



Helsinki
Center
of
Economic
Research

Discussion Papers

Productivity decompositions: Computation and inference

Ari Hyytinen
University of Jyväskylä and ETLA

Pekka Ilmakunnas
Aalto University and HECER

and

Mika Maliranta
ETLA and University of Jyväskylä

Discussion Paper No. 288
March 2010

ISSN 1795-0562

HECER – Helsinki Center of Economic Research, P.O. Box 17 (Arkadiankatu 7), FI-00014
University of Helsinki, FINLAND, Tel +358-9-191-28780, Fax +358-9-191-28781,
E-mail info-hecer@helsinki.fi, Internet www.hecer.fi

Productivity decompositions: Computation and inference *

Abstract

This paper shows how a moment-based estimation procedure can be used to compute point estimates and obtain standard errors for the two components of the productivity decomposition proposed by G. S. Olley and A. Pakes (The Dynamics of Productivity in the Telecommunications Equipment Industry, *Econometrica*, Vol. 64, No. 6, Nov. 1996). When applied to business-level microdata, the procedure allows for panel-robust inference and hypothesis testing about, e.g., the coevolution of the productivity components in different groups of firms. We provide an application to Finnish firm-level data and find that formal statistical inference is important for the interpretation of productivity dynamics and its sources.

JEL Classification: C10, O47.

Keywords: productivity, decomposition, inference.

Ari Hyytinen

School of Business and Economics
University of Jyväskylä
P.O. Box 35
FI-40014 University of Jyväskylä
FINLAND
e-mail: ari.hyytinen@econ.jyu.fi

Pekka Ilmakunnas

School of Economics
Aalto University
PO Box 21240
FI-00076 Aalto
FINLAND
e-mail: pekka.ilmakunnas@hse.fi

Mika Maliranta

The Research Institute of the Finnish Economy (ETLA)
Lönnrotinkatu 4 B
FI-00120 Helsinki
FINLAND
e-mail: mika.maliranta@etla.fi

* We would like to thank Anni Nevalainen and Otto Toivanen for useful comments and discussions. Data work and computations have been carried out at Statistics Finland following its terms and conditions of confidentiality. To obtain access to these data, contact the Research Laboratory of Business Structures Unit, Statistics Finland, FI-00022, Finland. Maliranta's work has in part been supported by the Finnish Funding Agency for Technology and Innovation (TEKES, project 441/31/08). The usual disclaimer applies.

1 Introduction

In their often-cited paper, Olley and Pakes (1996) show that when the level of industry productivity is measured by the weighted average of firm-level (or plant-level) productivity and computed using microdata, it can be decomposed to (i) the unweighted average of the productivity of firms and (ii) a covariance-like term between activity (i.e., output or input) shares and productivity. The within-industry covariance between size and productivity is of particular interest to economists: The smaller this cross-term is, the smaller the share of activity (or resources) that gets allocated to the most productive firms. Olley and Pakes found that changes in the covariance term may be due to policy. In particular, they argue that the deregulation of the U.S. telecommunications equipment industry may have increased the covariance term by increasing the allocation of resources to the most productive firms.

The original Olley-Pakes (OP) decomposition is cross-sectional and static.¹ However, when applied to a panel data of firms, it provides a window to the determinants of industry productivity growth that subsequent research has begun to utilize intensively (see, e.g., Eslava, Haltiwanger, Kugler and Kugler 2004, Van Biesebroeck, 2008, Bartlesman, Haltiwanger and Scarpetta 2009a, Eslava, Haltiwanger, Kugler and Kugler 2009a,b). Bartlesman et al. (2009a) argue, for example, that a low covariance term is a good indicator of misallocation of resources and (policy-induced) market distortions and provide evidence that its variation explains an important fraction of the cross-country differences in

¹Starting from Bailey, Hulten and Campbell (1992), there is a large literature on how different types of decompositions of industry productivity (growth) are able to capture its microeconomic sources. See, for example, Balk (2003) for a review and Foster, Haltiwanger and Krizan (2006) and Foster, Haltiwanger and Syverson (2008) for a couple of recent contributions that use decompositions other than the OP decomposition.

productivity. The most recent papers develop dynamic extensions of the OP decomposition and show how to allow for entry and exit (e.g., Melitz and Polanec 2009; see also Maliranta 2009 and Nevalainen 2010).

This paper builds on the observation that hypothesis testing and inference appear to be a neglected part of the decomposition literature. One of the few studies that obtain standard errors for the components of a productivity decomposition is Foster, Haltiwanger and Krizan (2006). They regress productivity on indicators of entry and exit, obtaining a regression analogue to a decomposition of productivity growth to entry and exit effects and growth in continuing firms. To the best of our knowledge, the estimation of the standard errors of the two components of the OP decomposition has not - despite its increasing popularity in applications - received attention in the prior studies.² The aim of this paper is to start filling this apparent gap in the literature. We outline, in particular, a moment-based procedure to the estimation of the OP components and their standard errors and illustrate how it leads to a simple two-step receipt that can be used for inference and hypothesis testing in applications.

Though it seems obvious that it would be worthwhile to have a procedure for inference and hypothesis testing for the OP components, a particular feature of the earlier applied literature is that the studies have often been based on register data that cover (nearly) the entire population of firms of a country (i.e., they are based on census data of some sort). These data are not samples in the traditional sense and the elements of the decompositions do not have stochastic variation due to sampling from a population. However, this does not mean that there is no need for statistical inference. Instead, we can think of the data as a sample drawn from the underlying data generating process (DGP) (see,

²Maliranta (2009) appears to be one of the first papers that considers statistical inference in this context.

e.g., Davidson and MacKinnon, 2004) and focus on estimating its parameters. What's more, a present tendency in the literature seems to be towards using finer decompositions and comparisons (by, e.g., firm cohort, geographic region, or firm type) in the hope that they help us to better understand the microeconomic sources of industry productivity growth. This tendency is likely to lead to denser slicing of the available microdata with a smaller number of observations, increasing thus the need for appropriate inference procedures.

We apply the estimation procedure to a Finnish firm-level panel data from 1995 to 2007. We focus on a single industry and cross-regional differences in its productivity dynamics. We find that in our application, there is a clear and statistically significant improvement in the level of industry productivity in one of the two regions that our data cover. However, formal statistical inference casts some doubt on the conclusions that one might draw about its sources based on a visual inspection of the dynamics of the two components of the OP decomposition. In particular, we find that (relative differences or changes in) the covariance term cannot be measured as accurately as (relative differences or changes in) the unweighted average of the productivity of firms.

The remainder of the paper is organized as follows: The next section presents the OP decomposition. In the third section, we develop the moment-based procedure using insights from the Generalized Method of Moments (GMM) estimation. The fourth section provides an application using large Finnish firm-level data. Section five concludes and discusses potential extensions.

2 Olley-Pakes Decomposition

To write down a formal expression for the OP decomposition, let s_{it} denote the activity share of firm i in period t and φ_{it} an index of productivity. How s_{it} and φ_{it} are measured depends on the application, as the decomposition can be

applied either to an industry-level index of total factor productivity (TFP) or to that of labour productivity, be done in levels or in log-units and computed using either input or output shares. For concreteness, we frame our analysis in terms of labour productivity and assume that the index of firm-level productivity is measured in log-units, i.e., $\varphi_{it} = \log \left[\frac{Y_{it}}{L_{it}} \right]$, where Y_{it} is a measure of value added and where L_{it} is the number of employees in firm i at time t . The activity shares are measured by labour inputs so that $s_{it} = L_{it} / \sum_{i=1}^{N_t} L_{it}$, where N_t refers to the number of firms in period t .

Taking a single cross-section of the data for period t , the OP decomposition of the aggregate productivity index of an industry is

$$\Phi_t = \bar{\varphi}_t + \sum_{i=1}^{N_t} (s_{it} - \bar{s}_t)(\varphi_{it} - \bar{\varphi}_t) \quad (1)$$

where $\Phi_t = \sum_{i=1}^{N_t} s_{it}\varphi_{it}$ is the weighted mean of firm-level productivity, $\bar{\varphi}_t$ refers to the unweighted mean and the last term is the covariance term. The covariance term consists of the deviations of input shares around their unweighted, cross-sectional mean, $(s_{it} - \bar{s}_t)$, and the deviations of firm productivity around their unweighted, cross-sectional mean, $(\varphi_{it} - \bar{\varphi}_t)$.³ To distinguish the second term from standard sample (cross-sectional) covariance, $\widehat{cov}_t(s_{it}, \varphi_{it})$, we denote $\hat{c}_t \equiv \sum_{i=1}^{N_t} (s_{it} - \bar{s}_t)(\varphi_{it} - \bar{\varphi}_t)$ in what follows. This means that $\hat{c}_t = \widehat{cov}_t(s_{it}, \varphi_{it}) \times N_t$.

A large part of the subsequent development in this paper is motivated by the simple observation that the two terms on the R.H.S. of equation (1) can be estimated jointly by regressing φ_{it} on a constant and an appropriately scaled s_{it} using Ordinary Least Squares (OLS). This insight relies on the anatomy of the population regression $E[\varphi_{it} | s_{it}] = E[\varphi_{it}] + \frac{cov(\varphi_{it}, s_{it})}{var(s_{it})} (s_{it} - E[s_{it}])$, which immediately suggests how the two components can be captured by a single moment condition. In particular, the moment condition allows us to obtain

³Note that $\bar{s}_t = 1/N_t$.

point estimates and panel-robust standard errors for the two components of (1) simultaneously, using a standard GMM procedure.

3 Computation and Inference

3.1 Moment-based Approach

To derive a moment-based representation of the OP decomposition, we assume for notational simplicity that the microdata used to compute the OP decomposition is a balanced panel and that the available sample period of interest is of length T . It should be obvious, however, that the approach can be generalized to many kinds of unbalanced panels (see, e.g., Wooldridge 2002, Ch. 17); see also the discussion below.

We start with the simplest case in which the aim is to compute the $2T$ decomposition terms, $(\bar{\boldsymbol{\varphi}}', \hat{\boldsymbol{c}}')' = (\bar{\varphi}_1, \dots, \bar{\varphi}_T, \hat{c}_1, \dots, \hat{c}_T)'$, and the associated standard errors. To this end, we define $s_{it}^* = \frac{s_{it} - \bar{s}_t}{\hat{\sigma}_t^2 N}$, where $\hat{\sigma}_t^2$ is the cross-sectional sample variance of s_{it} , i.e., $\hat{\sigma}_t^2 = \frac{1}{N} \sum_{i=1}^N (s_{it} - \bar{s}_t)^2$ in period t .⁴ We also let \mathbf{D}_i be a $(T \times T)$ period dummy matrix with typical element $d_{it,k}$, which is equal to one if $t = k$ and equal to zero otherwise. Using the period dummy matrix and collecting the scaled input share data, s_{it}^* , for firm i into a $(T \times T)$ diagonal matrix $\mathbf{S}_i^* = \text{diag}[s_{i1}^*, \dots, s_{iT}^*]$ allows us to define a $(T \times 2T)$ data matrix $\mathbf{X}_i = [\mathbf{D}_i \mathbf{S}_i^*]$. Using this matrix, we can write down population moment condition

$$E[\mathbf{X}_i'(\boldsymbol{\varphi}_i - \mathbf{X}_i\boldsymbol{\beta})] = \mathbf{0}_{(2T \times 1)} \quad (2)$$

where $\boldsymbol{\varphi}_i = (\varphi_{i1}, \dots, \varphi_{iT})'$ and $\boldsymbol{\beta} = (\boldsymbol{\theta}', \boldsymbol{\gamma}')' = (\theta_1, \dots, \theta_T, \gamma_1, \dots, \gamma_T)'$ is a $(2T \times 1)$ parameter vector.

⁴To simplify the presentation, we use N instead of the more usual $N - 1$ when computing sample variances. In large samples, $N \approx N - 1$.

The analogy principle says that a suitable estimator for unknown population parameters can be found by considering the sample counterpart of a population moment (see, Manski 1988). What we show next is that applying the principle to (2) results in an estimator for $\beta = (\theta', \gamma')'$ that is numerically equivalent to the two components on the R.H.S. of (1). Because moment-based estimators are naturally based on the analogy principle, we frame our discussion in terms of GMM. As explained in, e.g., Cameron and Trivedi (2005, pp. 744-745), a GMM estimator based on a moment like (2) results in a single-equation panel GMM estimator for parameter vector β . Besides the point estimates of the decomposition terms, the GMM procedure allows us to obtain their standard errors.

As we have specified moment (2), the number of instruments is equal to the number of parameters to be estimated. This property means that the model is just-identified and that the moment condition results in the familiar pooled OLS estimator of a linear panel model. To derive this pooled OLS estimator using the analogy principle, we replace the expectation operator in (2) by the corresponding sample average. The estimator therefore solves $\frac{1}{N} \sum_{i=1}^N [\mathbf{X}_i' (\varphi_i - \mathbf{X}_i \hat{\beta})] = \mathbf{0}$. Stacking all firms $\varphi' = (\varphi_1' \cdots \varphi_N')$ and $\mathbf{X}' = (\mathbf{X}_1' \cdots \mathbf{X}_N')$, the resulting pooled OLS estimator is $\hat{\beta} = (\hat{\theta}', \hat{\gamma}')' = (\mathbf{X}'\mathbf{X})^{-1} \mathbf{X}'\varphi$. This means, in other words, that to obtain the point estimates, we just regress φ_{it} on the complete set of period indicators and their interactions with s_{it}^* using OLS.

The pooled OLS estimator is numerically equivalent to $(\bar{\varphi}', \hat{\mathcal{C}}')'$. The result follows from standard results on partitioned regression and from the fact the model is completely saturated in terms of (orthogonal) period indicators. In particular, picking any $\hat{\theta}_t$, one can show that $\hat{\theta}_t = \bar{\varphi}_t - \hat{\gamma}_t \bar{s}_t^* = \bar{\varphi}_t$. The last step follows from $\bar{s}_t^* = 0$. Similarly, picking any $\hat{\gamma}_t$, one can establish that $\hat{\gamma}_t = \left(\sum_{i=1}^N (s_{it}^*) (\varphi_{it} - \bar{\varphi}_t) \right) \left(\sum_{i=1}^N (s_{it}^*)^2 \right)^{-1}$. This expression simplifies to $\widehat{cov}_t \times N =$

$\sum_{i=1}^N (s_{it} - \bar{s}_t)(\varphi_{it} - \bar{\varphi}_t)$ and is thus equal to \hat{c}_t , as desired. This establishes that for $t = 1, \dots, T$, the R.H.S. of (1) can be rewritten as $\hat{\boldsymbol{\theta}} + \hat{\boldsymbol{\gamma}}$.

In sum, obtaining point estimates for the two OP components from microdata consists of two steps: First, s_{it} is demeaned and scaled by N times its cross-sectional variance (separately for each period). Second, one regresses φ_{it} on a constant and the scaled s_{it} (using, e.g., OLS). The estimator for the constant gives $\bar{\varphi}_t$ and that of the slope $\sum_{i=1}^N (s_{it} - \bar{s}_t)(\varphi_{it} - \bar{\varphi}_t)$.

3.2 Panel-Robust Statistical Inference and Testing

The benefit of casting the estimation of the OP decomposition in terms of a population moment and GMM is that the GMM framework can be used to establish the asymptotic properties of the estimators, their asymptotic normality in particular, and to compute their standard errors.⁵

To derive an estimator for the standard errors, we start by noting that moment (2) implicitly defines a $(T \times 1)$ vector of regression errors $\mathbf{u}_i \equiv \boldsymbol{\varphi}_i - \mathbf{X}_i \boldsymbol{\beta}$ for each firm i . If it were the case that these errors were uncorrelated over time for a given firm and homoscedastic, using the classical OLS variance-covariance estimator (i.e., $\hat{\sigma}_u^2(\mathbf{X}'\mathbf{X})$), would lead to an estimator for the standard errors of $\bar{\varphi}_t$ and $\hat{\boldsymbol{\gamma}}_t$ that are similar (but not identical) to the conventional standard error estimators for the sample mean and covariance.⁶ However, there are strong reasons to suspect that the errors are both correlated over time for a given firm

⁵For a textbook treatment, see for example Cameron and Trivedi (2005, Ch. 6 and 22) and Wooldridge (2002, Ch. 14).

⁶The conventional (cross-sectional) estimator for the standard error of the sample mean is of course the square root of the sample analog of $\frac{1}{N}\sigma_\varphi^2$. The corresponding estimator for the sample covariance is the square root of the sample analog of $\frac{1}{N}(\mu_{jj,t} - \mu_{jj,t}^2)$, where $\mu_{jj,t} = E(s_{it} - \bar{s}_t)^j (\varphi_{it} - \bar{\varphi}_t)^j$. There is an efficiency gain from estimating both standard errors at the same time.

and heteroscedastic. The former should be allowed for, because shocks to the productivity of firm i are likely to be persistent over time. Heteroscedasticity is also expected in most microdata and should therefore be allowed for. For example, the cross-sectional variance of productivity shocks may vary over time, leading to a form of heteroscedasticity.

Assuming independence over i and $N \rightarrow \infty$, the panel-robust estimate of the asymptotic variance matrix of the estimator is

$$\widehat{V}[\widehat{\boldsymbol{\beta}}] = \left[\sum_{i=1}^N \mathbf{X}'_i \mathbf{X}_i \right]^{-1} \sum_{i=1}^N \mathbf{X}'_i \widehat{\mathbf{u}}_i \widehat{\mathbf{u}}'_i \mathbf{X}_i \left[\sum_{i=1}^N \mathbf{X}'_i \mathbf{X}_i \right]^{-1} \quad (3)$$

where $\widehat{\mathbf{u}}_i = \boldsymbol{\varphi}_i - \mathbf{X}_i \widehat{\boldsymbol{\beta}}$. This formula can be used to obtain a consistent estimate of the asymptotic covariance matrix for $\widehat{\boldsymbol{\beta}}$ that is robust to within-firm autocorrelation and heteroscedasticity of *unknown* form. This familiar variance estimator is a suitable choice when one is unwilling to make assumptions about the within-firm autocorrelation structure or the type of heteroscedasticity in the microdata. It is also suitable when the data are a short panel and thus have relatively few observations per each firm (small T) but includes many firms (large N); see Cameron and Trivedi (2005, Ch. 22 and 24) and Wooldridge (2003) for further discussion.

Expression (3) provides the basis for panel-robust statistical inference. The estimator is easy to implement, because it can be computed using a standard OLS command with an option for cluster-robust standard errors.⁷ To obtain confidence intervals for the components of the productivity decomposition follows from standard argumentation. One can draw on the asymptotic normality of the GMM estimators and use the standard errors that can be obtained as the

⁷Note, however, that some of the standard econometric softwares (such as *Stata*) make by default a small-sample correction when computing the cluster-robust standard errors. In large samples, this correction does not matter.

square root of the diagonal elements of (3). What is convenient is that standard regression output includes the confidence intervals automatically.

In applications, it may be of interest to test hypotheses about the components of the productivity decomposition. For example, to study whether the covariance term has remained stable over the sample period in a particular industry, one can formulate $H_0: \gamma_1 = \dots = \gamma_T$ and test for the joint hypothesis using standard joint testing procedures, such as the Wald-test. Similarly, testing for $H_0: \gamma_{T-s} = \dots = \gamma_T = 0$ corresponds to analysing the null hypothesis that the industry index of productivity during the last s years of the sample period is no higher than it would if the input shares were randomly allocated within the industry. As a final example, the null hypothesis of constant growth (rate) of the average firm productivity can be examined by testing $H_0: \theta_2 - \theta_1 = \dots = \theta_T - \theta_{T-1}$.

3.3 Discussion and Extensions

3.3.1 Mutually Exclusive Sub-groups

The first, perhaps most obvious, extension to the basic procedure builds on the observation that the aggregate productivity index for a group of firms can be computed as a weighted mean of the aggregate productivities of the sub-groups of firms. This observation suggests that one can assign all the firms of an industry to mutually exclusive sub-groups and estimate the productivity decompositions and the associated standard errors separately for each sub-group.

To illustrate how that could be done, we assume that there are J sub-groups ($j = 1, \dots, J$) and take the following four steps: First, we define sub-group indicator $q_{it,j}$, which is equal to one if firm i belongs to group j in period t and is zero otherwise. This implies that in each period, the number of members in sub-group j is $N_{t,j} = \sum_{i=1}^N q_{it,j}$. Second, we scale the input shares by period

and within each sub-group to obtain $s_{it,j}^* = \frac{s_{it} - \bar{s}_{t,j}}{\widehat{\sigma}_{t,j}^2 N_{t,j}}$, where $\bar{s}_{t,j}$ is the mean and $\widehat{\sigma}_{t,j}^2$ is the cross-sectional sample variance of the input share in sub-group j in period t . By definition, $s_{it,j}^*$ is zero for firm i in period t if it does not belong to group j during the period. Third, we let \otimes denote the Kronecker product and define $\widetilde{\mathbf{d}}_{it}' = (q_{it,1}, \dots, q_{it,J}) \otimes (d_{it,1}, \dots, d_{it,T})$ and $\widetilde{\mathbf{s}}_{it}^{*'} = (s_{it,1}^*, \dots, s_{it,J}^*) \otimes (d_{it,1}, \dots, d_{it,T})$, which are row vectors of length JT . Finally, we use population moment condition (2) and the GMM approach to estimate the two components of the OP decomposition for each sub-group by redefining matrix \mathbf{X}_i so that its t^{th} row is now $\mathbf{x}'_{it} = [\widetilde{\mathbf{d}}_{it}' \ \widetilde{\mathbf{s}}_{it}^{*'}]$. Of course, $\boldsymbol{\beta} = (\boldsymbol{\theta}', \boldsymbol{\gamma}')$ has to be redefined accordingly, i.e., to be a (column) vector of length $2JT$.

This extension is of potential interest in applications. For example, to study whether the (relative) importance of the covariance terms in an industry is similar in J geographic regions in a given period, we could use (2) and the GMM-procedure in estimation, pick the relevant parameters of the model (e.g., $\gamma_{t,j} = N_{t,j} \times \widehat{cov}_{t,j}(\varphi_{it}, s_{it})$) and test the hypothesis $H_0: \gamma_{t,1} = \dots = \gamma_{t,J}$ using a joint test. Implementing such a test is straightforward, because in each row of \mathbf{X}_i , the first JT terms are group-specific period indicators (i.e., the complete set of period indicators interacted with the complete set of sub-group indicators) and the next JT terms are the period and sub-group specific input shares $s_{it,j}^*$. We illustrate a variant of this extension in our application.

It is important to emphasize that $\widehat{\theta}_{t,1} + \widehat{\gamma}_{t,1} \dots + \widehat{\theta}_{t,J} + \widehat{\gamma}_{t,J}$ is *not* equal to Φ_t , i.e., the weighted mean of firm-level productivity in period t . However, if these estimates are weighted by the employment share of each sub-group in period t , $S_{t,j} = \sum_{i=1}^N q_{it,j} L_{it} / \sum_{i=1}^N L_{it}$, they total to Φ_t . That is, $\Phi_t = S_{t,1} (\widehat{\theta}_{t,1} + \widehat{\gamma}_{t,1}) + \dots + S_{t,J} (\widehat{\theta}_{t,J} + \widehat{\gamma}_{t,J})$, where $\sum_{j=1}^J S_{t,j} = 1$.

3.3.2 Industry-level Productivity

The procedure we have developed lends itself directly to making statistical inference about the L.H.S. of (1). It follows from the definition of the estimator that $Var[\Phi] = Var[\hat{\theta} + \hat{\gamma}]$, where $\Phi = (\Phi_1, \dots, \Phi_T)'$ is the vector containing the weighted mean of firm-level productivity for periods $t = 1, \dots, T$. This relation implies that it is easy to test hypotheses about, e.g., how industry productivity has developed over the sample period. For example, testing $H_0: (\theta_T - \theta_1) + (\gamma_T - \gamma_1) = 0$ would be a test of the hypothesis that there has been no (aggregate) productivity growth over the sample period (i.e., $\Phi_1 = \Phi_T$).

It is worth pointing out that the terms corresponding to the L.H.S. of (1) are periodic weighted averages and that they and their standard errors can also be obtained directly from a regression. To show how, let $\varphi_{it}^* = \varphi_{it} \sqrt{L_{it}}$, collect these weighted productivity indices for firm i into $\varphi_i^* = (\varphi_{i1}^*, \dots, \varphi_{iT}^*)'$, and define a $(T \times T)$ matrix \mathbf{X}_i^* with t^{th} row $\mathbf{x}_{it}^{*'} = (d_{1,it} \sqrt{L_{it}}, \dots, d_{T,it} \sqrt{L_{it}})$, where $d_{it,s}$ are, as before, period indicators. The rows of \mathbf{X}_i^* consist thus of "weighted" period dummies.

Using this notation, we can write down the following population moment condition for firm i :

$$E[\mathbf{X}_i^{*'}(\varphi_i^* - \mathbf{X}_i^* \boldsymbol{\alpha})] = \mathbf{0}_{(T \times 1)} \quad (4)$$

where $\boldsymbol{\alpha} = (\alpha_1, \dots, \alpha_T)'$ is a $(T \times 1)$ parameter vector. By the analogy principle, this moment condition results in the standard pooled OLS estimator of a linear panel model that regresses φ_{it}^* on the complete set of period indicators interacted with $\sqrt{L_{it}}$. Stacking all firms $\varphi^{*'} = (\varphi_1^{*'} \dots \varphi_N^{*'})$ and similarly for \mathbf{X}^* , this OLS estimator is $\hat{\boldsymbol{\alpha}} = (\mathbf{X}^{*'} \mathbf{X}^*)^{-1} \mathbf{X}^{*'} \varphi^*$. Equivalently, if we let $\mathbf{W}_i = \text{diag}[(L_{i1}, \dots, L_{iT})]$, (4) can be rewritten as $E[\mathbf{D}_i' \mathbf{W}_i (\varphi_i - \mathbf{D}_i \boldsymbol{\alpha})] = \mathbf{0}$. Stacking all firms $\mathbf{D} = (\mathbf{D}'_1 \dots \mathbf{D}'_N)$ and using the stacked \mathbf{D} and $\mathbf{W} = \text{diag}[\mathbf{W}_1, \dots, \mathbf{W}_N]$, the resulting estimator is $\hat{\boldsymbol{\alpha}} = (\mathbf{D}' \mathbf{W} \mathbf{D})^{-1} \mathbf{D}' \mathbf{W} \varphi$.

It takes a couple of steps of algebra to establish that both of the above OLS expressions are equivalent and that they are numerically equivalent to $\Phi = (\Phi_1, \dots, \Phi_T)'$. It immediately follows that using a variance estimator similar to (3), one can obtain a consistent estimate of the asymptotic covariance matrix for $\hat{\alpha}$ (and thus for Φ) that is robust to within-firm autocorrelation and heteroscedasticity.

3.3.3 Discussion and Further Extensions

We have above established a link between the moment-based estimators that can be derived from (2) and (4), because $\hat{\alpha} = \hat{\theta} + \hat{\gamma}$. This, of course, implies that $Var[\hat{\alpha}] = Var[\hat{\theta} + \hat{\gamma}]$. There is thus an indirect way to estimate the covariance term, as it can be obtained as the difference of the weighted and unweighted means, i.e., $\hat{\gamma} = \hat{\alpha} - \hat{\theta}$. The standard error of the covariance term is the square root of the diagonals of $Var[\hat{\gamma}] = Var[\hat{\alpha} - \hat{\theta}]$.

Allowing for an unbalanced panel data that is due to entry and exit of firms is possible. One way to do so is to focus on the dynamic OP decomposition introduced by Melitz and Polanec (2009). As shown in Nevalainen (2010), point estimates for the different terms of the dynamic OP decomposition can be obtained by focusing on two time periods and by regressing φ_{it} on appropriately scaled s_{it} using data on surviving incumbents from both periods, data on entrant firms from the latter of the two periods and data on exiting firms from the first period.

An alternative way to allow for entry and exit is to assume that there are 3 mutually exclusive sub-groups *in each period*, i.e., the sub-groups for surviving incumbents ($j = 1$), entrant firms ($j = 2$), and those firms that exit before the end of the next period ($j = 3$). One can then follow the steps outlined above for the estimation of the (static OP) productivity components in mutually exclusive sub-groups. The sub-group indicator, $q_{it,j}$, is defined as follows: If firm i neither enters at t nor exits at $t+1$, $q_{it,1} = 1$ and $= 0$ otherwise. If firm i enters the data

during period t , $q_{it,2} = 1$ and $= 0$ otherwise. For those firms that exit at $t + 1$, $q_{it,3} = 1$ and $= 0$ otherwise.⁸ Slicing data in this way one can, for example, compare productivity levels between entrants and surviving incumbents in a given period (e.g., test $H_0: \gamma_{t,2} + \theta_{t,2} - \gamma_{t,1} - \theta_{t,1} = 0$)⁹, study whether the (relative) productivity levels of entry vintages change over time (e.g., test $H_0: (\gamma_{t,2} + \theta_{t,2}) - (\gamma_{t-s,2} + \theta_{t-s,2}) = 0$ or $H_0: \gamma_{t,2} + \theta_{t,2} - \gamma_{t,1} - \theta_{t,1} = \gamma_{t-s,2} + \theta_{t-s,2} - \gamma_{t-s,1} - \theta_{t-s,1}$) and examine whether it is the change in the covariance component (e.g., test $H_0: (\gamma_{t,2} - \gamma_{t-s,2}) = 0$ or $H_0: \gamma_{t,2} - \gamma_{t,1} = \gamma_{t-s,2} - \gamma_{t-s,1}$), or changing average productivity of entrants (e.g., test $H_0: (\theta_{t,2} - \theta_{t-s,2}) = 0$ or $H_0: \theta_{t,2} - \theta_{t,1} = \theta_{t-s,2} - \theta_{t-s,1}$), that drives the change.

In some applications, there may be a benefit of not treating moment conditions (2) and (4) independently. Because the models are just-identified (this may however be relaxed; see below) and linked by definition, the benefit is for the present purposes more computational (and practical) than statistical: Stacking the two moment conditions and building GMM estimation on them gives point estimates and standard errors for the L.H.S. and the two R.H.S. components of (1) simultaneously. Such an estimation is easily implemented using modern software packages, such as *Stata*. Because the errors in (2) and (4) are corre-

⁸Using this notation, $\sum_{j=1}^3 q_{it,j} = 1$. There is, however, a remaining piece of ambiguity in how one should classify new plants that enter the data by the end of period t and exit by the end of period $t + 1$. For them, $q_{it,2} = 1$ and $q_{it,3} = 1$. When the time period becomes shorter, the share of such observations gets smaller. In applications that use annual data, the number of observations of this type may however be non-negligible. A practical solution to this problem is to introduce a fourth plant category for such "experimental", short-lived entrants.

⁹One could also study how a vintage of entrants contribute to the level of industry productivity in a given period. For such an analysis, an estimate of the activity shares (e.g., employment shares $S_{t,j} = \sum_{i=1}^N q_{it,j} L_{it} / \sum_{i=1}^N L_{it}$) is of course needed.

lated for firm i , the stacked GMM estimation should allow for (cross-moment) clustering of the errors.

Finally, it may sometimes be useful and possible to expand the instrument set in (2) with additional explanatory variables. In that case, $E[\mathbf{Z}_i'(\varphi_i - \mathbf{X}_i\boldsymbol{\beta})] = \mathbf{0}$, where \mathbf{Z}_i includes \mathbf{X}_i and the additional instruments. The model would then be over-identified and more efficient estimation is possible. It would call for using a two-step GMM with an appropriate weighting matrix. We leave it for future work to pursue extensions based on over-identified models and additional instrumental variables.

4 Application

4.1 Data

In our empirical application, we focus on the development of labour productivity in a single industry, "Computer and related activities" (NACE 72). The industry is an example of a dynamic service industry in Finland, with high net employment growth and intensive hiring and separation rates of the employees (see Maliranta and Nikulainen 2008).

Our firm-level data cover years from 1995 to 2007 and come from the Structural Business Statistics (SBS) data of Statistics Finland. The SBS data include all firms in the Finnish business sector. For larger firms, the SBS data are primarily obtained from the Financial Statements inquiry. For those firms not covered by the inquiry, typically employing less than 20 persons, data come from the Finnish Tax Administration's corporate taxation records and Statistics Finland's Register of Enterprises and Establishments.

We measure labour productivity, φ_{it} , by (the logarithm of) value added per person in year 2000 prices and activity shares, s_{it} , by the employment share of

firms.¹⁰ For deflation, we have used implicit price index of the industry obtained from the Finnish National Accounts.

Inspired by Bartelsman et al. (2009a), we focus on cross-regional differences in productivity.¹¹ We estimate, in particular, OP productivity decompositions for two Finnish regions that are in many ways not alike. The first is "Uusimaa", which is a region in Southern Finland province. Uusimaa consists of Helsinki, the capital of Finland, and 20 surrounding municipalities. The population of the region is 1.4 million that is a quarter of the total population of Finland. The second region is "Itä-Suomi" (the Eastern Finland). It is sparsely populated region whose area is 7.6 times larger than that of Uusimaa but its population is only 40% of that in Uusimaa. Uusimaa is much richer than Itä-Suomi; according to the statistics of Eurostat, in 2006 the GDP per inhabitant was 56.9% above the EU average in Uusimaa but 14.7% below in Itä-Suomi.

Some of the multi-unit firms have activities in several regions. In these cases, the location of the firm refers to the region that has the highest within-firm

¹⁰The number of persons refers to the average numbers of persons engaged in the activities of a firm during the accounting period. This convention means that a person who has been employed in the firm for six months corresponds to half an employee. On the other hand, part-time employees are not converted to full-time equivalents. The number of persons engaged include workers, salaried employees and entrepreneurs. It also includes, for example, employees on sick leave or on maternity leave and those laid-off for a fixed period. For a more detailed description of the SBS, see http://tilastokeskus.fi/til/tetipa/kas_en.html.

¹¹See also Böckerman and Maliranta (2007), who examine the plant-level sources of regional productivity divergence in Finnish manufacturing industries. By using a dynamic decomposition formula, they find that the main factor behind the widening productivity gap between the Eastern and Southern Finland since the mid-1980s has been the lack of productivity-enhancing plant-level restructuring within industries in the Eastern Finland. Further analysis of the regional productivity differences in Finland can be found from Ottaviano, Kangasharju, and Maliranta (2009).

employment share.¹² Using this information, we define two mutually exclusive sub-group indicators. The first, $q_{it,U}$, is equal to one if firm i is located in Uusimaa ($j = U$) in period t , and zero otherwise. The second indicator, $q_{it,I}$, is defined similarly for firms located in Itä-Suomi ($j = I$). The two indicators are complements, as $q_{it,I} = 1 - q_{it,U}$.

Descriptive statistics of the data can be found in Table 1.¹³ It displays for selected years the number of firms ($N_{t,j} = \sum_{i=1}^N q_{it,j}$), total employment ($\sum_{i=1}^{N_t} q_{it,j} L_{it}$), and the weighted average of labour productivity ($\sum_{i=1}^{N_t} q_{it,j} s_{it} \varphi_{it}$), separately for $j \in \{U, I\}$.

[Insert Table 1 about here]

4.2 Results

Our main results are displayed in Figures 1-3, obtained by the moment-based approach developed above. The figures display point estimates and the associated 95% confidence intervals (based on panel-robust standard errors) for the weighted average of labour productivity ($\hat{\alpha}$, Figure 1), the average of labour productivity ($\hat{\theta}$, Figure 2) and the covariance term ($\hat{\gamma}$, Figure 3), separately for the two regions (Uusimaa shown by the lines without dots and Itä-Suomi with dots) over the sample period from 1995 to 2007.

[Insert Figures 1-3 about here]

As can be seen from Figure 1, the productivity development has been somewhat erratic, especially in Itä-Suomi. However, the positive trend in the level

¹²The distribution of a firm's employment by region is computed by using Statistics Finland's Register of Enterprises and Establishments.

¹³We have excluded from the sample observations that have less than one (employed) person or that have negative value added.

of industry productivity is quite visible in Uusimaa. The same is not true for Itä-Suomi, which seems to have suffered from a dip in productivity development around 2001. However, the confidence intervals are much wider in Itä-Suomi than in Uusimaa.

Figures 2 and 3 suggest that while the covariance component has made a negative contribution in both regions, the positive trend in the average firm productivity has kept industry productivity increasing in Uusimaa and prevented it from falling in Itä-Suomi. Moreover, it appears that over the last six (or so) years of our sample period (i.e., years after the dot-com bubble period), the level of industry productivity has been higher in Uusimaa partly due to its larger covariance component. Interestingly, the confidence intervals of the average firm productivity are much narrower in Uusimaa than in Itä-Suomi, whereas those of the covariance term are of the same order of magnitude in the two regions.

We have formally tested a number of hypotheses about the regional development in productivity and its sources: First, the null hypothesis that the level of industry productivity has not changed from 1995 to 2007 (i.e., $H_0: \theta_{2007,j} + \gamma_{2007,j} - \theta_{1995,j} - \gamma_{1995,j} = 0$) is rejected for Uusimaa ($j = U$) but not for Itä-Suomi ($j = I$). The p -value for the (Wald) test of the former hypothesis is 0.001, whereas it is 0.966 for the test of the latter hypothesis. Looking at the sources of this difference, the OP decomposition shows that there is a statistically significant improvement from 1995 to 2007 in the average productivity of firms in Uusimaa. For Uusimaa, we reject $H_0: \theta_{2007,U} - \theta_{1995,U} = 0$ at better than the 1% significance level. However, the same null hypothesis for Itä-Suomi cannot be rejected at the 1% level (p -value is 0.042). Interestingly, we cannot reject $H_0: \gamma_{2007,j} - \gamma_{1995,j} = 0$ for either region (with p -values 0.255 and 0.108 for $j = U, I$, respectively). This finding suggests that long-term changes in the covariance term cannot be measured very accurately in our data.

Second, we clearly reject the null hypothesis that the level of industry productivity has, on average, been the same in the two regions during the period from 1995 to 2007. This hypothesis is equivalent to $H_0: \frac{1}{13} \sum_{t=1995}^{2007} (\theta_{t,U} + \gamma_{t,U} - \theta_{t,I} - \gamma_{t,I}) = 0$. The p -value of the associated test is less than 0.001. Looking at the OP components, there is a statistically significant difference between the two regions in the the average productivity of firms; the p -value of the test for $H_0: \frac{1}{13} \sum_{t=1995}^{2007} (\theta_{t,U} - \theta_{t,I}) = 0$ is less than 0.001. However, the difference between the two regions in the covariance term is not statistically significant, as we find that the p -value of the test for $H_0: \frac{1}{13} \sum_{t=1995}^{2007} (\gamma_{t,U} - \gamma_{t,I}) = 0$ is 0.146.

Third, when we focus on the last six years of the sample period, we find that $H_0: \frac{1}{6} \sum_{t=2002}^{2007} (\theta_{t,U} + \gamma_{t,U} - \theta_{t,I} - \gamma_{t,I}) = 0$ can be rejected at better than the 1% significance level (p -value < 0.001). This result appears to be due to two things: First, the average productivity of firms in Uusimaa has, on average, been higher during these years. The p -value of the test for $H_0: \frac{1}{6} \sum_{t=2002}^{2007} (\theta_{t,U} - \theta_{t,I}) = 0$ is less than 0.001. The covariance term has also been higher in Uusimaa than in Itä-Suomi. However, the difference cannot be measured as accurately: The p -value of the test for $H_0: \frac{1}{6} \sum_{t=2002}^{2007} (\gamma_{t,U} - \gamma_{t,I}) = 0$ is 0.024.

5 Conclusions

We show how a standard moment-based GMM procedure can be used to compute point estimates for the components of the Olley-Pakes productivity decomposition and to estimate their standard errors. The procedure provides applied researchers with a simple two-step receipt for panel-robust inference and allows for hypothesis testing about, e.g., the coevolution of the productivity components in different groups of firms.

We provide an application of the procedure to Finnish firm-level data from 1995 to 2007. We find that in our data, there is a clear and statistically sig-

nificant improvement in the level of industry productivity in one of the two regions that our empirical analysis covers. However, formal statistical inference reveals that we cannot in all cases measure the drivers of the change and the difference between the two regions accurately. In particular, we find that not all intertemporal changes and cross-regional differences in the covariance term are statistically significant even though they appear visible to a naked eye.

We have framed our analysis in terms of population moments and GMM because they immediately suggest a number of ways of how the estimation and inference procedure might be extended. For example, the procedure provides a starting point for the computation of, and inference about, dynamic productivity decompositions, such as those of Melitz and Polanec (2009) (see also Maliranta 2009 and Nevalainen 2010), that allow for differential productivity growth contributions by new, surviving and exiting firms.

Three other potential directions for extensions are also worth mentioning: The first is an instrumental variables application that might allow productivity researchers to account for measurement error in the raw data on productivity and activity shares; see Bartelsman, Haltiwanger and Scarpetta (2009b) for a discussion of measurement errors in this context. The second potential extension is to "decompose" the covariance term so as to better understand what drives it. Such a breakdown might be done by bringing in (appropriately scaled) additional regressors (and, if needed, additional population moments) into the model. The Frisch-Waugh regression anatomy formula can then be used to link the partial regression coefficients of the extended model to the covariance term. The third, but clearly more speculative, direction for an extension is the possibility of integrating industry-level (TFP) decomposition computations that the procedure developed in this paper enables with the regression-based estimation of firm-level production functions that precede the measurement of

firm-level TFP (e.g., Levinsohn and Petrin 2003).

References

- [1] Baily, M. N., Hulten C., and Campbell, D., 1992, Productivity Dynamics in Manufacturing Establishments, In: M. Baily and C. Winston (Eds.), *Brookings Papers on Economic Activity: Microeconomics*, 187–249, Washington: Brookings Institution.
- [2] Balk, B. M., 2003, The Residual: On Monitoring and Benchmarking Firms, Industries, and Economies with Respect to Productivity, *Journal of Productivity Analysis*, 20(1), 5-47.
- [3] Bartelsman, E. J., Haltiwanger J. C. and Scarpetta, S., 2009a, Cross-country Differences in Productivity: The Role of Allocation and Selection, NBER Working Paper Series, No. 15490.
- [4] Bartelsman, E. J., Haltiwanger J. C. and Scarpetta, S., 2009b, Measuring and Analyzing Cross-Country Differences in Firm Dynamics, In: T. Dunne, B. J. Jensen & M. J. Roberts (Eds.), *Producer Dynamics*, pp. 15-82, University of Chicago Press.
- [5] Böckerman, P. and Maliranta, M., 2007, The Micro-level Dynamics of Regional Productivity Growth: The Source of Divergence in Finland, *Regional Science and Urban Economics*, 37(2), 165-182.
- [6] Cameron, A. C. and Trivedi, P. K., 2005, *Microeconometrics: Methods and Applications*, Cambridge University Press.
- [7] Davidson, R. and MacKinnon, J.G., 2004, *Econometric Theory and Methods*, Oxford University Press.

- [8] Eslava, M., Haltiwanger J., Kugler, A. D. and Kugler, M., 2004, The Effects of Structural Reforms on Productivity and Profitability Enhancing Reallocation: Evidence from Colombia, *Journal of Development Economics*, 75(2), 333-371.
- [9] Eslava, M., Haltiwanger J., Kugler, A. D. and Kugler, M., 2009a, Trade Reforms and Market Selection: Evidence from Manufacturing Plants in Colombia, NBER Working Paper Series, No. 14935 (April 2009).
- [10] Eslava, M., Haltiwanger J., Kugler, A. D. and Kugler, M., 2009b, Factor Adjustments after Deregulation: Panel Evidence from Colombian Plants, *Review of Economics and Statistics*, forthcoming.
- [11] Foster L., Haltiwanger J. and Krizan C. J., 2006, Market Selection, Reallocation, and Restructuring in the U.S. Retail Sector in the 1990s, *Review of Economics and Statistics*, 88(4), 748-758.
- [12] Foster L., Haltiwanger J. and Syverson C., 2008, Reallocation, Firm Turnover, and Efficiency: Selection on Productivity or Profitability, *American Economic Review*, 98(1), 394-425.
- [13] Levinsohn, J. and Petrin, A., 2003, Estimating Production Functions Using Inputs to Control for Unobservables, *Review of Economic Studies*, 70(2), 317-342.
- [14] Maliranta, M., 2009, In Search of An Ideal Method for Analyzing Micro-level Dynamics of a Great Productivity Leap, Research Institute of the Finnish Economy (ETLA), Discussion Papers, to appear.
- [15] Maliranta, M. and Nikulainen, T., 2008, Labor Flow Paths as Industry Linkages: A Perspective on Clusters and Industry Life cycles, Research

Institute of the Finnish Economy (ETLA), Discussion Papers No. 1168.
Helsinki.

- [16] Manski, C. F., 1988, *Analog Estimation Methods in Econometrics*, Chapman and Hall.
- [17] Melitz, M. J. and Polanec, S., 2009, Dynamic Olley-Pakes Decomposition with Entry and Exit, University of Ljubljana, Working Paper no. 03/09.
- [18] Nevalainen, A., 2010, Estimating Dynamic Olley-Pakes Productivity Decomposition with an Application to Globalization, Research Institute of the Finnish Economy (ETLA), mimeo.
- [19] Ottaviano, G. I. P., Kangasharju, A. and Maliranta, M., 2009, Local Innovative Activity and Regional Productivity: Implications for the Finnish National Innovation Policy. In: R. Veugelers, K. Aiginger, D. Breznitz, C. Edquist, G. Murray, G. Ottaviano, A. Hyytinen, A., Kangasharju, M. Ketokivi, T. Luukkonen, M. Maliranta, M. Maula, P. Okko, P. Rouvinen, M. Sotarauta, T. Tanayama, O. Toivanen & P. Ylä-Anttila (Eds.), *Evaluation of the Finnish National Innovation System - Full Report*: Taloustieto Oy (on behalf of the Ministry of Education and the Ministry of Employment and the Economy), 203-238.
- [20] Olley, G. S. and Pakes, A., 1996, The Dynamics of Productivity in the Telecommunications Equipment Industry, *Econometrica*, 64(6), 1263-1297.
- [21] Van Biesebroeck, J., 2008, The Sensitivity of Productivity Estimates: Revisiting Three Important Productivity Debates, *Journal of Business and Economic Statistics*, 26(3), 321-238.
- [22] Wooldridge, J.M., 2002, *Econometric Analysis of Cross Section and Panel Data*, MIT Press.

- [23] Wooldridge, J.M., 2003, Cluster-Sample Methods in Applied Econometrics, *American Economic Review*, 93, 133-138.

Table 1: Descriptive Statistics

	Uusimaa -region			Itä-Suomi -region		
	N_t	$\sum_{i=1}^{N_t} L_{it}$	$\sum_{i=1}^{N_t} s_{it}\varphi_{it}$	N_t	$\sum_{i=1}^{N_t} L_{it}$	$\sum_{i=1}^{N_t} s_{it}\varphi_{it}$
1995	667	10 541.1	3.88	61	482.4	3.70
2000	855	21 604.6	4.02	109	886.8	3.68
2005	892	24 931.0	3.98	94	835.0	3.53
2007	1183	29 614.7	4.09	108	807.7	3.69

Figure 1: Industry productivity (by region)

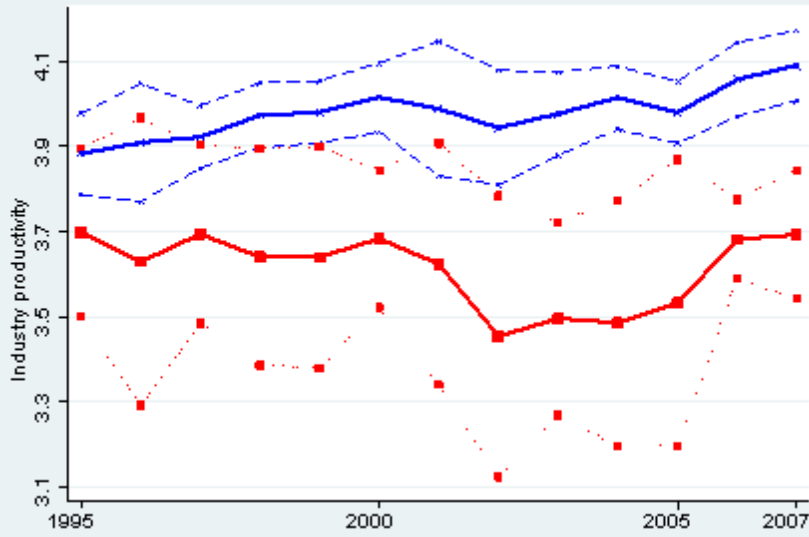


Figure 2: Average productivity (by region)



