{2} \left(1 - \sqrt{1 - \left(\frac{4(1-q)b}{qp_M^2} \right)^{1-1/k}} \right)$$

This implies that $dp_{\min}/dp_M < 1$, so that one can conclude from (35) that

$$(A23) \quad \frac{d^2 R_M(p_M)}{dp_M^2} = \frac{2}{1-q} \frac{dp_{\min}}{dp_M} - \frac{2}{1-q} < 0$$

This shows that if the derivative $dR_M(p_M)/dp_M$ is negative when $p_M = p_{M,0}$, it is negative for all values $p_M > p_{M,0}$. Hence, the monopolist has an incentive to raise the price above $p_M = p_{M,0}$ if and only if

$$(A24) \quad \frac{dR_M(p_{M,0})}{dp_M} > 0$$

Since $p_{\min} = p_{\max} = qp_{\max}/2$ when $p_M = p_{M,0}$, Proposition 5 and (A22) imply that

$$\frac{dR_M(p_{M,0})}{dp_M} = 1 - \frac{2p_{M,0}}{1-q} + \frac{qp_{M,0}}{1-q} \left(1 - \frac{2-q}{1-q} \sqrt{4(1-q)\frac{b}{q}} \right)$$

and this is positive if and only if

$$b > \frac{q(1-q)}{4(2-q)^2}$$

Together with (A22), this proves the former part of (34). Finally, the fact that the condition (35) suffices to determine equilibrium value of p_M when it is larger than $p_{M,0}$ follows from Proposition 5 and (A23).

PROOF OF PROPOSITION 7.

It is obvious that if F is increased and the monopolist does not change the value of p_M , the value of k determined by (3) either decreases or stays the same. Hence, Proposition 4 implies that the profit of the monopolist either increases or stays the same when F increases. Trivially, this must remain valid also when the monopolist chooses p_M optimally.

To prove the latter claim, let $\zeta > 0$ be arbitrary, assume that the fixed costs are given by $F(\zeta) = \zeta$, and compare two possible values of the advertising costs, $b_1(\zeta) = \zeta^2$

and $b_2(\zeta) = \zeta$. Clearly, if $b = b_1(\zeta)$ and at least two bootleggers pay the fixed cost $F(\zeta)$, their profit is at most

$$\sqrt{b_1(\zeta)r_{\max}} - b_1(\zeta) - F(\zeta) < \zeta - \zeta^2 - \zeta < 0$$

so that in this case there will be at most one bootlegger on the market, independently of the choice of p_M . It is clear that in this case the monopolist will let one bootlegger enter in equilibrium if ζ is sufficiently small. In this case the equilibrium price chosen by the monopolist is given by $p_{M,b1} = (1-q)/(2-q)$, and the equilibrium profit of the monopolist satisfies the condition

$$(A25) \quad \lim_{\zeta \rightarrow 0+} R_{M,b1}(p_{M,b1}) = \left(1 - \frac{2-q}{2(1-q)} p_M\right) p_M = \frac{1-q}{2(2-q)}$$

On the other hand, if $b = b_2(\zeta) = \zeta$ and there are at least two bootleggers on the market, in the limit in which $\zeta \rightarrow 0+$ the profit of the monopolist approaches

$$\lim_{\zeta \rightarrow 0+} R_{M,b2}(p_M) = p_M (1 - p_M / (1-q))$$

so that its maximum value approaches

$$(A26) \quad \lim_{\zeta \rightarrow 0+} R_{M,b2}(p_{M,b2}) = (1-q)/4$$

When $b = b_2(\zeta)$, there will be less than two bootleggers on the market only if

$$\sqrt{b_2(\zeta)r_{\max}} - b_2(\zeta) - F(\zeta) = \sqrt{\zeta r_{\max}} - 2\zeta < 0$$

This is equivalent with $r_{\max} < 2\sqrt{\zeta}$, and it is clear that if ζ is sufficiently small, it cannot be in the interest of the monopolist to choose such a small value of p_M that this condition would be valid for r_{\max} . Now it can be concluded from (A25) and (A26) that for sufficiently small values of ζ an increase of b from $b = b_1(\zeta)$ and $b = b_2(\zeta)$ will decrease the equilibrium profit of the monopolist.

REFERENCES

- Andrés A R. The relationship between copyright software protection and piracy: Evidence from Europe. *European Journal of Law and Economics* 2006; 21; 29-51.
- Banerjee D S. Software piracy: a strategic analysis and policy instruments. *International Journal of Industrial Organization* 2003; 21; 97-127.
- Banerjee D S. Lobbying and commercial software piracy. *European Journal of Political Economy* 2006a; 22; 139-155.
- Banerjee D S. Enforcement sharing and commercial piracy. *Review of Economic Research on Copyright Issues* 2006b; 3; 83-97.
- Butters G R. Equilibrium distributions of sales and advertising prices. *The Review of Economic Studies* 1977; 44; 465-491.
- Conner K R, Rumelt R P. Software piracy: an analysis of protection strategies. *Management Science* 1991; 37; 125-139.
- Eskicioglu A M, Delp J E. An overview of multimedia content protection in consumer electronics devices. *Signal Processing: Image Communication* 2001; 16; 681-699.
- Gayer A, Shy O. Publishers, artists and copyright infringement. *Information Economics and Policy* 2006; 18; 374-384.
- Kolm S. A note on optimum tax evasion. *Journal of Public Economics* 1973; 2; 265-270.
- Myles G D. *Public Economics*. Cambridge University Press: Cambridge; 1995.
- Park Y, Scotchmer S. Digital rights management and pricing of digital products. NBER Working Paper Series 2005; **11532**.
- Peitz M, Waelbroeck P. Piracy of digital products: a critical review of the theoretical literature. *Information Economics and Policy* 2006a; 18; 449-476.
- Peitz, M, Waelbroeck P. Why the music industry may gain from free downloading – the role of sampling. *International Journal of Industrial Organization* 2006b; 24; 97-913.
- Sandmo A. Income tax evasion, labour supply, and the equity-efficiency tradeoff. *Journal of Public Economics* 1981; 16; 265-288.
- Varian H R. Buying, sharing, and renting information goods. *The Journal of Industrial Economics* 2000; 48; 473-488.

Yao J-T. Counterfeiting and an optimal monitoring policy”. *European Journal of Law and Economics* 2005; 19; 95-114.