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New Attributes and Variables Control Charts under Repetitive Sampling

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ABSTRACT

New control charts under repetitive sampling are proposed, which can be used for variables and attributes quality characteristics. The proposed control charts have inner and outer control limits so that repetitive sampling may be needed if the plotted statistic falls between the two limits. Particularly, the new np and variable X-bar control charts under repetitive sampling are considered in detail. The in-control and out-of-control average run lengths are analyzed according to various process shifts. The performance of the proposed control charts is compared with the existing np and the X-bar control charts in terms of the average run lengths.

Keywords: Attribute Control Chart, Repetitive Sampling, Average Run Length, np Control Chart

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1. INTRODUCTION

Control charts are key techniques in statistical quality control to monitor a manufacturing process and prevent products from going out of the given specification limits. This is an important tool for detecting the assignable causes of variations in the process during the product manufacturing. Control charts can be divided into attributes and variables control charts, depending on whether the quality characteristic is attributed or measurable. Among the variables control charts, the X-bar control chart is simplest, and it is applied when the quality characteristic follows the normal distribution. In attributes control charts, the number of non-conforming items or the fraction non-conforming is obtained from a sample taken from a production process. These values are plotted in the chart over time, and the process is declared to be out of control when a point falls outside the lower control limit or the upper control limit. Studies on the attribute control charts have a long history, but they are still being developed by many authors recently in the literature including, for example, Chan *et al.* (2003), Epprecht and Costa (2001), Epprecht *et al.* (2003), Wu *et al.* (2001, 2006, 2009), Wu and Wang (2007), and Schoonhoven and Does (2012).

The idea of repetitive sampling was originally given by Sherman (1965) for an acceptance sampling plan. Repetitive sampling is considered as more efficient than single sampling in terms of the average sample number (ASN) required for reaching the decision on the lot deposition. Balamurali and Jun (2006) have also shown that their variables repetitive acceptance sampling plan performs better than a single or a double acceptance sampling plan in terms of the ASN. A control chart based on repetitive sampling has not been considered yet although double sampling has been implemented in developing an attributes control chart (De Araujo Rodrigues *et al.*, 2011).

It should be noted here that the idea of repetitive sampling is different from the double sampling. In double sampling, a second sample is selected and the decision will be made on the basis of combined samples if

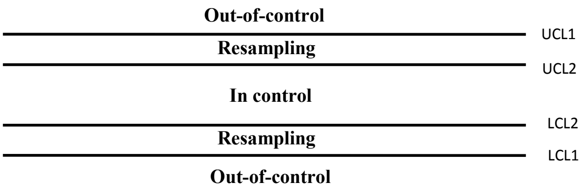

Figure 1. Operational procedure of the proposed chart.

the experimenter cannot reach a decision from the first sample. However, in the repetitive sampling, the process is repeated until a decision is made on the basis of the current sample if the experimenter cannot reach a decision from the previous samples (Sherman, 1965; Balamurali and Jun, 2006). Ahmad *et al*. (2013) and Aslam *et al*. (2014) introduced the repetitive sampling in the area of control charts.

Therefore, it may be interesting to introduce repetitive sampling in the area of control charts. The idea of applying repetitive sampling to a control chart is to use two pairs of control limits instead of one as shown in Figure 1. The process is declared to be in control if a plotted statistic falls within the inner control limits, while it is declared out-of-control if the statistic is plotted beyond the outer control limits. If the statistic is located between the inner and outer control limits, a new sample should be gathered and inspected. Since the sampling cost is relatively cheap these days, repetitive sampling is worth consideration.

In this paper, we propose a new (attributes) np control chart under repetitive sampling in Section 2. This control chart is just an example and other Shewhart types of attributes control charts can be considered similarly. In Section 3, we propose a variables X-bar control chart. The performance of the proposed control charts is compared with the existing Shewhart type control charts in terms of the average run length (ARL) in Section 4, and the concluding remarks are given in the last section of this paper.

2. NP CONTROL CHART UNDER REPETITIVE SAMPLING

We propose the following attributes control chart, which is called "the np control chart" under repetitive sampling.

Step 1: Take a sample of size n. Plot the number of nonconforming items (denoted by D).

Step 2: Declare the process as out-of-control if $D \geq UCL_1$ or $D \leq LCL_1$ (UCL_1 and LCL_1 are called the outer control limits). Declare the process as in-control if $LCL_2 \leq D \leq UCL_2$ (UCL_2 and LCL_2 are called the inner control limits). Otherwise, go to Step 1 and repeat the process.

We consider the following forms of the outer and inner control limits with positive constants k_1 and k_2 when assuming that the fraction non-conforming (p_0) is known when the process is in control:

$$UCL_1 = np_0 + k_1\sqrt{np_0(1-p_0)} \quad (1a)$$

$$LCL_1 = \max\left[0,\ np_0 - k_1\sqrt{np_0(1-p_0)}\right] \quad (1b)$$

$$UCL_2 = np_0 + k_2\sqrt{np_0(1-p_0)} \quad (2a)$$

$$LCL_2 = \max\left[0,\ np_0 - k_2\sqrt{np_0(1-p_0)}\right] \quad (2b)$$

Therefore, the proposed control chart is defined by two parameters k_1 and k_2 when the sample size is specified. Clearly, the proposed control chart reduces to the ordinary np control chart if $k_1 = k_2$.

The probability that the process is declared to be in control based on a single sample is given as

$$P(LCL_2 \leq D \leq UCL_2) = \sum_{d=\lfloor LCL_1 \rfloor+1}^{\lfloor LCL_2 \rfloor} \binom{n}{d} p^d (1-p)^{n-d}$$

where $\lfloor \cdot \rfloor$ indicates the greatest integer less than or equal to the argument and prepresents the fraction non-conforming. If the LCL_2 = 0, then the above sum should be evaluated from d = 0. The probability that the process is declared to be out-of-control based on a single sample is

$$P(D > UCL_1) + P(D < LCL_1)$$
$$= \sum\nolimits_{d=\lfloor UCL_1 \rfloor+1}^{n} \binom{n}{d} p^d (1-p)^{n-d}$$
$$+ \sum\nolimits_{d=0}^{\lfloor LCL_1 \rfloor} \binom{n}{d} p^d (1-p)^{n-d}$$

The probability that repetitive sampling is needed is obtained by

$$P_{\text{rep}}(p) = P\{LCL_1 \leq D < LCL_2\} + P\{UCL_2 \leq D < UCL_1\}$$
$$= \sum\nolimits_{d=\lfloor LCL_1 \rfloor+1}^{\lfloor LCL_2 \rfloor} \binom{n}{d} p^d (1-p)^{n-d}$$
$$+ \sum\nolimits_{d=\lfloor UCL_2 \rfloor+1}^{\lfloor UCL_1 \rfloor} \binom{n}{d} p^d (1-p)^{n-d} \quad (3)$$

where the above sum in the first term should be evaluated from d = 0 if the LCL_1 = 0. Then, the probability that the process is declared to be in control under the proposed control chart is given by

$$P_{\text{in}}(p) = \frac{P(LCL_2 \leq D \leq UCL_2)}{1 - P_{\text{rep}}(p)} \quad (4)$$

For the proposed control chart, the ASN is given as

$$\text{ASN}(p) = \frac{n}{1 - P_{\text{rep}}(p)} \quad (5)$$

Usually, the process is assumed to start in control ($p = p_0$) but sometime in the future an assignable cause may increase the fraction non-conforming to p_1. It is assumed that the control chart is set up when the process is in control. Further, it is assumed that items sampled are independent, and that once the process is gone to the out-of-control state ($p = p_1$), it remains in this condition until there is an intervention to bring it back to the in-control state ($p = p_0$).

The in-control *ARL*, denoted by ARL_0, for the proposed control chart is given by

$$ARL_0 = \frac{1}{1 - P_{\text{in}}(p_0)} \tag{6}$$

Similarly, the out-of-control ARL, ARL_1, is given by

$$ARL_1 = \frac{1}{1 - P_{\text{in}}(p_1)} \tag{7}$$

To find the values of k_1 and k_2 corresponding to the target ARL_0, r_0 we consider the following optimization problem:

$$\text{Minimize ASN } (p_0) \tag{8a}$$

subject to

$$ARL_0 \geq r_0 \tag{8b}$$
$$k_1 \geq k_2 \tag{8c}$$

The above optimization problem can be easily solved by a grid search. We have R program that can be obtained from the authors upon request. After determining the values of k_1 and k_2 corresponding to the target ARL_0, we calculate the values of ARL_1 for various process shifts. We consider the process shift in the form below:

Table 1. ARLs of the proposed np control charts when $n = 40$ and $r_0 = 100$

Shift(f)	$p_0 = 0.10$, $k_1 = 2.7$ $k_2 = 1.0$		$p_0 = 0.12$, $k_1 = 2.6$, $k_2 = 1.0$		$p_0 = 0.14$, $k_1 = 2.6$, $k_2 = 1.1$	
	ASN	ARL_1	ASN	ARL_1	ASN	ARL_1
0.0	69.45	113.76	58.64	110.85	54.12	106.03
0.1	70.52	57.78	61.19	52.19	53.64	50.99
0.2	73.40	31.16	65.39	26.87	54.79	26.38
0.3	77.70	17.73	70.95	14.60	57.16	14.58
0.4	82.99	10.63	77.35	8.44	60.35	8.57
0.5	88.67	6.71	83.75	5.21	63.79	5.37
1.0	94.88	1.57	83.32	1.34	64.75	1.38
1.5	66.37	1.08	55.95	1.03	48.49	1.04
2.0	49.23	1.01	44.16	1.00	41.65	1.00
2.5	42.70	1.00	40.79	1.00	40.18	1.00
3.0	40.63	1.00	40.00	1.00	40.01	1.00

ASN: average sample number, ARL: average run length.

$$p_1 = p_0 + fp_0 \tag{9}$$

where $f = 0$ means that the process is in control.

We completed Tables 1 and 2 for various shifts (f) in fraction non-conforming and specified values of ARL_0 using the above stated optimization problem when the sample size is 40. The average sample number is also reported in Tables 1 and 2.

From Tables 1 and 2, we note the following interesting trends in ARLs:

1) For the same values of k_1 and k_2, we note the decreasing trend in ARL_1 as f increases from 0 to 3.0.
2) As the gap between the values of k_1 and k_2 gets larger, the ASN increases but the ARL_1 decreases.
3) The values of ASN increase first and then decrease as the value of f increases. When the shift parameter f approaches to 3, the ASN reaches n.

Table 2. ARLs of np control charts when $n = 40$ and $r_0 = 200$ and 300

Shift(f)	$n = 40$ and $r_0 = 200$				$n = 40$ and $r_0 = 300$			
	$p_0 = 0.12$, $k_1 = 3.1$, $k_2 = 0.5$		$p_0 = 0.14$, $k_1 = 3.0$, $k_2 = 0.7$		$p_0 = 0.12$, $k_1 = 3.1$, $k_2 = 0.9$		$p_0 = 0.14$, $k_1 = 3.0$, $k_2 = 1.1$	
	ASN	ARL_1	ASN	ARL_1	ASN	ARL_1	ASN	ARL_1
0.0	105.63	195.51	81.70	210.14	59.01	349.97	54.46	315.26
0.1	111.11	83.25	80.59	91.51	61.96	149.30	54.31	135.78
0.2	120.90	37.96	83.14	42.61	66.93	68.57	56.05	63.20
0.3	134.47	18.46	88.67	21.09	73.90	33.59	59.42	31.47
0.4	150.61	9.60	96.43	11.10	82.73	17.47	64.22	16.67
0.5	166.66	5.38	105.15	6.25	92.90	9.66	70.03	9.39
1.0	139.48	1.22	99.18	1.27	112.01	1.51	82.23	1.54
1.5	70.50	1.01	57.33	1.02	68.61	1.04	55.88	1.04
2.0	48.15	1.00	43.56	1.00	48.02	1.00	43.46	1.00
2.5	41.77	1.00	40.47	1.00	41.77	1.00	40.46	1.00
3.0	40.27	1.00	40.03	1.00	40.27	1.00	40.03	1

ASN: average sample number, ARL: average run length.

3. PROPOSED X-BAR CONTROL CHART UNDER REPETITIVE SAMPLING

In this section, we propose a new X-bar control chart under repetitive sampling when the quality characteristic follows a normal distribution with mean μ and standard deviation σ. It is also assumed that the target mean is m when the process is in control. Similarly, the proposed control chart has two pairs of control limits, which will be operated as follows:

Step 1: Take a sample of size n. Measure each quality characteristic X and calculate $\bar{X}$ using this sample.

Step 2: Declare out-of-control if $\bar{X} > UCL_1$ or $\bar{X} < LCL_1$. Declare in-control if $LCL_2 < \bar{X} < UCL_2$. Otherwise, go to Step 1 and repeat the process.

The upper and lower outer control limit of the proposed $\bar{X}$ chart are given as follows:

$$UCL_1 = m + k_1\sigma/\sqrt{n} \tag{10a}$$

$$LCL_1 = m - k_1\sigma/\sqrt{n} \tag{10b}$$

Similarly, the upper and lower inner control limits are given by

$$UCL_2 = m + k_2\sigma/\sqrt{n} \tag{11a}$$

$$LCL_2 = m - k_2\sigma/\sqrt{n} \tag{11b}$$

The probability that the process is declared as in-control under the proposed X-bar chart is given as

$$P_{\text{in}} = \frac{P(LCL_2 < \bar{X} < UCL_2)}{1 - P_{\text{rep}}} \tag{12}$$

Here, P_{rep} is the probability of repetition when the process is in control, which is obtained by

$$P_{\text{rep}} = 2\left[\Phi(k_1) - \Phi(k_2)\right]$$

Also, the probability that the process is declared as in-control when the process is actually in control is given by

$$P\left\{LCL_2 < \bar{X} < UCL_2 \,\middle|\, \mu = m\right\} = \Phi(k_2) - \Phi(-k_2) = 2\Phi(k_2) - 1$$

Therefore, Eq. (12) can be written as follows

$$P_{\text{in}} = \frac{2\Phi(k_2) - 1}{1 - 2\left[\Phi(k_1) - \Phi(k_2)\right]} \tag{13}$$

Suppose now that the process mean has shifted from m to $m+c\sigma$. Then, the probability that the process is declared as in-control is obtained by

$$P_{\text{in}}^{*} = \frac{P(LCL_2 < \bar{X} < UCL_2 \mid \mu = m + c\sigma)}{1 - P_{\text{rep}}^{*}} \tag{14}$$

It turns out that Eq. (14) is rewritten by

$$P_{\text{in}}^{*} = \frac{\Phi(k_2 - c\sqrt{n}) + \Phi(k_2 + c\sqrt{n}) - 1}{\Phi(k_2 + c\sqrt{n}) - \Phi(k_1 + c\sqrt{n}) - \Phi(k_1 - c\sqrt{n}) + \Phi(k_2 - c\sqrt{n}) + 1} \tag{15}$$

Hence, the in-control and the out-of-control ARLs are respectively given by

$$ARL_0 = \frac{1}{1 - P_{\text{in}}} \tag{16}$$

$$ARL_1 = \frac{1}{1 - P_{\text{in}}^{*}} \tag{17}$$

We presented the several tables reporting ARL_1 of the proposed control chart for various values of sample size n and target in-control ARL according to shift parameters. The values of ASN are also presented in these tables. We considered the sample size of 10 (Table 3), 20 (Table 4), 30 (Table 5), and 40 (Table 6). Three target in-control ARLs are considered (100, 200, and 300). It should be reminded that ARL_1 when c = 0 indicates the in-control ARL.

From Tables 4–6, we note the following trends in ARLs and ASNs.

1) For the same values of n, k_1 and k_2, we note decreasing trend in ARL_1 as c changes from 0.0 to 3.0.
2) For the same values of c, ARL_1 increasesas n increases.
3) ASN increases first as c increases and then drops to the sample size n at last.

Table 3. ARLs of the proposed X-bar control charts when $n = 10$

Shift(c)	$k_1 = 2.8371$, $k_2 = 0.5988$, $r_0 = 100$		$k_1 = 2.9301$, $k_2 = 0.9825$, $r_0 = 200$		$k_1 = 3.0572$, $k_2 = 0.9699$, $r_0 = 300$	
	ASN	ARL_1	ASN	ARL_1	ASN	ARL_1
0.0	21.97	100.02	14.76	200.00	14.92	300.02
0.1	22.84	65.75	15.26	129.54	15.44	188.61
0.2	25.55	27.96	16.82	54.20	17.08	75.33
0.3	30.13	11.23	19.59	21.43	20.06	28.45
0.4	35.79	4.82	23.56	8.85	24.52	11.16
0.5	39.54	2.42	27.92	4.04	29.84	4.79
1.0	15.81	1.01	16.49	1.02	17.99	1.03
1.5	10.29	1.00	10.36	1.00	10.48	1.00
2.0	10.00	1.00	10.00	1.00	10.01	1.00
3.0	10.00	1.00	10.00	1.00	10.00	1.00

ASN: average sample number, ARL: average run length.

Table 4. ARLs of the proposed X-bar control charts when $n = 20$

Shift(c)	$k_1 = 2.7626$, $k_2 = 0.7852$, $r_0 = 100$		$k_1 = 2.9763$, $k_2 = 0.8072$, $r_0 = 200$		$k_1 = 3.1738$, $k_2 = 0.5975$, $r_0 = 300$	
	ASN	ARL_1	ASN	ARL_1	ASN	ARL_1
0.0	34.88	100.00	34.28	200.01	44.31	300.00
0.1	37.43	48.76	36.92	89.91	48.19	124.02
0.2	45.35	14.23	45.50	23.47	61.21	28.80
0.3	57.15	4.50	60.29	6.50	84.89	7.04
0.4	62.92	1.93	72.48	2.35	104.56	2.30
0.5	53.85	1.24	65.59	1.33	89.88	1.28
1.0	20.91	1.00	21.44	1.00	22.15	1.00
1.5	20.00	1.00	20.00	1.00	20.00	1.00
2.0	20.00	1.00	20.00	1.00	20.00	1.00
3.0	20.00	1.00	20.00	1.00	20.00	1.00

ASN: average sample number, ARL: average run length.

Table 5. ARLs of the proposed X-bar control charts when $n = 30$

Shift(c)	$k_1 = 2.8182$, $k_2 = 0.6402$, $r_0 = 100$		$k_1 = 2.9439$, $k_2 = 0.9245$, $r_0 = 200$		$k_1 = 3.0722$, $k_2 = 0.9062$, $r_0 = 300$	
	ASN	ARL_1	ASN	ARL_1	ASN	ARL_1
0.0	62.14	100.00	46.29	200.00	47.08	300.01
0.1	69.54	36.05	51.29	68.64	52.36	96.47
0.2	92.12	7.66	67.76	13.71	70.37	17.73
0.3	112.48	2.22	91.54	3.39	99.45	3.94
0.4	92.77	1.22	91.61	1.45	104.34	1.52
0.5	61.75	1.04	66.17	1.08	74.51	1.09
1.0	30.12	1.00	30.17	1.00	30.24	1.00
1.5	30.00	1.00	30.00	1.00	30.00	1.00
2.0	30.00	1.00	30.00	1.00	30.00	1.00
3.0	30.00	1.00	30.00	1.00	30.00	1.00

ASN: average sample number, ARL: average run length.

Table 6. ARLs of the proposed X-bar control charts when $n = 40$

Shift(c)	$k_1 = 2.8015$, $k_2 = 0.6801$, $r_0 = 100$		$k_1 = 2.9890$, $k_2 = 0.7670$, $r_0 = 200$		$k_1 = 3.1185$, $k_2 = 0.7444$, $r_0 = 300$	
	ASN	ARL_1	ASN	ARL_1	ASN	ARL_1
0.0	78.64	100.01	71.46	200.02	73.37	300.01
0.1	90.97	28.67	82.98	51.46	85.68	71.32
0.2	126.71	5.07	121.01	7.81	128.62	9.75
0.3	138.06	1.58	152.21	1.91	172.90	2.08
0.4	94.24	1.08	110.65	1.12	127.15	1.13
0.5	61.79	1.01	69.32	1.01	76.16	1.01
1.0	40.01	1.00	40.02	1.00	40.03	1.00
1.5	40.00	1.00	40.00	1.00	40.00	1.00
2.0	40.00	1.00	40.00	1.00	40.00	1.00
3.0	40.00	1.00	40.00	1.00	40.00	1.00

ASN: average sample number, ARL: average run length.

4. COMPARATIVE STUDY

In this section, we will discuss the advantage of the proposed control chart over the traditional np and X-bar charts in terms of ARLs. To compare the ARL_1 between two control charts, ARL_0 should be equal. Note that the usual traditional np chart having control constant k is a special case of the proposed control chart with $k_1 = k_2 = k$. So, we will find k for the traditional np chart such that the ARL_0s are similar for two charts. We found the values of ARL_1 for both control charts for various values of f and placed in Table 7.

From Table 7, we can see that the proposed control chart provides considerably smaller ARL_1 as compared to the traditional np chart for all shifts, various sample size and p_0. For example, when $p_0 = 0.21$, $n = 55$, and $f = 0.1$ the ARL for the proposed chart reduces to 74.5 from $ARL_0 = 230.6$, whereas the ARL for the existing np chart reduces to 110.2 from $ARL_0 = 242.9$. Therefore, it is clear that the proposed chart has the advantage over the existing control chart to detect the process shift.

Table 7. Comparison of ARLs between the proposed and the usual np charts

Shift(f)	ARL_0 around 200, $n = 55$, $p_0 = 0.21$		ARL_0 around 300, $n = 40$, $p_0 = 0.22$	
	Usual with $k = 2.8$	Proposed with $k_1 = 2.9$, $k_2 = 1.1$	Usual with $k = 2.8$	Proposed with $k_1 = 3.0$, $k_2 = 1.3$
0.0	242.87	230.62	281.73	273.26
0.1	110.21	74.48	117.88	97.10
0.2	43.42	24.83	50.18	37.90
0.3	19.30	9.26	23.89	16.26
0.4	9.82	4.02	12.70	7.69
0.5	5.63	2.15	7.45	4.06
1.0	1.32	1.01	1.58	1.08

ARL: average run length.

Table 8. Comparison of the proposed and the traditional X-bar control charts when $n = 20$

c	$r_0 = 100$		$r_0 = 200$		$r_0 = 300$	
	Existing $k = 2.5758$	Proposed $k_1 = 2.7626$, $k_2 = 0.7852$	Existing $k = 2.8070$	Proposed $k_1 = 2.9763$, $k_2 = 0.8072$	Existing $k = 2.9352$	Proposed $k_1 = 3.1738$, $k_2 = 0.5975$
	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1
0.0	100.00	100.00	200.00	200.01	300.02	300.00
0.1	55.88	48.76	102.98	89.91	147.44	124.02
0.2	21.46	14.23	35.71	23.47	48.31	28.80
0.3	9.21	4.50	14.00	6.50	18.01	7.04
0.4	4.64	1.93	6.48	2.35	7.95	2.30
0.5	2.72	1.24	3.52	1.33	4.13	1.28
1.0	1.03	1.00	1.05	1.00	1.07	1.00
1.5	1.00	1.00	1.00	1.00	1.00	1.00
2.0	1.00	1.00	1.00	1.00	1.00	1.00
3.0	1.00	1.00	1.00	1.00	1.00	1.00

ARL: average run length.

Table 9. Comparison of the proposed and the traditional X-bar control charts when $n = 30$

c	$r_0 = 100$		$r_0 = 200$		$r_0 = 300$	
	Existing $k = 2.5759$	Proposed $k_1 = 2.8182$ $k_2 = 0.6402$	Existing $k = 2.8070$	Proposed $k_1 = 2.9439$ $k_2 = 0.9245$	Existing $k = 2.9352$	Proposed $k_1 = 3.0722$ $k_2 = 0.9062$
	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1
0.0	100.02	100.00	200.01	200.00	300.01	300.01
0.1	45.12	36.05	81.11	68.64	114.55	96.47
0.2	14.39	7.66	22.97	13.71	30.37	17.73
0.3	5.70	2.22	8.18	3.39	10.19	3.94
0.4	2.86	1.22	3.72	1.45	4.38	1.52
0.5	1.77	1.04	2.12	1.08	2.37	1.09
1.0	1.00	1.00	1.00	1.00	1.01	1.00
1.5	1.00	1.00	1.00	1.00	1.00	1.00
2.0	1.00	1.00	1.00	1.00	1.00	1.00
3.0	1.00	1.00	1.00	1.00	1.00	1.00

ARL: average run length.

We presented Tables 8 and 9 for the comparison of the proposed X-bar chart and the traditional X-bar chart in terms of ARLs. Table 8 is for $n = 20$ and Table 9 is for $n = 30$. We considered three cases ($r_0 = 100, 200, 300$) of the target in-control ARL.

From these tables, we can see that the proposed X-bar control chart performs better than the traditional X-bar chart in term of ARL_1 for all mean shifts. For example, when $n = 20$, $r_0 = 300$ and $c = 0.1$, $ARL_1 = 1242.2$ from the proposed chart and it is 147.44 from the traditional Shewhart X-bar control chart. From Tables 8 and 9, we have the following trends:

1) For the same values of r_0, ARL_1 rapidly decreases as c increases from 0 to 3.0.
2) For the same values of r_0 and, as n changes from 20 to 30, ARL_1 decreases faster.

5. CONCLUDING REMARKS

A new attributes np control chart and an X-bar control chart based on repetitive sampling are proposed in this paper. The average run length properties are analyzed, and the tables of the ARLs are provided for various parameters. The proposed control charts provide the smaller values of ARL_1 as compared to the existing control charts when ARL_0 remains the same for both charts. It may be concluded that the proposed control charts perform better than the traditional np control chart and X-bar control chart in terms of the ARL. It may be an interesting future work to design other attributes or variables control charts under repetitive sampling.

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