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# Innovation, Imitation, Growth, and Capital Market Imperfections

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# Innovation, Imitation, Growth, and Capital Market Imperfections*

## Abstract

This paper analyzes the interaction of innovation and imitation in a growing economy. It is assumed that because firms cannot use their innovations or imitations as collateral, they finance R&D by issuing shares to the households. The main results are the following. When the mark-ups are close to uniform in all industries, a small subsidy to imitative R&D, a uniform subsidy to all R&D and the decrease in product market competition (PMC) are growth enhancing. In the first-best social optimum, PMC is welfare diminishing. If the government uses the uniform subsidy to all R&D optimally, then there is an "inverted-U" relationship between PMC and social welfare.

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# 1  Introduction

This paper examines technology policy in an economy with innovation and imitation. Through the development of new products, an innovator achieves a temporary advantage earning monopoly profits. This advantage ends when an imitator succeeds in copying the innovation, enters the market and starts competing with the innovator. In this paper, I assume that ($a$) firms cannot borrow without collateral, and ($b$) they cannot use their immaterial property (e.g. innovations or imitations) as collateral. It is instructive to see how these capital market imperfections affect the prospects of technology policy.

There is already a large literature concerning technology policy with imitation starting from Segerstrom (1991), who presents a model with the following properties: ($i$) Producers collude. ($ii$) R&D firms are subject to constant returns to scale. ($iii$) R&D firms can both innovate and imitate. ($iv$) Outsiders can innovate a new quality of product at the same cost as the incumbents. Segerstrom shows that innovation subsidies speed up growth, but promote welfare only if innovative effort is initially large enough.

Segerstrom's (1991) model is challenged by the following papers. Walz (1995) replaces cooperation ($i$) by Cournot competition and finds out that in some circumstances innovation subsidies may even retard economic growth. Davidson and Segerstrom (1998) replace constant returns ($ii$) by decreasing returns to scale and show that innovation subsidies promote growth but imitation subsidies do the opposite. Zeng (2001) obtains more or less similar results by rejecting ($iii$) and assuming that innovation improves product quality while imitation expands product variety.

Property ($iv$) in Segerstrom's model leads to leapfrogging: innovations will always be performed by outsiders and the current industry leaders will be replaced. The following papers eliminate this unrealistic outcome. Aghion et al. (1997, 2001) construct models where technological laggards must first catch up with the leading-edge technology before battling "neck-to-neck" for

technological leadership in the future. Mukoyama (2003) constructs a model in which only leaders can conduct next-round innovation, while outsiders can become leaders by imitation. These papers establish an "inverted-U" relationship between competition and growth, but with the following difference. Aghion et al. (1997, 2001) represent competition by the elasticity of substitution between the firms' products, while Mukoyama (2003) uses the proportion of two-producer industries in the economy for that purpose. In this paper, I preserve Mukoyama's assumption on cumulative technology, but measure competition by the elasticity of substitution.

All papers referred above assume that R&D firms can borrow any amount of capital and the household can make any investment at a given market interest rate. In such a case, firms decide on R&D and households are protected from uncertainly through diversification in the market portfolio. Because that assumption is in contradiction with the entire literature of venture capital,[1] it is instructive to assume for a change that firms cannot borrow without collateral and immaterial property cannot be used as collateral. Firms must then finance their R&D through issuing shares and households purchasing these shares face the uncertainly associated with investment.

The remainder of this paper is organized as follows. Section 2 introduces the basic structure of the model. Sections 3 and 4 consider firms in production and R&D. Section 5 examines households deciding on saving, section 6 general equilibrium, and section 7 the prospects of public policy. Optimal elasticity rules for subsidies and competition policy are presented.

## 2   The model

I extend Wälde's (1999a, 1999b) growth model with risk-averting households by replacing the sector of innovating firms by a large number of industries which innovate or imitate. Following Mukoyama (2003), I eliminate leapfrog-

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[1]A nice summary of this literature in given in Gompers and Lerner (1999).

ging through the assumption that in the product market only leaders can innovate. The model can then be characterized as follows:

(i) Labor is homogeneous and inelastically supplied. It is used in innovation, imitation or the production of the intermediate goods.

(ii) Competitive firms produce the consumption good from a great number of intermediate goods according to Cobb-Douglas technology.

(iii) All intermediate-good firms produce one unit of their output from one labor unit. Each intermediate good is produced by a separate industry and composed of the products of the firms in the industry through CES technology. The elasticity of substitution between any pair of the products is used as the measure of product market competition (PMC).

(iv) R&D firms innovate or imitate. Imitation is necessary for an outsider to become an innovator. A successful imitator enters the product market and starts an innovation race with the old producers. A successful innovator becomes a monopolistic producer of the latest technology until its technology is imitated.

(v) R&D firms finance their expenditure by issuing shares. The households save only in these shares. Each R&D firm distributes its profit among those who had financed it in proportion to their investment in the firm. The subsidies to R&D are financed by lump-sum taxes.

# 3 Production

There is a great number of intermediate-good industries that are placed over the limit $[0, 1]$. The representative consumption-good firm makes its output $y$ from the products of all intermediate-good firms through technology

$$\log y = \int_0^1 \log[B_j x_j] dj, \quad x_j = \left[\sum_{\kappa=1}^{a_j} x_{j\kappa}^{1-1/\varepsilon}\right]^{\varepsilon/(\varepsilon-1)}, \quad \varepsilon > 1, \qquad (1)$$

where $B_j$ is the productivity parameter in industry $j$, $a_j$ the number of firms in industry $j$, $x_j$ the quantity of intermediate good $j$, $x_{j\kappa}$ the output of firm $\kappa$ in industry $j$, and $\varepsilon$ the elasticity of substitution between any pair of the products of the firms in the same industry. The consumption-good firm maximizes its profit

$$\Pi \doteq Py - \int_{j \in [0,1]} \sum_{\kappa=1}^{a_j} p_{j\kappa} x_{j\kappa} dj$$

by its inputs $x_j$, taking the price $P$ of the consumption good and the input prices $\{p_{j\kappa}\}$ as fixed. I normalize total consumption expenditure $Py$ at unity. Because the consumption-good firm is subject to constant returns to scale, we then obtain

$$Py = 1, \quad \Pi = 0, \quad \sum_{\kappa=1}^{a_j} p_{j\kappa} x_{j\kappa} = 1 \text{ for all } j,$$

$$p_{j\kappa} = P \frac{\partial y}{\partial x_j} \frac{\partial x_j}{\partial x_{j\kappa}} = P \frac{y}{x_j} \frac{\partial x_j}{\partial x_{j\kappa}} = x_j^{1/\varepsilon - 1} x_{j\kappa}^{-1/\varepsilon} \text{ for all } j \text{ and } \kappa. \qquad (2)$$

All intermediate-good firms produce one unit of their output from one labor unit. Technological change is random. I assume that a successful innovator in industry $j$ is able to make a perfect substitute for intermediate good $j$, which is a composite product of the outputs of all incumbent firms in industry $j$. I assume furthermore that the innovator's product provides exactly the constant $\mu > 1$ times as many services as the intermediate good of earlier generation. To push old producers out of the market, the innovator in industry $j$ chooses the price $p_{j1} = \mu w$ for its product, where $w$ is the wage for labor.[2] From $p_j = \mu w$ and (2) it then follows that

$$x_j = x_{j1} = \frac{1}{p_{j1}} = \frac{1}{\mu w} \text{ and } \pi_j = \left(1 - \frac{1}{\mu}\right) p_{j1} x_{j1} = 1 - \frac{1}{\mu} \text{ for } a_j = 1. \qquad (3)$$

The innovator is always the first leader in the industry. A successful imitator of the state-of-art good is able to make a close substitute for the

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[2]Because the productivity of the old producers is equal to $1/\mu$, with the innovator's mark-up rule $p_{j1} = \mu w$ the price index of their composite product $x_j$ must be equal to $\mu$. Thus, none of the old producers can charge a price greater than its marginal cost $\mu$.

product of the innovator. Thus with each imitation, the number of leaders and products increases by one. I assume that all leaders $1, ..., a_j$ in industry $j$ behave in Cournot manner, taking each other's output levels as given.[3] Given (1) and (2), leader $\kappa$ in industry $j$ maximizes its profit

$$\pi_{j\kappa} = p_{j\kappa} x_{j\kappa} - w x_{j\kappa} = x_j^{1/\varepsilon - 1} x_{j\kappa}^{1-1/\varepsilon} - w x_{j\kappa}, \qquad (4)$$

by its output $x_{j\kappa}$, assuming that the output levels $x_{j\imath}$ for the other leaders $\imath \neq \kappa$ in industry $j$ are constant. It therefore sets the wage $w$ equal to the marginal product of labor:

$$w = p_{j\kappa} + x_{j\kappa} \left[ \frac{\partial p_{j\kappa}}{\partial x_j} \frac{\partial x_j}{\partial x_{j\kappa}} + \frac{\partial p_{j\kappa}}{\partial x_{j\kappa}} \right] = p_{j\kappa} + x_{j\kappa} \left[ \left( \frac{1}{\varepsilon} - 1 \right) \frac{p_{j\kappa}}{x_j} \left( \frac{x_j}{x_{j\kappa}} \right)^{1/\varepsilon} - \frac{1}{\varepsilon} \frac{p_{j\kappa}}{x_{j\kappa}} \right]$$
$$= p_{j\kappa} \left[ 1 + \left( \frac{1}{\varepsilon} - 1 \right) \left( \frac{x_j}{x_{j\kappa}} \right)^{1/\varepsilon - 1} - \frac{1}{\varepsilon} \right] = p_{j\kappa} \left( 1 - \frac{1}{\varepsilon} \right) \left( 1 - \frac{1}{a_j} \right).$$

Because this condition holds for all competitors $\kappa = 1, ..., a_j$, noting (2) and (4), we obtain the symmetry $x_{j\kappa} = x_{j1}$ for all $\kappa$, and

$$p_{j\kappa} = \Phi(a_j)w, \quad \Phi(a_j) \doteq (1 - 1/\varepsilon)^{-1}(1 - 1/a_j)^{-1}, \quad \Phi' < 0, \quad a_j p_{j\kappa} x_{j\kappa} = 1,$$
$$\pi_{j\kappa} = (p_{j\kappa} - w)x_{j\kappa} = \left[ 1 - \frac{1}{\Phi(a_j)} \right] p_{j\kappa} x_{j\kappa} = \left[ 1 - \frac{1}{\Phi(a_j)} \right] \frac{1}{a_j} \doteq \pi(a_j), \quad \pi' < 0,$$
$$x_j = a_j^{\varepsilon/(\varepsilon-1)} x_{j\kappa} = a_j^{1/(\varepsilon-1)} / [\Phi(a_j)w]. \qquad (5)$$

I assume that the entry of the second leader decreases the first leader's mark-up $p_{j1}/w$ from $\mu$ to $\Phi$:

$$\Phi(a_j) < \mu. \qquad (6)$$

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[3]Alternatively, one could introduce a more general framework through the assumption that leader $\kappa$ estimates the response of the other leaders $\ell \neq \kappa$ by $dx_{j\ell}/dx_{j\kappa} = \varpi \, x_{j\ell}/x_{j\kappa}$, where $\varpi < 1$ is a constant. In that model, the special case $\varpi = 0$ corresponds to Cournot competition. The general case $\varpi < 1$ would with some complication produce the same results as this paper. Kreps and Scheinkman (1983) provide a rather convincing argument for the Cournot assumption. They show that the Cournot game can be interpreted as a result of a two-stage game in which the firms first choose their capacities and then sell their products at the market-clearing prices. The unique equilibrium is that in the first stage the firms fix their capacities at the Cournot output levels.

Anyone investing in firms attempts to maximize his expected profit. The innovator is the first leader in an industry. If one invests in imitation to enter an industry with one leader, then its prospective profit is $\pi_{j\kappa}\big|_{a_j=2}$, but if it invests (with the same cost) in imitatation to enter an industry with more than two leaders, then its prospective profit is $\pi_{j\kappa}\big|_{a_j>2}$. Because, by (5), the profit is smaller with more than two leaders, $\pi_{j\kappa}\big|_{a_j=2}> \pi_{j\kappa}\big|_{a_j>2}$, investors invest in imitation only in one-leader industries. I summarize:

**Proposition 1** *Each industry has one or two leaders. In one-leader industries the followers imitate and in two-leader industries the leaders innovate.*

I denote the set of one-leader industries by $\Theta \subset [0,1]$, the relative proportion of one-leader industries (two-leader industries), $\alpha$ ($\beta$) by

$$\alpha = \int_{j\in\Theta} dj, \quad \beta \doteq \int_{j\notin\Theta} dj = 1 - \alpha. \tag{7}$$

Noting this, $a_j = 2$, (3), (5) and (6), we obtain the following result:

**Proposition 2** *A firm's profit is $\pi_\alpha \doteq 1 - \frac{1}{\mu}$ in one-leader industries $j \in \Theta$ and $\pi_\beta \doteq \frac{1}{2}\left(1 - \frac{1}{\phi}\right)$ in two-leader industries $j \notin \Theta$, and the total output of the industry is $x_\alpha \doteq \frac{1}{\mu w}$ for one-leader industries $j \in \Theta$ and $x_\beta = \frac{1}{\phi w}$ for two-leader industries $j \notin \Theta$, where $\phi \doteq \Phi(2) \in (0,\mu)$ is the mark-up factor $\phi$ in the two-leader industries $j \notin \Theta$. The higher the elasticity of substitution, $\varepsilon$, the closer the mark-up factor $\phi$ is to one.*

Noting proposition 2 and equations (1), (3) and (7), and summing up throughout all firms and industries, we obtain that the employment of labor in production, $x$, and total output $y$ are determined as follows:

$$x \doteq \alpha x_\alpha + (1-\alpha)x_\beta = \frac{\varphi}{w}, \quad \varphi(\alpha,\phi) \doteq \frac{\alpha}{\mu} + \frac{1-\alpha}{\phi} < \frac{1}{\phi}, \quad \frac{\partial\varphi}{\partial\alpha} = \frac{1}{\mu} - \frac{1}{\phi} < 0,$$

$$\frac{\partial\varphi}{\partial\phi} = \frac{\alpha-1}{\phi^2} < 0, \quad x_\alpha = \frac{x}{\mu\varphi}, \quad \frac{\partial}{\partial\phi}\left(\frac{x_\alpha}{x}\right) > 0, \quad x_\beta = \frac{x}{\phi\varphi} > 0, \quad \frac{\partial}{\partial\phi}\left(\frac{x_\beta}{x}\right) < 0,$$

$$y = Bx_\alpha^\alpha x_\beta^{1-\alpha} = \chi(\alpha,\phi)xB, \quad \chi(\alpha,\phi) \doteq \frac{\mu^{-\alpha}\phi^{\alpha-1}}{\varphi(\alpha,\phi)}, \quad \log B \doteq \int_0^1 \log B_j dj,$$
$$\tag{8}$$

where $x$ is employment, $\varphi = wx$ wage expenditure and $B$ the average level of productivity in production. More intense competition (i.e. a smaller $\phi$) increases employment $x$ and total wages in production, $\partial \varphi / \partial \phi < 0$. Because innovating two-leader industries $j \notin \Theta$ employ more than imitating one-leader industries $j \in \Theta$, a decrease in the proportion $\alpha$ of imitating industries raises employment $x$ and total wages $\varphi$ in production, $\partial \varphi / \partial \alpha < 0$.

Because innovating two-leader industries $j \notin \Theta$ employ more than imitating one-leader industries $j \in \Theta$, a smaller proportion $\alpha$ of imitating industries raises employment $x$ and total wages $\varphi$ in production. Because by (8),

$$\frac{1}{\chi}\frac{\partial \chi}{\partial \phi} = \frac{\partial \log \chi}{\partial \phi} = \frac{\alpha - 1}{\phi} - \frac{1}{\varphi}\frac{\partial \varphi}{\partial \phi} = \frac{1 - \alpha}{\phi}\Big(\frac{1}{\phi\varphi} - 1\Big) > 0,$$

we obtain the following result:

**Proposition 3** *More intense PMC (i.e. a lower mark-up $\phi$ in the two-leader industries) decreases the productivity $\chi$ of efficient labor, $\partial \chi / \partial \phi > 0$.*

Proposition 3 can be explained as follows. The problem is the maximization of total output $y = B x_\alpha^\alpha x_\beta^{1-\alpha}$ subject to the allocation of labor between innovation and imitation, $x = \alpha x_\alpha + (1 - \alpha)x_\beta$, keeping total employment in production, $x$, constant. Output $y$ is at the maximum, if all industries employ the same amount of labor, $x_\alpha = x_\beta$, and this is possible only if two-leader industries collude and set monopoly prices. More intense competition (i.e. a smaller $\epsilon$) transfers labor from one-leader into two-leader industries (i.e. $x_\alpha$ falls and $x_\beta$ rises by (8)). The greater the difference $x_\beta - x_\alpha$, the lower total output $y$ for given $x$.

## 4  Research

Given proposition 1, there are three types of R&D firms: the first leader (successful innovator), which I call firm 1, the second leader (successful imitator), which I call firm 2, and followers, which we call firm 0. In two-leader

industry $j \notin \Theta$, firms 1 and 2 innovate and no firm imitates. The techno-
logical change of firm $\kappa \in \{1, 2\}$ is characterized by a Poisson process $q_{j\kappa}$ in
which the arrival rate of innovations is given by

$$\Lambda_{j\kappa} = \lambda l_{j\kappa} + \xi l \text{ for } j \notin \Theta \text{ and } \kappa \in \{1, 2\}, \tag{9}$$

where $l_{j\kappa}$ is the firm's own input, $l$ total employment in R&D in the economy
and $\lambda > 0$ and $\xi > 0$ are constants. In the production function (9), the
term $\xi l$ characterizes the spillover of knowledge between R&D firms. During
a short time interval $d\nu$, there is an innovation $dq_{j\kappa} = 1$ in firm $\kappa$ with
probability $\Lambda_{j\kappa}d\nu$, and no innovation $dq_{j\kappa} = 0$ with probability $1 - \Lambda_{j\kappa}d\nu$.

In one-leader industry $j \in \Theta$, the representative follower (firm 0) imitates
and no firm innovates. The technological change of firm 0 is characterized
by a Poisson process $Q_j$ in which the arrival rate of imitations is given by

$$\Gamma_j = \gamma l_{j0} \text{ for } j \in \Theta, \tag{10}$$

where $l_{j0}$ is total imitative input in industry and $\gamma > 0$ a constant. During a
short time interval $d\nu$, there is an imitation $dQ_j = 1$ with probability $\Gamma_j d\nu$,
and no imitation $dQ_j = 0$ with probability $1 - \Gamma_j d\nu$.

The invention of a new technology in industry $j$ raises the number of
technology in that industry, $t_j$, by one and the level of productivity, $B_j^{t_j}$, by
$\mu > 1$. Given this and (8), the average productivity in the economy, $B$, is a
function of the technologies of all industries, $\{t_k\}$, as follows:

$$\log B^{\{t_k\}} \doteq \int_0^1 \log B_j^{t_j} dj, \quad B^{t_j+1}/B_j^{t_j} = \mu. \tag{11}$$

The arrival rate of innovations in industry $j \notin \Theta$ is the sum of the arrival
rates of both firms in the industry, $\Lambda_{j1} + \Lambda_{j2}$. The average growth rate of $B_j$
due to technological change in industry $j$ in the stationary state is then given
by $E\left[\log B_j^{t_j+1} - \log B_j^{t_j}\right] = (\Lambda_{j1} + \Lambda_{j2})\log\mu$, where $E$ is the expectation
operator.[4] Because only industries $j \notin \Theta$ innovate, then, noting (9), the

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[4]For this, see Aghion and Howitt (1998), p. 59.

average growth rate of the average productivity $B$ in the stationary state is

$$g \doteq \int_{j \notin \Theta} E\big[\log B_j^{t_j+1} - \log B_j^{t_j}\big] dj = (\log \mu) \int_{j \notin \Theta} (\Lambda_{j1} + \Lambda_{j2}) dj$$

$$= (\log \mu) \int_{j \notin \Theta} \big[\lambda(l_{j1} + l_{j2}) + 2\xi l\big] dj. \tag{12}$$

I use $g$ above as a measure of the growth rate of the economy.

Total employment in R&D is given by

$$l \doteq \int_{j \notin \Theta} (l_{j1} + l_{j2}) dj + \int_{j \in \Theta} l_j dj. \tag{13}$$

There exists a fixed number $N$ of households, each supplying one labor unit. Total labor supply $N$ is equal to inputs in production, $x$, and R&D, $l$:

$$N = x + l. \tag{14}$$

The government subsidizes R&D expenditures, but possibly at different rates in innovating and imitating industries. Given proposition 2, we obtain total expenditures from these subsidies as follows:

$$R \doteq \tau_\alpha \int_{j \in \Theta} w l_{j0} dj + \tau_\beta \int_{j \notin \Theta} (w l_{j1} + w l_{j2}) dj, \tag{15}$$

where $w l_{j0}$ is expenditure on imitation in firm 0 industry $j \in \Theta$, $w l_{j\kappa}$ expenditure on innovation in firm $\kappa \in \{1, 2\}$ in industry $j \notin \Theta$ and $\tau_\alpha \in (-\infty, 1)$ $\big(\tau_\beta \in (-\infty, 1)\big)$ is the subsidy to imitation (innovation). If the government cannot discriminate between innovation and imitation, then $\tau_\alpha = \tau_\beta$.

In industry $j \in \Theta$ firm 0 and in industry $j \notin \Theta$ firms 1 and 2 issue shares to finance their labor expenditure in R&D, net of government subsidies. Because the households invest in these shares, we obtain

$$\sum_{\iota=1}^{N} S_{\iota j0} = (1 - \tau_\alpha) w l_{j0} \text{ for } j \in \Theta,$$

$$\sum_{\iota=1}^{N} S_{\iota j\kappa} = (1 - \tau_\beta) w l_{j\kappa} \text{ for } \kappa \in \{1, 2\} \text{ and } j \notin \Theta, \tag{16}$$

where $wl_{j0}$ is the imitative expenditure of firm 0 in industry $j \in \Theta$, $\tau_\alpha$ the subsidy to it, $wl_{j\kappa}$ the innovative expenditure of firm $\kappa \in \{1, 2\}$ in industry $j \notin \Theta$, $\tau_\beta$ subsidy to it, $S_{\iota j0}$ ($S_{\iota j\kappa}$) household $\iota$'s investment in firm 0 in industry $j \in \Theta$ (firm $\kappa$ in industry $j \notin \Theta$), and $\sum_{\iota=1}^{N} S_{\iota j0}$ $\left( \sum_{\iota=1}^{N} S_{\iota j\kappa} \right)$ aggregate investment in firm 0 in industry $j \in \Theta$ (firm $\kappa$ in industry $j \notin \Theta$). Household $\iota$'s relative investment shares in the firms are given by

$$i_{\iota j0} \doteq \frac{S_{\iota j0}}{(1 - \tau_\alpha)wl_j} \text{ for } j \in \Theta; \quad i_{\iota j\kappa} \doteq \frac{S_{\iota j\kappa}}{(1 - \tau_\beta)wl_{j\kappa}} \text{ for } j \notin \Theta. \quad (17)$$

I denote household $\iota$'s income by $A_\iota$. Total income throughout all households $\iota \in \{1, ..., N\}$ is then equal to income earned in the production of consumption goods, $Py$, plus income earned in R&D, $wl$, minus government expenditures $R$ (= lump-sum taxes). Since $Py = 1$ by (2), this yields

$$\sum_{\iota=1}^{N} A_\iota = Py + wl - R = 1 + wl - R. \quad (18)$$

# 5 Households

The utility for risk-averting household $\iota \in \{1, ..., N\}$ from an infinite stream of consumption beginning at time $T$ is given by

$$U(C_\iota, T) = E \int_T^\infty C_\iota^\sigma e^{-\rho(\nu - T)} d\nu \text{ with } 0 < \sigma < 1 \text{ and } \rho > 0, \quad (19)$$

where $\nu$ is time, $E$ the expectation operator, $C_\iota$ the index of consumption, $\rho$ the rate of time preference and $1/(1-\sigma)$ is the constant relative risk aversion.

Because investment in shares in R&D firms is the only form of saving in the model, the budget constraint of household $\iota$ is given by

$$A_\iota = PC_\iota + \int_{j \in \Theta} S_{\iota j0} dj + \int_{j \notin \Theta} (S_{\iota j1} + S_{\iota j2}) dj, \quad (20)$$

where $A_\iota$ is the household's total income, $C_\iota$ its consumption, $P$ the consumption price, and $S_{\iota j0}$ ($S_{\iota j\kappa}$) the household's investment in firm 0 in industry

$j \in \Theta$ (firm $\kappa$ in industry $j \notin \Theta$). When household $\iota$ has financed a success-ful R&D firm, it acquires the right to the firm's profit in proportion to its relative investment share. Thus, I define:

$s_{\iota j \kappa}$    household $\iota$'s true profit from firm $\kappa$ in industry $j$ when the uncertainty in R&D is taken into account;

$i_{\iota j \kappa}$    household $\iota$'s investment share in firm $\kappa$ in industry $j$ [Cf. (17)];

$\pi_\alpha i_{\iota j \kappa}$   household $\iota$'s expected profit from firm $\kappa \in \{1, 2\}$ in industry $j \notin \Theta$ after innovation in firm $\kappa$ changes $j$ from a two-leader into a one-leader industry;

$\pi_\beta i_{\iota j 0}$   household $\iota$'s expected profit from firm 0 in industry $j \in \Theta$ after imitation in firm 0 changes $j$ from a one-leader into a two-leader industry.

The changes in the profits of firms in industry $j$ are functions of the increments $(dq_{j1}, dq_{j2}, dQ_j)$ of Poisson processes $(q_{j1}, q_{j2}, Q_j)$ as follows:[5]

$$ds_{\iota j \kappa} = (\pi_\alpha i_{\iota j \kappa} - s_{\iota j \kappa})dq_{j\kappa} - s_{\iota j \kappa}dq_{j(\zeta \neq \kappa)} \text{ when } j \notin \Theta;$$
$$ds_{\iota j 0} = (\pi_\beta i_{\iota j 0} - s_{\iota j 0})dQ_j \text{ when } j \in \Theta. \tag{21}$$

These functions can be explained as follows. Consider first industry $j \notin \Theta$ in which there are two innovating leaders 1 and 2. If a household invests in firm $\kappa$, then, in the advent of a success for the firm, $dq_{j\kappa} = 1$, the amount of its share holdings rises up to $\pi_\alpha i_{\iota j \kappa}$, $ds_{\iota j \kappa} = \pi_\alpha i_{\iota j \kappa} - s_{\iota j \kappa}$, but in the advent of success for the other firm $\zeta \neq \kappa$ in the industry, its share holdings in the firm fall down to zero, $ds_{\iota j \kappa} = -s_{\iota j \kappa}$. Next, consider industry $j \in \Theta$ in which firm 0 imitates. If a household invests in that firm, then, in the advent of a success for the firm, $dQ_j = 1$, the amount of its share holdings rises up to $\pi_\beta i_{\iota j 0}$, $ds_{\iota j 0} = \pi_\beta i_{\iota j 0} - s_{\iota j 0}$.

Household $\iota$'s total income $A_\iota$ consists of its wage income $w$ (the household supplies one labor unit), its profits $s_{\iota j 1}$ from the single leader in each industry

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[5]This extends the idea of Wälde (1999a, 1999b).

$j \in \Theta$, its profits $s_{\iota j1}$ and $s_{\iota j2}$ from the two leaders 1 and 2 in each industry $j \notin \Theta$, minus its share $1/N$ in the government's expenditures $R$ (= the household's lump-sum tax). Given this and proposition 2, we obtain

$$A_\iota = w + \int_{j \in \Theta} s_{\iota j1} dj + \int_{j \notin \Theta} (s_{\iota j1} + s_{\iota j2}) dj - \frac{R}{N}. \tag{22}$$

Household $\iota$ maximizes its utility (19) by its investment, $\{S_{\iota j0}\}$ for $j \in \Theta$ and $\{S_{\iota j1}, S_{\iota j2}\}$ for $j \notin \Theta$, subject to its budget constraint (20), the stochastic changes (21) in its profits, the composition of its income, (22), and the determination of its relative investment shares, (17), given the arrival rates $\{\Lambda_{j\kappa}, \Gamma_j\}$, the wage $w$, the consumption price $P$, the subsidies $(\tau_\alpha, \tau_\beta)$ and the government's expenditures $R$. In the households' stationary equilibrium in which the allocation of resources is invariable across technologies, this maximization yields the following results (see Appendix A):

$l_{j\kappa} = \ell_\beta$ for $j \notin \Theta$,
$l_{j0} = \ell_\alpha$ for $j \in \Theta$,

$$\frac{\ell_\beta}{l} = \psi(\phi, \tau_\alpha, \tau_\beta) \doteq \xi \left[ \frac{(1-\tau_\beta)\pi_\beta \gamma}{(1-\tau_\alpha)\pi_\alpha \mu^\sigma} - \lambda \right]^{-1} = \xi \left[ \frac{(1-\tau_\beta)(1-1/\phi)\gamma}{2(1-\tau_\alpha)(1-1/\mu)\mu^\sigma} - \lambda \right]^{-1},$$

$$\frac{\partial \psi}{\partial \phi} < 0, \quad \frac{\partial \psi}{\partial \tau_\alpha} < 0, \quad \frac{\partial \psi}{\partial \tau_\beta} > 0, \quad \psi(\phi, \tau, \tau) = \psi(\phi, 1, 1), \tag{23}$$

$$\ell_\alpha = [1 - 2(1-\alpha)\psi]l/\alpha, \tag{24}$$

$$g \doteq (\log \mu)(1-\alpha)(\Lambda_{j1} + \Lambda_{j2}) = (2\log \mu)(1-\alpha)(\lambda\psi + \xi)l, \tag{25}$$

$$\rho + \frac{1-\mu^\sigma}{\log \mu} g = \frac{hz}{1-\tau_\alpha} \doteq \nabla(l, \alpha, \phi, \tau_\alpha, \tau_\beta), \quad \frac{\partial \nabla}{\partial l} < 0, \quad \left.\frac{\partial \nabla}{\partial \phi}\right|_{\tau_\alpha = \tau_\beta} > 0,$$

$$\left.\frac{\partial \nabla}{\partial \alpha}\right|_{\tau_\alpha = \tau_\beta} > 0, \quad \lim_{\phi \to \mu} \left.\frac{\partial \nabla}{\partial \alpha}\right|_{\tau_\alpha = \tau_\beta} = 0, \quad \left.\frac{\partial \nabla}{\partial \tau_\alpha}\right|_{\tau_\alpha = \tau_\beta = 0} > 0, \quad \left.\frac{\partial \nabla}{\partial \tau_\beta}\right|_{\tau_\alpha = \tau_\beta = 0} > 0,$$

$$\left.\frac{\partial \nabla}{\partial \tau}\right|_{\tau_\alpha = \tau_\beta = \tau} > 0, \tag{26}$$

where $z \doteq \pi_\beta \Gamma_j / (wl_{j0})$ is the rate of return to investment in imitative R&D,[6] $h$ is the households' propensity to consume and $\nabla$ the rate of return paid to

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[6]Because a successful imitator obtains the profit $\pi_\beta$, the expected revenue from imitation is the profit times the arrival rate of imitations, $\pi_\beta \Gamma_j$. Dividing this by total imitation cost $wl_{j0}$ yields the rate of return to investment in imitative R&D.

savings. Result (23) says that with a smaller subsidy $\tau_\alpha$ to imitative R&D, a bigger subsidy $\tau_\beta$ to innovative R&D or with more intense PMC (i.e. a smaller $\phi$), R&D firms spend relatively more in innovative than imitative R&D (i.e. a higher $\ell_\beta/\ell_\alpha$). With a uniform R&D subsidy $\tau_\alpha = \tau_\beta = \tau$, the relative investment in imitation is independent of the subsidy. Result (25) says that the larger the proportion $1-\alpha$ of innovating industries or the more each innovating firm invests (i.e. the bigger $\ell_\beta$), the higher growth rate $g$.

# 6 General equilibrium

To close the system, I now specify how the proportion $\alpha$ of imitating industries is determined. When innovation occurs in an industry, this industry switches from the group of two-leader industries to that of one-leader industries, and when imitation occurs in an industry, this industry switches from one-leader industries to two-leader industries. In a steady-state equilibrium, every time a new superior-quality product is discovered in some industry, imitation must occur in some other industry.[7] Thus, the rate at which industries leave the group of two-leader industries, $\beta(\Lambda_{j1} + \Lambda_{j2})d\nu$, is equal to the rate at which industries leave the group of one-leader industries, $\alpha\Gamma_j d\nu$. This, (7), (23) and (24) yield

$$2(1-\alpha)(\lambda\psi + \xi)l = 2(1-\alpha)(\lambda\ell_\beta + \xi l) = 2\beta(\lambda\ell_\beta + \xi l) = \alpha\gamma\ell_\alpha$$
$$= \gamma[l - 2(1-\alpha)\ell_\beta] = \gamma[l - 2(1-\alpha)\psi]l,$$

from which I solve for the proportion of one-leader industries as follows:

$$\alpha = \Psi(\psi) \doteq 1 - \frac{\gamma/2}{(\lambda + \gamma)\psi + \xi}, \quad \Psi' = \frac{d\Psi}{d\psi} > 0. \tag{27}$$

Inserting (27) into (25), the following equation can be defined:

$$l(\psi, g) = 2\left[\frac{1}{\gamma} + \frac{1}{\lambda + \xi/\psi}\right]g, \quad \frac{\partial l}{\partial \psi} > 0, \quad \frac{\partial l}{\partial g} = \frac{l}{g} > 0. \tag{28}$$

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[7]Cf. Segerstrom (1991), p. 817.

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The four equations (23), (26), (27) and (28) form a system of four unknown variables: employment in R&D, $l$, the proportion of innovative labor in R&D, $\psi$, the proportion of one-leader industries, $\alpha$, and the growth rate $g$. The comparative statics of this system yields the result [Appendix B]

$$g(\phi, \tau_\alpha, \tau_\beta), \quad \left.\frac{\partial g}{\partial \tau}\right|_{\tau_\alpha=\tau_\alpha=\tau} > 0, \quad \lim_{\phi \to \mu} \left.\frac{\partial g}{\partial \tau_\alpha}\right|_{\tau_\alpha=\tau_\alpha=0} > 0, \quad \lim_{\phi \to \mu} \left.\frac{\partial g}{\partial \phi}\right|_{\tau_\alpha=\tau_\beta} > 0.$$

(29)

This can be rephrased as follows:

**Proposition 4** *A small uniform subsidy $\tau$ to all R&D boost economic growth. If the mark-up factor in the two-leader industries is initially high enough (i.e. $\phi \to \mu$), then less intense price competition (i.e. a higher mark-up $\phi$) or a small targeted subsidy $\tau_\alpha$ to imitative R&D is growth enhancing.*

A subsidy to imitative R&D and a higher producer's market power in the two-leader industries are equivalent in the sense that they both increase the expected profit of a successful imitation. This has two opposing effects on the extent of innovation. First, it increases the overall investment in R&D, a proportion of which is used in innovation. Second, it increases the proportion of investment being used in imitation and decreases that being used in innovation. If the mark-up in the two leader industries, $\phi$, is already close to that in the one-leader industries, $\mu$, the latter effect is weak and outweighed by the former. In such a case, investment in innovative R&D and the growth rate will increase. Otherwise, the outcome remains ambiguous. If a uniform subsidy to all R&D is used, then the second reallocating effect disappears altogether and the growth rate increases.

# 7   Optimal public policy

The symmetry across the households $\iota = 1, ..., n$ yields $C_\iota = y/N$. Noting $C_\iota = y/N$, (8), (14), (27) and (53), a single household's consumption relative

to the level of productivity, $c$, can be written as follows:

$$c(g, \alpha, \phi) \doteq \frac{C_\iota}{B^{\{t_k\}}} = \frac{y/N}{B^{\{t_k\}}} = \frac{x}{N}\chi = \left[1 - \frac{l(\psi, g)}{N}\right]\chi$$

$$= \chi(\alpha, \phi)\left[1 - \frac{1}{N}l\big(\Psi^{-1}(\alpha), g\big)\right]_, \quad \frac{\partial c}{\partial g} = -\frac{\chi}{N}\frac{\partial l}{\partial g} = -\frac{cl}{xg} < 0, \qquad (30)$$

where $\Psi^{-1}$ is the inverse function of $\Psi$. Given this, a single household's utility function (19) takes the form

$$U(C_\iota, T) = E\int_T^\infty c(g, \alpha, \phi)^\sigma \big(B^{\{t_k\}}\big)^\sigma e^{-\rho(\nu-T)}d\nu. \qquad (31)$$

On the assumption that the government is benevolent, it maximizes the representative household's welfare (31). I consider two cases:

(a) *First-best policy.* The government can discriminate between innovation and imitation, $\tau_\alpha \neq \tau_\beta$. Because there is one-to-one correspondence from $(\tau_\alpha, \tau_\beta)$ to $(g, \alpha)$ through (23), (27) and (29), the government can control the growth rate $g$ and the proportion of imitating industries, $\alpha$, by the subsidies $(\tau_\alpha, \tau_\beta)$. It maximizes social welfare (31) by the growth rate $g$ and the proportion of imitating industries, $\alpha$.

(b) *Second-best policy.* The government cannot discriminate between innovation and imitation, $\tau_\alpha = \tau_\beta = \tau$. Given (27) and (29), the proportion of imitating industries, $\alpha$, is wholly exogenous and the growth rate $g$ can be controlled by the uniform subsidy $\tau$. The government then maximizes social welfare (31) by $g$.

I denote:

$\Upsilon(\{t_k\})$ the value of each industry $k$ using current technology $t_k$.

$\Upsilon(t_j + 1, \{t_{k\neq j}\})$ the value of industry $j$ using technology $t_j + 1$, when other industries $k \neq j$ use current technology $t_k$.

The maximization problems in both the first-best $(a)$ and second-best $(b)$ cases above lead to the Bellman equation

$$\rho \Upsilon(t) = \begin{cases} \max_{g,\alpha} \mathcal{F} & \text{in the first-best policy (a),} \\ \max_g \mathcal{F} & \text{in the second-best policy (b),} \end{cases} \quad \text{where}$$

$$\mathcal{F} \doteq c(g,\alpha,\phi)^\sigma \big(B^{\{t_k\}}\big)^\sigma + \int_{j \notin \Theta} (\Lambda_{j1} + \Lambda_{j2}) \big[ \Upsilon(t_j + 1, \{t_{k \neq j}\}) - \Upsilon(\{t_k\}) \big] dj$$

$$= \frac{c(g,\alpha,\phi)^\sigma}{\big(B^{\{t_k\}}\big)^{-\sigma}} + \frac{g}{(1-\alpha)\log\mu} \int_{j \notin \Theta} \big[ \Upsilon(t_j + 1, \{t_{k \neq j}\}) - \Upsilon(\{t_k\}) \big] dj. \quad (32)$$

*(a) First-best policy.* The socially optimal levels for the growth rate $g$ and the proportion of imitating industries, $\alpha$, are given by [see Appendix C]

$$g^* = \frac{\rho\sigma\log\mu}{(\mu^\sigma - 1)(\sigma + x/l)}, \quad \alpha^* = \frac{\eta}{\eta + l/x}, \quad (33)$$

where

$$\eta(g,\alpha,\phi) \doteq -\frac{\alpha}{c}\frac{\partial c}{\partial \alpha} \quad (34)$$

is the elasticity of consumption with respect to imitation.

Inserting $g = g^*$ from (33) into (26) yields

**Proposition 5** *The welfare-maximizing subsidy to imitative R&D is*

$$\tau_\alpha^* = 1 - \frac{hz}{\rho + \frac{1-\mu^\sigma}{\log\mu}g^*} = 1 - \left(\sigma\frac{l}{x} + 1\right)\frac{hz}{\rho}.$$

The starting point is that $\tau_\alpha^*$ determines the welfare-maximizing levels for both subsidies $\tau_\alpha$ and $\tau_\beta$. In the next proposition, I examine how much $\tau_\alpha$ and $\tau_\beta$ should be differentiated. The lower the propensity to consume, $h$, the average rate of return to investment in imitative R&D, $z$, or the relative proportion of workers in R&D, $l/x$, the more R&D should be subsidized. The promotion of R&D by subsidies speeds up growth and increases future consumption and welfare. On the other hand, it crowds out the production of consumption goods through higher wages and decreases welfare. The

subsidies to R&D should be increased as long as the former growth effect dominates over the latter current-consumption effect. The lower the propensity to consume, $h$, the weaker the current-consumption effect and the higher the optimal subsidy. The lower the "private" rate of return $z$ to imitative R&D, the higher subsidy is needed to cover the gap between it and the social rate of return to imitative R&D. Finally, the lower the relative proportion of workers in R&D, $l/x$, the less a proportional increase in R&D crowds out current consumption and the higher the optimal subsidy.

Inserting (25) and (33) into (23), we obtain [see Appendix D]:

**Proposition 6** *If the government can discriminate between innovation and imitation, $\tau_\alpha \neq \tau_\beta$, then the welfare-maximizing subsidy to innovative R&D, $\tau_\beta^*$, is determined by*

$$\frac{1 - \tau_\beta^*}{1 - \tau_\alpha} = \left[\frac{\lambda}{\gamma} + \frac{\left(\frac{\lambda}{\gamma} + 1\right)\xi}{\frac{\gamma}{2}\left(1 + \frac{1}{\eta}\frac{l}{x}\right) - \xi}\right]\mu^\sigma \frac{\pi_\alpha}{\pi_\beta}, \quad \frac{\partial}{\partial(\eta\frac{x}{l})}\left(\frac{1 - \tau_\beta^*}{1 - \tau_\alpha}\right) > 0.$$

The bigger the relative profit in the two-leader industries, $\pi_\beta/\pi_\alpha$, or the less workers there are in R&D (i.e. the smaller $l$), the more the government should prefer innovation to imitation (i.e. the higher $\tau_\beta^*$ relative to $\tau_\alpha$ and the lower the ratio $(1 - \tau_\beta^*)/(1 - \tau_\alpha)$). The profit in the two-leader industries, $\pi_\beta$, and the subsidy to imitative R&D, $\tau_\alpha$, are strategic substitutes, for they both increase the incentives to imitate. Therefore, at the optimum, the increase in $\pi_\beta$ relative to $\pi_\alpha$ should lead to the decrease in $\tau_\alpha$ relative to $\tau_\beta$.

Given proposition 3, PMC causes inefficiency. Noting (30), (32) and proposition 3, we obtain:

**Proposition 7** *In the first-best case $\tau_\alpha \neq \tau_\beta$, the increase of PMC (i.e. a smaller $\phi$) is welfare diminishing, $\partial\mathcal{F}/\partial\phi > 0$.*

*(b) Second-best policy.* The optimal level $\alpha^*$ of $\alpha$ is given by (33). Because $\alpha$ is an decreasing function of $\phi$ through $\psi$ [cf. (23) and (27)], there is a

socially optimal level $\phi^*$ for the mark-up factor $\phi$ as well. This result can be rephrased as follows:

**Proposition 8** *If the government cannot discriminate between innovation and imitation but uses the uniform subsidy $\tau$ optimally, then there is an "inverted-U" relationship between PMC and welfare. When PMC is weak enough for $\phi < \phi^*$ (strong enough for $\phi > \phi^*$), it should be strengthened (weakened) to raise (lower) $\phi$.*

PMC has two opposing effects. It decreases the consumption price and thereby increases current consumption. On the other hand, it transfers labor from R&D to the production of goods and thereby hampers economic growth. These opposing effects are balanced for $\phi = \phi^*$ and $\alpha = \alpha^*$.

# 8  Conclusions

This paper examines a multi-industry economy in which growth is generated by creative destruction: a firm creating the newest technology by a successful R&D project crowds out the other firms with older technologies from the market so that the latter lose their value. A research firms can innovate to produce better versions of the products or imitate to copy existing innovations. Firms finance their R&D by issuing shares, and households save only in these shares. The government subsidizes R&D, possibly discriminating between innovation and imitation, and promotes collusion or product market competition (PMC). The main findings of this paper are as follows.

Each industry has either ($i$) one leader and a number of imitating followers, or ($ii$) two leaders which both innovate and no followers which would imitate. In the first-best, one labor unit in the consumption-good sector produces the largest amount of consumption. PMC produces a distortion through allocating too much labor in the two-leader and too little labor in the one-leader industries. In terms of consumption, lower output in the one-leader industries outweighs higher output in the two-leader industries.

The lower the propensity to consume, the average rate of return to investment in imitative R&D or the relative proportion of workers in R&D, the more R&D should be subsidized. The promotion of R&D by subsidies speeds up growth and increases future consumption and welfare. On the other hand, it crowds out the production of consumption goods through higher wages and decreases welfare. The subsidies to R&D should be increased as long as the former growth effect dominates over the latter current-consumption effect. The lower the propensity to consume, the weaker the current-consumption effect and the higher the optimal subsidy. The lower the "private" rate of return to imitative R&D, the higher subsidy is needed to cover the gap between it and the social rate of return to imitative R&D. Finally, the lower the relative proportion of workers in R&D, the less a proportional increase in R&D crowds out current consumption and the higher the optimal subsidy.

The bigger relative profit in the two-leader industries, the more the government should subsidize innovation relative to imitation. The profit in the two-leader industries and the subsidy to imitative R&D are strategic substitutes, for they both increase the incentives to imitate. Therefore, at the optimum, the increase in the former should lead to the decrease in the latter.

In the second-best case in which the government cannot discriminate between innovation and imitation, there is an "inverted-U" relationship between PMC and social welfare. PMC has two opposing effects. PMC has two opposing effects. It decreases the consumption price and thereby increases current consumption. On the other hand, it transfers labor from R&D to the production of goods and thereby hampers economic growth. PMC is at the optimum when these opposing effects are balanced. When PMC is below (above) its optimum level, it should be increased (increased).

While a great deal of caution should be exercised when a highly stylized dynamic model is used to explain the relationship of growth, product market competition and public policy, the following judgement nevertheless seems to be justified. With the exclusion of the second-best case, there

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seems to be no support to the assertion that imitation-induced PMC would be growth enhancing. In the literature, a common explanation of such result is that competition reduces the rewards from innovation and thereby incentives to engage innovative activity,[8] but this paper provides a different story. PMC reduces incentives to imitative, not to innovative R&D. In such a case, households transfer their investment from imitating to innovating firms, firms spend longer time in the imitative stage, the proportion of innovative industries decreases and the growth rate falls.

# Appendix

*A. Results (23)-(29)*

I denote:

$\Omega\big(\{s_{\iota k \upsilon}\}, \{t_k\}\big)$ the value of receiving profits $s_{\iota k \upsilon}$ from all firms $\upsilon$ in all industries $k$ using current technology $t_k$.

$\Omega\big(\pi_\alpha i_{\iota j \kappa}, 0, \{s_{\iota(k \neq j)\upsilon}\}, t_j + 1, \{t_{k \neq j}\}\big)$ the value of receiving the profit $\pi_\alpha i_{\iota j \kappa}$ from firm $\kappa$ in industry $j \notin \Theta$ using technology $t_j + 1$, but receiving no profits from the other firm which was a leader in that industry when technology $t_j$ was used, and receiving profits $s_{\iota(k \neq j)\upsilon}$ from all firms $\upsilon$ in other industries $k \neq j$ with current technology $t_k$.

$\Omega\big(\pi_\beta i_{\iota j 1}, \pi_\beta i_{\iota j 2}, \{s_{\iota(k \neq j)\upsilon}\}, \{t_k\}\big)$ the value of receiving profits $\pi_\beta i_{\iota j \kappa}$ from firms $\kappa \in \{1, 2\}$ in industry $j \in \Theta$, but receiving profits $s_{\iota(k \neq j)\upsilon}$ from all firms $\upsilon$ in the other industries $k \neq j$ with current technology $t_k$.

The Bellman equation associated with the household's maximization is[9]

$$\rho\Omega\big(\{s_{\iota k \upsilon}\}, \{t_k\}\big) = \max_{S_{\iota j} \geq 0 \text{ for all } j} \Xi_\iota, \tag{35}$$

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[8]Cf. Barro and Sala-i-Martin (1995), pp. 223-226.
[9]Cf. Dixit and Pindyck (1994).

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where

$$\Xi_\iota \doteq C_\iota^\sigma + \int_{j\in\Theta} \Gamma_j \Big[ \Omega\big(\pi_\beta i_{\iota j 1}, \pi_\beta i_{\iota j 1}, \{s_{\iota(k\neq j)\upsilon}\}, \{t_k\}\big) - \Omega\big(\{s_{\iota k\upsilon}\}, \{t_k\}\big) \Big] dj$$

$$+ \int_{j\notin\Theta} \sum_{\kappa=1,2} \Lambda_{j\kappa} \Big[ \Omega\big(\pi_\alpha i_{\iota j\kappa}, 0, \{s_{\iota(k\neq j)\upsilon}\}, t_j+1, \{t_{k\neq j}\}\big) - \Omega\big(\{s_{\iota k\upsilon}\}, \{t_k\}\big) \Big] dj. \tag{36}$$

Because $\partial C_\iota / \partial S_{\iota j\kappa} = -1/P$ by (20), the first-order conditions are given by

$$\Lambda_{j\kappa} \frac{d}{dS_{\iota j\kappa}} \Big[ \Omega\big(\pi_\alpha i_{\iota j\kappa}, 0, \{s_{\iota(k\neq j)\upsilon}\}, t_j+1, \{t_{k\neq j}\}\big) - \Omega\big(\{s_{\iota k\upsilon}\}, \{t_k\}\big) \Big] = \frac{\sigma}{P} C_\iota^{\sigma-1}$$

$$\text{for } j \notin \Theta \text{ and } \kappa \in \{1,2\}, \tag{37}$$

$$\Gamma_j \frac{d}{dS_{\iota j 0}} \Big[ \Omega\big(\pi_\beta i_{\iota j 1}, \pi_\beta i_{\iota j 2}, \{s_{\iota(k\neq j)\upsilon}\}, \{t_k\}\big) - \Omega\big(\{s_{\iota k\upsilon}\}, \{t_k\}\big) \Big] = \frac{\sigma}{P} C_\iota^{\sigma-1}$$

$$\text{for } j \in \Theta. \tag{38}$$

I try the solution that for each household $\iota$ the propensity to consume, $h_\iota$, and the subjective interest rate $r_\iota$ are independent of income $A_\iota$, i.e. $PC_\iota = h_\iota A_\iota$ and $\Omega = C_\iota^\sigma / r_\iota$.

Let us denote variables depending on technology $t_k$ by superscript $t_k$. Since according to (22) income $A_\iota^{\{t_k\}}$ depends directly on variables $\{s_{\iota k}^{t_k}\}$, I denote $A_\iota^{\{t_k\}}(\{s_{\iota k}^{t_k}\})$. Assuming that $h_\iota$ is invariant across technologies yields

$$P^{\{t_k\}} C_\iota^{\{t_k\}} = h_\iota A_\iota^{\{t_k\}}(\{s_{\iota k}^{t_k}\}). \tag{39}$$

The share in the next innovator $t_j + 1$ is determined by investment under the present technology $t_j$, $s_{\iota j\kappa}^{t_j+1} = \pi_\alpha i_{\iota j\kappa}^{t_j}$ for $j \notin \Theta$. The share in the next imitator is determined by investment under the same technology $t_j$, $s_{\iota j\kappa}^{t_j} = \pi_\beta i_{\iota j\kappa}^{t_j}$ for $j \in \Theta$. The value functions are then given by

$$\Omega\big(\{s_{\iota k\upsilon}\}, \{t_k\}\big) = \Omega\big(\pi_\beta i_{\iota j 1}, \pi_\beta i_{\iota j 2}, \{s_{\iota(k\neq j)\upsilon}\}, \{t_k\}\big) = \frac{1}{r_\iota}\big(C_\iota^{\{t_k\}}\big)^\sigma,$$

$$\Omega\big(\pi_\alpha i_{\iota j\kappa}, 0, \{s_{\iota(k\neq j)\upsilon}\}, t_j+1, \{t_{k\neq j}\}\big) = \frac{1}{r_\iota}\big(C_\iota^{t_j+1,\{t_{k\neq j}\}}\big)^\sigma. \tag{40}$$

Given this, we obtain

$$\frac{\partial \Omega\big(\{s_{\iota k\upsilon}\}, \{t_k\}\big)}{\partial S_{\iota j}^{t_j}} = 0. \tag{41}$$

From (17), (22), (39), (40), $s_{\iota j\kappa}^{t_j+1} = \pi_\alpha i_{\iota j\kappa}^{t_j}$ for $j \notin \Theta$, and $s_{\iota j\kappa}^{t_j} = \pi_\beta i_{\iota j\kappa}^{t_j}$ for $j \in \Theta$ it follows that

$$\frac{\partial s_{\iota j\kappa}^{t_j+1}}{\partial i_{\iota j\kappa}^{t_j}} = \pi_\alpha \text{ for } j \notin \Theta, \quad \frac{\partial s_{\iota j0}^{t_j}}{\partial i_{\iota j0}^{t_j}} = \pi_\beta \text{ for } j \in \Theta, \quad \frac{\partial A_\iota^{t_j+1,\{t_{k\neq j}\}}}{\partial s_{\iota j\kappa}^{t_j+1}} = \frac{\partial A_\iota^{\{t_k\}}}{\partial s_{\iota j\kappa}^{t_j}} = 1,$$

$$\frac{\partial i_{\iota j0}^{t_j}}{\partial S_{\iota j0}^{t_j}} = \frac{1}{(1-\tau_\alpha)w^{\{t_k\}}l_{j0}^{\{t_k\}}} \text{ for } j \in \Theta, \quad \frac{\partial i_{\iota j\kappa}^{t_j}}{\partial S_{\iota j\kappa}^{t_j}} = \frac{1}{(1-\tau_\beta)w^{\{t_k\}}l_{j\kappa}^{\{t_k\}}} \text{ for } j \notin \Theta,$$

$$\frac{\partial \Omega\big(\pi_\alpha i_{\iota j\kappa}, 0, \{s_{\iota(k\neq j)\upsilon}\}, t_j+1, \{t_{k\neq j}\}\big)}{\partial S_{\iota j\kappa}^{t_j}}$$

$$= \frac{\sigma}{r_\iota}\big(C_\iota^{t_j+1,\{t_{k\neq j}\}}\big)^{\sigma-1} \underbrace{\frac{\partial C_\iota^{t_j+1,\{t_{k\neq j}\}}}{\partial A_\iota^{t_j+1,\{t_{k\neq j}\}}}}_{h_\iota/P^{t_j+1,\{t_{k\neq j}\}}} \underbrace{\frac{\partial A_\iota^{t_j+1,\{t_{k\neq j}\}}}{\partial s_{\iota j\kappa}^{t_j+1}}}_{=1} \underbrace{\frac{\partial s_{\iota j\kappa}^{t_j+1}}{\partial i_{\iota j\kappa}^{t_j}}}_{=\pi} \frac{\partial i_{\iota j\kappa}^{t_j}}{\partial S_{\iota j\kappa}^{t_j}}$$

$$= \frac{\pi_\alpha \sigma h_\iota \big(C_\iota^{t_j+1,\{t_{k\neq j}\}}\big)^{\sigma-1}}{r_\iota P^{t_j+1,\{t_{k\neq j}\}}} \frac{\partial i_{\iota j\kappa}^{t_j}}{\partial S_{\iota j\kappa}^{t_j}} = \frac{\pi_\alpha h_\iota \sigma \big(C_\iota^{t_j+1,\{t_{k\neq j}\}}\big)^{\sigma-1}}{(1-\tau_\beta)r_\iota w^{\{t_k\}}P^{t_j+1,\{t_{k\neq j}\}}l_{j\kappa}^{\{t_k\}}} \quad \text{for } j \notin \Theta,$$

(42)

$$\frac{\partial \Omega\big(\pi_\beta i_{\iota j1}, \pi_\beta i_{\iota j2}, \{s_{\iota(k\neq j)\upsilon}\}, \{t_k\}\big)}{\partial S_{\iota j0}^{t_j}} = \frac{\sigma}{r_\iota}\big(C_\iota^{\{t_k\}}\big)^{\sigma-1} \underbrace{\frac{\partial C_\iota^{\{t_k\}}}{\partial A_\iota^{\{t_k\}}}}_{=h_\iota/P^{\{t_k\}}} \underbrace{\frac{\partial A_\iota^{\{t_k\}}}{\partial s_{\iota j0}^{t_j}}}_{=1} \underbrace{\frac{s_{\iota j0}^{t_j}}{\partial i_{\iota j0}^{t}}}_{=\pi_\beta} \frac{\partial i_{\iota j0}^{t}}{\partial S_{\iota j0}^{t}}$$

$$= \frac{\pi_\beta \sigma h_\iota}{r_\iota P^{\{t_k\}}}\big(C_\iota^{\{t_k\}}\big)^{\sigma-1} \frac{\partial i_{\iota j0}^{t}}{\partial S_{\iota j0}^{t}} = \frac{\pi_\beta h_\iota \sigma \big(C_\iota^{\{t_k\}}\big)^{\sigma-1}}{(1-\tau_\alpha)r_\iota w^{\{t_k\}}P^{\{t_k\}}l_{j0}^{\{t_k\}}} \quad \text{for } j \in \Theta. \tag{43}$$

I focus on a stationary equilibrium where the growth rate $g$ and the allocation of labor, $(l_{j\kappa}, x)$, are invariant across technologies. Given (2), (8), (11) and (14), this implies

$$l_{j\kappa}^{\{t_k\}} = l_{j\kappa}, \quad x^{\{t_k\}} = x = N - l, \quad w^{\{t_k\}} = w = x/\varphi,$$

$$\frac{P^{\{t_k\}}}{P^{t_j+1,\{t_{k\neq j}\}}} = \frac{C_\iota^{t_j+1,\{t_{k\neq j}\}}}{C_\iota^{\{t_k\}}} = \frac{A_\iota^{t_j+1,\{t_{k\neq j}\}}}{A_\iota^{\{t_k\}}} = \frac{y^{t_j+1,\{t_{k\neq j}\}}}{y^{\{t_k\}}} = \frac{B^{t_j+1,\{t_{k\neq j}\}}}{B^{\{t_k\}}} = \mu.$$

(44)

Inserting (36), (39), (40), (44) and $g \doteq \int_{j \notin \Theta} l_j dj$ into equation (35) yields

$$
\begin{aligned}
0 &= \Big[\rho + \int_{j \notin \Theta}(\Lambda_{j1} + \Lambda_{j2})dj + \int_{j \in \Theta}\Gamma_j dj\Big]\Omega\big(\{s_{\iota k \upsilon}\},\{t_k\}\big) - \big(C_\iota^{\{t_k\}}\big)^\sigma \\
&\quad - \int_{j \notin \Theta}\sum_{\kappa=1,2}\Lambda_{j\kappa}\Omega\big(\pi_\alpha i_{\iota j \kappa}, 0, \{s_{\iota(k \neq j)\upsilon}\}, t_j + 1, \{t_{k \neq j}\}\big)dj \\
&\quad - \int_{j \in \Theta}\Gamma_j\Omega\big(\pi_\beta i_{\iota j1}, \pi_\beta i_{\iota j2}, \{s_{\iota(k \neq j)\upsilon}\}, \{t_k\}\big)dj \\
&= \Big[\rho + \int_{j \notin \Theta}(\Lambda_{j1} + \Lambda_{j2})dj\Big]\frac{\big(C_\iota^{\{t_k\}}\big)^\sigma}{r_\iota} - \big(C_\iota^{\{t_k\}}\big)^\sigma \\
&\quad - \int_{j \notin \Theta}\sum_{\kappa=1,2}\frac{\Lambda_{j\kappa}}{r_\iota}\big(C_\iota^{\{t_j+1\},\{t_{k \neq j}\}}\big)^\sigma dj \\
&= \Big[\rho + \int_{j \notin \Theta}(\Lambda_{j1} + \Lambda_{j2})dj\Big]\frac{\big(C_\iota^{\{t_k\}}\big)^\sigma}{r_\iota} - \big(C_\iota^{\{t_k\}}\big)^\sigma - \int_{j \notin \Theta}\sum_{\kappa=1,2}\Lambda_{j\kappa}\frac{\mu^\sigma}{r_\iota}\big(C_\iota^{\{t_k\}}\big)^\sigma dj \\
&= \frac{1}{r_\iota}\big(C_\iota^{\{t_k\}}\big)^\sigma\Big[\rho + (1 - \mu^\sigma)\int_{j \notin \Theta}(\Lambda_{j1} + \Lambda_{j2})dj - r_\iota\Big] \\
&= \frac{1}{r_\iota}\big(C_\iota^{\{t_k\}}\big)^\sigma\Big[\rho - r_\iota + \frac{1 - \mu^\sigma}{\log \mu}g\Big].
\end{aligned}
$$

This equation is equivalent to

$$
r_\iota = \rho + \frac{1 - \mu^\sigma}{\log \mu}g. \tag{45}
$$

Because there is symmetry throughout all households $\iota$, their propensity to consume is equal, $h_\iota = h$. From $h_\iota = h$, (15), (16), (18), (20), (22) and (39) it follows that

$$
\begin{aligned}
wl - R &= w\int_{j \in \Theta}l_{j0}dj + w\int_{j \notin \Theta}(l_{j1} + l_{j2})dj - R \\
&= (1 - \tau_\alpha)w\int_{j \in \Theta}l_{j0}dj + (1 - \tau_\beta)w\int_{j \notin \Theta}(l_{j1} + l_{j2})dj \\
&= \sum_{\iota=1}^N\Big[\int_{j \in \Theta}S_{\iota j0}dj + \int_{j \notin \Theta}(S_{\iota j1} + S_{\iota j2})dj\Big] = \sum_{\iota=1}^N(A_\iota - PC_\iota) \\
&= (1 - h)\sum_{\iota=1}^N A_\iota = (1 - h)(1 + wl - R).
\end{aligned}
$$

Solving for the propensity to consume, we obtain

$$h_\iota = h = \frac{1}{1 + wl - R}.\tag{46}$$

Given (8) and (14), we obtain the wage

$$w = \frac{\varphi}{x} = \frac{\varphi(\alpha, \pi_\beta)}{N - l}.\tag{47}$$

I define the rate of return to imitative R&D by $z \doteq \pi_\beta \Gamma_j / (wl_{j0})$. Inserting this, (9), (10), (41), (42), (43) and $\pi_\alpha = 1 - 1/\mu$ and $\pi_\beta = (1 - 1/\phi)/2$ from proposition 2 into (37) and (38), we obtain

$$\frac{(1 - 1/\mu)h\lambda\sigma\mu^\sigma \big(C_\iota^{\{t_k\}}\big)^{\sigma-1}(\lambda + \xi l/l_{j\kappa})}{(1 - \tau_\beta)\Big(\rho + \frac{1-\mu^\sigma}{\log\mu}g\Big)wP^{\{t_k\}}} = \frac{\sigma\pi_\alpha h_\iota \mu^\sigma \Lambda_{j\kappa}\big(C_\iota^{\{t_k\}}\big)^{\sigma-1}}{(1 - \tau_\beta)r_\iota wl_{j\kappa}P^{\{t_k\}}}$$

$$= \frac{\sigma\pi_\alpha h_\iota \Lambda_{j\kappa}\big(C_\iota^{t_j+1,\{t_{k\neq j}\}}\big)^{\sigma-1}}{(1 - \tau_\beta)r_\iota wl_{j\kappa}P^{t_{t_j+1},\{t_{k\neq j}\}}} = \Lambda_{j\kappa}\frac{d}{dS_{\iota j\kappa}}\Omega\big(\pi_\alpha i_{\iota j}, \{s_{\iota(k\neq j)}\}, t_j + 1, \{t_{k\neq j}\}\big)$$

$$= \frac{\sigma}{P^{\{t_k\}}}\big(C_\iota^{\{t_k\}}\big)^{\sigma-1} \quad \text{for } j \notin \Theta \text{ and } \kappa \in \{1,2\},\tag{48}$$

$$\frac{\frac{1}{2}(1 - 1/\phi)h\gamma\sigma\big(C_\iota^{\{t_k\}}\big)^{\sigma-1}}{(1 - \tau_\alpha)\Big(\rho + \frac{1-\mu^\sigma}{\log\mu}g\Big)wP^{\{t_k\}}} = \frac{\sigma h\big(C_\iota^{\{t_k\}}\big)^{\sigma-1}z}{(1 - \tau_\alpha)\Big(\rho + \frac{1-\mu^\sigma}{\log\mu}g\Big)P^{\{t_k\}}}$$

$$= \frac{\sigma\pi_\beta h_\iota \Gamma_j\big(C_\iota^{\{t_k\}}\big)^{\sigma-1}}{(1 - \tau_\alpha)r_\iota wl_{j0}P^{\{t_k\}}} = \Gamma_j\frac{d}{dS_{\iota j0}}\Omega\big(\{\pi_\beta i_{\iota j1}, \pi_\beta i_{\iota j2}, \{s_{\iota m(k\neq j)}\}, \{t_k\}\big)$$

$$= \frac{\sigma}{P^{\{t_k\}}}\big(C_\iota^{\{t_k\}}\big)^{\sigma-1} \quad \text{for } j \in \Theta.\tag{49}$$

Given equations (48) and (49) and proposition 2, we obtain

$$\begin{array}{ll} l_{j\kappa} = \ell_\beta & \text{for } j \notin \Theta, \\ l_{j0} = \ell_\alpha & \text{for } j \in \Theta, \end{array} \quad \frac{\ell_\beta}{l} = \psi(\phi, \tau_\alpha, \tau_\beta) \doteq \xi\left[\frac{(1 - \tau_\beta)(1 - 1/\phi)\gamma/2}{(1 - \tau_\alpha)(1 - 1/\mu)\mu^\sigma} - \lambda\right]^{-1},$$

$$\partial\psi/\partial\phi < 0, \quad \partial\psi/\partial\tau_\alpha < 0, \quad \partial\psi/\partial\tau_\beta > 0, \quad [\partial\psi/\partial\tau]_{\tau_\alpha=\tau_\beta=\tau} = 0.\tag{50}$$

Equations (2), (7), (8), (9), (12), (13), (14), (15), (46), (49) and (50) yield

$$w = \frac{\varphi(\alpha, \phi)}{x} = \frac{\varphi(\alpha, \phi)}{N - l},$$

$$l = \int_{j \notin \Theta}(l_{j1} + l_{j2})dj + \int_{j \in \Theta}l_j dj = \ell_\beta\int_{j \notin \Theta}dj + \ell_\alpha\int_{j \in \Theta}dj$$

$$= \alpha\ell_\alpha + 2(1 - \alpha)\ell_\beta = \alpha\ell_\alpha + 2(1 - \alpha)\psi l,$$

$$\ell_\alpha = [1 - 2(1 - \alpha)\psi]l/\alpha,$$

$$R = \tau_\alpha \int_{j \in \Theta} w l_{j0} dj + \tau_\beta \int_{j \notin \Theta} (w l_{j1} + w l_{j2}) dj$$

$$= \tau_\alpha w \ell_\alpha \int_{j \in \Theta} dj + 2\tau_\beta w \ell_\beta \int_{j \notin \Theta} dj = \tau_\alpha w \ell_\alpha \alpha + 2\tau_\beta w \ell_\beta (1-\alpha)$$

$$= \big\{ \tau_\alpha [1 - 2(1-\alpha)\psi] + 2\tau_\beta (1-\alpha)\psi \big\} w l$$

$$= \big[ \tau_\alpha + 2(1-\alpha)\psi(\tau_\beta - \tau_\alpha) \big] w l,$$

$$h = \frac{Py}{\sum_\iota A_\iota} = \frac{1}{1 + wl - R} = \frac{1}{1 + \big[ 1 - \tau_\alpha + 2(1-\alpha)\psi(\tau_\alpha - \tau_\beta) \big] wl}$$

$$= \frac{N - l}{N - l + \big[ 1 - \tau_\alpha + 2(1-\alpha)\psi(\tau_\alpha - \tau_\beta) \big] \varphi(\alpha, \phi)},$$

$$\Lambda_{j\kappa} = \lambda l_{j\kappa}^\xi l^{1-\xi} = \lambda \psi^\xi l \text{ for } j \notin \Theta \text{ and } \kappa \in \{1, 2\},$$

$$g = (\log \mu) \int_{j \notin \Theta} (\Lambda_{j1} + \Lambda_{j2}) dj = (\log \mu)(1-\alpha)(\Lambda_{j1} + \Lambda_{j2})$$

$$= (2\lambda \log \mu)(1-\alpha)\psi^\xi l, \tag{51}$$

$$\rho + \frac{1 - \mu^\sigma}{\log \mu} g = \frac{hz}{1 - \tau_\alpha} = \frac{(1 - 1/\phi)\gamma h}{2(1 - \tau_\alpha)w} = \frac{(1 - 1/\phi)\gamma h(N - l)}{2(1 - \tau_\alpha)\varphi(\alpha, \phi)}$$

$$= \frac{(1 - 1/\phi)\gamma}{2(1 - \tau_\alpha)} \frac{(N - l)/\varphi(\alpha, \phi)}{1 + \big[ 1 - \tau_\alpha + 2(1-\alpha)\psi(\phi, \tau_\alpha, \tau_\beta)(\tau_\alpha - \tau_\beta) \big] \varphi(\alpha, \phi) l/(N - l)}$$

$$\doteq \nabla(l, \alpha, \phi, \tau_\alpha, \tau_\beta), \quad \frac{\partial \nabla}{\partial l} < 0, \quad \left. \frac{\partial \nabla}{\partial \phi} \right|_{\tau_\alpha = \tau_\beta} > 0, \quad \left. \frac{\partial \nabla}{\partial \alpha} \right|_{\tau_\alpha = \tau_\beta} > 0,$$

$$\lim_{\phi \to \mu} \left. \frac{\partial \nabla}{\partial \alpha} \right|_{\tau_\alpha = \tau_\beta} > 0, \quad \left. \frac{\partial \nabla}{\partial \tau_\alpha} \right|_{\tau_\alpha = \tau_\beta = 0} > 0, \quad \left. \frac{\partial \nabla}{\partial \tau_\beta} \right|_{\tau_\alpha = \tau_\beta = 0} > 0, \quad \left. \frac{\partial \nabla}{\partial \tau} \right|_{\tau_\alpha = \tau_\beta = \tau} > 0. \tag{52}$$

Equations (50), (51) and (52) define (23)-(26).

*B. Results (29)*

Inserting the functions (23), (27) and (28) into (26), we obtain

$$\rho + \frac{1 - \mu^\sigma}{\log \mu} g = \Delta(\phi, \tau_\alpha, \tau_\beta, g) \doteq \nabla\big(l(\psi(\phi, \tau_\alpha, \tau_\beta), g), \alpha(\psi(\phi, \tau_\alpha, \tau_\beta)), \phi, \tau_\alpha, \tau_\beta\big), \tag{53}$$

in which

$$\frac{\partial \Delta}{\partial g} = \underbrace{\frac{\partial \nabla}{\partial l}}_{-} \underbrace{\frac{\partial l}{\partial g}}_{+} < 0, \quad \left. \frac{\partial \Delta}{\partial \tau} \right|_{\tau_\alpha = \tau_\beta = \tau} = \underbrace{\left. \frac{\partial \nabla}{\partial \tau} \right|_{\tau_\alpha = \tau_\beta = \tau}}_{+} > 0,$$

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$$\left.\frac{\partial \Delta}{\partial \phi}\right|_{\tau_\alpha=\tau_\beta=0} = \underbrace{\frac{\partial \nabla}{\partial l}}_{-}\underbrace{\frac{\partial l}{\partial \psi}}_{+}\underbrace{\frac{\partial \psi}{\partial \phi}}_{-} + \underbrace{\frac{\partial \nabla}{\partial \alpha}}_{+}\underbrace{\frac{\partial \alpha}{\partial \psi}}_{+}\underbrace{\frac{\partial \psi}{\partial \phi}}_{-} + \underbrace{\frac{\partial \nabla}{\partial \phi}}_{+},$$

$$\lim_{\phi\to\mu}\left.\frac{\partial \Delta}{\partial \phi}\right|_{\tau_\alpha=\tau_\beta=0} = \underbrace{\frac{\partial \nabla}{\partial l}}_{-}\underbrace{\frac{\partial l}{\partial \psi}}_{+}\underbrace{\frac{\partial \psi}{\partial \phi}}_{-} + \underbrace{\frac{\partial \nabla}{\partial \phi}}_{+} > 0,$$

$$\left.\frac{\partial \Delta}{\partial \tau_\alpha}\right|_{\tau_\alpha=\tau_\beta=0} = \underbrace{\frac{\partial \nabla}{\partial l}}_{-}\underbrace{\frac{\partial l}{\partial \psi}}_{+}\underbrace{\frac{\partial \psi}{\partial \tau_\alpha}}_{-} + \underbrace{\frac{\partial \nabla}{\partial \alpha}}_{+}\underbrace{\frac{\partial \alpha}{\partial \psi}}_{+}\underbrace{\frac{\partial \psi}{\partial \tau_\alpha}}_{-} + \underbrace{\frac{\partial \nabla}{\partial \tau_\alpha}}_{+},$$

$$\lim_{\phi\to\mu}\left.\frac{\partial \Delta}{\partial \tau_\alpha}\right|_{\tau_\alpha=\tau_\beta=0} = \underbrace{\frac{\partial \nabla}{\partial l}}_{-}\underbrace{\frac{\partial l}{\partial \psi}}_{+}\underbrace{\frac{\partial \psi}{\partial \tau_\alpha}}_{-} + \underbrace{\frac{\partial \nabla}{\partial \tau_\alpha}}_{+} > 0, \qquad \left.\frac{\partial \Delta}{\partial \tau}\right|_{\tau_\alpha=\tau_\beta=\tau} = \underbrace{\frac{\partial \nabla}{\partial \tau}}_{+} > 0,$$

$$\left.\frac{\partial \Delta}{\partial \tau_\beta}\right|_{\tau_\alpha=\tau_\beta=0} = \underbrace{\frac{\partial \nabla}{\partial l}}_{-}\underbrace{\frac{\partial l}{\partial \psi}}_{+}\underbrace{\frac{\partial \psi}{\partial \tau_\beta}}_{+} + \underbrace{\frac{\partial \nabla}{\partial \alpha}}_{+}\underbrace{\frac{\partial \alpha}{\partial \psi}}_{+}\underbrace{\frac{\partial \psi}{\partial \tau_\beta}}_{+} + \underbrace{\frac{\partial \nabla}{\partial \tau_\beta}}_{+}.$$

The growth rate $g$ and employment in R&D, $l$ are determined by the two equations (28) and (53). In the $(g,l)$-plane, the equation (28) defines the increasing line $OL$ that goes through the origin, and the equation (53) the increasing curve $GG$ [see Fig. 1]. The equilibrium for $(g,l)$ is in the intersection $Q$ of these. If $\frac{1-\mu^\sigma}{\log \mu} < \frac{\partial \Delta}{\partial g} < 0$, then the curve $GG$ were steeper than the line $OL$ at equilibrium $Q$ and it is plausible that the equilibrium is unstable. Assume for instance that a household adjusts its investment in R&D (i.e. $l$) along line $OL$ towards the curve $GG$ on which its subjective discount factor $\rho + \frac{1-\mu^\sigma}{\log \mu}g$ is equal to the rate of return to savings, $\nabla$. The system then escapes from equilibrium $Q$ along line $OL$ when $GG$ is steeper, but converges to $Q$ when $OL$ is steeper. I therefore assume $0 > \frac{1-\mu^\sigma}{\log \mu} > \frac{\partial \Delta}{\partial g}$. The line $OL$ is then steeper at $Q$ and the system converges to $Q$. Equation (53) defines the

function $g(\alpha, \phi, \tau_\alpha, \tau_\beta)$ with the properties $\lim_{\phi \to \mu} \partial g / \partial \phi > 0$ and

$$\lim_{\phi \to \mu} \frac{\partial g}{\partial \phi} = \underbrace{\left( \frac{1 - \mu^\sigma}{\log \mu} - \frac{\partial \Delta}{\partial g} \right)^{-1}}_{+} \lim_{\phi \to \mu} \frac{\partial \Delta}{\partial \phi} > 0,$$

$$\left. \frac{\partial g}{\partial \tau} \right|_{\tau_\alpha = \tau_\beta = \tau} = \underbrace{\frac{\partial \Delta}{\partial \tau}}_{+} \underbrace{\left( \frac{1 - \mu^\sigma}{\log \mu} - \frac{\partial \Delta}{\partial g} \right)^{-1}}_{+} > 0.$$

*C. Results (33)*

Noting (30), the first-order conditions for $g$ and $\alpha$ in the government's maximization are given by

$$\frac{\partial \mathcal{F}}{\partial g} = \sigma c^{\sigma-1} \big( B^{\{t_k\}} \big)^\sigma \frac{\partial c}{\partial g} + \frac{1}{(1-\alpha) \log \mu} \int_{j \notin \Theta} \big[ \Upsilon(t_j + 1, \{t_{k \neq j}\}) - \Upsilon(\{t_k\}) \big] dj$$

$$= 0, \tag{54}$$

$$\frac{\partial \mathcal{F}}{\partial \alpha} = \sigma c^{\sigma-1} \big( B^{\{t_k\}} \big)^\sigma \frac{\partial c}{\partial \alpha} + \frac{g}{(1-\alpha)^2 \log \mu} \int_{j \notin \Theta} \big[ \Upsilon(t_j + 1, \{t_{k \neq j}\}) - \Upsilon(\{t_k\}) \big] dj$$

$$= 0. \tag{55}$$

I try the solution

$$\Upsilon(\{t_k\}) \doteq \vartheta c^\sigma \big( B^{\{t_k\}} \big)^\sigma, \tag{56}$$

where $\vartheta$ is independent of the endogenous variables of the system. Noting (11) and (56), we then obtain

$$\Upsilon(t_j + 1, \{t_{k \neq j}\}) = \vartheta c^\sigma \big( B^{t_j + 1, \{t_k\}} \big)^\sigma = \vartheta \mu^\sigma c^\sigma \big( B^{\{t_k\}} \big)^\sigma = \mu^\sigma \Upsilon(\{t_k\}). \tag{57}$$

Inserting (56) and (57) into the Bellman equation (32), we obtain

$$0 = c^\sigma \big( B^{\{t_k\}} \big)^\sigma + \frac{g/(1-\alpha)}{\log \mu} \int_{j \notin \Theta} \Big[ \Upsilon(t_j + 1, \{t_{k \neq j}\}) - \Upsilon(\{t_k\}) \Big] dj - \rho \Upsilon(\{t_k\})$$

$$= \Upsilon(\{t_k\}) [1/\vartheta - \rho + (\mu^\sigma - 1) g/(\log \mu)]$$

and

$$1/\vartheta = \rho - (\mu^\sigma - 1) g/(\log \mu) < \rho. \tag{58}$$

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Given (30), (34)-(56), (57) and (58), we obtain

$$
\begin{aligned}
\frac{\partial \mathcal{F}}{\partial g} &= \sigma c^{\sigma-1}\big(B^{\{t_k\}}\big)^\sigma \frac{\partial c}{\partial g} + \frac{\mu^\sigma - 1}{\log \mu}\Upsilon(\{t_k\}) = \Big(\frac{\sigma}{\vartheta c}\frac{\partial c}{\partial g} + \frac{\mu^\sigma - 1}{\log \mu}\Big)\Upsilon(\{t_k\}) \\
&= \Big(\frac{\mu^\sigma - 1}{\sigma \log \mu} - \frac{l}{\vartheta x g}\Big)\sigma \Upsilon(\{t_k\}) = \Big[\frac{\mu^\sigma - 1}{\sigma \log \mu} - \Big(\rho - \frac{\mu^\sigma - 1}{\log \mu}g\Big)\frac{l}{x g}\Big]\sigma \Upsilon(\{t_k\}) \\
&= 0,
\end{aligned}
\tag{59}
$$

$$
\begin{aligned}
\frac{\partial \mathcal{F}}{\partial \alpha} &= \sigma c^{\sigma-1}\big(B^{\{t_k\}}\big)^\sigma \frac{\partial c}{\partial \alpha} + \frac{(\mu^\sigma - 1)g}{(1-\alpha)\log \mu}\Upsilon(\{t_k\}) \\
&= \Big(\frac{\sigma}{c\vartheta}\frac{\partial c}{\partial \alpha} + \frac{\mu^\sigma - 1}{\log \mu}\frac{g}{1-\alpha}\Big)\Upsilon(\{t_k\}) = \Big(\frac{1}{c}\frac{\partial c}{\partial \alpha} + \frac{\mu^\sigma - 1}{\sigma \log \mu}\frac{\vartheta g}{1-\alpha}\Big)\frac{\sigma}{\vartheta}\Upsilon(\{t_k\}) \\
&= \Big[-\frac{\eta}{\alpha} + \frac{l}{(1-\alpha)x}\Big]\frac{\sigma}{\vartheta}\Upsilon(\{t_k\}) = 0.
\end{aligned}
\tag{60}
$$

Noting (59), we obtain

$$
g = \frac{\rho \sigma \log \mu}{(\mu^\sigma - 1)(\sigma + x/l)}.
$$

Given (34) and (60), $\partial c/\partial \alpha < 0$, $\eta > 0$ and $\alpha \doteq \eta/(\eta + l/x)$ hold.

*D. Proposition 6*

Inserting $\alpha = \alpha^*$ and (33) into (27) and noting yields

$$
\frac{\gamma/2}{(\lambda + \gamma)\psi + \xi} = \alpha = \alpha^* = \frac{\eta}{\eta + l/x}.
$$

From this and (23) it follows that

$$
\xi\left[\frac{(1-\tau_\beta)\pi_\beta\gamma}{(1-\tau_\alpha)\pi_\alpha\mu^\sigma} - \lambda\right]^{-1} = \psi = \frac{1}{\lambda + \gamma}\left[\frac{\gamma}{2}\Big(1 + \frac{1}{\eta}\frac{l}{x}\Big) - \xi\right].
$$

Solving for the ratio $(1 - \tau_\beta^*)/(1 - \tau_\alpha)$ and noting (14), we obtain

$$
\frac{1 - \tau_\beta^*}{1 - \tau_\alpha} = \left\{\frac{\lambda}{\gamma} + \Big(\frac{\lambda}{\gamma} + 1\Big)\xi\left[\frac{\gamma}{2}\Big(1 + \frac{1}{\eta}\frac{l}{x}\Big) - \xi\right]^{-1}\right\}\mu^\sigma\frac{\pi_\alpha}{\pi_\beta}.
$$

# References:

Aghion, P., Harris, C. and Vickers, J. (1997). Competition and growth with step-by-step innovation: an example. *European Economic Review 41*, 771-782.

Aghion, P., Harris, C., Howitt, P. and Vickers, J. (2001). Competition, Imitation and growth with step-by-step innovation. *Review of Economic Studies 68*, 467-492.

Aghion, P. and Howitt, P. (1998). *Endogenous Growth Theory*. Cambridge (Mass.): MIT Press.

Barro, R.J. and Sala-i-Martin, X. (1995). Economic Growth. New York: MacGraw-Hill.

Cheng, L.K. and Tao, Z. (1999). The impact of public policies on innovation and imitation: the role of R&D technology in growth models. *International Economic Review 40*, 187-207.

Davidson, C. and Segerstrom, P. (1998). R&D subsidies and economic growth. *RAND Journal of Economics 29*, 548-577.

Dixit, A. and Pindyck, K. (1994). *Investment under Uncertainty*. Princeton: Princeton University Press.

Gompers, P. and Lerner, J. (1999). *The Venture Capital Cycle*. Cambridge (Mass.): The MIT Press.

Kreps, D.M. and Scheinkman, J.A. (1983). Quantity precommitment and Bertrand competition yields Cournot outcomes. *Bell Journal of Economics 14*, 326-337.

Mukoyama, T. (2003). Innovation, imitation, and growth with cumulative technology. *Journal of Monetary Economics 50*, 361-380.

Segerstrom, P.S. (1998). Innovation, imitation, and economic growth. *Journal of Political Economy 99*, 807-827.

Segerstrom, P.S. (1991). Endogenous growth without scale effects. *The American Economic Review 88*, 1290-1310.

Zeng, J. (2001). Innovative vs. imitative R&D and economic growth. *Journal of Development Economics 64*, 499-528.

Walz, U. (1995). Endogenous innovation and imitation in a model of equilibrium growth. *European Journal of Political Economy 11*, 709-723.

Wälde, K. (1999a). A model of creative destruction with undiversifiable risk and optimizing households. *The Economic Journal 109*, C156-C171.

Wälde, K., (1999b). Optimal saving under poisson uncertainty. *Journal of Economic Theory 87*, 194–217.