Title	Capital Accumulation and Employment Cycles in a Model of Creative Destruction
Author(s)	Palokangas, Tapio
Date	2004
URL	http://hdl.handle.net/10138/16544

Helsinki Center of Economic Research

Discussion Papers

Capital Accumulation and Employment Cycles in a Model of Creative Destruction

Tapio Palokangas
University of Helsinki and HECER

Discussion Paper No. 35
November 2004

ISSN 1795-0562

HECER – Helsinki Center of Economic Research, P.O. Box 17 (Arkadiankatu 7), FI-00014 University of Helsinki, FINLAND, Tel +358-9-191-28780, Fax +358-9-191-28781,
E-mail info-hecer@helsinki.fi, Internet www.hecer.fi

HECER
Discussion Paper No. 35

Capital Accumulation and Employment Cycles in a Model of Creative Destruction*

Abstract

A Schumpeterian growth model is constructed for an economy with wage bargaining. It is shown that the economy is subject to cycles in which capital, output and employment vary in fixed proportion. These increase through saving and capital accumulation until a new technology is introduced, at which moment they fall sharply due to obsolescence of capital. If the labour market is deregulated to weaken workers' position in bargaining, then the labour-capital ratio increases but the average growth rate of the economy decreases. The growth cycle can be socially optimal. An elasticity rule is given for when the labour market should be regulated and when deregulated.

JEL Classification: O41, E32

Keywords: growth, cycles, labour unions, creative destruction

Tapio Palokangas

Department of Economics
P.O. Box 17 (Arkadiankatu 7)
University of Helsinki
FI-00014 University of Helsinki
FINLAND

e-mail: tapio.palokangas@helsinki.fi

*This is a substantially revised version of *CESifo Working Paper 855 (2003)*.

1. Introduction

The purpose of this paper is to construct a model that would explain economic growth with fluctuations in output and employment. The study is therefore closely related to theories of endogenous growth and real business cycles (*RBC*). Aghion and Howitt (1992) shows that the introduction of jump processes into general equilibrium models leads to endogenous business cycles. In their original model, however, there is a perfect labour market, no real capital, and the households were risk neutral. Aghion and Howitt (1998) incorporated capital accumulation and Wälde (1999) risk averse households into this model. Despite of these generalizations, it is still typical for this theory that the economy generates output and employment cycles only *outside* the balanced-growth path. We construct a model which generates such cycles *on* the balanced-growth path, but in which there are constant equilibrium levels for the labour-capital ratio and the productivity-adjusted wages.

Introducing endogenous shocks into a *RBC* model, Wälde (2002) showed that the 'laissez faire' economy and the social planner generate different outcomes. In his model, however, the economy is characterized by 'bang-bang' development: because R&D is subject to constant returns to scale and the same good is used in both R&D and capital accumulation, the firms either do R&D or invest in real capital, but do not both. We assume that because the firms also learn from each other, technological change in a single firm is a function of R&D inputs of all firms in the economy. This means that firms invest in R&D and real capital simultaneously and the economy holds on a stationary state despite of endogenous technological shocks.

All papers mentioned above assume a perfectly competitive labour market. To explain real business cycles, one should however focus on labour market imperfections. This can be supported by the following stylized facts:

- The level of employment adjusts faster than real wages to a shock, not vice versa as suggested by models with a perfect labour market.
- A shock that makes some of capital obsolete reduces the level of employment, rather than increases the labour-capital ratio to maintain full employment as suggested by models with a perfect labour market.
- There is no trend for the rate of unemployment.

Because at least in European countries wage bargaining is a major form of labour market imperfection, we take it as a starting point. Following Blanchard and Giavazzi (2001), we call any measures of public policy that increase (decrease) the workers' relative bargaining power as *labour market regulation* (*deregulation*). In addition to the construction of a business cycle model, we also explain why it may not be in the government's interest to eliminate unemployment by deregulation. The paper is organized as follows. The structure of the economy is specified in section 2, technological change in 3 and production and capital accumulation in 4. Section 5 introduces agents, section 6 wage bargaining and section 7 constructs general equilibrium. Welfare evaluations are carried out in section 8.

2. The sectors

The economy comprises of three sectors – two producing consumption and investment goods from labour and capital, and a third sector doing R&D by labour only. Firms of all sectors are subject to constant returns to scale. A key feature of our model is the existence of two simultaneous forms of technological change. First, the productivity of labour in the consumption-good and investment-good sectors is a product of learning by investment at the level of the whole economy. This eliminates the trend in the rate of unemployment. Second, *total factor productivity (TFP)* in the consumption-good sector is a random process in which a single firm can increase the probability of change by its own R&D. This generates employment cycles.

There are two separate labour markets:[1] one for the consumption-good and investment-good sectors, and the other for R&D. In Wälde's (2002) model, the economy produces from labour and capital one good which is used for consumption, for capital accumulation and as an input for R&D. Resources can then be transferred between investment and R&D without cost and the economy grows in a bang-bang manner, with savings being allocated in either investment or R&D but not in both. Our study starts from the assumption that R&D is less capital intensive than the production of

[1]Separate labour markets for the consumption-good and investment-good sectors would not make any difference in the results.

investment goods. We bring this specification to the extreme, for simplicity, and ignore the use of capital in R&D.

Another key feature of our model is that each household must decide *ex ante* in which market it is going to supply labour. This discrete choice of occupation implies that in equilibrium the expected wage (i.e., the wage times the probability of employment) must be uniform in the economy. Wage bargaining is possible in the production of consumption or investment goods, because the marginal product of labour is there falling for given capital stock. In the R&D sector, the marginal product of labour is constant, there are no profits, wages are competitively determined and there is no unemployment.

We assume that households hold shares only in those firms in which they are not working.[2] Given this, firms can be aggregated with their owners into consumer-producer agents and the whole analysis can be carried out in an extensive-game framework as follows. There is a fixed number n of agents that consume, produce, do R&D, invest in real capital and supply labour to the other agents, taking wages and all macroeconomic variables as given. At stage I, agents choose the sector where they supply labour; at stage II, union-employer bargaining determines wages for the production of consumption and investment goods; and at stage III, agents make the rest of their decisions. This game is solved by backward induction in sections 5-7.

3. Technology

The productivity of labour in R&D is unity. Because of learning by investment and the spillover of this knowledge, the productivity of labour in the production of consumption and investment goods, a, increases in in proportion to the expected accumulation of aggregate capital stock:[3]

$$\frac{\dot{a}}{a} = E\left(\sum_{k=1}^{n} dK_k \Big/ \sum_{k=1}^{n} K_k\right) = E\left(\frac{dK}{K}\right), \tag{1}$$

[2] Alternatively, to obtain the same results, one could assume that there is a large number of households which hold an equal but ignorable share of all firms.

[3] This assumption ensures that there is no trend for the rate of unemployment. The economy would converge to full employment for $\dot{a}/a < E(dK/K)$, and unemployment would increase indefinitely for $\dot{a}/a > E(dK/K)$. See section 7.

where $\dot{a} = da/dt$, K_j the capital stock of agent j, $K = \frac{1}{n}\sum_k K_k$ aggregate capital stock in the economy, and E is the expectations operator. A single agent takes the productivity of labour, a, as fixed.

In the investment-good sector total factor productivity (TFP) is kept constant, but in the consumption-good sector it is determined so that each new technology increases the level of productivity by constant $A > 1$.[4] In other respects, the production function is the same for these two sectors. This means that consumption and investment goods can be aggregated into a single product so that with technology γ, TFP in the consumption-good sector is given by A^γ but that in the investment-good sector is equal to unity. We normalize the price of this product at unity.

Because there is externality in the R&D sector, agent j outcome in R&D depends on both its own demand for R&D services, Z_j, and the other agents' demands, Z_k for $k \neq j$. We specify this dependence in a CES form:[5]

$$G(Z_j, Z_{-j}) \doteq n\Big[\frac{1}{n}Z_j^{1-1/\mu} + \Big(1-\frac{1}{n}\Big)Z_{-j}^{1-1/\mu}\Big]^{\mu/(\mu-1)}, \qquad \mu > 0,$$
$$Z_{-j} \doteq \Big[\frac{1}{n-1}\sum_{k\neq j} Z_k^{1-1/\mu}\Big]^{\mu/(\mu-1)}, \qquad \frac{\partial G}{\partial Z_j} = \Big(\frac{G}{nZ_j}\Big)^{1/\mu}, \tag{2}$$

where n is the number of agents and μ the constant elasticity of substitution.

In a small period of time dt, the probability that R&D leads to development of a new technology is given by $G\,dt$, while the probability that R&D remains without success is given by $1 - G\,dt$:

$$dq = \begin{cases} 1 & \text{with probability } G\,dt, \\ 0 & \text{with probability } 1 - G\,dt, \end{cases} \tag{3}$$

where q is the Poisson process resulting from R&D and dq is the increment of this process. From this property it follows that $\ln A^{\gamma+1} - \ln A^\gamma = (\ln\gamma)\chi(t)$, where $\chi(t)$ is the number of innovations between γ and $\gamma+1$. Because variable $\chi(t)$ is Poisson distributed with parameter G, the average growth rate of the

[4]This discontinuous technological progress mechanism and the R&D technology presented later are borrowed from Aghion and Howitt (1992). Our specification of the process is to a large extent based on Wälde (2001).

[5]Given this specification, the marginal product of R&D input, $\partial G/\partial Z_j$, is independent of the number of agents, n, in the symmetric equilibrium $Z_j = Z_{-j} = G/n$.

level of productivity A^{γ} in the stationary state is given by[6]

$$E[\log A^{\gamma+1} - \log A^{\gamma}] = G \log A, \tag{4}$$

where E is the expectations operator.

The convenient feature of the model is that because the function (2) is similar for all agents $j = 1, ..., n$, there is only one stochastic process. This does not however mean that a single agent j would ignore the effect of its R&D on the level of productivity. If the elasticity of substitution, μ, is small enough, then agent j's demand for R&D services, Z_j, has a significant impact on the probability of technological change, G, given the other agents' demand for R&D services, Z_{-j}, even when the number of firms, n, is large. In a symmetric equilibrium $Z_j = Z_{-j} = Z = nG$, the demand for R&D services, Z, can be used as a proxy of the growth rate (4).

4. Capital accumulation

Because each agent j can change its members' occupation from a worker to an researcher at some cost and the abilities of all individuals in economy j differ, there is a decreasing and convex transformation function between the supply of workers, N_j, and the supply of researchers, L_j, as:

$$N_j = aN(Z_j), \quad N' < 0, \quad N'' < 0, \tag{5}$$

where a is the productivity of labour in the production of consumption and investment goods. Given $N'' < 0$, more and more workers must be transformed in order to create one more research input.

In the production of consumption and investment goods, agent j pays the wage w_j per effective labour input, agents $k = 1, ..., n$ supply $\sum_k N_k$ physical labour units, agent j employs L_j labour units and the probability of being employed by agent j is $L_j / \left(\sum_k N_k\right)$. The expected wage in the production of consumption and investment goods, p, is then the sum of the wages, w_j, weighed by the probabilities of being employed, $L_j / \left(\sum_k N_k\right)$. Noting this

[6] For this, see Aghion and Howitt (1998), p. 59.

and (5), we obtain

$$p \doteq \sum_{j=1}^{n} \frac{w_j L_j}{\sum_{k=1}^{n} N_k} = \frac{1}{a} \sum_{j=1}^{n} \frac{w_j L_j}{\sum_{k=1}^{n} N(Z_k)}. \tag{6}$$

Given TFP in the consumption-good sector, A^{γ}, we obtain agent j's budget constraint as:

$$A^{-\gamma} C_j + I_j = \Pi_j + pN_j, \tag{7}$$

where C_j consumption, I_j investment in capital, Z_j the demand for R&D, p the expected wage in the production of consumption and investment goods, Π_j profits from the production of consumption and investment goods and pN_j expected labour income. We assume, for simplicity, that capital is a stock of goods that does not depreciate. Investment per unit of time dt then equals deterministic capital accumulation, $I_j dt = dK_j^d$. Solving for I_j from (7) and noting (5), we obtain

$$dK_j^d = I_j dt = [\Pi_j + paN(Z_j) - A^{-\gamma} C_j] dt. \tag{8}$$

R&D is directed at developing new production units. We assume that after a successful development of new technology, a certain share s of the previous vintage can be upgraded which therefore has the higher productivity.[7] The remaining share $1 - s$ of capital stock becomes obsolete. The capital stock after successfully finishing an R&D project, $\widetilde{K}_j$, is then given by the current capital stock K_j as follows:

$$\widetilde{K}_j = sK_j, \quad 0 < s < 1. \tag{9}$$

Given this definition, the entire capital stock belongs to the same vintage.

Noting (15), capital accumulation for agent j is given by

$$dK_j = I_j dt + (\widetilde{K}_j - K_j) dq = \left[\Pi_j + paN(Z_j) - A^{-\gamma} C_j\right] dt + (\widetilde{K}_j - K_j) dq. \tag{10}$$

This is a stochastic differential equation where uncertainty results from a Poisson process q. During a small period of time dt, the capital stock of

[7] This idea is from Wälde (2002).

vintage γ increases deterministically by investment in capital accumulation. With a successful R&D project, $dq = 1$, capital stock jumps by $\widetilde{K}_j - K_j$ and the level of productivity rises by A. When no investment in R&D takes place or when R&D fails, the increment dq is zero, the level of productivity does not change and there is no jump in capital stock K_j.

5. Agents

Agent j employs L_j units of effective labour at wage w_j from the other agents $k \neq j$ and produces output Y_j from input L_j and capital K_j through a twice-differentiable production function $Y_j = F(K_j, L_j)$ with constant returns to scale. Agent j's profit Π_j is equal to output Y_j minus labour costs $w_j L_j$:

$$\Pi_j = Y_j - w_j L_j = F(K_j, L_j) - w_j L_j. \tag{11}$$

Agent j maximizes its expected utility over time by choosing its streams of consumption, R&D and labour input, $\{C_j(\tau), Z_j(\tau), L_j(\tau)\}$, subject to the accumulation of capital (10) and the stochastic process (3), given the wage for its workers, w_j, and the price for R&D services p. We denote the constant rate of time preference by $\rho > 0$, the constant rate of risk aversion by $1/(1-\sigma)$, and define the value of the optimal program at time t as:

$$\Gamma(K_j, w_j, p, \gamma) = \max_{C_j,\, Z_j,\, L_j} E \int_t^\infty e^{-\rho(\tau - t)} C_j^\sigma d\tau \ \text{ s.t. (15) and (10).} \tag{12}$$

Because the agent is a risk averter, $0 < \sigma < 1$ holds. Let $\widetilde{K_j}$, $\widetilde{w_j}$ and $\widetilde{\Gamma} = \Gamma(\widetilde{K}_j, \widetilde{w}_j, p, \gamma + 1)$ be the values of K_j, w_j and Γ after successfully finishing an R&D project. Denoting $\Gamma_K \doteq \partial \Gamma / \partial K_j$ and noting (3), (10) and (11), the Bellman equation of the optimal program of agent j obtains[8]

$$\rho \Gamma(K_j, w_j, p, \gamma) = \max_{C_j,\, Z_j,\, L_j} \Phi(C_j, Z_j, L_j, K_j, w_j, p, \gamma), \tag{13}$$

where

$$\begin{aligned} \Phi(C_j, Z_j, L_j, K_j, w_j, p, \gamma) &\doteq C_j^\sigma + G[\widetilde{\Gamma} - \Gamma] + \Gamma_K I_j \\ &= C_j^\sigma + G(Z_j, Z_{-j})[\Gamma(\widetilde{K_j}, \widetilde{w_j}, \widetilde{p}, \gamma + 1) - \Gamma(K_j, w_j, p, \gamma)] \\ &\quad + \big[F(K_j, L_j) - w_j L_j + paN(Z_j) - A^{-\gamma} C_j\big] \Gamma_K(K_j, w_j, p, \gamma). \end{aligned} \tag{14}$$

[8]Cf. Dixit and Pindyck (1994).

Maximizing (14) by labour input L_j is equivalent to maximizing profits $\Pi_j = Y_j - w_j L_j$ by L_j. Given this, duality and the properties of the production function $Y_j = F(K_j, L_j)$, profit, output and labour input become functions of capital K_j and the wage w_j as:

$$\Pi_j = \max_{L_j}[Y_j - w_j L_j] = \max_{L_j}[F(K_j, L_j) - w_j L_j] = \pi(w_j)K_j, \quad \pi' < 0,$$
$$\pi'' > 0, \quad L_j = -\pi'(w_j)K_j, \quad Y_j/K_j = y(w_j) \doteq \pi(w_j) - w_j\pi'(w_j). \tag{15}$$

We denote the ratio of wages to profits by δ and the elasticity of employment with respect to the wage in production, when capital K is held constant, by ϵ. Given (15), we then obtain

$$\delta(w) \doteq \frac{wL}{\Pi} = -\frac{w\pi'(w)}{\pi(w)}, \quad \epsilon(w) \doteq \left|\frac{w}{L}\frac{dL}{dw}\right| = -\frac{w\pi''(w)}{\pi'(w)} > 0. \tag{16}$$

Maximizing (14) by consumption C_j yields

$$\sigma C_j^{\sigma-1} = A^{-\gamma}\Gamma_K. \tag{17}$$

We try the solution that consumption expenditure $A^{-\gamma}C_j$ is a share $c_j \in (0,1)$ of income net of R&D, $\pi(w_j)K_j + pN(Z_j)$, and the value function is given by $\Gamma = C_j^\sigma/(c_j r_j)$, where c_j and r_j are constants. This and (17) imply

$$C_j = c_j A^\gamma[\pi(w_j)K_j + paN(Z_j)], \ \ \partial C_j/\partial K_j = c_j A^\gamma \pi(w_j), \ \ \Gamma = C_j^\sigma/(c_j r_j),$$
$$\Gamma_K = \frac{1}{c_j r_j}\frac{\partial C_j^\sigma}{\partial K_j} = \frac{\sigma C_j^{\sigma-1}}{c_j r_j}\frac{\partial C_j}{\partial K_j} = \frac{A^{-\gamma}\Gamma_K}{c_j r_j}\frac{\partial C_j}{\partial K_j} = \frac{\Gamma_K}{r_j}\pi, \ \ r_j = \pi(w_j). \tag{18}$$

Noting (2), (17) and (18), maximizing (14) by Z_j yields

$$\begin{aligned}
\frac{\partial\Phi}{\partial Z_j} &= (\widetilde{\Gamma} - \Gamma)\frac{\partial G}{\partial Z_j} + pa\Gamma_K N' = (\widetilde{\Gamma} - \Gamma)\Big(\frac{G}{nZ_j}\Big)^{1/\mu} + pa\Gamma_K N' \\
&= \Gamma_K\Big\{\Big(\frac{\widetilde{\Gamma}}{\Gamma} - 1\Big)\Big(\frac{G}{nZ_j}\Big)^{1/\mu}\frac{\Gamma}{\Gamma_K} + paN'\Big\} \\
&= \Gamma_K\Big\{\Big(\frac{\widetilde{\Gamma}}{\Gamma} - 1\Big)\Big(\frac{G}{nZ_j}\Big)^{1/\mu}\frac{\Gamma}{\sigma C_j^{\sigma-1}A^\gamma} + paN'\Big\} \\
&= \Gamma_K\Big\{\Big(\frac{\widetilde{\Gamma}}{\Gamma} - 1\Big)\Big(\frac{G}{nZ_j}\Big)^{1/\mu}\frac{C_j}{A^\gamma\sigma c_j r_j} + paN'\Big\} = 0.
\end{aligned} \tag{19}$$

6. Wage bargaining

In a bargain over the wage w_j, the workers employed by agent j are organized in a union which attempts to maximize their total wages $W_j \doteq w_j L_j$, while the management representing agent j attempts to maximize profits Π_j. We assume, for simplicity, that both parties in bargaining take capital stock K_j as given.[9] The Generalized Nash product of an asymmetric bargaining is then given by $\Lambda_j \doteq W_j^\alpha \Pi_j^{1-\alpha}$, where constant $\alpha \in (0,1)$ is the union's relative bargaining power. Given (15), this product takes the form

$$\Lambda_j(w_j, K_j, \alpha) \doteq W_j^\alpha \Pi_j^{1-\alpha} = w_j^\alpha [-\pi'(w_j)]^\alpha \pi(w_j)^{1-\alpha} K_j, . \tag{20}$$

The outcome of bargaining is obtained through maximizing the product (20), given capital stock K_j. We specify the production function so that there exists a wage rate $\overline{w}$ which maximizes total wages $w_j L_j$, given capital stock K_j.[10] Otherwise, unions would have no incentive to raise wages above the level that corresponds to full employment. This implies:

$$\begin{aligned} &\overline{w} = \arg\max_{w_j}[w_j L_j] = \arg\max_{w_j}[-w_j \pi'(w_j)] = \arg\min_{w_j}[w_j \pi'(w_j)] > 0, \\ &\pi'(w_j) + w_j \pi''(w_j) = \frac{\partial [w_j \pi'(w_j)]}{\partial w_j} \begin{cases} > 0 \text{ for } w_j > \overline{w}, \\ < 0 \text{ for } w_j < \overline{w}. \end{cases} \end{aligned} \tag{21}$$

Maximizing (20) by w_j is equivalent to maximizing $(1/\alpha)\log \Lambda_j$ by w_j. This produces first-order and second-order conditions:

$$\frac{1}{\alpha K_j}\frac{\partial \log \Lambda_j}{\partial w_j} = \frac{\pi'(w_j) + w_j \pi''(w_j)}{w_j \pi'(w_j)} + \Big(\frac{1}{\alpha} - 1\Big)\frac{\pi'(w_j)}{\pi(w_j)} = 0, \quad \frac{\partial^2 \log \Lambda_j}{\partial w_j^2} < 0.$$

Differentiating the first-order condition totally and noting (15), (21) and the second-order condition, we obtain that wages are uniform and that they

[9] If these parties took also the effect of the wage w_j through capital accumulation into account, then the union's (management's) target would be the expected value of the stream of wages (profits). Because in our model capital stock follows a cycle, the mathematic solutions for such expected values would be very difficult to obtain.

[10] A good example of such technology is that the elasticity of substitution is one between labour and capital, but raw materials (produced by other firms) are used in fixed proportion b to labour. This defines the production function $F(K_j, L_j) \doteq \chi K_j^{1-\beta} L_j^\beta - bL_j$, where $\chi > 0$, $0 < \beta < 1$ and $b > 0$ are parameters. Then $\overline{w} = b/\beta > 0$ obtains. For an ordinary Cobb-Douglas function with $b = 0$, $\overline{w} = 0$ obtains.

increase with the unions' relative bargaining power:

$$w_j = w(\alpha) \in (0, \overline{w}) \quad \text{with} \quad \frac{dw}{d\alpha} \doteq \left[\frac{1}{\alpha K_j}\frac{\partial^2 \log \Lambda_j}{\partial w_j^2}\right]^{-1}\frac{\pi'}{\alpha^2 \pi} > 0. \tag{22}$$

7. General equilibrium

Given symmetry across agents $j = 1, ..., n$ and equations (2), (5), (6), (9), (15) and (18), we obtain

$$\begin{aligned}
&K_j = K, \quad \widetilde{K}_j = \widetilde{K} = sK, \quad L_j = L = -\pi'(w)K, \quad Z_j = Z_{-j} = Z = G/n,\\
&w_i = w, \quad pa = wL/[N(Z)] = -w\pi'(w)K/[N(Z)], \quad r_j = r = \pi(w),\\
&c_j = c, \quad \pi K + pN(Z) = (\pi - w\pi')K = y(w)K = Y,\\
&C_j = C = cA^\gamma[\pi K + pN(Z)] = cA^\gamma(\pi - w\pi')K = cy(w)A^\gamma K,\\
&\frac{\widetilde{C}}{C} = A\frac{\widetilde{K}}{K}, \quad pa = \frac{(1-\widetilde{\Gamma}/\Gamma)C}{N'(Z_j)A^\gamma \sigma c r} = \Big(1 - \frac{\widetilde{\Gamma}}{\Gamma}\Big)\frac{y(w)K}{N'(Z)\sigma\pi(w)}.
\end{aligned} \tag{23}$$

Noting this, (9) and (18), we obtain

$$\widetilde{\Gamma}/\Gamma = (\widetilde{C}/C)^\sigma = (A\widetilde{K}/K)^\sigma = (sA)^\sigma. \tag{24}$$

We assume that a technological change leads to the increase in welfare, $\widetilde{\Gamma} > \Gamma$, since otherwise, there would be no incentive to do R&D. Given this and (24), we can define a constant

$$\theta \doteq \widetilde{\Gamma}/\Gamma - 1 = (sA)^\sigma - 1 > 0. \tag{25}$$

Noting (15), (23) and (25), we obtain

$$w\pi'\frac{N'(Z)}{N(Z)} = -\frac{pa}{K}N'(Z) = \Big(\frac{\widetilde{\Gamma}}{\Gamma} - 1\Big)\frac{y}{\sigma\pi} = \frac{\theta}{\sigma}\frac{y}{\pi} = \frac{\theta}{\sigma}\Big(1 - \frac{w\pi'}{\pi}\Big)$$

and

$$-\frac{N'(Z)}{N(Z)} = \frac{\theta}{\sigma}\frac{w\pi' - \pi}{w\pi'\pi}. \tag{26}$$

We assume that the ratio of wages to profits in the production of consumption and investment goods, δ, is greater than one. Differentiating the equation (26) totally and noting (16), we then obtain

$$\underbrace{\Big(\frac{N''}{N'} - \frac{N'}{N}\Big)}_{+}\frac{dZ}{dw} = \underbrace{\frac{w\pi''}{(w\pi' - \pi)^2} - \frac{\pi''}{\pi'}}_{+} - \frac{\pi'}{\pi} - \frac{1}{w} > -\frac{\pi'}{\pi} - \frac{1}{w} = \frac{\delta - 1}{w} > 0$$

and define the function $Z(w)$ with $Z' > 0$. This result can be rephrased as:

Proposition 1 *Labour market regulation (deregulation), i.e., the increase (decrease) in union power α, increases (decreases) the wage w and speeds up (slows down) R&D and economic growth, $Z(w)$ with $Z' > 0$.*

Inserting (14), (18), (23) and (25) into (13) yields

$$\begin{aligned}
\rho &= C^\sigma/\Gamma + (\widetilde{\Gamma}/\Gamma - 1)G + [Y - wL + p(N - Z) - A^{-\gamma}C]\Gamma_K/\Gamma \\
&= C^\sigma/\Gamma + \theta G + (Y - A^{-\gamma}C)\Gamma_K/\Gamma = cr + \theta G + (1/c_j - 1)A^{-\gamma}C\Gamma_K/\Gamma \\
&= cr + \theta G + (1/c - 1)\sigma C^\sigma/\Gamma = cr + \theta G + (1/c - 1)\sigma cr \\
&= cr + \theta G + (1 - c)\sigma r = (1 - \sigma)c\pi(w) + \theta n Z(w) + \sigma\pi(w).
\end{aligned} \tag{27}$$

We assume that the propensity to consume c is less than one. Given proposition 1, equation (27) defines c as a function of the wage:

$$c(w) = \frac{1}{1-\sigma}\Big[\frac{\rho - \theta n Z(w)}{\pi(w)} - \sigma\Big] \in (0, 1). \tag{28}$$

From (23) and (28) it follows that consumption C is a function of the wage w, capital K and vintage γ:

$$C(w, K, \gamma) = cy(w)KA^\gamma = \frac{KA^\gamma}{1-\sigma}\Big[\frac{\rho - \theta n Z(w)}{\pi(w)} - \sigma\Big]y(w). \tag{29}$$

The wage elasticity of consumption, when capital stock K and the number of technology γ are kept constant, is given by

$$\varepsilon(w) \doteq \frac{w}{C}\frac{\partial C}{\partial w}. \tag{30}$$

The sign of this elasticity is ambiguous.

Given (10), (15), (23), (28) and (29), we obtain capital accumulation

$$\begin{aligned}
dK &= [\Pi + paN(Z) - A^{-\gamma}C]dt + (\widetilde{K} - K)dq \\
&= [Y - wL + paN(Z) - A^{-\gamma}C]dt + (\widetilde{K} - K)dq \\
&= (Y - A^{-\gamma}C)dt + (\widetilde{K} - K)dq \\
&= [y(w)K - A^{-\gamma}C(w, K, \gamma)]dt + (\widetilde{K} - K)dq \\
&= [1 - c(w)]y(w)K\,dt - (1 - s)K\,dq.
\end{aligned} \tag{31}$$

This shows that because between moments of technological change the wage $w(\alpha)$ is kept constant, capital stock grows at as fixed rate. At the occurrence of a technological change, total factor productivity TFP in the consumption-good sector rises from A^{γ} to $A^{\gamma+1}$, capital stock falls from K to $\widetilde{K} = sK$, and also employment falls in proportion to the decrease in capital K.

Noting (3), (23) and (31), the expected rate of capital accumulation reads:

$$E(dK/K) = (1-c)y(w)dt - (1-s)E(dq) = (1-c)y(w)dt - (1-s)G\,dt$$
$$= \Big\{[1-c(w)]y(w) - (1-s)nZ(w)\Big\}dt. \quad (32)$$

If the rate $E[dK/K]$ were greater than the growth rate of the productivity of labour, $\dot{a}/a = (1/a)(da/dt)$, then both K/a and the expected wage $p = -w\pi'(w)K/[aN(Z)]$ would increase indefinitely. If $E[dK/K] < \dot{a}/a$, then both K/a and the expected wage p would decrease indefinitely. Hence, an equilibrium stationary state, in which the expected wage p is constant, exists only when the condition (1) holds. We assume that s is small enough for unemployment to persist forever.[11] The results can then be summarized as:

Proposition 2 *The economy is subject to a business cycle where output, capital stock and the level of employment increase in fixed proportions until a new technology is introduced, at which moment they sharply fall in proportion to the share of capital stock that becomes obsolete. The average rate of unemployment and the growth rate of the economy are kept constant.*

8. Social welfare

The government can determine the wage $w(\alpha)$ by regulating union power α. Given (23) and (29), total income is determined by $Y = y(w)K$, aggregate consumption by $C(w, K, \gamma)$. The social planner maximizes the representative agent's expected utility over time by choosing w subject to capital accumulation (31) and the stochastic process (3). The value of the planner's optimal program at time t can be defined as

$$\Omega(K,\gamma) = \max_{w \text{ s.t. } (31)} E \int_t^{\infty} e^{-\rho(\tau - t)} C^{\sigma} d\tau. \quad (33)$$

[11]The feasible values for s are calculated in the Appendix.

Defining $\Omega_K \doteq \partial\Omega/\partial K$ and noting $G = nZ(w)$ from (23) and proposition 1, the Bellman equation for the social planner's program is given by[12]

$$\rho\Omega(K,\gamma) = \max_{w \text{ s.t. } (31)} \Psi(w, K, \gamma), \tag{34}$$

where

$$\begin{aligned}\Psi(w, K, \gamma) \doteq C(w, K, \gamma)^\sigma + nZ(w)[\Omega(\widetilde{K}, \gamma + 1) - \Omega(K, \gamma)] \\ + \Omega_K(K,\gamma)\big[y(w)K - A^{-\gamma}C(w, K, \gamma)\big].\end{aligned} \tag{35}$$

Denoting $\widetilde{\Omega} \doteq \Omega(\widetilde{K}, \gamma + 1)$, we obtain the first-order condition

$$\partial\Psi/\partial w = [\sigma C^{\sigma-1} - \Omega_K A^{-\gamma}]\partial C/\partial w + \Omega_K K y' + (\widetilde{\Omega} - \Omega)nZ'. \tag{36}$$

Noting (29), we try the solution

$$\Omega(K, \gamma) = C^\sigma/m = c^\sigma y^\sigma K^\sigma A^{\gamma\sigma}/m, \tag{37}$$

where m is a constant. Equations (23), (25) and (37) produce

$$\Omega_K = \sigma\Omega/K, \quad \widetilde{\Omega}/\Omega = (A\widetilde{K}/K)^\sigma = (sA)^\sigma = \theta + 1. \tag{38}$$

Inserting (38) into the Bellman equation (34) and (35), and noting (23), (28), (37) and (38) yield

$$\begin{aligned}\rho &= C^\sigma/\Omega + nZ[\widetilde{\Omega}/\Omega - 1] + [yK - A^{-\gamma}C]\Omega_K/\Omega \\ &= C^\sigma/\Omega + nZ[\widetilde{\Omega}/\Omega - 1] + (1-c)yK\Omega_K/\Omega = m + \theta nZ + (1-c)\sigma y.\end{aligned}$$

Solving for m and noting proposition 1 and (28), we obtain

$$m(w) \doteq \rho - \theta nZ(w) - [1 - c(w)]\sigma y(w). \tag{39}$$

Inserting (37), (38) and (39) into equation (36) and noting (15), (16), (23), (27), (29) and (30), we obtain

$$\begin{aligned}\frac{1}{\sigma\pi'\Omega}\frac{\partial\Psi}{\partial w} &= \frac{1}{\pi'}\left\{\left[\frac{C^{\sigma-1}}{\Omega} - \frac{\Omega_K}{\sigma\Omega}A^{-\gamma}\right]\frac{\partial C}{\partial w} + \frac{\Omega_K K}{\sigma\Omega}y' + (\widetilde{\Omega}/\Omega - 1)nZ'/\sigma\right\} \\ &= \frac{1}{\pi'}\left\{\left[\frac{C^{\sigma-1}}{\Omega} - \frac{cy}{C}\right]\frac{\partial C}{\partial w} + y' + \frac{\theta}{\sigma}nZ'\right\} = \frac{1}{\pi'}\left\{\frac{m - cy}{C}\frac{\partial C}{\partial w} + y' + \frac{\theta}{\sigma}nZ'\right\}\end{aligned}$$

[12] Cf. Dixit and Pindyck (1994).

$$
\begin{aligned}
&= \frac{1}{\pi'}\left\{\{\rho - \theta n Z - [(1-\sigma)c+\sigma]y\}\frac{1}{C}\frac{\partial C}{\partial w} + y' + \frac{\theta}{\sigma}nZ'\right\} \\
&= \frac{1}{\pi'}\left\{[(1-\sigma)c+\sigma]\frac{\pi - y}{C}\frac{\partial C}{\partial w} + y' + \frac{\theta}{\sigma}nZ'\right\} \\
&= \frac{1}{\pi'}\left\{[(1-\sigma)c+\sigma]\frac{w\pi'}{C}\frac{\partial C}{\partial w} - w\pi'' + \frac{\theta}{\sigma}nZ'\right\} \\
&= [(1-\sigma)c+\sigma]\varepsilon + \epsilon + \frac{1}{1+\delta}\left[1 - \delta - \frac{\epsilon}{1+\delta}\right]. \qquad (40)
\end{aligned}
$$

Given $\pi' < 0$ and proposition 1, α and $w(\alpha)$ should be increased (i.e., $\partial\Psi/\partial w > 0$) if and only if (40) is negative. This result can be rephrased as:

Proposition 3 *The labour market should be regulated (deregulated), if*

$$
[(1-\sigma)c(w)+\sigma]\varepsilon(w) + \left\{1 - \frac{1}{[1+\delta(w)]^2}\right\}\epsilon(w) + \frac{1-\delta(w)}{1+\delta(w)} < 0 \;\; (> 0).
$$

Given the propensity of consume, $c(w)$, and the ratio of wages to profits in the production of consumption and investment goods, $\delta(w) \doteq wL/\Pi$, the labour market should be regulated (deregulated) the more likely, the lower (higher) the wage elasticity of consumption, $\varepsilon(w) \doteq (w/C)\partial C/\partial w$, or the lower (higher) the wage elasticity of employment, $\epsilon(w) \doteq |(w/L)\partial L/\partial w|$.

This proposition is explained in the final section. It shows that the existence of union power and involuntary unemployment can be optimal policy.

9. Conclusions

This paper examines growth and business cycles in an economy with imperfect labour markets. The theory of creative destruction, in which a new technology renders an old technology obsolete, is taken as a starting point. In other respects, the particular features of the model are the following:

- There exist separate labour markets for R&D and the rest of the economy. Labour suppliers choose between these two markets *ex ante*.

- In the production of consumption and investment goods, there is union-employer bargaining over wages. The government can regulate or deregulate the labour market to increase (decrease) union power.

- The firms in production can increase the probability of a technological change for themselves by R&D.
- Learning-by-investment increases the productivity of labour in production in proportion to the expected accumulation of capital. Given this 'razor-edge' condition, there is no trend for the rate of unemployment.

It is assumed, for simplicity, that there is perfect symmetry over the firms. The main results and their interpretations are as follows.

Labour market deregulation decreases the rate of unemployment in the consumption-good and investment-good sectors. This increases the expected wage rate in these sectors and encourages the agents to shift their labour supply from R&D to these sectors. Consequently, the level of R&D, the number of innovations and the average growth rate of the economy will fall. Vice versa with labour market regulation.

Capital stock swings up and down due to endogenous technological shocks. Because wages are set by bargaining, the labour-capital ratio in production is fixed and output and employment swing in proportion to capital stock. A typical cycle of the economy is as follows. Starting with some level of real capital, new capital will be accumulated through savings in the economy and some of the labour force will be allocated to R&D. At some point of time, a new technology will be found, total factor productivity in the consumption-good sector will rise but some of the outstanding capital stock will become obsolete. Capital stock (as measured in terms of the newest technology) will fall sharply and start accumulating again.

Labour market regulation has two opposite effects on welfare. First, it decreases employment and the expected wage in the production of consumption and investment goods, which makes people to supply more labour to R&D. With larger R&D, there will be faster economic growth. On the other hand, regulation decreases output and the level of current consumption the more, the higher the wage elasticities of consumption and employment are. If these elasticities are low enough, then the latter effect will be weak enough to be outweighed by the former and labour market regulation is welfare enhancing. Otherwise, it would be socially optimal to deregulate the labour market.

While a great deal of caution should be exercised when a highly stylized mathematical model is used to draw conclusions about wage bargaining, economic growth and business cycles, the following judgement nevertheless seems to be justified. With labour market regulation, a stationary state equilibrium with involuntary unemployment, employment cycles and stable real wages can be possible. If growth and income effects of regulation are properly taken into account, such an equilibrium may even be socially optimal.

Appendix

Assume that the earlier technological change has occurred at time 0. Because G in (3) is the probability that R&D leads to development of a new technology in one unit of time, then $T(w) \doteq 1/G = 1/[nZ(w)]$ is the time in which the technological change occurs with probability one. The economy never attains full employment, if $L_j < N_j$ holds at time $T(w)$. Given (5), (15), (23) and proposition 1, this condition is equivalent to

$$\frac{K(T(w))}{a(T(w))} < \frac{N(Z(w))}{-\pi'(w)}. \tag{41}$$

Now we prove (41).

Because at time $t = 0$ there is unemployment, it is true that

$$\frac{K(0)}{a(0)} < \frac{N(Z(w))}{-\pi'(w)}.$$

Noting this, $GT = 1$, (1), (3), (31) and (32), we obtain that between two technological changes, $\dot{K}/K - \dot{a}/a = (1-s)G$ and

$$\begin{aligned}\log\left[\frac{K(T(w))}{a(T(w))}\right] &= \log\left[\frac{K(0)}{a(0)}\right] + \int_0^T \left(\frac{\dot{K}}{K} - \frac{\dot{a}}{a}\right) dt = \log\left[\frac{K(0)}{a(0)}\right] + (1-s)GT \\ &= \log\left[\frac{K(0)}{a(0)}\right] + 1 - s < \log\left[\frac{N(Z(w))}{-\pi'(w)}\right] + 1 - s\end{aligned}$$

hold. Hence, the condition (41) holds, if s is small enough for

$$s < 1 + \log\left[\frac{N(Z(w))}{-\pi'(w)}\right] - \log\left[\frac{K(T(w))}{a(T(w))}\right] = 1 + \log\left[\frac{N(Z(w))a(T(w))}{-\pi'(w)K(T(w))}\right].$$

References:

Aghion, P. and Howitt, P. (1992). A model of growth through creative destruction. *Econometrica 60*, 323-351.

Aghion, P. and Howitt, P. (1998). *Endogenous Growth Theory.* Cambridge (Mass.): MIT Press.

Blanchard, O. and Giavazzi, F. (2001). Macroeconomic effect of regulation and deregulation in goods and labor markets. *NBER Working paper 8120.*

Dixit, A. and Pindyck, K. (1994). *Investment under Uncertainty.* Princeton: Princeton University Press.

Grossman, G. and Helpman, E. (1991). *Innovation and Growth.* Cambridge (Mass.): The MIT Press.

Palokangas, T. (2000). *Labour Unions, Public Policy and Economic Growth.* Cambridge (U.K.): Cambridge University Press.

Wälde, K. (1999). A model of creative destruction with undiversifiable risk and optimizing households. *The Economic Journal 109*, C156-C171.

Wälde, K. (2002). The economic determinants of technology shocks in a real business cycle model. *Journal of Economic Dynamics and Control 27*, 1-28.