Language and the Scientific Imagination
The 11th International Conference of ISSEI
Language Centre, University of Helsinki (Finland)
28 July – 2 August 2008

Making "men see clearly": Physical imperfection and mathematical order in Ptolemy's *Syntaxis*

In *Syntaxis*, Ptolemy tells us that our inquiry into the unchanging nature of the uniform motion of the heavenly bodies through the zodiac will help regulate the natural disorder of our souls and will, "most of all things, concerning that which is most beautifully good with respect to actions and to character, make men see clearly."[1] Why can he be so confident of this? He claims a causal connection between the *mathematical* study of the heavenly appearances and the state of a human's soul, finding that "from the contemplation of the divine beings such as this, in its good-orderliness, symmetry and calm, those being carried after are made lovers of this divine beauty, and are made accustomed to, and re-natured, as it were, to a similar spiritual state" (*Syntaxis*: H7). Such claims regarding a close tie between a love of wisdom, especially a love of the highest objects of knowing, and a well-ordered character are not uncommon. This one is of especial interest because Ptolemy emphasizes that it is only as one undertakes the study of the heavens as a mathematician, and *not* as a theologian *or* a physicist, that one can so improve the state of one's soul. I will investigate here what exactly about the *mathematic* study of the heavenly motion underwrites this claim. In so doing, I will suggest that the heart of Ptolemy's project—standing as it does on a non- or even anti-mechanistic, but also non-theological, life-world—is lost on us today in our having

divided accounts of the universe into "scientific" and "religious," with no possibility of bridging these two realms.

Ptolemy's account of the uniform motion of the sun, and in particular, his claim of the indifference between the two hypotheses he develops to account for the apparent anomaly in its motion, will be the occasion of our investigation. By working carefully through what he calls the "demonstration of the apparent anomaly of the sun" (*Syntaxis*: H216), we will find that Ptolemy aims to offer not an account of the path (or orbit) of the sun, or even some such mathematically possible, if "unreal," orbit.[2] Mathematical understanding means precisely *not* looking for such an orbit, but rather somehow seeing the uniformity in the unchanging character of the motion as a whole, not as an unfolding movement from here to there and back again, but as one unchanging thing: the very being of order. It is in coming to understand this about the sun and its motion that Ptolemy means for his work to contribute to the ordering of our souls as students of mathematics, and not of physics or theology. Having worked through this account, I will ask what it might mean to engage in such an activity today, if such is even possible. What might such a recasting of the study of order in the cosmos mean for our sense of ourselves, and the "war of gods and monsters" often held to be the only possible principle behind human action in a desacralized, if somehow law-governed, universe?

The full version of the paper from which this presentation is drawn progresses in three parts. I give here only the briefest synopsis of the "technical" work of the first two sections before spending the remainder of my time more patiently developing the theoretical and political ramifications of that technical work. Following this, we reflect

on what these mixed results, with respect to the motion of the sun and the motion of the planets, means for Ptolemy's project as a strictly mathematical one, and on what that means for our contemporary conversations which clearly subordinate the mathematical study of physical phenomena to their proper ("correct") physical study. We will then turn to a final consideration of our broader concern for the relationship between physics and theology, as understood by Ptolemy, and as very differently cast for us today.

1. §1,2. The simple hypothesis that "saves" the simplicity of the sun vs. the inherently compound nature of any hypothesis to explain planetary motion

The core of what we must understand from *Syntaxis*—in order to have some sense of what I am arguing are the theoretical and political ramifications of a reconsideration of his work—is a methodological difference between (a) the analytic tools Ptolemy is able to deploy in "saving" the "true" regular and circular motion of the sun around the earth from the "apparent" irregularities and non-circularity of that motion and (b) the tools he deploys in attempting to similarly hypothesize regularity in the motions of the planets. What is essential in this difference, structurally, is that the tools deployed in the first instance are "simple," while those for the planets are not. This "simplicity" is, for Ptolemy, both a feature of the analytic work that a mathematician does in approaching the subject matter, and also of the subject matter itself. Thus, if a mathematical account fails to meet the test of simplicity, it means either that the subject matter itself is not simple—this is impossible in this case, as the motion *must* be regular and circular (hence simple)—or that the mathematical analysis has failed to convey the full truth.

The content of this difference—which the standard account of Ptolemy as presenting a "geocentric model of the universe" elides or ignores (e.g., Kuhn, 67-8)—is this: whereas, with the motion of the sun, Ptolemy is able to account for any "apparent irregularity" with *either* the hypothesis of either an epicyclic or an eccentric circle, any regularity in the motions of the planets can only be "saved" using a single "deferent-epicycle" hypothesis, which not only relies on combining the an epicyclic and an eccentric circle (by "putting" the epicycle "on" the circumference of the eccentric circle), but also needs to specify the precise position of that eccentric circle with respect to some point on which the regular motion "ought" to be centered, but from which the "actual" center of the eccentric circle differs a certain precise amount.[3] What's more, the regularity to be found in any of the motions of the planets needs to be "saved" with an independent deferent, and (as it happens) the deferents are also in different planes, and differ from the "true" center along axes which fall in different dimensions. This means that in order to "hypothesize" a regular motion for any one of planets—and surely to do so for all of them—we must place the arcs of the circles they theoretically trace in definite relation in a certain physical place.

It should be clear from even this highly condensed account, that while the account of the sun (allowing Ptolemy his assumptions, and eschewing for the present the hard work of following his account in detail for ourselves) passes the "simplicity test" described above, this account of the planets fails.

2. §3. Mathematizing a world which both is and is not both physical and theological

In this concluding section, I intend to show that this difference between the two "demonstrations of the anomaly" is one that truly makes a difference, because it is precisely in requiring us to exercise our imagination in such a way as to "place" the heavenly bodies whose regular motion is saved despite apparent irregularity that the account of the planets—crucially unlike the account of sun—edges Ptolemy's account toward a physical one, and thus dispels the promise of a purely mathematical astronomy, an astronomy indifferent to physical positioning of the bodies under theoretical study. Why does it even matter that such promise is compromised? It might well be necessary for any actual astronomy of this sort to be replaced by one that cannot be indifferent to the positioning of bodies immediately from the outset of astronomy as an enterprise—indeed, it might very well be that an astronomy indifferent to physical laws and realities is itself a physical, if not a logical, impossibility. What I wish to show is beyond doubt, though, is that once a purely mathematical conception has been so replaced, the contest between theology and physics to explain of the *meaning* of the heavens—which Ptolemy already saw and believed a mathematics to be the only cure for—is on. Given the intensity and undecidability of this contest today, I hope in this conclusion to help us see how recovering the successful attempt to give a purely mathematical account of the sun's anomaly, in contrast to Ptolemy's failure with the planets, might help us to push the boundaries of what is possible today in our own accounting of the meaning of the heavens.

Having seen in the previous sections how the account of the anomaly of the planets differs from that of the sun, we can now answer why this difference points to something as pivotal as a possible resolution, or better dissolution, of the contemporary antinomy between science and theology. The answer rests in the meaning of the mathematical failure of the demonstration of the planetary anomaly. Since Ptolemy is forced to place the planetary hypothesis within the heavens physically, and to theorize an epicyclic motion around an eccentric deferent, and a deferent particular to the given planet, it becomes impossible to conceive of his account as anything other than a story—however plausible—of what is "actually happening" with this particular heavenly body and its motion. If Ptolemy, or his generous reader, must accept this as the meaning of his mathematical work, then there is nothing to do but understand his planetary hypothesis as an alternative to the heliocentric one and to weigh its merits as an explanation of the physical processes at work in heavenly motions. And, if that's the conversation, then clearly there is no interest in Ptolemy's work except an historical interest in prior, inaccurate explanations. Put another way, if the demonstrations of Book IX, as presented, are the best picture of a "mathematical" approach to the planets, then there really is no alternative but to abandon this notion of astronomy for the more accurate and consistent physical astronomy that we know today, and let that astronomy contest with theological accounts of the heavens for the right to speak definitively regarding cosmology, or the meaning of the heavenly bodies for human understanding.

Our hope—not as defenders of Ptolemy or some such, but as theorists of a possible third, mathematical, way between theological and physical explanations of the heavens today—

is that there is some way to mobilize the account of the anomaly of the sun given in Book III. This requires that we take that account seriously. How can we do so today, as we reject the Ptolemaic system as a physical explanation, and as we acknowledge Ptolemy's failure to provide—indeed, the impossibility for anyone to provide—the purely mathematical astronomy he promises in his preface? Even if we have been convinced that there is this crucial difference between the accounts of the solar and planetary anomalies, what impact can this have on our overall assessment of the meaning or value of a truly "mathematical" astronomy?

A full answer can only be provided if and when a thoroughgoing mathematical realism takes hold—when the objects of mathematical understanding are taken not to be a model of reality, or a symbolization of physical laws, or wholly theoretical and hence unreal in any meaningful sense. The present reflection on the meaning of the account of the anomaly of the sun can only be seen as a case study on what that understanding might look like. That is, what is true of the seeming disorder, and deeper order of the motion of the sun—that being the mathematical object under consideration in this work—will be true of all mathematical objects when we engage in this broader project. Only as we see the fruit of this larger work will we truly be able to assess the viability of any "mathematical" enterprise, be it astronomy, geometry, arithmetic or whatever. Why should such a project, radical in its nature and seemingly hopeless in its ambitions, be taken seriously?

Because what is true of our mathematical object here is nothing less than the true meaning of order: order is not the periodicity of the "actual" motion we "see" the sun making; order is also not the abstraction of that motion into some orbit which exists as a model of the "true" perfect motion of the body that is hidden in the appearances; order is not the formula that gives us the quantitative measurement of that motion. Rather, order is the condition that exists in us, as we reflect on the wholeness of the motion of the sun; not the fact that it starts "somewhere" and "returns" there after so many units of measurement, but the fact that the account we have provided of the motion that we see (or believe we see) itself has resolved its inner tension with the facts of our recording and the presuppositions of our theorizing. This order, to be sure, is not a feature of the universe—one need not hold that the sun never came to be or will never pass away, nor that the earth is the center of anything so as to see this order. Instead, this order—which is a mathematical object on the understanding we develop here—is a feature of our understanding itself, and as such it has an ontological status we need to understand better. And the way to do that is to articulate an account of the class of objects to which this object belongs: that is, to determine the kind of being a mathematical object, on our present understanding, is.

Having thus made explicit that this exploration, as it concludes, is only a prolegomena, let us return to our original question so as to take heart with respect to what makes the work before us worth doing. We wondered, you will recall, how it is that Ptolemy could be so certain that if one engages in the work of developing a mathematical account of the motion of the heavens—and not a theological or physical one—one will be good. This

seems to be a statement (a striking and highly controversial one) about a necessary connection not only between ontology and ethics, but also between theory practice: if one gets the account of being right, then one will get the account of being good right, and if one gets the account of being good right, one will get the being good right. What does this statement have to do with the necessity of developing a thoroughgoing mathematical realism today, and the possible resolution of the debate between theology and physical science over the nature and provenance of the universe?

What we can now see is that it is because (in *Syntaxis*) he understands order to be a feature of the reflective intellect in which order dwells, and which dwells on that order. If this—and not some feature of the cosmos as a system of motion, or some series of phenomena, or something else, or nothing at all—*is* the being of order, then it follows that it is impossible to properly understand this being and not be in accordance with it. That is, if order is the dwelling of a stable whole in the intellect, and if a person successfully directs their intellect to an understanding of that stable wholeness (that is, if they get the ontological question right), then it is impossible for them not to have understood how they need to pattern themselves on that stable wholeness (that is, not to get the ethical question right)—for, in order to have successfully understood that stable wholeness, they have had to demonstrate the anomaly of the motion of the sun, and in order to do so, they have had to carefully work through the observations and also attended to the hypothesis, and then to have proven how the hypothesis can explain the contradictions between the suppositions and the observations. And this demonstration, in turn, is impossible, if one has not already dwelt with one's intellect upon that order which

must underlie the deceiving appearances. And, if one has genuinely understood how they need to pattern themselves on the stable wholeness, it is impossible for them not to. In this way, for these reasons, Ptolemy is certain both of a necessary connection between ethics and ontology and between theory and practice. And thus, he is sure that mathematics, and mathematics alone can "make men see clearly."

The preceding chain of deductions will surely not convince anyone of the fundamental views which are glaringly clear in their importance to Ptolemy's argument. But what our account has done, if successful, is demonstrate that the conviction Ptolemy has regarding the salutary effects of astronomical reasoning is inseparable from his understanding of what a mathematical object *is*. And that this understanding, which can be excavated from his account of the order to be found by "demonstrating the anomaly of the motion of the sun," can serve as a case study for the class of objects to be found in a new and needed robust mathematical realism, which just might be an avenue out of our intractable and ugly impasse about fundamental questions today.

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[1] *Syntaxis*: H7. This translation, and all herein are ultimately my responsibility, though I have relied on (if deviated not infrequently from): Toomer, G.J, *Ptolemy's Syntaxis* (Princeton, NJ: Princeton University Press; 1998). For the Greek, I consulted: Halma, N, *De La Composition Mathématique de Claude Ptolemee* (Paris: Albert Blanchard; 1988).

[2] In this respect chiefly our work will vary greatly from the usual approach, where Ptolemy's "two-sphere system" is described as an alternative to Copernicus's, with the actual nature of the universe taken for granted. See, for example, Kuhn, Thomas S., *The Copernican Revolution* (Cambridge, MA: Harvard University Press; 1957), 67-73.

[3] The work that I am describing is found in *Syntaxis*, III.3-6, and IX.1-9.