



Helsinki
Center
of
Economic
Research

Discussion Papers

Peace-building and local conflicts in developing countries

Pertti Haaparanta
Helsinki School of Economics and HECER

and

Kirsti Kuisma
Helsinki School of Economics

Discussion Paper No. 113
August 2006

ISSN 1795-0562

HECER – Helsinki Center of Economic Research, P.O. Box 17 (Arkadiankatu 7), FI-00014
University of Helsinki, FINLAND, Tel +358-9-191-28780, Fax +358-9-191-28781,
E-mail info-hecer@helsinki.fi, Internet www.hecer.fi

Peace-building and local conflicts in developing countries*

Abstract

This paper extends the basic international trade and conflict model to analyze peace-building by outside countries in local conflicts (either civil wars or international war restricted to few countries). This paper argues that if peace building is taken into account, the intensity of conflicts can decline when the conflicting country's income increases, and conflicts can even be completely prevented. The paper distinguishes between ex ante and ex post peace-building measures. It is shown successful pre-conflict peace-building requires that the measures are taken at large enough scale. The peace-building countries have an incentive to take such measures only if both they (peace-builders) and the conflicting regions are sufficiently rich. Finally, it turns out that the post-conflict operations are time-consistent only if the outside country is rich enough. Repairing war damages turns out to be counterproductive, because it increases military investments.

JEL Classification: F1, O1

Keywords: International trade, conflict, peace-building.

Pertti Haaparanta

Department of Economics
Helsinki School of Economics
P.O. Box 1210
FI-00101 Helsinki
FINLAND

e-mail: pertti.haaparanta@hse.fi

Kirsti Kuisma

Department of Economics
Helsinki School of Economics
P.O. Box 1210
FI-00101 Helsinki
FINLAND

e-mail: kirsti.kuisma@hse.fi

* This work is part of the project No. 298850, "Institutions, Aid, and Development" financed by the Academy of Finland. We thank Dr. Mansoob S. Murshed and participants in EEA 2002 for comments.

1 Introduction

Why do rich countries seem to be much more concerned with stability in oil-producing countries than poor countries? Can they credibly commit themselves to actions that guarantee peace? Or can they only commit to actions that in fact increase the extent of conflicts? These are among the questions that we study in the paper. Among the results we provide a new explanation for one of the major results in the recent empirical research on conflicts: there is a non-linear relationship between income per capita and probability of conflicts, at low levels of income the probability is high, but the probability begins to decline when income increases (Collier et. al. 2003). Here the explanation is that incentives for countries outside the conflict to provide help in peace-keeping are credible only if the conflicting regions/countries have high enough income.

We extend the basic international trade and conflict model by Haaparanta and Kuisma (2005) and Becsi and Lahiri (2005) to analyze the role of peace-building by outside countries in local conflicts in developing countries¹. This is an important question as most wars nowadays are intrastate and take place in the third world, thus significantly weakening their already poor growth prospects. In this paper we find that if peace-building is taken into account, the intensity of conflicts can decline when the conflict country's income increases (which is consistent with empirical evidence as pointed out above). This paper also argues that incomplete pre-conflict peace-building operations will in fact increase military buildup. In contrast, successful pre-conflict peace-building requires that the outside countries are sufficiently rich, and that the conflicting countries have a high enough income as well (which conforms with the existing empirical evidence). Finally, it turns out that post-conflict operations are time-consistent only if the outside country is sufficiently rich. On the other hand, repairing war damages

¹See also Skaperdas and Syropoulos (1996; 2001)

is counterproductive, because it increases military investments. To the best of our knowledge these results are new in the economic research of conflicts.

Traditionally, conflicts and civil wars have been assumed to have their roots in political, geographical or religious issues and/or inequality among different groups, i.e. the so called "grievance-approach"². Recently, however, a growing interest has emerged to also examine the economic aspects of conflicts. It is clear that conflicts have enormous economic consequences, in particular for countries already poor, as hostilities destroy scarce resources and convert funding away from productive uses³. Conflicts also have many other indirect channels through which they significantly reduce growth prospects. For example, conflicts have negative effects on financial development (Addison et al., 2004), they destroy social capital (Colletta and Cullen, 2000) and significantly reduce tourism (Dhariwal, 2004) to mention only a few. Brück (2001) gives an account of the impacts of the civil war on the post-war Mozambican economy.

The literature on the economics of conflict has proposed that, in addition to their economic consequences, conflicts can have economic causes, and that conflicts may at least partly be a result of greed instead of grievances⁴. It is noteworthy that grievances have an economic dimension, too. For example, we can think of the economic consequences of discrimination and inter-group

²See for example Collier and Hoeffler (2001) for discussion of the different motives for rebellion. On the implications of inequality, see also Stewart (2002), who focuses on horizontal inequality as a source of political instability. Yet another view of inequality is presented by Dutta and Mischra (2003) who propose that anticipated future inequality (which has to be significant), in particular, is an important factor in generating conflict.

³On the costs of conflicts see for example new results by Pottebaum and Kanbur (2004). See also Collier et al. (2003).

⁴On the economics of conflict in developing countries, see for example Collier and Hoeffler (2001), Collier (2000) and Collier et al. (2003). On the economics of conflict in general, see example Sandler and Hartley (1995) for a thorough survey and Grossman (1991) as an introduction of modelling conflicts in economics.

inequalities. Furthermore, the role of trade in conflicts⁵ has received attention. While it is an accepted fact that conflicts do have economic aspects, the importance of these is a matter of debate. In a nutshell, economists tend to emphasize the role of economic factors, while other scholars focus more on sociological, religious and other non-economic aspects⁶. In the economics literature a high incidence of conflicts has been associated with low income and slow growth rate, and heavy reliance on primary commodities. The latter effect emerges as nonlinear, first increasing and peaking when the share of primary commodity exports approaches to 30 percent of GDP. (Collier et al. 2003)⁷.

Even though the fundamental causes for conflicts are somewhat debatable, it is a fact that conflicts and civil wars are extremely detrimental for development. The consequences of a (often long lasting) civil war can be disastrous both to the population of the country in war and internationally. It is clear that civil war not only affects the countries at war, but also has serious spillover effects on neighboring countries and to the international community⁸. It is becoming increasingly recognized that intrastate wars demand action from outside countries. As has been proposed in research, trade can be an indirect means of affecting the situation in conflict-prone countries, even though the role of trade in mitigating conflicts is somewhat controversial⁹. A direct means of influencing the incidence and duration of

⁵See e.g. Polachek (1997), Skaperdas and Syropoulos (2001), Mc Donald (2004) and Haaparanta and Kuisma (2005)

⁶The ongoing discussion is reviewed in Malone and Nitzschke (2004)

⁷Nafziger (2004) lists the following factors (that partly coincide with the findings of Collier) as the root causes of war: stagnation and decline in real (inflation-adjusted GDP), slow growth in food production, high income inequality, failure to adjust to chronic international balance of goods and services deficits, a high ratio of military expenditure to national income, competition for control of mineral exports, and a tradition of violent conflict.

⁸These spillover effects may include, for example, excessive flow of refugees, civil war contagion to nearby areas, international terrorism (Collier et al.; 2003)

⁹According to the interdependence argument, trade decreases the motivation for con-

conflict is peace-building, which is the main focus of this paper. We use the same framework as Haaparanta and Kuisma (2005), which analyzes how the possibility of trade with outside countries affects the intensity of civil conflicts in developing countries. Here we include intervention of a third country (interpreted as the rest of the world) to a local conflict among two countries/regions. This paper is organized such that chapter 2 begins by presenting the framework, section 2.1 examines pre-conflict peace-building and section 2.2 focuses on post-conflict peace-building. Chapter 3 concludes.

2 Model of conflicts and peace-building

The model analyzes the motives and actions of an outside country (countries) engaging in peace-keeping operations in the conflict zones. As in Collier (2000), we distinguish between *ex ante* (pre-conflict) and *ex post* (post-conflict) measures. *Ex ante* measures refer to action taken before the conflict. These can include measures to reduce the efficiency of resources devoted to conflict, or the formation of safe zones by sending in peace-keeping forces as the conflict begins to intensify. Respectively, *ex post* measures, such as rebuilding the country, repairing war damages, take place after the conflict has ceased. The economic research on peace-building and external intervention in civil conflicts has recently received increasing attention. For example, World Bank's research program "Economics of civil war, conflict and violence" and UNU-Wider Research Project on humanitarian emergencies (in collaboration with Queen Elizabeth House, Oxford) have contributed to the understanding of the effectiveness of international peace-building¹⁰. The findings of these studies seem, however, quite contradictory. Doyle and

flict. On the other hand, trade may facilitate the financing of conflict, thus increasing the likelihood of conflict.

¹⁰See e.g. Elbadawi and Sambanis (2000), Doyle and Sambanis (2000) for results from the World Bank Program and Nafziger (2004) for the UNU-Wider Program. See also Regan (2001).

Sambanis (2000) conclude that UN peace-keeping is positively correlated with democratization following a civil war, and that multilateral operations are often successful in ending the violence. On the other hand, Regan (2002) finds that neutral third party interventions tend to lengthen the duration of civil war. According to that study, intervention will work to shorten the duration only if it is biased in favour of one party. Similarly, Elbadawi and Sambanis (2000) conclude that external intervention is positively associated with the duration of conflict. Finally, Addison and Murshed (2002) point out that when war provides economic gains, peace is not necessarily incentive compatible and peace agreements can become time inconsistent. The role of outside agents is to enhance conflict parties' commitment to peace. Outside countries' policies (commitment technologies) can thus lower the risk of civil war reigniting.

The results from the UNU-Wider research project indicate that ex post interventions are generally less efficient than ex ante actions. The study also shows that ex ante measures, however, should extend to long term strategies (such as macroeconomic stabilization, structural adjustment programs and reduction in trade barriers against low income countries) to reduce the risk of conflicts¹¹. In general, short term strategies (e.g. military and diplomatic operations) are not that successful, as the willingness of the third parties to participate is often poor and/or their policies ineffectual. (Nafziger, 2004). It is important to note that a third country's willingness to participate is by no means certain; they must derive some utility from their involvement. Here we pay special attention to these third country incentives. However, it is important to note that there may be other arguments for peace-building abroad. For example, western countries may engage in peace-building perceiving peace to be an international public good. But it is important also to understand that the familiar collective action problems are present.

¹¹Note that the relation between trade barriers and conflict is not necessarily a straightforward one. See Haaparanta and Kuisma (2005).

Recent research by Gershenson (2002) also relates to our work. Gershenson examines the implications of sanctions imposed by outside parties on civil war contestants. He examines the specific kind of sanctions whose purpose is to influence the outcome of civil conflict in favor of challenger (against the incumbent). Possible outcomes include deterrence, engagement and surrender (in case the challenger is powerful enough). In short, it is found that strong sanctions benefit the challenger, but weak sanctions can actually hurt them. Sanctions are more likely to work when income for incumbent in case of defeat is large. This could explain why sanctions were effective in the case of South Africa and Rhodesia but ineffective in Cuba. Our ex ante peace-building framework is to some extent analogous to Gershenson's work. Sanctions are seen as a one measure of pre-conflict peace-building, even though the explicit goal of sanctions is somewhat different¹².

The general framework we adopt is the following. Let us assume that the world comprises 3 regions/countries of which regions/countries 1 and 2 engage into a mutual conflict. Country 3, representing the rest of the world, has made a credible commitment of non-hostility to countries 1 and 2. We assume that countries 1 and 2 are developing countries/regions while country 3 represents the developed world. Since nowadays almost all conflicts take place in developing regions, we postulate that the developed world does not initiate conflict against developing regions.

The timing in the model is following: First, countries 1 and 2 invest in arms and a war erupts. After the cease of hostilities, goods will be exchanged in the world market with country 3 participating in the exchange. Peace-keeping actions by the third country can take place either in the pre-conflict phase (before the decision how much to arm), or in the post-conflict era (when arms investments by countries 1 and 2 are already made). For

¹²The aim of sanctions is to support the challenger and to force the incumbent to surrender. In our model the third country tries to prevent conflict i.e. discourage military investment by establishing non-conflict zones or protecting existing property rights.

analytical simplicity we abstain from production considerations and study exchange economies only.

It is also assumed that all countries initially own only one type of endowment. Hence, there are three commodities: x , y , and z . The total world endowment of x is X and is initially owned by region 1. Respectively, the total world endowment of y is Y and of z is Z and the first is owned by region 2 and the latter by country 3. All sellers and buyers in the world markets act competitively and take the world market prices as given. Let us denote the world market price for good j by p_j , where $j = x, y, z$. Clearly, countries 1 and 2 engaging in conflict must devote resources to the war. We assume that x and y are perfect substitutes in the production of arms. The investment in conflict by country i is W_i , $i = 1, 2$. Accordingly, the endowments available after arms investment are $X - W_1$ in country 1 and $Y - W_2$ in country 2¹³. For simplicity's sake, we are ignoring the destruction of resources during the fighting, although we acknowledge that it evidently does occur and can rise up to significant levels.

As the war breaks out, the investments in arms result in success. We follow Hirshleifer (199?) by modelling the consequences of the conflict in terms of a success function. War success is determined by the following success function ϕ , which determines the success of country 1. The success of country 2 is determined by the success function $1 - \phi$. We refrain from offence/defence considerations and assume that military success depends only on investments in arms as follows:

$$\phi = \phi(W_1, W_2), \quad 0 \leq \phi \leq 1, \quad \phi_{W_1} > 0, \quad \phi_{W_2} < 0 \quad (1)$$

The success function can be interpreted as the probability of winning the war and capturing the existing endowment of the other country. Equivalently, as we assume that the countries decide on war investment to maximize the welfare of the representative agent, we simplify the analysis by

¹³The military build-up is assumed be financed by a lump sum tax on endowment.

omitting all the issues related to risk aversion. Thus, assuming risk neutrality, the success function can be interpreted to give the available shares of total endowments available accruing to each participant¹⁴. In our model, after conflict the relative endowments of countries 1 and 2 of commodities x and y are identical, post conflict endowments for country $i = 1, 2$ are given by \hat{x}_i, \hat{y}_i and are: $\hat{x}_1 = \phi(X - W_1), \hat{y}_1 = \phi(Y - W_2), \hat{x}_2 = (1 - \phi)(X - W_1)$ and $\hat{y}_2 = (1 - \phi)(Y - W_2)$. The aggregate welfare levels of countries 1 and 2 are determined by the following equations¹⁵:

$$\begin{aligned} e(p_x, p_y, p_z, u_1) &= \phi [p_x(X - W_1) + p_y(Y - W_2)], \\ e(p_x, p_y, p_z, u_2) &= (1 - \phi) [p_x(X - W_1) + p_y(Y - W_2)] \end{aligned} \quad (2)$$

where $e(p_x, p_y, p_z, u_i)$ denotes the expenditure function having the usual properties (see e.g. Jehle and Reny 2001), $u_i =$ aggregate welfare in country i . It is straightforward to see that welfare is increasing in the national share in endowments, in net endowments and in p_x and p_y while decreasing in p_z . Country 3 welfare is similarly given by

$$e(p_x, p_y, p_z, u_3) = p_z Z \quad (3)$$

and is increasing in p_z while it decreases with p_x and p_y .

We assume that the national utility functions are of Cobb-Douglas form, $u_i = C_{xi}^\alpha C_{yi}^\beta C_{zi}^\gamma$. Consumers maximize this with respect to budget constraint (2). From this problem we get the consumption levels C_{ji} as a function of prices and incomes. C_{ji} denotes the consumption of good j in country i . Next, we maximize the indirect utility function with respect to decisions to arm W_i taking into account the success function (1), and prices to calculated below (and given in (7)). Country 3, i.e. the rest of the world, engages

¹⁴This form of contest success function is widely used in the conflict literature. The risk neutrality assumption is also standard, see e.g. Hirshleifer (1995; 2000).

¹⁵It is easy to note that as $\frac{\hat{x}_1}{\hat{y}_1} = \frac{\hat{x}_2}{\hat{y}_2}$, prospects for trade among countries 1 and 2 have disappeared.

in peace-building operations, either ex ante or ex post. Let S denote the measure taken by country 3. Country 3 is assumed to act selfishly and to maximize u_3 when deciding on S . This approach seems appropriate given e.g. the attention paid to the Middle East to ensure that energy markets remain stable and oil prices do not increase. Accordingly, we are able to define $\psi = \psi(W_1, W_2, S)$, i.e. the aggregate income of country 1 depends on the peace-building operations of the third country. If S is an ex ante measure, then also W_i depends on S .

We have chosen the Cobb-Douglas presentation of national welfare levels consciously to strengthen the points we make. Haaparanta and Kuisma (2005) show that in a 3-country model with trade between exchange economies, the extent of conflict between two of the countries increases with their incomes¹⁶. This is inconsistent with the empirical evidence. The point here is to show that with peace-building operations by the third country possible one reaches a conclusion consistent with the empirical evidence.

To take action S country 3 has to spend its own resources. We make a distinction between ex ante conflict prevention (actions taken before the conflict has erupted) and ex post peacebuilding (e.g. reparation of war damages). Special focus will be on the conditions under which the ex ante peacebuilding is credible. The distinction also helps to highlight conditions under which peacebuilding will increase conflicts.

We assume that the resource cost of action S is S . The ex ante action is always taken before the markets open, leaving $(Z - S)$ as the marketable endowment of country 3. Thus, the peace-building operation will also have an effect on world market prices. If S is an ex ante peace-building measure, then country 3 is a Stackelberg leader vis a vis countries 1 and 2, i.e. takes into account the responses of the conflict parties. On the other hand, if S

¹⁶Haaparanta and Kuisma (2005) show that with Stone-Geary function the relationship can be non-monotonous and consistent with empirical evidence.

is a post-conflict action, country 3 takes the military investments as given. In this case countries 1 and 2 act as Stackelberg leaders towards country 3. Finally, to keep the analytics as simple as possible (since we are now studying a game with a sequence of moves) we assume throughout that citizens everywhere have Cobb-Douglas utility functions, i.e. country i , $i = 1, 2, 3$ maximizes

$$u_i = C_{xi}^\alpha C_{yi}^\beta C_{zi}^\gamma, \quad \alpha + \beta + \gamma = 1 \quad (4)$$

The Cobb-Douglas specification, despite its obvious shortcomings, is widely used in conflict literature and serves well in the current context, as the main point is to highlight the incentives for various types of actions, and the assumption helps to fix the focal point. In order to obtain specific results we also use the following form of the success function:

$$\phi = \frac{W_1}{W_1 + W_2} \quad (5)$$

This form of success function implies that there is always some military buildup, but with diminishing returns (with buildup). The above form is used widely in the literature, usually having parameters representing the technology of insurgents¹⁷. These are left out in the current context, however, as the main interest is in peace-keeping operations and because the differences in military technology are not usually decisive in intrastate conflicts in developing countries .

2.1 Pre-conflict peace-building

Let us first consider ex ante measures. The main focus is on measures that (partially) protect existing property rights or establish non-conflict zones.

¹⁷This form of success function is typically used in conflict models. For example, Hirshleifer (1995) uses contest success function of type $p_1 = \frac{F_1^m}{F_1^m + F_2^m}$, where p_1 is the success ratio for party 1, F_1 and F_2 are the fighting efforts and m is a decisiveness parameter. The decisiveness parameter has been excluded from the current discussion for analytical simplicity.

We postulate that the measure S taken by country 3 helps to put a share $\xi_i S$, $\xi_i \geq 0$, of country i 's endowment beyond the conflict, $i = 1, 2$. Thus, this share is not available for contention between countries 1 and 2. We assume that this secure endowment cannot be used for military investment either. Accordingly, the post-conflict income for country 1 is:

$$\psi_1 = p_x [\xi_1 S + \phi (X - \xi_1 S - W_1)] + p_y \phi (Y - \xi_2 S - W_2) \quad (6)$$

which leaves $(X - W_1)$ as the total supply of commodity x in the post-conflict exchange. Analogous expressions hold for country/region 2 and commodity y . Due to the peace-keeping investment, the total supply of commodity z is $(Z - S)$. This implies that the world market prices for the conflict countries' commodities are, obtained by equating demands with supplies using the utility functions and budget constraints:

$$p_x = \frac{\alpha (Z - S)}{\gamma (X - W_1)}, \quad p_y = \frac{\beta (Z - S)}{\gamma (Y - W_2)}. \quad (7)$$

Country 1's optimal arms building can be calculated using (6) and (7). The problem for country 1 is:

$$\begin{aligned} \max_{W_1} \quad & V(p_y, p_z \Psi_1) \\ \text{s.t.} \quad & \end{aligned} \quad (8)$$

$$p_x [\xi_1 S + \phi (X - \xi_1 S - W_1)] + p_y \phi (Y - \xi_2 S - W_2) - p_x C_{x1} - p_y C_{y1} - C_{z1} = 0$$

From the above problem we get the following first order condition for country 1:

$$\begin{aligned} & \frac{\partial \phi}{\partial W_1} \frac{\alpha (Z - S)}{\gamma (X - W_1)} \left[\tilde{X} + \frac{\beta (X - W_1)}{\alpha (Y - W_2)} \tilde{Y} \right] - \frac{\alpha (Z - S)}{\gamma (X - W_1)} \phi + \\ & \left\{ (1 - \alpha) \xi_1 S + \phi \left[(1 - \alpha) \tilde{X} - \beta \frac{X - W_1}{Y - W_2} \tilde{Y} \right] \right\} \frac{\alpha (Z - S)}{\gamma (X - W_1)^2} = 0 \end{aligned} \quad (9)$$

where $\tilde{X} \equiv (X - \xi_1 S - W_1)$ and $\tilde{Y} \equiv (Y - \xi_2 S - W_2)$. Focusing on symmetric equilibrium, i.e. setting $X = Y$, $W_1 = W_2$, condition (9) simplifies to:

$$\frac{\partial \phi / \partial W_1}{\phi} \frac{(1 - \gamma)(X - W)}{\alpha} = \frac{X - W}{\tilde{X}} - \frac{[(1 - \alpha)(\xi S / \phi) + \gamma \tilde{X}]}{\tilde{X}} \quad (10)$$

Taking into account that due to symmetry $\alpha = (1 - \gamma) / 2$ and $\phi = 1/2$, equation (10) can be expressed in following relatively simple form:

$$\frac{X - W}{W} = \frac{(1 - \gamma)(X - W) - \xi S}{X - W - \xi S} \quad (11)$$

Since this is a quadratic equation it has, in general, two solutions. We can also note that in case of no peace-building (with $S = 0$), the solution becomes $\frac{X - W}{W} = (1 - \gamma)$. It is straightforward to calculate that a solution with positive level of investment in conflict exists only if

$$S < \frac{3 - \gamma - 2\sqrt{2 - \gamma}}{2\xi} \quad (12)$$

The upper bound is decreasing in γ . We call these levels of peace-building *imperfect*, as $W > 0$. In other words, conflict is mitigated but not prevented. The solution we look at is the smaller root¹⁸.

To evaluate the consequences of the peace-building action let us note that the LHS of (11) does not depend on S. It is straightforward to see that the RHS is smaller than $(1 - \gamma)$ when $S > 0$. This means that military buildup is always higher with pre-conflict peace-keeping than without it. Furthermore, the RHS decreases with S, implying that the more extensive the pre-conflict peace-keeping operation is, the larger will be the military buildup. Since S and ξ have a symmetric effect, we also now know that the more intensive (bigger ξ) the peace-keeping operation is, the larger will be the investment in arms. This gives us the first proposition.

¹⁸Since it satisfies the condition that arms building increases with γ which is the ordinary Cobb-Douglas solution when $S = 0$.

Proposition 1 *Incomplete pre-conflict peace-building operations, i.e. operations with $S < \frac{3-\gamma-2\sqrt{2-\gamma}}{2\xi}$ and $W > 0$, will increase military buildup.*

Thus, interestingly enough, ex ante peace-building operations can have a perverse effect by increasing rather than reducing the buildup of military strength. To find out the intuition we need to examine the four effects that are present, all of which are not pointing in the same direction. First, peace-building reduces the direct marginal benefit of the increased military investment $[p_x(X - \xi_1 S - W_1) + p_y(Y - \xi_2 S - W_2)] \frac{\partial \phi}{\partial W_1}$, because the amount of "lootable" resources as well as their prices (see price equations in (7)) are reduced. It is easy to see that prices in Cobb-Douglas model without peace-building would be $p_x = \frac{\alpha Z}{\gamma(X-W_1)}$, $p_y = \frac{\beta Z}{\gamma(Y-W_2)}$ which are clearly higher. Second, peace-building simultaneously reduces the direct cost of looting $p_x \phi$. Third, peace-building reduces the terms-of-trade gain from the buildup of arms by reducing the responsiveness of prices $\frac{\alpha(Z-S)}{\gamma(X-W_1)^2}$ to it. Lastly, it increases the base on which the gain from higher world market price (because of the positive terms-of-trade effect of conflict) applies. Peace-building operation increases the amount of available goods, ceteris paribus, of which only a proportion is spent in the home country and thus more is exported to world markets. With Cobb-Douglas preferences the impacts increasing incentives for arms investment are greater than the impacts working in the opposite direction.

It must be noted that the result in Proposition 1 is only local. This is to say that if the peace-building operation is massive enough (or efficient enough), no incentive for investing in conflict remains. To state it analytically, if $S = \frac{X}{\xi}$, then property rights are perfectly protected and looting is not profitable¹⁹, for the simple reason that there is nothing to loot. In fact, as can be derived from condition (12), conflict can be made unprofitable provided that the following condition holds:

¹⁹Looting still has the terms-of-trade effect but it is not strong enough to make pure destruction ("mindless" looting) profitable.

$$S > \frac{3 - \gamma - 2\sqrt[3]{2 - \gamma}}{2\xi} \quad (13)$$

We call these peace-keeping operations *perfect*, because they completely prevent conflicts. The right hand side of the above condition is decreasing in γ . This implies that the larger the share of goods from the non-conflict region is (in the conflicting countries), the smaller the peace-keeping operation needed to ensure peace. This leads to an important conclusion: peace-building is less costly in an open world. Thus we have obtained

Proposition 2 *Peace can be fully ensured even without complete enforcement of property rights provided that $S > \frac{3 - \gamma - 2\sqrt[3]{2 - \gamma}}{2\xi}$. If peace-building operations make endowments safe enough (secure property rights almost in full), no conflicts arise. As S is decreasing in γ , peace-building is less costly in an open world.*

In addition to the effectiveness of peace-building, it is important to consider what are the incentives of country 3 to engage in peace-keeping, i.e. why should it become involved in these costly operations²⁰. In this paper we exclude all altruistic considerations and assume that the third country maximizes its own welfare only. Let us begin by considering the welfare of country 3 by and focusing on the peace-building operation that has no effect on conflict, i.e. $\frac{\partial W_i}{\partial S} = 0$. In this case it is straightforward to calculate that $\frac{\partial u_3}{\partial S} = -\gamma$, which clearly is negative. This is to say that peace-building without any effect on conflict reduces the welfare of the peace-building country unambiguously²¹. Under the conditions of Proposition 1 we know that in this model $\frac{\partial W_i}{\partial S} > 0$, i.e. military build-up is increased by incomplete

²⁰This point was left out by Gershenson (2002) who does not consider motives of an outside for peace-keeping.

²¹This also substantiates the claim made above that pure looting (destruction of one's own endowments without any gain) for manipulation of terms-of-trade is not beneficial for countries 1 and 2.

peace-building operations. This implies that small-scale peace-building operations (with $S < \frac{3-\gamma-2\sqrt[2]{2-\gamma}}{2\xi}$) will definitely reduce welfare even more than completely ineffective peace-building operations (those with $\xi = 0$), since they will also deteriorate the terms-of-trade of the peace-building country. Therefore, it is clear that there are no incentives for country 3 to engage in incomplete peace-building operations.

We are then left with perfect peace-building operations. We know that the indirect welfare of country 3 is proportional to $(p_x)^{-\alpha} (p_y)^{-\beta} Z$, which can in general be written as $\left(\frac{\alpha}{\gamma}\right)^{-\alpha} \left(\frac{\beta}{\gamma}\right)^{-\beta} \left(\frac{Z-S}{X-W_1}\right)^{-\alpha} \left(\frac{Z-S}{Y-W_2}\right)^{-\beta} Z$. In the symmetric case without peace-building this is equal to $\left(\frac{\alpha}{\gamma}\right)^{-2\alpha} \left[\frac{(1-\gamma)X}{2-\gamma}\right]^{2\alpha} Z^\gamma$ where $\alpha = \frac{1-\gamma}{2}$. With perfect peace-keeping in symmetric equilibrium, it equals $\left(\frac{\alpha}{\gamma}\right)^{-2\alpha} X^{2\alpha} (Z-S)^\gamma$. It is easy to see that perfect peace-building is beneficial if the resource requirement for it satisfies the condition:

$$S < \frac{[f(\gamma) - 1] Z}{f(\gamma)}, \text{ where } f(\gamma) \equiv \left(\frac{2-\gamma}{1-\gamma}\right)^{\frac{1-\gamma}{\gamma}}. \quad (14)$$

Combining (12) and (14) and noticing also that the feasibility condition $S \leq \frac{X}{\xi}$ must hold, it turns out that perfect peace-building increases the welfare of country 3 if $\frac{3-\gamma-2\sqrt[2]{2-\gamma}}{2\xi} < S < \min\left\{\frac{X}{\xi}, \frac{[f(\gamma)-1]Z}{f(\gamma)}\right\}$

Proposition 3 *Outside countries are more likely to eliminate conflict perfectly the more efficient the peace-building operations are (ξ high), and the more dependent the consumer welfare is on goods supplied by the non-conflict countries (γ large). Perfect peace-building is also more likely the richer the non-conflict countries are. Poor non-conflict countries will not engage in pre-conflict peace-building. Perfect peace-building requires that conflicting countries' income is high enough, i.e. $X > \xi S$.*

Proposition 3 can be used to highlight several issues. First, as such it provides an explanation for peace-building conducted by outside countries. This holds especially for wealthy outside countries. It also shows that the

attempts of poor outside countries to build peace are likely to fail, i.e. they cannot provide enough resources for the operations to be successful. This is the recent experience in Africa (Andreatta et. al., 2000), where peace-building operations have been allocated to smaller countries without the burden not being shared by all African countries.

One must also consider the fact that in reality there is not only one single outside country but a large number of them. Proposition 3 tells that as a they group have an incentive to engage in ex ante peace-bulding. However, due to well-known Olsonian collective action problems, they may not be able to coordinate the process. One way for them to coordinate their decisions and commit to the peace-building process is to establish separate peace-keeping organizations with clearly defined national contributions to ensure the financial viability of these organizations. Perhaps one way to understand UN role and current EU plans to establish joint peace-building forces is exactly this.

Another interesting implication of Proposition 3 is that perfect peace-building is not feasible in the poorest countries (i.e. if X is small). The intuition is that if conflicting countries are very poor, there is not much resources for the trade anyhow, and thus no incentives for the third country to be involved. Again referring to the African experience it may be a reason why the conflicts there seem to be hard to mitigate. Interestingly, Doyle and Sambanis (2000) have found that an index of the level of development has a positive impact on the success of peace-building processes: The higher the index the higher the probability of success. Their index contains the level of GDP of the region in conflict as one building block. It is also important to note that unless pre-conflict peace-building operations are controlled for, an empirical study on conflicts shows that an increase in national income of the conflicting country decreases the probability of conflict (Collier and Hoeffler, 2000). Since peace building seems to be more effective the more developed the country is, it may be suggested that at least part of the effect

that investment in conflict decreases as income increases comes through peace-building²².

There are many other means of pre-conflict peace-building than just establishing conflict-free zones or securing property rights (Collier, 2000; Doyle and Sambanis 2000 and Nafziger, 2004 give examples). For example, the efficiency of arms investment in conflict areas can be directly reduced by employing the peace-keeping forces with superior weaponry. This can be analyzed e.g. by specifying that the effective arms investment with a gross outlay of W_i is just $(1 - \nu_i S) W_i$. It is straightforward to see that this reduces incentives to invest in arms and thus mitigates conflict even for small investment in peace-keeping. This effect also provides incentives for outside countries actually to engage in the operation. One interesting possibility is that arms investments require as inputs all the goods, but there is some substitutability. With this specification one can analyze arms embargoes and such type of policies. This, however, requires some modifications to the model and is excluded from the present discussion.

2.2 Post-conflict peace-building

Let us next move on to post-conflict peace-building. Although pre-conflict peace-keeping would be superior so as to avoid both human casualties and material damages, unfortunately pre-conflict peace-keeping operations often fail and post-conflict measures are needed. We analyze two types of post-conflict peace-keeping operations. First we look at policies that try to re-establish the pre-conflict property rights (e.g borders between countries/regions) on the endowments now partly destroyed by the war. Secondly, we consider policies that repair war damages. Repairing war damages could mean for example direct transfers to the people impoverished by the

²²It must be noted that peace-building also makes the Cobb-Douglas function consistent with the empirical evidence when discussing on the problem how possibilities for international trade affect conflicts (see Haaparanta and Kuisma, 2005).

war or clearing up of mine-fields after the conflict.

2.2.1 Reestablishment of property rights

Consider country 1 in the post-conflict situation. It now owns share ϕ of all the post-conflict resources in countries 1 and 2. In particular, after conflict it owns only share ϕ of the resource X it owned initially completely. Let us assume that a policy S chosen by country 3 distributes back a share λS , $\lambda > 0$ from the resource looted by country 2. Country 1's post conflict share of X thus becomes $\phi + \lambda S(1 - \phi)$ which is equivalent to $(1 - \lambda S)\phi + \lambda S$. Obviously, at the same time it loses partly its share in resource Y by getting to own share $(1 - \lambda S)\phi$. Thus, country 1's total post-conflict income including the peace-keeping effect is:

$$\psi = p_x [(1 - \lambda S)\phi + \lambda S](X - W_1) + p_y (1 - \lambda S)\phi(Y - W_2) \quad (15)$$

Since the total post-conflict endowments of x and y do not depend on the peace-keeping operation, the world market prices are given by (7). The first order condition for the military investment by country 1 can be written (after a slight manipulation) in the following simple form:

$$(1 - \lambda S)(1 - \gamma) \frac{\partial \phi}{\partial W_1} - [(1 - \gamma)(1 - \lambda S)\phi + \alpha \lambda S] \frac{\alpha}{X - W_1} = 0 \quad (16)$$

We focus again on the symmetric equilibrium assuming that $X = Y$ and $W_1 = W_2$. From (16) one can directly calculate that the investment in arms in symmetric case is:

$$W = \frac{X}{1 + g(S)}, g(S) \equiv (1 - \gamma) \left(1 + \frac{\lambda S}{1 + \lambda S} \right) \quad (17)$$

It is clear that investment is definitely smaller than it would be without the peace-keeping operation; we may recall that the solution in symmetric Cobb-Douglas case without peace-keeping operations is $W = \frac{X}{(2-\gamma)}$ which is always higher than the above solution. The arms investment decreases when the operation becomes more extensive, i.e. when S grows. The intuition is

straightforward: incentives to conflict diminish because reestablishment of property rights reduces the catch one can get through the conflict. This holds for both one's own endowment and the foreign endowment.

The problem with this policy is that country 3 does not have any incentive to exercise it *ex post*. From country 3's point of view peace-keeping just uses its resources which are redistributed to conflicting countries and does not change the amount (increase) them. Evidently, this improves its terms-of-trade, but as was argued above, terms-of-trade manipulation via destruction of own resources is never beneficial. Hence, this policy is not credible in the post-conflict situation.

Viewed from the pre-conflict situation the policy may be beneficial because it will increase the supply of x and y in the post-conflict exchange, as conflict investment is decreased. It is easy to show that in the symmetric equilibrium:

$$\frac{\partial u_3}{\partial S} = -\gamma - 2\alpha \left(\frac{Z - S}{X - W} \right) \frac{\partial W}{\partial S}. \quad (18)$$

Evaluated at $S = 0$ the above equation becomes:

$$\frac{\partial u_3}{\partial S}_{S=0} = -\gamma + \frac{\lambda(1-\gamma)(2-\gamma)Z}{X} \quad (19)$$

which is positive if

$$\frac{Z}{X} > \frac{\gamma}{\lambda(1-\gamma)(2-\gamma)} \quad (20)$$

We have now established the following result

Proposition 4 *Post-conflict restoration of property rights is not a time-consistent policy without pre-commitment, because the third country does not have any incentive to exercise that policy ex post. However, by committing to the policy ex ante it is beneficial for third country if $\frac{Z}{X} > \frac{\gamma}{\lambda(1-\gamma)(2-\gamma)}$.*

Proposition 3 together with Proposition 4 highlights the importance of commitment to peace-keeping actions, if the actions protect property rights.

They both underline the incentives of rich countries to provide efficient peace-keeping. Ex ante commitment to post-conflict peace-building is beneficial if the outside country (countries) are rich enough compared to the conflicting countries, but it is important that outside countries commit to the policy beforehand. That way they are able to increase the post-conflict amounts of x and y , as the commitment to restore property rights decreases the conflict investments of countries 1 and 2. It is important to note that ex post the restoration of property rights only uses outside country's resources and is not profitable. For poor outside countries it is not advantageous to use their own resources at all to make more x and y available after conflict.

2.2.2 Reparation of war damages

Typically in civil conflicts, war damages can become severe while the resources available to consumption and production are low. The reparation of war damages can be one means of post-conflict peace-building. When war damages are repaired, it effectively means that some of the resources that were destroyed during the war will become available again for market exchange after the conflict. There are two ways postulate this, both of which lead to equivalent formulations. Let us begin by assuming that the authorities of the conflicting countries cannot use the future anticipated reparation payments to finance the military investment. In other words, they will only get a share of the destroyed resources back after the conflict. We also assume that countries get war damages paid in proportion of the post-conflict endowments, which means that country 1's post-conflict income is:

$$\phi [p_x ((X - W_1) + \varphi_1 S W_1) + p_y ((Y - W_2) + \varphi_2 S W_2)] \quad (21)$$

where $\varphi_i S$, is the share of war damages repaired such that $\varphi_i > 0, 0 \leq S \leq \frac{1}{\varphi}$. Similar expression holds for country 2.

The other possibility is to assume that the authorities can "borrow" against the war reparation payments. In this case we can capture the ef-

fects of peace-keeping action by setting $(1 - \varphi_i S) W_i$ to be the damage after reparation. Country 1's post-conflict income in this case is:

$$\phi [p_x (X - (1 - \varphi_1 S) W_1) + p_y ((Y - (1 - \varphi_2 S) W_2))] \quad (22)$$

It is obvious that the two cases are identical. The first order condition for the military investment by country 1 is now:

$$\frac{\partial \phi}{\partial W_1} - \phi \alpha \left(\frac{1 - \varphi_1 S}{X - \widehat{W}_1} \right) = 0, \text{ where } \widehat{W}_1 \equiv (1 - \varphi_1 S) W_1 \quad (23)$$

In the symmetric case this gives the solution:

$$W = \frac{X}{2 - \gamma - \varphi S} \quad (24)$$

This implies immediately that war reparations will increase the intensity of military conflict. The intuition is straightforward: war reparations, when anticipated, reduce the marginal cost of arms build-up, since the effective endowments are not reduced by the full amount of investment in arms²³. One might expect that country 3 would never want to commit to war reparations policy before the conflict. But, it turns out to be a time consistent policy. Country 3 has incentive to war reparations after the conflict, provided that it is rich enough. The impact of a marginal increase in peace-keeping expense on country 3 welfare can be expressed as:

$$\frac{\partial u_3}{\partial S} = -\gamma + \frac{2\alpha (Z - S) \varphi W}{X - (1 - \varphi S) W}$$

When this is evaluated at $S = 0$, the condition for there to exist an incentive for country 3 to run the peace-keeping operation becomes:

$$Z > \frac{\gamma}{\varphi}$$

²³This is somewhat a similar problem as with foreign aid to conflicting countries. Due to the fungibility of aid, it can be used to peaceful or warlike purposes. Thus, foreign aid or debt relief to conflicting countries may, in fact, increase military investments. See Addison and Murshed (2003).

Hence, once again, if the outside country is rich enough it has an incentive to secure peace in conflicting countries. It is noteworthy that the requirement is more tighter the smaller is the size of the operation and the more important good z is in consumption. We have now established

Proposition 5 *Repairing war damages is a time consistent policy for a rich enough non-conflict country. As a tool to mitigate conflicts it is counterproductive, since it increases military investments.*

The intuition is that war reparations increase the supply of goods imported by the non-conflict country. Thus, the policy improves its terms of trade both directly and indirectly. The problem with this policy is that if conflicting countries see that country 3 finds the post-conflict reparations advantageous, they will obviously increase their conflict investments. If a country 3 could credibly tie its hands so that conflicting countries would not expect these kind of operations, the magnitude of conflict would be decreased.

Proposition 5 and Proposition 4 point out some difficulties in peace-keeping. Outside countries (if rich enough) have an incentive to secure peace by securing the conflicting parties' initial property rights. Proposition 4 shows that this works perfectly if it is the only policy used and if policy measures are announced ex ante. Proposition 5, however, tells that it is not the only policy that the outside countries engage in, as the ex post reparation of war damages can be profitable for them too. As long as the initial endowments are not completely secured, though, conflicting countries have an incentive to invest in arms, since war reparations expand the resources they can use in the conflict. Non-conflicting countries can prevent this by extending the protection of property rights before the conflict beyond those stated in Proposition 4.

3 Conclusions

This paper has focused on the peace-building by outside countries in local conflicts in the third world. The issue is extremely important, since presently almost all conflicts take place in developing countries and it seems highly probable that conflicts are a significant factor prohibiting growth and development in these regions. Furthermore, the detrimental consequences of civil wars are not restricted to the areas in war, but conflicts often affect neighboring regions too. Our work aims to contribute to the understanding of the role of the external countries from economic standpoint. The model used in this paper is an extension of the basic model of international trade and conflicts to a three-country world. We have argued that the intensity of conflicts can decline when the conflicting country's income increases if peace building is taken into account. This provides a new, and complementary to other explanations, explanation for the empirical observation that the relationship between conflicts and income levels is non-monotonous. We also found that incomplete pre-conflict peace-building operations will increase military buildup. Successful pre-conflict peace-building requires that the outside countries are sufficiently rich, and that the conflicting countries also have a high enough income (which conforms with the existing empirical evidence). Finally, it turns out that post-conflict re-establishment of property rights is not time-consistent unless the peacebuilding countries are rich enough. Repairing war damages is counterproductive as it increases military investments if announced in advance.

In our framework the outside countries have an incentive to either ex ante or ex post peacebuilding purely from selfish interests. These interests do not always help to mitigate conflicts but can do so (in case of ex ante peacebuilding). The incentives work through terms of trade, peacebuilding keeps the supplies of imported goods at higher level. In this we have managed to formalise the notion that concern for supplies of some goods may dictate foreign policy actions. At the same time the model may help to

understand why peacekeeping by poor countries may not be effective.

The model we have used omits many important aspects relevant for conflicts. The extension to include production (like in Becsi and Lahiri 2005) would be quite straightforward. Inclusion of uncertainty and organisation of conflict activities, like e.g. modelling of political conflict and organisation of guerilla groups, would certainly add much to the analysis and should be taken into account in future research.

References

Addison, T. and Murshed, S.M. (2002), "Credibility and Reputation in Peacemaking", *Journal of Peace Research*, Vol. 39, No. 4, pp. 487-501

Addison, T. and Murshed, S.M. (2003), "Debt Relief and Civil War", *Journal of Peace Research*, Vol. 40, No. 2, pp. 159-176

Addison, T., Chowdhury, A.R. and Murshed, S.M. (2004), "By How Much Does Conflict Reduce Financial Development?", UNU/WIDER Discussion Paper no. 2002/48

Andreatta, F., Ardeni P. G. and Pallotti A. (2000), "*Swords and Plowshares: Regional Trade Agreements and Political Conflict in Africa*", paper presented at the workshop on New Forms Of Integration In Emerging Africa organized by the OECD Development Centre, 13 October, Genova

Becsi Z. and S. Lahiri (2005) Bilateral War in a Multilateral World, mimeo Southern Illinois University.

Colletta, N.J. and Cullen, M.L. (2000), "*Violent conflict and the transformation of social capital : lessons from Cambodia, Rwanda, Guatemala, and Somalia*", World Bank report

Collier P., (1999), "*On the Economic Consequences of Civil War*", Working Paper, The World Bank

Collier P., (2000), "*Economic Causes of Civil Conflict and their Implications for Policy*", Working Paper, The World Bank.

Collier P. and Hoeffler, A., (1998), "On Economic Causes of Civil War", *Oxford Economic Papers*, Vol 50, No. 4, pp. 563-573

Collier P. and Hoeffler, A., (2001), "*Greed and Grievance in Civil War*", Working Paper, The World Bank.

Collier P., Elliott, V., Hegre, H., Hoeffler, A., Reynal-Querol, M and Sambanis, N., (2003), "*Breaking the Conflict Trap: Civil War and Development Policy*", A World Bank Policy Research Report, Oxford University Press.

Dhariwal, R., (2004), "*Tourist Arrivals in India: How Important are Domestic Disorders*", paper presented at the UNU-WIDER Conference "Making Peace Work", 4-5 June 2004, Helsinki

Doyle M. and Sambanis N., (2000), "International Peace Building: A Theoretical and Quantitative Analysis", *American Political Science Review*, 94:4, pp. 779-801

Dutta, I. and Mishra, A. (2003), "*Does Inequality lead to Conflict?*", paper presented in UNU Wider conference Making Peace Work, 4-5 June 2004, Helsinki

Elbadawi, I.A. and Sambanis, N., (2000), "*External Interventions and the Duration of Civil Wars*", Working Paper, The World Bank

Gershenson, D., (2002), "Sanctions and Civil Conflict", *Economica*, Vol. 69, Iss. 274, pp. 185-206

Grossman, H. (1991), "A General Equilibrium Model of Insurrections", *The American Economic Review*, Vol. 81, no. 4, pp. 912-921

Grossman H. (2001) The Creation of Effective Property Rights, *American Economic Review, Papers and Proceedings*, Vol. 91, No. 2, pp. 347-352

Haaparanta P. and Kuisma, K. (2005), "*Trade, War and Peace*", Working Paper, Helsinki School of Economics

Hirshleifer, J. (1995), "Anarchy and Its Breakdown", *Journal of Political Economy*, February, Vol. 103, No. 1, pp. 26-52.

Hirshleifer, J. (2000), "The Macrotechnology of Conflict", *The Journal of Conflict Resolution*, Vol. 44, No. 6, pp. 773-792

Jehle G . and Reny, P. (2001), *Advanced Microeconomic Theory*, Addison-

Wesley.

Malone, D.M. and Nitzsche, H. (2004), "*Economic Agendas in Civil Wars: What We Know, What We Need to Know*", paper presented at the UNU-WIDER Conference "Making Peace Work, 4-5 June 2004, Helsinki

McDonald, P.J. (2004), "Peace through Trade or Free Trade?", *Journal of Conflict Resolution*, Vol. 48, Iss. 4, pp. 547-573

Nafziger, W.E. (2004), "*Peacemaking as Preventing War before It Starts: Economic Development, Inequality, and Humanitarian Emergencies*", paper presented at UNU/WIDER conference on "Making Peace Work", Helsinki, Finland, 4-5 June 2004

Polachek, S.W.(1997), "Why Democracies Cooperate More and Fight Less: The Relationship Between International Trade and Cooperation", *Review of International Economics*, 5(3), pp. 295-309

Pottebaum, D.A. and Kanbur, R., (2004), "*Civil War, Public Goods and the Social Wealth of Nations*", paper presented at UNU/WIDER conference on "Making Peace Work", Helsinki, Finland, 4-5 June 2004

Regan, P.M. (2002), "Third Party Intervention and the Duration of Intra-state Conflict", *Journal of Conflict Resolution*, Vol. 46, No. 1, February 2002, pp. 55-73.

Sandler, T. and Hartley, K., (1995), "*The Economics of Defence*", Cambridge University Press, Cambridge, Great Britain

Skaperdas, S. and Syropoulos, C. (1996), "Competitive trade with conflict", in Garfinkel M. and S. Skaperdas (eds.) *The political economy of conflict and appropriation*, Cambridge University Press.

Skaperdas, S. and Syropoulos, C. (2001), "Guns, Butter, and Openness: On the Relationship between Security and Trade", *American Economic Review*, Papers and Proceedings, Vol. 91, No. 2, pp. 353-357.

Stewart, F. (2002), "*Horizontal Inequalities: A Neglected Dimension of Development*", QEHWPS81, University of Oxford

3.1 Appendix

The first order condition (9) is generated by maximizing the indirect utility function $V(p_i, \Psi_i)$ where $i = 1, 2$. We obtain this function by maximizing $u_i = C_{xi}^\alpha C_{yi}^\beta C_{zi}^\gamma$, with respect to budget constraint. From this problem we get the consumption levels C_{ji} as a function of prices and incomes. C_{ji} denotes the consumption of good j in country i . Next, we maximize the indirect utility function with respect to arming decisions W_i taking into account the success function (1), budget equation (2) and prices (7). The problem for country 1 is:

$$\begin{aligned} \max_{W_1} V(p_y, p_z, \Psi_1) \\ s.t. \end{aligned} \tag{25}$$

$$p_x [\xi_1 S + \phi (X - \xi_1 S - W_1)] + p_y \phi (Y - \xi_2 S - W_2) - p_x C_{x1} - p_y C_{y1} - C_{z1} = 0$$

Using the envelope theorem the first order condition for this problems can be written as equation (8) in the text.