



Discussion Papers

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Stabilizing Endogenous Cycles with Balanced Budget Distortionary Taxation*

Abstract

We study the effects of distortionary taxation on endogenous cycles, and the determinacy of equilibria, in a competitive overlapping generations model with a balanced budget rule. Under proportional taxation there is a critical tax rate above which cycles will vanish, while with linearly progressive taxation there is a critical level of exemption below which cycles will vanish as well. If the utility function is quasi-linear, increasing tax rate can cause the economy to become determinate both with proportional and linearly progressive taxation so that tax exemption does not matter. But tax exemption might matter if the utility function is more general. Finally, a policy with the price level target can completely eliminate fluctuations.

JEL Classification: D 91, E32, H30

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1. Introduction

Macroeconomists have long been interested in studying the stabilizing effects of different policies. More than fifty years ago Friedman (1948) advocated a non-discretionary monetary and fiscal framework for stability. It was recognized in the early literature on fiscal policy e.g. by Musgrave and Miller (1948) that also progressive taxation operates as an automatic stabilizer to smooth business fluctuations.¹

It is not always the case that fluctuations are bad for welfare, since the equilibria associated with stable cycles can be efficient. The resulting indeterminacy, i.e. the multiplicity of equilibria, however, can be an independent reason for stabilization policy as argued e.g. by Woodford (1984). An appropriate policy can render the equilibrium determinate, and thus possibly rid the economy from the effects of sunspots and bubbles.² If fluctuations are chaotic, as sometimes is the case even in simple overlapping generations models, policies, which stabilize the economy, can also help agents to coordinate their actions more easily.

The possibility of endogenous cycles in overlapping generations models was observed by Gale (1973) and Cass, Okuno and Zilcha (1979). Grandmont (1985) elaborated their findings, and analyzed precisely the conditions for the existence of cycles. From the policy point of view Grandmont (1986a) pointed out that simple fiscal and monetary policies involving proportional transfers and lump-sum taxes (or transfers) can abolish cycles completely. In his demonstration of sunspot equilibria Aiyagari (1988) showed that there is a simple policy of proportional tax rate and lump-sum transfers, which can stabilize the asset price completely and thus rid the economy from the effects of sunspots.³

Woodford (1986) studied the model with an infinitely lived agent with similar preferences to what we have below, and with a finance constraint. Because of that constraint it turns out that consumption and labor supply decision in his model are identical to the decisions, which would be made by two-period lived agents in an

¹ See also Vickrey (1945) and Slitor (1948).

² See especially section V in Woodford's (1984) survey, where he also discusses the policy responses to indeterminacy.

overlapping generations model. There are equilibrium fluctuations and sunspot equilibria in his model. Woodford showed that government expenditures adjusted to changes in private investment demand can stabilize the price level. Smith (1994) utilized a two-period overlapping generations model with money, storage and reserve requirements to study the effects of many types of monetary and fiscal policies on indeterminacy. He showed that indeterminacy is a pervasive feature under many policies, and furthermore that a certain inflation target can be achieved but often with a welfare cost meaning that the same target can be achieved by other policy measures with better welfare properties.

Ghiglini and Tvede (2000) studied an overlapping generations model with a prescribed objective function for the government. They showed that if the discount factor is close to one, the optimal policy can completely stabilize the economy. Goenka (1994) advocated a role for discretionary policies in a general equilibrium model with public goods to abolish the sunspot equilibria, and thus stabilize the economy. Keister (1998) studied the effects of redistribution on the volatility of the economy and argued that models with indeterminacies can be useful vehicles for certain types of policy analyses. He showed e.g. that larger transfers lead to higher fluctuations in consumption.

There is a related literature, though not in an OG framework, in which the role of various tax schemes as stabilizing or destabilizing devices has been analyzed in models with inefficiencies and/or externalities. In these models the potential impacts of progressive, proportional and regressive taxation on cycles and indeterminacy have been studied. This literature includes Guo (1999), Guo and Lansing (1998, 2001), Guo and Harrison (2001, 2004), and Schmitt-Grohe and Uribe (1997). Giannitsarou (2004) revisits the issue of indeterminacy and aggregate instability when government expenditures may be financed by consumption taxes as well.

Aloi, Lloyd-Braga and Whitta-Jacobsen (2003) utilize an overlapping generations monetary model to study the impacts of fiscal policy rules on the determinacy of rational expectations equilibrium. This paper is closely related to our

³ Guesnerie and Woodford (1992) in their survey on endogenous fluctuations touch on the issues of stabilization policy, but mainly from the point of view of preventing sunspot equilibria.

analysis. Their emphasis is not on taxation, but on discretionary fiscal policy rules, where government expenditures depend on the current and last period's outputs. They show e.g. that stabilization at the monetary steady state can be obtained by using a sufficiently 'countercyclical' rule for government spending. Moreover, they demonstrate that a sufficiently 'procyclical' rule will create indeterminacy.

In both the OG and real business cycle models summarized briefly above, the stabilizing role of proportional and non-proportional taxes is sensitive to the details of model specifications. The source for cycles in our model is the same as in many overlapping generations models; the income effect dominates the substitution effect in an old agent's utility function. We re-examine the stabilizing effects of fiscal policies in a simple overlapping generations model by assuming, in contrast to Grandmont (1986) and Aiyagari (1988), that government uses distortionary taxes to maintain a balanced budget. In particular, we analyze the impact of proportional and linearly progressive taxes on cycles and the indeterminacy of equilibria, and briefly study the price level targeting as a stabilizing device.

We show the following results. Under proportional taxation the steady state supply exceeds (falls short of) the one without taxation, if the elasticity of the marginal utility of the second period consumption, is higher (lower) than one. This is because the higher (lower) steady state supply in the presence of taxation is due to the fact that the positive income effect of the tax rate dominates (is dominated by) the negative substitution effect. Moreover, and importantly, in the presence of endogenous cycles, there is a critical level of tax rate above which there are no cycles. Under progressive taxation the steady state supply is less than the one with proportional taxation due to the negative income effect of tax exemption. In this case we show that there is a critical level of tax exemption, such that for all tax exemptions below that level there are no cycles in the economy. This is due to the fact that when tax exemption is low enough, the slope of the offer curve will become positive and cycles will vanish.

We also characterize the effects of tax policy on the indeterminacy of equilibria, and demonstrate the following results. First, if the utility function is quasi-linear, increasing the tax rate can cause the equilibrium to become determinate both with proportional and linearly progressive taxation so that tax exemption does not matter,

because the income effect of tax exemption is zero for consumption. Second, if the utility function is not quasi-linear, a relatively high tax exemption might make the equilibrium indeterminate due to non-zero income effect. Following the approaches of Woodford (1986), Aiyagari (1988), and Smith (1994) we also consider a policy, which completely eliminates fluctuations. We show that by fixing the target for the price level this can be done by either taxing or subsidizing young workers.

We proceed as follows. In section 2 we present an overlapping generations model with a balanced budget distortionary taxation. Section 3 characterizes competitive equilibrium with proportional taxation, the relationship between the level of taxes and endogenous cycles, and the impact of tax policy on the indeterminacy and determinacy of equilibria. In section 4 we ask what the implications of linearly progressive taxation are for cycles and indeterminacy. In section 5 we characterize price level targeting as a stabilizing policy. Finally, there is a concluding section.

2. An Overlapping Generations Model with a Balanced Budget Fiscal Policy

We consider a perfect foresight overlapping generations model with money and zero population growth. We assume the stock of money to be constant. Producer-consumers consume when old and produce when young. The person born at t has the following additively separable lifetime utility function

$$(1) \quad U(c_{t+1}, n_t) = u(c_{t+1}) - v(n_t),$$

where c_{t+1} denotes consumption when old, and n_t labor supply in youth. Labor is transformed to output, y_t , in a linear fashion, i.e. $y_t = n_t$. $u(c)$ is an increasing strictly concave function, and $v(n)$ an increasing strictly convex function. We denote by \bar{L} the upper bound for the available time, and make the following assumptions: $\lim_{c \rightarrow 0} u'(c) = +\infty$, $\lim_{c \rightarrow \infty} u'(c) = 0$, $\lim_{n \rightarrow 0} v'(n) = 0$ and $\lim_{n \rightarrow \bar{L}} v'(n) = +\infty$. Given our assumptions it follows that $v'(0)/u'(0) < 1$. This means that the slope of the indifference curve at the

endowment point is less than unity. Using the terminology in Gale (1973) we consider here a Samuelson case, which is needed for monetary equilibria in this model.

We consider a fiscal policy with a balanced budget, where government taxes the output (income) produced by the young, and uses the revenues to buy output from the market. We study the potential effects of taxation both in terms of the relationship between the level of the tax rate and cycles as well as in terms of the determinacy of equilibria. We also compare the results of proportional taxation with those of linearly progressive taxation.

To study the effects of progressive taxation in a simple manner we assume that there is a nominal exemption on the taxable income and a constant marginal tax rate, i.e. $\tau_t = \tau \forall t$.⁴ Total nominal tax revenues in period t are thus

$$(2) \quad T_t = \tau(p_t n_t - E_t),$$

where $p_t n_t$ is the nominal tax base (i.e. price times output of the young), and E_t denotes the nominal exemption, which we assume to be constant as well, i.e. $E_t = E \forall t$. The average tax rate, $T_t / p_t n_t = \tau(1 - E / p_t n_t)$, is less than the marginal tax rate τ . The tax schedule (2) is progressive in the sense that even though the marginal tax rate is constant, the average tax rate increases with the tax base. An increase in the average tax rate is higher the higher is the marginal tax rate and the tax exemption when the tax base goes up.

The government budget constraint in the absence of debt financing is

$$(3) \quad p_t g_t = T_t = \tau(p_t n_t - E).$$

We emphasize that the real government expenditure, g_t , is not a given sequence, but it adjusts every period to the level necessary to maintain balanced budget. With proportional taxation the government budget constraint is $g_t = \tau n_t$.

3. Competitive Equilibrium with Proportional Taxation

In this section we study the effects of proportional taxation, and provide answers to the following questions. First, given the existence of endogenous cycles, can the proportional taxation eliminate them? And second, what is the effect of the tax rate on the determinacy of equilibrium.

We first study the properties of competitive equilibrium in the presence of proportional taxation. The private sector periodic budget constraints are

$$(4i) \quad M_t^d = p_t(1-\tau)n_t$$

$$(4ii) \quad p_{t+1}c_{t+1} = M_t^d,$$

and the lifetime constraint is

$$(5) \quad c_{t+1} = \frac{p_t(1-\tau)}{p_{t+1}}n_t.$$

The young producers accumulate money by selling their output to the old, and part of it to the government. The first-order condition for the utility maximization subject to the periodic budget constraints (4i) and (4ii) is

$$(6) \quad \frac{p_t(1-\tau)}{p_{t+1}}u'\left(\frac{p_t(1-\tau)}{p_{t+1}}n_t\right) = v'(n_t).$$

The solution to (6) gives the young's supply function $n_t = n[p_t(1-\tau)/p_{t+1}]$. If supply is increasing in the after tax real wage ($p_t(1-\tau)/p_{t+1}$), a rise in the tax rate will decrease supply. If supply is downward sloping, the reverse happens so that an increase in the tax rate will increase supply.

The equilibrium condition for the goods market is

$$(7) \quad \frac{M_{t-1}^d}{p_t} + g_t = n_t.$$

Taking into account the government budget constraint, $g_t = \tau n_t$, and the fact that the nominal money supply is constant, we rewrite (7) as

⁴ For the definition of progressive taxation, see the seminal paper by Musgrave and Thin (1948). See also Lambert (2001), chapters 7-9 for further analyses. Sandmo (1983) and Koskela and Vilmunen (1995) provide different applications.

$$(8) \quad \frac{M}{p_t} = (1 - \tau)n_t.$$

Using equation (8) in the first-order condition (6) we can re-express it as

$$(9) \quad (1 - \tau)n_{t+1}u'[(1 - \tau)n_{t+1}] = n_t v'(n_t),$$

This equation determines the equilibrium sequence of supplies for a given tax rate, and implicitly defines the reflected generational offer curve.⁵ The steady state equilibrium is determined from $(1 - \tau)u'[(1 - \tau)n] = v'(n)$. Given the Inada conditions and the feasible tax rates, $0 < \tau < 1$, it is straightforward to see that the steady state is unique.

In order to explore the impact of tax policy we first ask: How does the steady state solution to (9) (denoted by \hat{n}) compare to the steady state supply without proportional taxation (denoted by n^*)? The answer is given in

Proposition 1. Under proportional taxation the steady state supply exceeds (falls short of) the one without taxation, if the elasticity of the marginal utility of the second period consumption (or the Arrow-Pratt measure of the relative risk aversion), denoted by σ , is higher (lower) than one, i.e.

$$\hat{n} \begin{cases} > \\ = \\ < \end{cases} n^* \text{ as } \sigma \begin{cases} > \\ = \\ < \end{cases} 1.$$

Proof: Rewriting equation (9) in the steady state as $LHS(n; \tau) \equiv (1 - \tau)u'[(1 - \tau)n] = v'(n) \equiv RHS(n)$, and given the Inada conditions, we have $RHS(0) = 0$, $\lim_{n \rightarrow \bar{L}} RHS(n) = \infty$, and $RHS'(n) > 0$. We also have $LHS'_n(n; \tau) < 0$.

We calculate $LHS'_\tau(n; \tau) = -u'[(1 - \tau)n] - (1 - \tau)nu''[(1 - \tau)n]$, and re-express it as $LHS'_\tau(n; \tau) = -u'[(1 - \tau)n](1 - \sigma(n))$, where $\sigma(n) = -(1 - \tau)nu''[(1 - \tau)n]/u'[(1 - \tau)n]$.

Note that consumption in steady state is $(1 - \tau)n$. When $\sigma(n) > 1$, the curve $LHS(n; \tau)$ shifts up, if the tax rate is raised and vice versa when $\sigma(n) < 1$. The results of Proposition 1 follow from these observations. Q.E.D.

⁵ The geometric techniques of the reflected generational offer curves, developed by Cass, Okuno and Zilcha (1979), are useful for characterizing properties of equilibria in many overlapping generations models.

Proposition 1 follows from the Slutsky equation. The higher (lower) steady state supply with than without taxation is due to the fact that the positive income effect of the tax rate dominates (is dominated by) the negative substitution effect (for a precise elaboration of this intuition, see Appendix). The Slutsky equation can be written as $n_\tau = n_\tau^c - nn_m$, where n_τ^c is the negative substitution effect and $-nn_m$ the positive income effect. The latter (former) effect - evaluated at $m=0$ - dominates if $\sigma(n) > (<)1$.

In order to derive the slope of the offer curve we differentiate (9) to get

$$(10) \quad \frac{dn_{t+1}}{dn_t} = \frac{v'(n_t) + n_t v''(n_t)}{(1-\tau)u'[(1-\tau)n_{t+1}] + (1+\tau)^2 n_{t+1} u''[(1-\tau)n_{t+1}]}$$

We drop the subscripts and note that the second period consumption equals $(1-\tau)n$. Equation (10) cannot be signed generally since the sign of the denominator is a priori ambiguous. Defining $D(n) = (1-\tau)\{u'[(1-\tau)n] + (1-\tau)nu''[(1-\tau)n]\}$ we can express it as

$$(11) \quad D(n) = (1-\tau)u'((1-\tau)n) \left[1 + \frac{(1-\tau)nu''[(1-\tau)n]}{u'[(1-\tau)n]} \right]$$

We define $\sigma(n) = -[(1-\tau)nu''((1-\tau)n)/u'((1-\tau)n)]$, which is the elasticity of the marginal utility of the second period consumption. Using this definition equation (11) can be rewritten as $D(n) = u'(n)[1 - \sigma(n)]$. So we conclude that $D(n) > 0$, when $\sigma(n) < 1$. This means that it is necessary for backward bending offer curve (and indeed for endogenous cycles) to have $\sigma(n) > 1$.

To explore the stabilizing effect of proportional taxation on cycles we use the following notation for the two parts of equation (9), presented above

$$(12) \quad U[n; \tau] \equiv (1-\tau)nu'[(1-\tau)n] \text{ and } G(n) \equiv nv'(n).^6$$

Since function $G(n)$ is monotone increasing we can invert it, and obtain the following relation

$$(13) \quad n_t = G^{-1}(U[n_{t+1}; \tau]) \equiv \Phi_\tau(n_{t+1}).$$

⁶ Here we follow Grandmont (1986a), and partly Aiyagari (1988).

which is actually the inverted reflected generational offer curve. Note that this equation describes the backward dynamics of equilibrium. If there is no τ subscript in (13), it then refers to the case without taxation.

Given (13) we say that there is indeterminacy, if the absolute value of the slope of Φ_τ at the steady state is greater than unity. This also means that the corresponding forward dynamics is locally stable, i.e. equilibria are indeterminate. And, if that slope is less than unity, then the steady state is determinate.⁷

To make this problem interesting from the point of view of endogenous cycles and the potential tax effects on them we assume that there can be cycles in our model economy even in the absence of exogenous shocks. For the existence of a two-cycle (or a periodic point with period two) it is necessary that function $\Phi(n)$ (economy without taxation) is downward sloping. That property, however, is not sufficient for periodic solutions of higher order. To have periodic points with period three and more, it is necessary that the curve, $n_t = \Phi(n_{t+1})$, must be hump-shaped. More precisely, if there are at least three cycles in the economy without taxation (see Figure 1), there must be periodic solutions of any order higher than three according to Sarkovskii's theorem.⁸ It is also well known that cycles in overlapping generations models are intimately connected with sunspot equilibria.⁹ Below we present a parametric specification of our model and provide conditions under which we get a hump-shaped offer curve.

Next we ask: Is there a proportional tax policy, which can eliminate cycles? We provide a positive answer in the following proposition.

⁷ Guesnerie and Woodford (1992), especially chapter 5, discuss thoroughly the concept of indeterminacy in OG models.

⁸ An elementary discussion and elaboration of Sarkovskii's theorem can be found e.g. in Holmgren (1996), in particular chapter 5. On the conditions for the existence of endogenous cycles of more than two periods in economic models, see e.g. Grandmont (1986b).

⁹ Azariadis and Guesnerie (1986) showed that a two-cycle is enough for the existence of stationary sunspot equilibrium.

Proposition 2. There is a critical level of tax rate, $\hat{\tau}$, such that for all $\tau > \hat{\tau}$ there are no endogenous cycles in the economy.

PROOF: Let n' be the maximum of the function $\Phi(n)$ such that $\Phi(n') > n'$. We see that the maximum for the function $\Phi_\tau(n)$ is at $n'/(1-\tau)$. The maximizing point of $\Phi_\tau(n_{t+1})$ can then be increased in such a way that ultimately we have for some τ , say $\hat{\tau}$, that $\Phi_{\hat{\tau}}[n'/(1-\hat{\tau})] < n'/(1-\hat{\tau})$. It follows that $1 > \Phi_{\hat{\tau}}'(\hat{n}) > 0$, where \hat{n} is the respective steady state, which in turn means that there can be no cycles. Q.E.D.

According to Proposition 2 a sufficiently high proportional tax rate will eliminate cycles by changing the location and slope of the inverted reflected generational offer curve. In Figure 1 we have described an economy without taxation but with cycles (curve $n_t = \Phi(n_{t+1})$) and an economy with high enough proportional taxation, and with such a policy that there can be no cycles (curve $n_t = \Phi_\tau(n_{t+1})$). The steady state is stable in backward dynamics, and is denoted by \hat{n} in Figure 1.

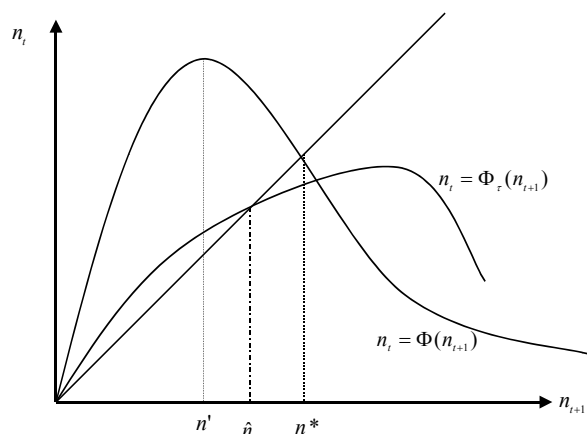


Figure 1.

To be able to study more explicitly the relationship between proportional taxation, endogenous cycles and indeterminacy, we consider a parametric example, which allows for backward-bending reflected generational offer curve. We specify the

quasi-linear utility function¹⁰ as follows: The consumption preferences are described by $u(c) = (c + a)^{1-\sigma} / (1-\sigma)$, where a denotes a luxury of consumption¹¹, and the preferences for disutility of labor are linear, $v(n) = n$. We assume here that $\sigma > 1$, which lies in conformity with empirics¹² and yields possibly interesting dynamics. Note that now the elasticity of marginal utility of consumption is not constant, but equals $\sigma c / (c + a)$, and is thus an increasing function of consumption. Under these specifications the supply function can be written as

$$(14) \quad n_t = R_{t+1}^{\frac{1}{\sigma}-1} (1-\tau)^{\frac{1}{\sigma}-1} - \frac{a}{R_{t+1}(1-\tau)},$$

where we have used the notation: $R_{t+1} = p_t / p_{t+1}$. Differentiating (14) with respect to the interest factor and the tax rate we get

$$(15i) \quad \frac{\partial n_t}{\partial R_{t+1}} = (1-\tau)^{-1} R_{t+1}^{-2} \left[\left(\frac{1-\sigma}{\sigma} \right) R_{t+1}^{\frac{1}{\sigma}} (1-\tau)^{\frac{1}{\sigma}} + a \right]$$

$$(15ii) \quad \frac{\partial n_t}{\partial \tau} = - \left(\frac{R_{t+1}}{1-\tau} \right) \frac{\partial n_t}{\partial R_{t+1}}.$$

These equations show that the interest factor and the tax rate have an opposite effect on supply. Clearly, the supply function can be backward bending with respect to the interest factor, if $\sigma > 1$. Hence, given the interest factor, for all the tax rates, which fulfill the condition $\tau < 1 - \left(\frac{\sigma}{\sigma-1} \right)^{\sigma} a^{\sigma} R_{t+1}^{-1}$, the supply function is backward bending with respect to the tax rate. This means that decreasing the tax rate will decrease the supply and vice versa if $\tau > 1 - \left(\frac{\sigma}{\sigma-1} \right)^{\sigma} a^{\sigma} R_{t+1}^{-1}$.

Next we analyze the properties of competitive equilibrium for this example with the assumption that $\sigma > 1$. Utilizing the equilibrium condition (9) we get

$$(16) \quad \frac{(1-\tau)n_{t+1}}{[(1-\tau)n_{t+1} + a]^{\sigma}} = n_t,$$

¹⁰ Quasi-linearity is often used in the welfare analyses to simplify the presentation, see e.g. Laffont (1988), 158-161.

¹¹ For a further discussion of this specification, see e.g. Auerbach and Hines (2002).

¹² See e.g. the survey by Attanasio (1999).

and in the steady state $n_t = \hat{n} = (1 - \tau)^{\frac{1}{\sigma-1}} - a/(1 - \tau)$. Note that the steady state without taxes is $n^* = 1 - a$. By direct differentiation with respect to the tax rate we see that $\hat{n} > n^*$, if $(1 - \tau)^{\frac{1}{\sigma}} - a > 0$. This latter condition is fulfilled naturally, since it equals consumption ($= (1 - \tau)n$) in the steady state.¹³ Differentiating (16) we get

$$(17) \quad \frac{\partial n_t}{\partial n_{t+1}} = \frac{(1 - \sigma)(1 - \tau)^2 n_{t+1} + (1 - \tau)a}{[(1 - \tau)n_{t+1} + a]^{\sigma+1}}.$$

To get the hump-shaped offer curve described in Figure 1, and thus to allow for periodic solutions of order higher than two we need to have the maximum of the offer curve, $a/(\sigma - 1)(1 - \tau)$, to be less than the steady state, $(1 - \tau)^{\frac{1}{\sigma-1}} - a/(1 - \tau)$. This will lead to the following inequality for the proportional tax rate, $\tau < 1 - \left(\frac{a\sigma}{\sigma - 1}\right)^\sigma = \hat{\tau}$. If the tax rate exceeds $\hat{\tau}$, there can be no cycles in this economy as noted in Proposition 2.

Finally we explore the indeterminacy of equilibrium. Evaluating the slope of (17) at the steady state yields

$$(18) \quad \frac{\partial n_t}{\partial n_{t+1}}(n_t = n^*) = \frac{(1 - \sigma)(1 - \tau)^{\frac{1}{\sigma}} + \sigma a}{(1 - \tau)^{\frac{1}{\sigma}}},$$

where the numerator should be negative for cycles. For the determinacy of perfect foresight dynamics we need $\partial n_t / \partial n_{t+1} > -1$, and thus get the condition

$$(19) \quad \sigma \left[1 - \frac{a}{(1 - \tau)^{\frac{1}{\sigma}}} \right] - 1 < 1,$$

which implies that the tax rate must fulfill the following inequality $\tau > 1 - \left(\frac{a\sigma}{\sigma - 2}\right)^\sigma = \tilde{\tau}$, and $\hat{\tau} > \tilde{\tau}$. According to (19) increasing the tax rate above $\tilde{\tau}$

guarantees the determinacy of equilibrium. Whether this happens depends also on the magnitudes of the luxury of consumption, a , and the parameter, σ , which affects the elasticity of the marginal utility of consumption. It is important to emphasize that to

¹³ See also Proposition 1.

completely abolish the endogenous cycle, i.e. when the offer curve is upward sloping, we need a rather high tax rate ($\hat{\tau}$). To make the backward dynamics stable (i.e. the requirement for determinacy), but still maintaining the downward sloping offer curve, it is enough to have a lower tax rate than $\hat{\tau}$.

4. Competitive Equilibrium with Linearly Progressive Taxation

In this section we study the effects of linearly progressive taxation from the same perspective as we did for proportional taxation, and compare the results to those presented above.

The private sector periodic budget constraints are now

$$(20i) \quad M_t^d = p_t(1-\tau)n_t + \tau E$$

$$(20ii) \quad p_{t+1}c_{t+1} = M_t^d,$$

so that the lifetime constraint is

$$(21) \quad c_{t+1} = \frac{p_t(1-\tau)n_t + \tau E}{p_{t+1}}.$$

The young producers accumulate money by selling their output again to the old and the government. The first-order condition for the utility maximization subject to budget constraints (20i) and (20ii) is

$$(22) \quad \frac{p_t(1-\tau)}{p_{t+1}} u' \left(\frac{p_t(1-\tau)n_t + \tau E}{p_{t+1}} \right) = v'(n_t),$$

which implicitly defines the young's supply function, $n_t = n \left(\frac{p_t(1-\tau)}{p_{t+1}}, \frac{\tau E}{p_{t+1}} \right)$. The

equilibrium condition in the goods market is again

$$(23) \quad \frac{M_{t-1}^d}{p_t} + g_t = n_t.$$

Taking into account the government budget constraint and the fact that the nominal money supply is constant we can rewrite (23) for periods t and t+1 as

$$(24) \quad M = p_t(1-\tau)n_t + \tau E, \quad M = p_{t+1}(1-\tau)n_{t+1} + \tau E.$$

It follows that $p_t n_t = p_{t+1} n_{t+1}$, which means that the tax base stays constant also outside the steady states. Note also that we must have the following natural condition for the policy parameters: $\tau E / M < 1$. Now we can develop the first-order condition above as

$$(25) \quad \frac{(1-\tau)n_{t+1}}{n_t} u' \left(\frac{M}{p_{t+1}} \right) = v'(n_t).$$

From (24) we solve $p_{t+1} = \frac{M - \tau E}{(1-\tau)n_{t+1}}$, and plug into (25) to obtain

$$(26) \quad (1-\tau)n_{t+1} u' \left(\frac{(1-\tau)n_{t+1}}{1 - \frac{\tau E}{M}} \right) = n_t v'(n_t),$$

which determines the equilibrium sequence of supplies. If the second period preferences are logarithmic, the dynamics is determined from the condition $n_t v'(n_t) = 1 - \tau E / M$, i.e. the economy stays forever at the steady state, \tilde{n} , determined from $\tilde{n} v'(\tilde{n}) = 1 - \tau E / M$. It is also interesting to note that the level of nominal money supply affects the intertemporal allocation in the presence of progressive taxation.

Next we ask: How does the steady state solution to (26) (denoted by \tilde{n}) under progressive taxation compare to the steady state with proportional taxation, \hat{n} ? The answer is given in

Proposition 3. The steady state supply with progressive taxation is less than the supply with proportional taxation.

Proof: We rewrite equation (26) in the steady state as $LHS(n; \tau, E) \equiv (1-\tau)u'[(1-\tau)n/(1-(\tau E/M))] = v'(n) \equiv RHS(n)$. Given the Inada conditions we have $RHS(0) = 0$, $\lim_{n \rightarrow \bar{L}} RHS(n) = \infty$, and $RHS'(n) > 0$. We also have

$$LHS_n(n; \tau, E) < 0, \quad LHS_E(n; \tau, E) = (1-\tau) \left(1 - \frac{\tau E}{M}\right)^{-2} \frac{\tau}{M} n u'' \left[\frac{(1-\tau)n}{1 - (\tau E/M)} \right] < 0$$

so that the $LHS(n; \tau, E)$ shifts down, when tax exemption is increased. Proposition 3 follows from this finding. Q.E.D.

Proposition 3 is natural, since in competitive models progressive taxation is more distortionary than proportional taxation. This is because a higher level of tax exemption decreases the steady state supply due to the negative income effect of tax exemption on supply.

To explore the stabilizing effect of taxation on cycles we use the following notation in equation (26)

$$(27) \quad \hat{U}[n; \tau, E] \equiv (1 - \tau)nu' \left(\frac{(1 - \tau)n}{1 - \frac{\tau E}{M}} \right) \text{ and } \hat{G}(n) = nv'(n).$$

Since function $\hat{G}(n)$ is monotone increasing we can invert it, and obtain from (27) the following relation

$$(28) \quad n_t = \hat{G}^{-1}(\hat{U}[n_{t+1}; \tau, E]) \equiv \Phi_E(n_{t+1}),$$

which is again the inverted reflected generational offer curve, where subscript E refers to the case of linearly progressive taxation. If there is no subscript in (28), it then refers to the case without taxation. Obviously the same definition for indeterminacy as above for Φ_τ , applies here for Φ_E . Again we assume that there is at least a two-cycle in the economy without taxation (see Figure 1). As with proportional taxation the hump-shaped form of equation (28) is not necessary for two cycles, but it is necessary for three cycles.¹⁴

Is there a progressive tax policy, which can stabilize the economy by eliminating endogenous cycles? We provide the answer in the following proposition.

Proposition 4. Given the marginal tax rate there is a critical level of tax exemption, \hat{E} , such that for all $E < \hat{E}$, there are no endogenous cycles in the economy.

Proof: Let n' be the maximum of the function $\Phi(n)$ such that $\Phi(n') > n'$. The maximum for the function $\Phi_\tau(n)$ is thus at $n'/(1 - \tau)$, and for the function $\Phi_E(n)$ at $(1 - \frac{\tau E}{M})n'/(1 - \tau)$. The maximizing point of $\Phi_E(n_{t+1})$ can then be increased by

¹⁴ See footnote 7 above.

decreasing E in such a way that ultimately we have for some E , say \hat{E} , that $\Phi_E \left[\left(1 - \frac{\tau E}{M}\right) n' / (1 - \hat{\tau}) \right] < \left(1 - \frac{\tau E}{M}\right) n' / (1 - \hat{\tau})$. This also means that $1 > \Phi_E'(\tilde{n}) > 0$, where \tilde{n} is the respective steady state, which in turn means that there can be no cycles. Q.E.D.

Hence, according to Proposition 4 a sufficiently low progressive taxation – meaning that the average tax rate does not increase with the tax base too much - will eliminate endogenous cycles. This is because lower tax exemption will decrease the income effect and thus will change both the slope and location of the offer curve. When exemption is low enough, the slope will become positive and cycles will vanish.

Next we consider our parametric example specified in section 3 above to explore the issue of indeterminacy. Analogously to the derivation of the supply function with proportional taxes (c.f. equation 15) we get

$$(29) \quad n_t = R_{t+1}^{\frac{1}{\sigma}-1} (1-\tau)^{\frac{1}{\sigma}-1} - \frac{a}{R_{t+1}(1-\tau)} - \frac{\tau E}{(1-\tau)p_t},$$

where the term, E/p_t , is the real exemption. Differentiating (29) with respect to the tax rate, and the tax exemption we get

$$(30i) \quad \frac{\partial n_t}{\partial \tau} = -(1-\tau)^{-2} R_{t+1}^{-1} \left[\left(\frac{1-\sigma}{\sigma} \right) R_{t+1}^{\frac{1}{\sigma}} (1-\tau)^{\frac{1}{\sigma}} + a \right] - (1-\tau)^{-2} \frac{E}{p_t}$$

$$(30ii) \quad \frac{\partial n_t}{\partial E} = -\frac{\tau}{(1-\tau)p_t}.$$

For a given interest factor decreasing the tax rate will decrease the supply (i.e. $\partial n_t / \partial \tau > 0$), if the marginal tax rate and tax exemption fulfill the following condition

$$(31) \quad \tau \leq 1 - \left(\frac{\sigma}{\sigma-1} \right)^{\sigma} \left(\frac{a}{R_{t+1}} + \frac{E}{p_t} \right)^{\sigma} R_{t+1}^{\sigma-1}.$$

Now we analyze the determinacy of competitive equilibrium for this specification again with the assumption that $\sigma > 1$. Using equation (26) we obtain

$$(32) \quad \frac{(1-\tau)n_{t+1}}{\left[\frac{(1-\tau)n_{t+1}}{1-\frac{\tau E}{M}} + a \right]^\sigma} = n_t,$$

where we calculate the steady state $\tilde{n} = (1 - \frac{\tau E}{M})(1 - \tau)^{\frac{1}{\sigma}-1} - (1 - \frac{\tau E}{M})\frac{a}{1-\tau}$. This can be

expressed in terms of the steady state with proportional taxation as $\tilde{n} = (1 - \frac{\tau E}{M})\hat{n}$.

Hence, introducing tax exemption, which makes taxation progressive, decreases the steady state supply compared to the case with proportional taxation (see Proposition 3).

Differentiating (32) yields

$$(33) \quad \frac{\partial n_t}{\partial n_{t+1}} = \frac{\frac{(1-\sigma)(1-\tau)^2 n_{t+1}}{1-\frac{\tau E}{M}} + (1-\tau)a}{\left[\frac{(1-\tau)n_{t+1}}{1-\frac{\tau E}{M}} + a \right]^{\sigma+1}}.$$

Evaluating the slope at the steady state we get

$$(34) \quad \frac{\partial n_t}{\partial n_{t+1}}(n_t = \tilde{n}) = \frac{(1-\sigma)(1-\tau)^{\frac{1}{\sigma}} + \sigma a}{(1-\tau)^\sigma},$$

where the numerator must be negative. We get the condition for determinacy as

$$(35) \quad \sigma \left[1 - \frac{a}{(1-\tau)^{\frac{1}{\sigma}}} \right] - 1 < 1.$$

Decreasing the tax rate can cause indeterminacy, i.e. if τ decreases enough the inequality sign turns around depending on the relative size of parameters a and σ .¹⁵ Note that this condition is exactly the same (c.f. equation (19)) as with the presence of

¹⁵ This can also be seen by differentiating the left-hand side of (35) with respect to the tax rate, and noting that the partial derivative will be negative.

proportional taxation. This is due to the quasi-linear specification of the utility function, which is strictly concave in consumption and linear in leisure and labor supply. Therefore the income effect of tax exemption is zero for consumption so that tax progressivity does not matter here.

Finally we consider an example where – unlike in the case of quasi-linear utility function - the income effect for consumption is not zero. We assume that disutility function ($v(n)$) is of the form $v(n) = (1/4)n^2$. This gives the following equilibrium dynamics

$$(36) \quad \frac{(1-\tau)n_{t+1}}{\left[\frac{(1-\tau)n_{t+1}}{1-\frac{\tau E}{M}} + a \right]^\sigma} = \frac{1}{2}n_t^2.$$

While we cannot explicitly solve for the steady state, we, however, get from (36) the following relation at the steady state

$$(37) \quad LHS(n; \tau, E, M) \equiv \left[\frac{(1-\tau)n}{1-\frac{\tau E}{M}} + a \right]^\sigma = \frac{2(1-\tau)}{n} \equiv RHS(n; \tau).$$

The left-hand side of (37) is an increasing, and the right-hand side a decreasing function of supply, n . Furthermore, we see from (37) that $LHS_E(n; \tau, E, M) > 0$. This means that an increase in the level of exemption will shift the $LHS(n; \tau, E, M)$ curve up, so that there is a negative relationship between the steady state employment and exemption, i.e. $\partial n / \partial E < 0$.

Using (36) we get

$$(38) \quad \frac{\partial n_t}{\partial n_{t+1}} = \frac{(1-\tau) \left[\frac{(1-\sigma)(1-\tau)n_{t+1}}{1-\frac{\tau E}{M}} + a \right]}{\left[\frac{(1-\tau)n_{t+1}}{1-\frac{\tau E}{M}} + a \right]^{\sigma+1}} \times \frac{1}{n_t},$$

Again we concentrate on the case, where $\sigma > 1$, so that the above slope can be negative. Next we evaluate the slope at the steady state. Using the steady state characterization from (37) and evaluating (38) at the steady state we obtain

$$(39) \quad \frac{\partial n_t}{\partial n_{t+1}} \Big|_{n_t = n^*} = \frac{\left[\frac{(1-\tau)n}{1 - \frac{\tau E}{M}} + a - \sigma \frac{(1-\tau)n}{1 - \frac{\tau E}{M}} \right]}{2 \left[\frac{(1-\tau)n}{1 - \frac{\tau E}{M}} + a \right]} = \frac{(1-\sigma)(1-\tau)n + a(1 - \frac{\tau E}{M})}{2(1-\tau)n + 2a(1 - \frac{\tau E}{M})}.$$

Hence (39) provides the following condition for the determinacy of equilibrium

$$(40) \quad \frac{2(1-\tau)n + 2a(1 - \frac{\tau E}{M})}{(1-\sigma)(1-\tau)n + a(1 - \frac{\tau E}{M})} < -1.$$

Taking into account the fact that, in the case we are considering, the left-hand side of (40) must be negative, we get from the denominator the following inequality for exemption

$$(41) \quad E > \frac{M}{\tau} \left[1 - \frac{(\sigma-1)(1-\tau)n}{a} \right].$$

We can re-express (40) as

$$(42) \quad E < \frac{M}{\tau} \left[1 - \frac{(\sigma-3)(1-\tau)n}{3a} \right].$$

Hence (41) and (42) provide bounds for the level of exemption, and thereby for determinacy to hold. In this example it is necessary for determinacy that $\sigma > 3$. In particular, if σ is just slightly greater than three (42) holds for sure and we have determinacy. Thus, there is an upper bound for tax exemption as already suggested by Proposition 4 above. Tax exemption matters for determinacy because of non-zero income effect. Since the steady state employment is a decreasing function of exemption, both sides of (42) are increasing functions of exemption. Increasing the level of exemption might switch the inequality around, and make the steady state indeterminate, if the effect of a change in exemption on supply is not very large.

5. Price Level Targeting as a Stabilizing Policy

We briefly discuss here an example of a policy, which will stabilize the economy completely, i.e. we look for a policy under which the only equilibrium is stationary. We partly follow Woodford (1986), Aiyagari (1988), and Smith (1994), who discuss different policies in the same sense. The young are taxed and the proceeds are directly transferred to the old. This means that the amount of the nominal money supply stays constant as above. Consider now the following tax policy: the policy authority chooses a benchmark price level, p^* , and taxes the young workers by a proportional rate $1 - p^*/p_t$. If $p^*/p_t > 1$, the workers are subsidized and the old are taxed. The decision problem of the young is to maximize the lifetime utility function, $u(c_{t+1}) - v(n_t)$ subject to the budget constraints

$$(42i) \quad M_t^d = p_t n_t - \left(1 - \frac{p^*}{p_t}\right) p_t n_t$$

$$(42ii) \quad p_{t+1} c_{t+1} = M_t^d + S_{t+1},$$

where S_{t+1} is the subsidy (if positive) they get in the second period of their lives. The first-order condition is

$$(43) \quad \frac{p^*}{p_{t+1}} u' \left(\frac{p^* n_t + S_{t+1}}{p_{t+1}} \right) = v'(n_t).$$

The goods market equilibrium condition in period t is

$$(44) \quad \frac{M_{t-1}^d + S_t}{p_t} = n_t.$$

Subsidy in equilibrium will be $S_t = (p_t - p^*)n_t$, and the nominal money supply is constant, i.e. $M_t = M$ for all t . It then follows that $M = p^* n_t$. We can now rewrite the first-order condition, (43), as

$$(45) \quad \frac{p^*}{p_{t+1}} u' \left(\frac{M}{p^*} \right) = v' \left(\frac{M}{p^*} \right),$$

which can be solved for a unique price level. Thus the only equilibrium is stationary.

6. Conclusions

We have studied the effects of distortionary taxation on endogenous cycles and the determinacy of equilibria in a competitive overlapping generations model with money and a balanced budget rule for fiscal policy. In particular, we have explored the implications of proportional and progressive tax systems.

We have shown that under proportional taxation the steady state supply exceeds (falls short of) the one without taxation, if the elasticity of the marginal utility of the second period consumption, is higher (lower) than one. This is because the higher (lower) steady state supply in the presence of taxation is due to the fact that the positive income effect of the tax rate dominates (is dominated by) the negative substitution effect. Moreover, and importantly, in the presence of cycles there is a critical level of tax rate such that for all higher tax rates there are no cycles in the economy. The steady state supply under progressive taxation is less than the one with proportional taxation due to the negative income effect of tax exemption on supply. In this case there is a critical level of tax exemption, such that for all smaller tax exemptions there are no cycles in the economy. When tax exemption is low enough, the slope of the offer curve will become positive and cycles will vanish.

We have also characterized the effects of tax policy on the determinacy of equilibria by providing the following results. First, if the lifetime utility function is quasi-linear, increasing the tax rate can make the equilibrium determinate both with proportional and linearly progressive taxation so that tax exemption does not matter. This is because the income effect of tax exemption is zero for consumption. Second, if the lifetime utility function is more general, then tax exemption might matter because of the income effect. But for a small tax rate an increase in progression can bring about determinate equilibria. We have also shown that policy, which fixes a target for the price level, can completely eliminate fluctuations in the economy.

Appendix:

Derivation of the Slutsky equation for the supply function

Maximizing the utility function $U = u(c) - v(n)$ subject to $c = (1 - \tau)n + m$, where m is non-labor income, gives $U_n = 0 = u'(c)(1 - \tau) - v'(n)$, which implicitly defines the supply function $n = n(\tau, m)$. Substituting this for n in U gives the indirect utility function $U^*(\tau, m) = U^o$ with the following properties: $U_m^* = u'(c^*) > 0$ and $U_\tau^* = -nU_m^* < 0$. Given the monotone U_m^* we can invert the indirect utility function for m so that we have the following expenditure function, $m = h(\tau, U^o)$. Substituting this for m in $U^*(\tau, m) = U^o$ yields the compensated indirect utility function (see Diamond and Yaari, 1972) $U^*(h(\tau, U^o), \tau) = U^o$ with the following property $U_m^* h_\tau + U_\tau^* = 0$ so that $h_\tau = -U_\tau^* / U_m^* = n$. According to the duality theorem we can write the relationship between the uncompensated and the compensated supply as follows $n(h(\tau, U^o), \tau) = n^c(\tau, U^o)$. Differentiating this with respect to the tax rate gives $n_\tau + n_m h_\tau = n_\tau^c$ which can be written as the Slutsky equation $n_\tau = n_\tau^c - n n_m$, where we have the negative substitution effect, $n_\tau^c (= \frac{u'(c)}{u''(c)(1-\tau)^2 - v''(n)} < 0)$, and the positive income effect, $-n n_m (= \frac{c u''(c)}{u''(c)(1-\tau)^2 - v''(n)} > 0)$. The latter (former) effect evaluated at $m = 0$ dominates if $\sigma(n) > (<) 1$.

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