Industrial Engineering
& Management Systems
Vol 14, No 2, June 2015, pp.183-195
ISSN 1598-7248 │ EISSN 2234-6473 │

http://dx.doi.org/10.7232/iems.2015.14.2.183

Sampling Plans Based on Truncated Life Test for a Generalized Inverted Exponential Distribution

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(Received: April 24, 2015 / Revised: June 2, 2015 / Accepted: June 4, 2015)

ABSTRACT

In this paper, we propose a two-stage group acceptance sampling plan for generalized inverted exponential distribution under truncated life test. Median life is considered as a quality parameter. Design parameters are obtained to ensure that true median life is longer than a given specified life at certain level of consumer's risk and producer's risk. We also explore situations under which design parameters based on median lifetime can be used for other percentile points. Tables and specific examples are reported to explain the proposed plans. Finally a real data set is analyzed to implement the plans in practical situations and some suggestions are given.

Keywords: Consumer's Risk, Double Acceptance Sampling Plan, Group Acceptance Sampling Plan, Producer's Risk, Single Acceptance Sampling Plan, Truncated Life Test, Two-Stage Group Acceptance Sampling Plan

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1. INTRODUCTION

In various statistical quality control and reliability studies, acceptance sampling plans are conducted to gain adequate inferential knowledge about a product. In acceptance sampling plans consumers make decision about whether to accept or reject a lot of product under consideration. Usually in a single acceptance sampling plan, based on a truncated life test, a random sample of n units drawn from a lot under consideration is placed on a life testing experiment for pre-assigned time duration t_0. If during the experimentation the total number of observed failures exceeds to a prescribed number c (acceptance number), the test is terminated and the consumer rejects the lot. On the other hand, if the total number of observed failures is less than or equal to c, then the consumer accepts the lot. Here n and c are known as design parameters of the single acceptance sampling plan. Several researchers have proposed single acceptance sampling plans for various lifetime distributions. One may refer to, among others, Epstein (1954) for exponential distribution, Goode and Kao (1961) for Weibull distribution, Gupta (1962) for normal and lognormal distributions, Rosaiah and Kantam (2005) for inverse Rayleigh distribution, Rosaiah *et al*. (2006) for exponentiated log-logistic distribution, Tsai and Wu (2006) for generalized Rayleigh distribution, Balakrishnan *et al*. (2007) and Lio *et al*. (2010) for generalized Birnbaum-Saunders distribution and Aslam *et al*. (2010) for generalized exponential distribution. A generalization of the single acceptance sampling plan is known as the double acceptance sampling plan. Such plans have also found wide applications in the area of quality control and reliability analysis and one may refer to Aslam and Jun (2010) and Aslam *et al*. (2011a) for some related results.

Balasooriya (1995) suggested that when the test units are highly reliable but relatively cheap and the test facilities are scarce, a life test experiment can be conducted by placing g independent groups on the test where each group contains r number of units. This gives rise to group acceptance sampling plan. In this plan, a random sample of size n units is drawn from the proposed lot and then these units are allocated to g number of different groups, where each group contains r units such that $n = g \times r$. In group acceptance sampling, all the groups are simultaneously subjected to a certain life test for a prescribed time period t_0. If during the experimentation more than c number of failures is observed, the experiment is terminated and consumer rejects the lot, otherwise, the lot is accepted. Note that g, r and c are the design parameters of the associated group acceptance sampling plan. In literature, group acceptance sampling plans have been discussed by Aslam *et al.* (2009) for gamma distribution, Aslam and Jun (2009a) for Weibull distribution, Aslam and Jun (2009b) for inverse Rayleigh and log-logistic distributions, Rao (2010) for generalized exponential distribution and Aslam *et al.* (2011a) for Birnbaum-Saunders distribution. It is to be noticed that two-stage group acceptance sampling plan is a further generalization of the group acceptance sampling plan. One may refer to the works of Aslam *et al.* (2011), (2011b), (2013) and cited references there-in for various results on such acceptance sampling plans.

In this paper, we establish two-stage group acceptance sampling plan for a generalized inverted exponential distribution based on the truncated life test. The rest of this paper is organized as follows. In Section 2, we review some useful properties of the generalized inverted exponential distribution. Sampling plans are explained with an example in Section 3. In Section 4, a real data set is used to illustrate the implementation of sampling plans in practical situations. Sampling plans for 100pth percentile points are discussed in Section 5. Finally, we present a conclusion in Section 6.

2. GENERALIZED INVERTED EXPONENTIAL DISTRIBUTION

A two-parameter generalized inverted exponential distribution has the probability density function (PDF) and cumulative distribution function (CDF) of the form

$$f(t;\gamma,\lambda)=\gamma\lambda t^{-2}\mathrm{e}^{-\frac{\lambda}{t}}\left(1-\mathrm{e}^{-\frac{\lambda}{t}}\right)^{\gamma-1},\quad t>0,\ \gamma>0,\lambda>0.$$

$$F(t;\gamma,\lambda)=1-\left(1-e^{-\frac{\lambda}{t}}\right)^{\gamma}. \tag{1}$$

We denote this distribution as GIE (γ, λ) where γ is a shape parameter and λ is a scale parameter. Abouammoh and Alshingiti (2009) introduced this distribution in literature as a generalization to the inverted exponential (IE) distribution. Some interesting properties of inverted exponential distribution can also be found in Killer and Kamath (1982), Lin *et al.* (1989) and Dey (2007). In fact GIE (γ, λ) distribution with $\gamma = 1$ reduces to an IE distribution. Nadarajah and Kotz (2003) have discussed some interesting properties of GIE distribution and its various applications in the field of accelerated life testing, queue theory and modeling wind speeds etc. In many situations it has been suggested that this distribution provides a better fit than models like exponential, gamma, Weibull, generalized exponential and inverted exponential. We refer to Abouammoh and Alshingiti (2009), Krishna and Kumar (2013) and Dey and Pradhan (2013) for further details. Observe that mean μ of a GIE (γ, λ) distribution is given by

$$\mu=\lambda\int_0^\infty y^{-2}(1-e^{-y})^{\gamma}\,dy,$$

which exists for $\gamma > 1$. The corresponding 100pth percentile, say $\theta_p = F^{-1}(p)$ is of the form

$$\theta_p=-\frac{\lambda}{\log\left(1-(1-p)^{\frac{1}{\gamma}}\right)}. \tag{2}$$

and is defined for all parameter values. The median is of the form

$$m=-\frac{\lambda}{\log\left(1-0.5^{\frac{1}{\gamma}}\right)}. \tag{3}$$

In the next section, we construct various acceptance sampling plans by treating median lifetime as a quality parameter for the test units. A generalization of these acceptance sampling plans for the other percentile points is discussed in Section 5.

3. TWO-STAGE GROUP ACCEPTANCE SAMPLING PLAN

Suppose that a producer submits a lot of units and claims that the specified median lifetime of the units is m_0. Assume that the lifetime of these units follows a GIE (γ, λ) distribution where the quality of the units is measured in terms of median lifetime m. We are interested in making inference whether the true median lifetime m of a unit is larger than the specified lifetime m_0. The common practice is to draw a random sample from the lot and then perform a truncated life test for t_0 units of time. Here t_0 can be taken as a multiple of m_0. That is for any positive constant a, we may take $t_0 = am_0$. Then a lot under investigation will be accepted if there is enough evidence that $m \geq m_0$, at certain level of producer's risk α and consumer's risk β. In this regard, we suggest the following two-stage group acceptance sampling plan.

(1) (*First stage*) Draw a random sample of size n_1 units and then allocate r units to each of the g_1 number of groups, so that $n_1 = r\times g_1$. Put all the groups simultaneously on a life test for t_0 units of time. Accept the lot if c_1 or less number of units fail during the test and reject the lot and terminate the test as soon as c_2+1 or more number of units fail. Otherwise, go to next step.

(2) (*Second stage*) Draw a second random sample of size n_2 units and then allocate r units to each of the g_2 groups, so that $n_2 = r\times g_2$. Put all the groups simultaneously on a life test for t_0 units of time. Accept the lot if at most c_2 number of units failed from the two samples and reject the lot, otherwise.

For a given $r(r \neq 1)$, a two-stage group acceptance sampling plan can be characterized by the design parameters (g_1, g_2, c_1, c_2). Further for $c_1 = c_2 = c$, the proposed sampling plan reduces to a (single-stage) group acceptance sampling plan in which only the first stage will be taken into consideration with $g = g_1$. Observe that the design parameters of a group acceptance sampling plan are simply (g, c). Notice that for $r = 1$, the proposed sampling plan reduces to a double acceptance sampling plan characterized by the design parameters (n_1, n_2, c_1, c_2). For $c_1 = c_2 = c$, it further reduces to a single acceptance sampling plan with $n = n_1$ and equivalently, (n, c) denotes the corresponding design parameters. Now the probability of accepting a lot for the proposed sampling plan is

$$P_a(p) = P_a^{(1)}(p) + P_a^{(2)}(p) \tag{4}$$

Here, $P_a^{(1)}(p)$ and $P_a^{(2)}(p)$ are the probabilities of accepting a lot from the *first stage* and *second stage* respectively and are given by

$$P_a^{(1)}(p) = \sum_{i=0}^{c_1}\binom{n_1}{i}p^i(1-p)^{r-i},$$

and

$$P_a^{(2)}(p) = \sum_{j=c_1+1}^{c_2}\binom{n_1}{j}p^j(1-p)^{n_1-j}\left[\sum_{i=0}^{c_2-j}\binom{n_2}{i}p^i(1-p)^{n_2-i}\right]$$

Here $p(t_0;\gamma,\lambda)$ is the probability that a test unit fails before the termination time point t_0 and using Eq. (3), it can be written as

$$p = 1-\left[1-\left(1-0.5^{\frac{1}{\gamma}}\right)^{\frac{m}{am_0}}\right]^{\gamma}. \tag{5}$$

Thus p can be computed if a, m, m_0 and γ are known. Notice that p is independent of the scale parameter λ. However, median lifetime of the product accounts the information of both shape and scale parameter (see, Eq. (3)). Assume that $\frac{m}{m_0}$, the ratio of median lifetime to the prescribed lifetime of the unit, represent a quality level. Notice that from a consumer perspective when $m = m_0$, the probability of accepting a lot should be smaller than the probability of accepting a bad lot. On the other hand, producer needs that the probability of rejecting a lot should be smaller than the probability of rejecting a good lot when $m > m_0$. So, for a given quality level $\frac{m}{m_0}$ at consumer's risk β and producer's risk α and for given a, the design parameters can be obtained by solving the following two inequalities simultaneously

$$P_a\left(p_1\middle|\frac{m}{m_0}=r_1\right) \le \beta, \tag{6}$$

$$P_a\left(p_2\middle|\frac{m}{m_0}=r_2\right) \ge 1-\alpha, \tag{7}$$

Here p_1 is the probability that a test unit fails before the termination time t_0 when r_1 is the quality level corresponding to consumer's risk β and p_2 is the probability that a test unit fails before the termination time t_0 when r_2 is the quality level corresponding to producer's risk α. Observe that multiple combinations of design parameters may satisfy inequalities (6) and (7). Here we are interested in finding the minimum sample size satisfying the above two inequalities. The minimum average sample number (ASN) required to make a decision about the lot is obtained as

$$ASN(p) = n_1 + n_2P_d(p). \tag{8}$$

where $P_d(p) = \sum_{j=c_1+1}^{c_2}\binom{n_1}{j}p^j(1-p)^{n_1-j}$ is the probability that a decision is taken from second stage. Consequently, the desired design parameters having minimum sample size can be obtained by solving the following optimization problem:

Minimize ASN $(p_1) = n_1 + n_2P_d(p_1)$
Subject to

$$P_a\left(p_1\middle|\frac{m}{m_0}=r_1\right) \le \beta,$$

$$P_a\left(p_2\middle|\frac{m}{m_0}=r_2\right) \ge 1-\alpha,$$

$$1 \le n_2 \le n_1,$$

n_1, n_2 : Integers.

Recall that for $r = 1$, the proposed sampling plan reduces to the double acceptance sampling plan. We first obtain design parameters (n_1, n_2, c_1, c_2) under double acceptance sampling plan for some arbitrary values of γ, say $\gamma = 1, 2$. We consider three different levels of a as 0.5, 0.7, 1.0 and four level of consumer's risk β as 0.25, 0.10,

Table 1. Double acceptance sampling plan for $\gamma = 1$

		$a = 0.5$						$a = 0.7$						$a = 1.0$					
β	r_2	n_1	n_2	c_1	c_2	ASN	p_α	n_1	n_2	c_1	c_2	ASN	p_α	n_1	n_2	c_1	c_2	ASN	p_α
0.25	1.5	33	19	5	10	46.04	0.9516	28	27	7	17	51.48	0.9506	35	28	13	28	60.54	0.9508
	2.0	12	9	1	3	16.41	0.9675	14	6	3	5	16.40	0.9607	16	6	6	8	18.22	0.9519
	2.5	6	6	0	1	8.13	0.9587	7	4	1	2	8.13	0.9555	8	4	2	4	9.96	0.9599
	3.0	6	6	0	1	8.13	0.9886	4	3	0	1	5.10	0.9597	4	3	0	2	5.87	0.9548
	3.5	6	6	0	1	8.13	0.9970	4	3	0	1	5.10	0.9840	4	3	0	2	5.87	0.9820
	4.0	6	6	0	1	8.13	0.9992	4	3	0	1	5.10	0.9938	3	2	0	1	3.75	0.9688
0.10	1.5	48	34	4	15	77.70	0.9541	48	36	10	25	83.13	0.9513	54	44	19	42	97.11	0.9512
	2.0	19	11	0	4	24.07	0.9633	22	8	4	7	24.74	0.9572	22	11	6	12	29.83	0.9532
	2.5	11	10	0	2	15.12	0.9755	11	6	1	3	12.93	0.9549	13	6	3	6	15.72	0.9650
	3.0	9	8	0	1	10.80	0.9771	8	5	0	2	9.76	0.9746	8	4	1	3	9.31	0.9500
	3.5	9	8	0	1	10.80	0.9938	6	4	0	1	6.87	0.9674	5	4	0	2	6.87	0.9629
	4.0	9	8	0	1	10.80	0.9984	6	4	0	1	6.87	0.9871	5	4	0	2	6.87	0.9852
0.05	1.5	59	48	6	19	102.96	0.9581	57	50	12	31	106.51	0.9510	63	63	22	53	125.28	0.9525
	2.0	25	15	1	5	30.56	0.9639	25	11	4	8	28.95	0.9508	26	16	7	15	39.15	0.9599
	2.5	14	10	0	2	16.63	0.9641	14	8	0	4	16.85	0.9670	16	7	3	7	18.73	0.9627
	3.0	11	11	0	1	12.70	0.9644	9	6	0	2	10.64	0.9629	10	6	1	4	12.19	0.9604
	3.5	11	11	0	1	12.70	0.9902	7	6	0	1	7.96	0.9501	8	5	0	3	9.79	0.9773
	4.0	11	11	0	1	12.70	0.9974	7	6	0	1	7.96	0.9799	6	5	0	2	7.64	0.9739
0.01	1.5	83	72	9	26	149.59	0.9540	83	78	18	45	160.81	0.9539	93	85	29	73	177.98	0.9501
	2.0	33	21	1	6	38.16	0.9502	34	22	4	12	44.70	0.9614	35	24	8	20	55.25	0.9547
	2.5	20	18	0	3	23.99	0.9708	18	13	0	5	21.72	0.9581	23	10	5	9	24.97	0.9504
	3.0	18	15	0	2	19.94	0.9864	13	11	0	3	15.46	0.9685	13	9	1	5	15.59	0.9525
	3.5	17	10	0	1	17.42	0.9834	11	10	0	2	12.55	0.9755	11	8	0	4	13.19	0.9782
	4.0	17	10	0	1	17.42	0.9956	11	5	0	1	11.19	0.9663	9	8	0	3	11.01	0.9816

Table 2. Double acceptance sampling plan for $\gamma = 2$

		$a = 0.5$						$a = 0.7$						$a = 1.0$					
β	r_2	n_1	n_2	c_1	c_2	ASN	p_α	n_1	n_2	c_1	c_2	ASN	p_α	n_1	n_2	c_1	c_2	ASN	p_α
0.25	1.5	26	14	2	4	31.54	0.9617	22	8	4	7	25.84	0.9559	21	10	7	13	29.10	0.9587
	2.0	10	8	0	1	12.61	0.9767	6	6	0	2	9.66	0.9722	8	4	2	4	9.96	0.9699
	2.5	10	8	0	1	12.61	0.9977	5	4	0	1	6.38	0.9834	4	3	0	2	5.87	0.9807
	3.0	10	8	0	1	12.61	0.9998	5	4	0	1	6.38	0.9969	3	2	0	1	3.75	0.9798
	3.5	10	8	0	1	12.61	0.9999	5	4	0	1	6.38	0.9994	3	2	0	1	3.75	0.9937
	4.0	10	8	0	1	12.61	0.9999	5	4	0	1	6.38	0.9999	3	2	0	1	3.75	0.9981
0.10	1.5	40	23	2	6	50.99	0.9653	30	16	3	10	40.50	0.9533	29	19	8	19	47.18	0.9565
	2.0	14	14	0	1	17.12	0.9509	13	7	1	3	15.24	0.9749	11	6	2	5	13.80	0.9553
	2.5	14	14	0	1	17.12	0.9950	7	6	0	1	8.35	0.9673	5	4	0	2	6.87	0.9604
	3.0	14	14	0	1	17.12	0.9995	7	6	0	1	8.35	0.9937	4	3	0	1	4.75	0.9620
	3.5	14	14	0	1	17.12	0.9999	7	6	0	1	8.35	0.9988	4	3	0	1	4.75	0.9879
	4.0	14	14	0	1	17.12	0.9999	7	6	0	1	8.35	0.9998	4	3	0	1	4.75	0.9963
0.05	1.5	48	29	1	7	61.23	0.9630	37	25	5	13	55.29	0.9586	35	25	10	23	59.27	0.9534
	2.0	21	18	0	2	26.09	0.9821	13	10	0	3	16.64	0.9572	15	6	3	6	16.71	0.9574
	2.5	18	15	0	1	20.10	0.9927	9	6	0	1	9.81	0.9548	8	5	0	3	9.79	0.9753
	3.0	18	15	0	1	20.10	0.9993	9	6	0	1	9.81	0.9911	6	5	0	2	7.64	0.9859
	3.5	18	15	0	1	20.10	0.9999	9	6	0	1	9.81	0.9983	5	4	0	1	5.62	0.9805
	4.0	18	15	0	1	20.10	0.9999	9	6	0	1	9.81	0.9997	5	4	0	1	5.62	0.9939
0.01	1.5	65	45	2	9	81.13	0.9532	50	40	4	18	81.80	0.9611	48	39	10	32	86.73	0.9506
	2.0	28	26	0	2	31.44	0.9592	18	15	0	4	22.19	0.9577	20	10	3	8	22.50	0.9539
	2.5	26	25	0	1	27.20	0.9840	14	11	0	2	15.39	0.9782	11	8	0	4	13.19	0.9759
	3.0	26	25	0	1	27.20	0.9985	13	8	0	1	13.34	0.9828	8	6	0	2	8.84	0.9719
	3.5	26	25	0	1	27.20	0.9998	13	8	0	1	13.34	0.9967	7	5	0	1	7.27	0.9654
	4.0	26	25	0	1	27.20	0.9999	13	8	0	1	13.34	0.9994	7	5	0	1	7.27	0.9890

0.05, 0.01. Producer's risk α is assigned a value of 0.05. Further, the quality level r_1 at the consumer's risk is taken as 1, while, the quality level r_2 at the producer's risk are considered as $(\frac{m}{m_0} = 1.5, 2.0, 2.5, 3.0, 3.5, 4.0)$. The design parameters (n_1, n_2, c_1, c_2) and the ASN along with the lot acceptance probability (p_α) at the producer's risk are reported in Table 1 and Table 2. The tabulated values indicate that with the decrease in consumer's risk β, the sample size and acceptance number tend to increase. It is also observed that an increase in the value of shape parameter γ or r_2 leads to the smaller sample size. Further, we report a comparison between double and single acceptance sampling plans in Table 3. It is to be noticed that a sampling plan with minimum sample size would be more economical from practical context. From Table 3 it is observed that sample sizes obtained using double acceptance sampling plans are smaller than those obtained using single acceptance sampling plans. This holds true for all the cases reported in the table except possibly the case when no failure is allowed, that is when c = 0. Such a plan is called zero acceptance sampling plan.

Further, we considered two choices of r given as r = 3, 5 and obtain the design parameters (g_1, g_2, c_1, c_2) under two-stage group acceptance sampling plan. The design parameters along with ASN and p_α are presented in Table 4 and Table 5.

It is observed that if an experimenter desires to minimize the total number of units then in that case a different group size may be suggested. For an example, with $\gamma = 1$, $\beta = 0.25$, $\alpha = 0.7$, $r_2 = 1.5$, we observed that for r = 3, average sample size of 50.25 units are required to make a decision about the proposed lot. However, under the same conditions when r is fixed as 5, then an average sample of size 55.66 units are required to make a decision. Therefore, in this case a group of size 3 would be preferred but a group of size 5 would be preferred if r_2 = 2. A similar behaviour is also observed in case of group acceptance sampling plans.

In Table 6, a comparison of two-stage and group acceptance sampling plans is made on the basis of sample size. Tabulated values suggest that the two-stage group acceptance sampling plans require reasonably smaller sample size than the group acceptance sampling plans. Further illustration of the proposed plans is presented in the following example.

Example 1: Suppose that a producer submits a lot of units and makes a claim that the specified lifetime of a unit is 1,000 hours. Further assume that lifetime of a unit follows a GIE (2, λ) distribution. It is known that the consumer's

Table 3. Comparison between double and single acceptance sampling plans

		$\gamma=1$						$\gamma=2$					
		$a=0.5$		$a=0.7$		$a=1.0$		$a=0.5$		$a=0.7$		$a=1.0$	
β	r_2	ASN	$n(c)$	ASN	$n(c)$	ASN	$n(c)$	ASN	$n(c)$	ASN	$n(c)$	ASN	$n(c)$
0.25	1.5	46.04	51(10)	51.48	54(17)	60.54	65(29)	31.54	38(4)	25.84	30(7)	29.10	31(13)
	2.0	16.41	20(3)	16.40	19(5)	18.22	23(9)	12.61	16(1)	9.66	12(2)	9.96	12(4)
	2.5	8.13	10(1)	8.13	10(2)	9.96	12(4)	12.61	8(0)	6.38	8(1)	5.87	7(2)
	3.0	8.13	10(1)	5.10	7(1)	5.87	7(2)	12.61	8(0)	6.38	4(0)	3.75	5(1)
	3.5	8.13	5(0)	5.10	7(1)	5.87	7(2)	12.61	8(0)	6.38	4(0)	3.75	5(1)
	4.0	8.13	5(0)	5.10	7(1)	3.75	5(1)	12.61	8(0)	6.38	4(0)	3.75	3(0)
0.10	1.5	77.70	82(15)	83.13	84(25)	97.11	100(43)	50.99	62(6)	40.50	46(10)	47.18	48(19)
	2.0	24.07	30(4)	24.74	30(7)	29.83	33(12)	17.12	23(1)	15.24	20(3)	13.80	17(5)
	2.5	15.12	20(2)	12.93	16(3)	15.72	19(6)	17.12	23(1)	8.35	11(1)	6.87	9(2)
	3.0	10.80	15(1)	9.76	13(2)	9.31	14(4)	17.12	13(0)	8.35	11(1)	4.75	7(1)
	3.5	10.80	15(1)	6.87	9(1)	6.87	9(2)	17.12	13(0)	8.35	7(0)	4.75	7(1)
	4.0	10.80	9(0)	6.87	9(1)	6.87	9(2)	17.12	13(0)	8.35	7(0)	4.75	7(1)
0.05	1.5	102.96	107(19)	106.51	107(31)	125.28	126(53)	61.23	77(7)	55.29	62(13)	59.27	60(23)
	2.0	30.56	40(5)	28.95	39(9)	39.15	42(15)	26.09	37(2)	16.64	22(3)	16.71	21(6)
	2.5	16.63	23(2)	16.85	22(4)	18.73	23(7)	20.10	27(1)	9.81	14(1)	9.79	13(3)
	3.0	12.70	18(1)	10.64	15(2)	12.19	16(4)	20.10	17(0)	9.81	14(1)	7.64	11(2)
	3.5	12.70	18(1)	7.96	11(1)	9.79	13(3)	20.10	17(0)	9.81	8(0)	5.62	8(1)
	4.0	12.70	11(0)	7.96	11(1)	7.64	11(2)	20.10	17(0)	9.81	8(0)	5.62	8(1)
0.01	1.5	149.59	155(26)	160.81	161(45)	177.98	178(73)	81.13	110(9)	81.80	90(18)	86.73	87(32)
	2.0	38.16	60(7)	44.70	56(12)	55.25	59(20)	31.44	48(2)	22.19	33(4)	22.50	30(8)
	2.5	23.99	37(3)	21.72	31(5)	24.97	35(10)	27.20	38(1)	15.39	24(2)	13.19	19(4)
	3.0	19.94	31(2)	15.46	24(3)	15.59	22(5)	27.20	26(0)	13.34	18(1)	8.84	14(2)
	3.5	17.42	24(1)	12.55	20(2)	13.19	19(4)	27.20	26(0)	13.34	18(1)	7.27	11(1)
	4.0	17.42	24(1)	11.19	15(1)	11.01	17(3)	27.20	26(0)	13.34	13(0)	7.27	11(1)

Table 4. Two-stage group acceptance sampling plan for $\gamma = 1$

			$a = 0.5$						$a = 0.7$						$a = 1.0$					
r	β	r_2	g_1	g_2	c_1	c_2	ASN	p_α	g_1	g_2	c_1	c_2	ASN	p_α	g_1	g_2	c_1	c_2	ASN	p_α
3	0.25	1.5	13	4	6	10	45.10	0.9542	13	5	10	17	50.25	0.9540	11	10	12	28	60.55	0.9507
		2.0	4	3	1	3	16.41	0.9675	6	2	4	6	19.98	0.9675	5	2	5	8	18.27	0.9511
		2.5	2	2	0	1	8.13	0.9587	3	2	1	3	11.74	0.9718	3	1	2	4	10.23	0.9559
		3.0	2	2	0	1	8.13	0.9886	2	2	0	2	9.24	0.9808	2	2	1	3	9.28	0.9597
		3.5	2	2	0	1	8.13	0.9970	2	1	0	1	6.65	0.9719	2	1	1	2	6.70	0.9691
		4.0	2	2	0	1	8.13	0.9992	2	1	0	1	6.65	0.9890	1	1	0	1	4.12	0.9597
	0.10	1.5	18	10	8	15	74.63	0.9501	17	11	11	25	82.56	0.9513	17	17	16	44	101.72	0.9584
		2.0	6	4	0	4	24.15	0.9634	7	3	3	7	24.89	0.9548	9	2	9	12	28.73	0.9560
		2.5	4	3	0	2	15.23	0.9748	4	3	1	4	16.40	0.9744	5	2	4	6	16.46	0.9512
		3.0	3	3	0	1	11.02	0.9754	3	2	0	2	10.64	0.9629	3	2	1	4	11.88	0.9701
		3.5	3	3	0	1	11.02	0.9933	2	2	0	1	7.31	0.9587	2	1	0	2	6.98	0.9617
		4.0	3	3	0	1	11.02	0.9982	2	2	0	1	7.31	0.9835	2	1	0	2	6.98	0.9847
	0.05	1.5	22	14	8	19	99.47	0.9549	22	15	16	32	109.17	0.9523	21	21	22	53	125.28	0.9525
		2.0	9	5	2	5	31.17	0.9586	9	4	4	9	31.95	0.9652	10	4	8	15	36.77	0.9588
		2.5	5	3	0	2	17.00	0.9635	5	3	1	4	17.52	0.9553	5	3	3	7	19.34	0.9549
		3.0	4	3	0	1	13.14	0.9646	3	2	0	2	10.64	0.9629	4	2	1	5	14.30	0.9814
		3.5	4	3	0	1	13.14	0.9903	3	2	0	2	10.64	0.9899	3	2	1	3	10.40	0.9669
		4.0	4	3	0	1	13.14	0.9974	3	1	0	1	9.24	0.9798	2	2	0	2	7.96	0.9677
	0.01	1.5	30	22	11	26	144.92	0.9515	30	24	21	45	161.39	0.9508	31	30	32	75	182.84	0.9522
		2.0	11	7	1	6	38.16	0.9502	13	6	6	12	43.59	0.9573	13	8	11	21	56.61	0.9533
		2.5	7	6	0	3	24.40	0.9681	7	5	2	6	25.17	0.9736	8	4	5	10	27.20	0.9604
		3.0	6	5	0	2	19.94	0.9864	5	3	0	3	16.18	0.9680	5	4	2	6	18.59	0.9598
		3.5	6	3	0	1	18.30	0.9830	4	3	0	2	13.03	0.9748	4	3	1	4	13.71	0.9691
		4.0	6	3	0	1	18.30	0.9954	4	2	0	1	12.16	0.9587	3	3	0	3	11.26	0.9778
5	0.25	1.5	9	2	6	11	50.01	0.9633	9	3	12	19	55.66	0.9631	9	4	18	29	62.31	0.9559
		2.0	3	1	1	3	16.90	0.9678	3	1	3	5	16.80	0.9570	4	1	7	10	22.28	0.9722
		2.5	2	1	0	2	12.34	0.9897	1	1	0	2	8.16	0.9578	2	1	3	5	12.25	0.9726
		3.0	2	1	0	1	10.93	0.9796	1	1	0	2	8.16	0.9886	1	1	0	3	8.90	0.9730
		3.5	2	1	0	1	10.93	0.9945	1	1	0	1	6.44	0.9706	1	1	1	2	6.56	0.9720
		4.0	2	1	0	1	10.93	0.9985	1	1	0	1	6.44	0.9884	1	1	1	2	6.56	0.9889
	0.10	1.5	12	6	9	16	79.03	0.9522	12	6	14	27	87.00	0.9605	11	9	19	43	99.32	0.9550
		2.0	4	2	0	4	24.11	0.9633	4	2	3	7	24.91	0.9556	6	1	10	13	31.21	0.9652
		2.5	3	1	0	2	16.11	0.9768	3	1	2	4	16.22	0.9789	3	1	4	6	16.22	0.9583
		3.0	2	2	0	1	11.87	0.9701	2	1	1	2	10.75	0.9697	2	1	2	4	11.61	0.9732
		3.5	2	2	0	1	11.87	0.9918	2	1	1	2	10.75	0.9918	1	1	0	2	7.34	0.9521
		4.0	2	2	0	1	11.87	0.9978	2	1	0	1	10.28	0.9705	1	1	0	2	7.34	0.9805
	0.05	1.5	15	7	11	19	94.78	0.9503	14	8	16	32	107.41	0.9556	13	13	22	55	129.59	0.9591
		2.0	5	3	1	5	30.56	0.9639	6	2	5	9	32.57	0.9597	7	2	12	15	37.05	0.9506
		2.5	3	2	0	2	17.22	0.9598	4	1	3	4	20.27	0.9565	4	2	5	9	23.91	0.9748
		3.0	3	1	0	1	15.33	0.9633	2	1	0	2	11.04	0.9621	3	1	3	5	15.66	0.9720
		3.5	3	1	0	1	15.33	0.9899	2	1	0	2	11.04	0.9897	2	1	1	3	10.80	0.9647
		4.0	3	1	0	1	15.33	0.9973	2	1	0	1	10.28	0.9705	2	1	1	2	10.21	0.9500
	0.01	1.5	19	12	11	26	139.71	0.9539	19	14	22	46	164.15	0.9543	19	18	32	76	184.91	0.9552
		2.0	7	5	1	7	43.04	0.9675	8	4	6	13	46.61	0.9691	9	4	13	22	54.93	0.9619
		2.5	4	4	0	3	24.43	0.9659	4	3	1	6	25.08	0.9757	5	2	5	10	27.10	0.9662
		3.0	4	3	0	2	21.32	0.9838	3	2	0	3	16.31	0.9636	3	2	2	6	17.99	0.9717
		3.5	4	1	0	1	20.10	0.9842	3	1	0	2	15.21	0.9768	2	2	0	4	13.75	0.9732
		4.0	4	1	0	1	20.10	0.9958	3	1	0	2	15.21	0.9938	2	2	0	3	11.70	0.9682

risk is 25%, when the true median life of a unit is 1,000 hours and the producer's risk is 5%, when the true median life of a unit is 1,500 hours. We are interested in finding the design parameters of a double acceptance sampling plan for the case where an experimenter would like to run the experiment for 700 hours. Accordingly, we have $\gamma = 2$, $m_0 = 1000$ hours, $\beta = 0.25$, $\alpha = 0.05$, $r_1 = 1$, $r_2 = 1.5$ and $a = 0.7$. Then design parameters from Table 2 is obtained as

Table 5. Two-stage group acceptance sampling plan for $\gamma = 2$

			$a = 0.5$						$a = 0.7$						$a = 1.0$					
r	β	r_2	g_1	g_2	c_1	c_2	ASN	p_α	g_1	g_2	c_1	c_2	ASN	p_α	g_1	g_2	c_1	c_2	ASN	p_α
3	0.25	1.5	9	4	2	4	31.57	0.9628	7	3	3	7	26.41	0.9535	9	2	11	13	28.67	0.9522
		2.0	4	2	0	1	13.64	0.9744	2	2	0	2	9.66	0.9722	3	1	2	4	10.23	0.9669
		2.5	4	2	0	1	13.64	0.9975	2	1	0	1	6.85	0.9819	2	1	1	2	6.70	0.9669
		3.0	4	2	0	1	13.64	0.9997	2	1	0	1	6.85	0.9966	1	1	0	1	4.12	0.9738
		3.5	4	2	0	1	13.64	0.9999	2	1	0	1	6.85	0.9993	1	1	0	1	4.12	0.9918
		4.0	4	2	0	1	13.64	0.9999	2	1	0	1	6.85	0.9998	1	1	0	1	4.12	0.9975
	0.10	1.5	14	7	2	6	51.03	0.9648	12	5	6	11	43.36	0.9587	11	5	10	19	45.51	0.9566
		2.0	5	4	0	1	17.39	0.9510	4	3	0	3	15.89	0.9682	4	2	3	5	13.88	0.9528
		2.5	5	4	0	1	17.39	0.9950	3	1	0	1	9.40	0.9671	2	1	0	2	6.98	0.9591
		3.0	5	4	0	1	17.39	0.9995	3	1	0	1	9.40	0.9937	2	1	0	2	6.98	0.9919
		3.5	5	4	0	1	17.39	0.9999	3	1	0	1	9.40	0.9988	2	1	0	1	6.28	0.9786
		4.0	5	4	0	1	17.39	0.9999	3	1	0	1	9.40	0.9997	2	1	0	1	6.28	0.9933
	0.05	1.5	17	9	2	7	61.23	0.9609	14	7	7	13	52.80	0.9560	13	7	12	23	57.65	0.9537
		2.0	7	6	0	2	26.09	0.9821	5	3	1	3	17.02	0.9555	5	2	3	6	16.71	0.9574
		2.5	6	5	0	1	20.10	0.9927	3	2	0	1	9.81	0.9548	3	2	1	3	10.40	0.9640
		3.0	6	5	0	1	20.10	0.9993	3	2	0	1	9.81	0.9911	2	2	0	2	7.96	0.9824
		3.5	6	5	0	1	20.10	0.9999	3	2	0	1	9.81	0.9983	2	1	0	1	6.28	0.9786
		4.0	6	5	0	1	20.10	0.9999	3	2	0	1	9.81	0.9997	2	1	0	1	6.28	0.9933
	0.01	1.5	23	14	3	9	80.84	0.9515	19	11	6	18	75.50	0.9611	17	12	14	32	86.08	0.9510
		2.0	10	7	0	2	32.19	0.9632	6	5	0	4	22.19	0.9577	7	3	4	8	22.69	0.9553
		2.5	9	6	0	1	27.75	0.9861	5	3	0	2	15.88	0.9799	4	3	1	4	13.71	0.9660
		3.0	9	6	0	1	27.75	0.9987	5	2	0	1	15.13	0.9816	3	2	0	2	9.52	0.9658
		3.5	9	6	0	1	27.75	0.9998	5	2	0	1	15.13	0.9965	3	1	0	1	9.05	0.9614
		4.0	9	6	0	1	27.75	0.9999	5	2	0	1	15.13	0.9993	3	1	0	1	9.05	0.9876
5	0.25	1.5	4	4	1	4	32.83	0.9600	4	2	3	7	26.38	0.9544	5	2	10	14	30.75	0.9591
		2.0	2	2	0	1	13.26	0.9734	2	1	1	2	11.07	0.9567	2	1	3	4	11.02	0.9546
		2.5	2	2	0	1	13.26	0.9974	1	1	0	1	6.72	0.9809	1	1	1	2	6.56	0.9701
		3.0	2	2	0	1	13.26	0.9997	1	1	0	1	6.72	0.9964	1	1	1	2	6.56	0.9941
		3.5	2	2	0	1	13.26	0.9999	1	1	0	1	6.72	0.9993	1	1	0	1	5.78	0.9776
		4.0	2	2	0	1	13.26	0.9999	1	1	0	1	6.72	0.9998	1	1	0	1	5.78	0.9929
	0.10	1.5	9	4	3	6	51.54	0.9620	8	2	7	11	43.20	0.9631	8	3	15	21	49.07	0.9579
		2.0	4	3	0	2	24.67	0.9863	2	2	0	3	15.83	0.9736	3	1	4	6	16.22	0.9709
		2.5	3	3	0	1	17.99	0.9943	2	1	0	1	10.51	0.9525	2	1	2	3	10.58	0.9735
		3.0	3	3	0	1	17.99	0.9994	2	1	0	1	10.51	0.9900	1	1	0	2	7.34	0.9896
		3.5	3	3	0	1	17.99	0.9999	2	1	0	1	10.51	0.9983	1	1	0	1	5.78	0.9776
		4.0	3	3	0	1	17.99	0.9999	2	1	0	1	10.51	0.9996	1	1	0	1	5.78	0.9929
	0.05	1.5	10	6	2	7	62.04	0.9560	8	5	7	13	54.65	0.9503	9	3	14	23	54.13	0.9529
		2.0	4	4	0	2	26.23	0.9814	3	2	1	3	17.25	0.9507	3	2	3	7	19.82	0.9626
		2.5	4	2	0	1	21.08	0.9933	2	1	0	1	10.51	0.9525	2	1	1	3	10.80	0.9616
		3.0	4	2	0	1	21.08	0.9993	2	1	0	1	10.51	0.9906	2	1	1	2	10.21	0.9721
		3.5	4	2	0	1	21.08	0.9999	2	1	0	1	10.51	0.9983	1	1	0	1	5.78	0.9776
		4.0	4	2	0	1	21.08	0.9999	2	1	0	1	10.51	0.9996	1	1	0	1	5.78	0.9929
	0.01	1.5	14	8	2	9	80.67	0.9529	12	6	7	18	73.64	0.9611	11	7	17	33	88.05	0.9528
		2.0	6	5	0	2	32.60	0.9565	4	3	1	4	22.81	0.9509	4	2	3	8	22.50	0.9539
		2.5	6	2	0	1	30.27	0.9877	3	2	0	2	15.98	0.9778	2	2	0	4	13.75	0.9704
		3.0	6	2	0	1	30.27	0.9988	3	1	0	1	15.11	0.9829	2	1	0	2	10.26	0.9651
		3.5	6	2	0	1	30.27	0.9999	3	1	0	1	15.11	0.9968	2	1	0	2	10.26	0.9931
		4.0	6	2	0	1	30.27	0.9999	3	1	0	1	15.11	0.9994	2	1	0	1	10.04	0.9819

$(n_1, n_2, c_1, c_2) = (22, 8, 4, 7)$ and ASN = 25.84. The implication following this sampling plan can be explained as follows. A random sample of 22 units will be drawn from the submitted lot and will be placed on a life test for 700 hours. During the experiment, if at most 4 failures are observed then consumer will accept that lot and if more than 7 failures are observed then that lot will be rejected. Further, if observed failures are between 5 and 7 then a

Table 6. Comparison between two-stage and group acceptance sampling plans

			$\gamma=1$						$\gamma=2$					
			$a=0.5$		$a=0.7$		$a=1.0$		$a=0.5$		$a=0.7$		$a=1.0$	
r	β	r_2	ASN	$n(c)$	ASN	$n(c)$	ASN	$n(c)$	ASN	$n(c)$	ASN	$n(c)$	ASN	$n(c)$
3	0.25	1.5	45.10	51(10)	50.25	54(17)	60.55	69(31)	31.57	39(4)	26.41	30(7)	28.67	33(14)
		2.0	16.41	21(3)	19.98	24(6)	18.27	27(11)	13.64	18(1)	9.66	12(2)	10.23	12(4)
		2.5	8.13	15(2)	11.74	15(3)	10.23	12(4)	13.64	9(0)	6.85	9(1)	6.70	9(2)
		3.0	8.13	12(1)	9.24	12(2)	9.28	12(4)	13.64	9(0)	6.85	9(1)	4.12	6(1)
		3.5	8.13	6(0)	6.65	9(1)	6.70	9(2)	13.64	9(0)	6.85	6(0)	4.12	6(1)
		4.0	8.13	6(0)	6.65	9(1)	4.12	6(1)	13.64	9(0)	6.85	6(0)	4.12	3(0)
	0.10	1.5	74.63	87(16)	82.56	84(25)	101.72	102(44)	51.03	63(6)	43.36	51(11)	45.51	48(19)
		2.0	24.15	30(4)	24.89	30(7)	28.73	33(12)	17.39	24(1)	15.89	21(3)	13.88	21(6)
		2.5	15.23	21(2)	16.40	21(4)	16.46	21(7)	17.39	24(1)	9.40	12(1)	6.98	9(2)
		3.0	11.02	15(1)	10.64	15(2)	11.88	15(4)	17.39	15(0)	9.40	12(1)	6.98	9(2)
		3.5	11.02	15(1)	7.31	9(1)	6.98	9(2)	17.39	15(0)	9.40	9(0)	6.28	9(1)
		4.0	11.02	9(0)	7.31	9(1)	6.98	9(2	17.39	15(0)	9.40	9(0)	6.28	9(1)
	0.05	1.5	99.47	108(19)	109.17	114(33)	125.28	126(53)	61.23	78(7)	52.80	63(13)	57.65	60(23)
		2.0	31.17	42(5)	31.95	39(9)	36.77	42(15)	26.09	39(2)	17.02	24(3)	16.71	21(6)
		2.5	17.00	24(2)	17.52	24(4)	19.34	24(7)	20.10	27(1)	9.81	18(2)	10.40	15(3)
		3.0	13.14	18(1)	10.64	15(2)	14.30	18(5)	20.10	18(0)	9.81	15(1)	7.96	12(2)
		3.5	13.14	18(1)	10.64	15(2)	10.40	15(3)	20.10	18(0)	9.81	9(0)	6.28	9(1)
		4.0	13.14	12(0)	9.24	12(1)	7.96	12(2)	20.10	18(0)	9.81	9(0)	6.28	9(1)
	0.01	1.5	144.92	156(26)	161.39	165(46)	182.84	183(75)	80.84	111(9)	75.50	90(18)	86.08	87(32)
		2.0	38.16	60(7)	43.59	57(12)	56.61	66(22)	32.19	48(2)	22.19	33(4)	22.69	30(8)
		2.5	24.40	39(3)	25.17	36(6)	27.20	36(10)	27.75	39(1)	15.88	24(2)	13.71	21(4)
		3.0	19.94	33(2)	16.18	24(3)	18.59	27(6)	27.75	27(0)	15.13	18(1)	9.52	15(2)
		3.5	18.30	24(1)	13.03	21(2)	13.71	21(4)	27.75	27(0)	15.13	18(1)	9.05	12(1)
		4.0	18.30	24(1)	12.16	15(1)	11.26	18(3)	27.75	27(0)	15.13	15(0)	9.05	12(1)
5	0.25	1.5	50.01	55(11)	55.66	60(19)	62.31	65(29)	32.83	40(4)	26.38	30(7)	30.75	35(15)
		2.0	16.90	20(3)	16.80	20(5)	22.28	25(10)	13.26	20(1)	11.07	20(3)	11.02	15(5)
		2.5	12.34	10(1)	8.16	10(2)	12.25	15(5)	13.26	10(0)	6.72	10(1)	6.56	10(3)
		3.0	10.93	10(1)	8.16	10(2)	8.90	10(3)	13.26	10(0)	6.72	10(1)	6.56	5(1)
		3.5	10.93	5(0)	6.44	10(1)	6.56	10(3)	13.26	10(0)	6.72	5(0)	5.78	5(1)
		4.0	10.93	5(0)	6.44	10(1)	6.56	5(1)	13.26	10(0)	6.72	5(0)	5.78	5(1)
	0.10	1.5	79.03	95(17)	87.00	90(27)	99.32	100(43)	51.54	65(6)	43.20	50(11)	49.07	55(22)
		2.0	24.11	30(4)	24.91	30(7)	31.21	35(13)	24.67	35(2)	15.83	20(3)	16.22	20(6)
		2.5	16.11	20(2)	16.22	20(4)	16.22	20(6)	17.99	25(1)	10.51	20(2)	10.58	15(3)
		3.0	11.87	15(1)	10.75	15(2)	11.61	15(4)	17.99	15(0)	10.51	15(1)	7.34	10(2)
		3.5	11.87	15(1)	10.75	10(1)	7.34	15(3)	17.99	15(0)	10.51	10(0)	5.78	10(1)
		4.0	11.87	10(0)	10.28	10(1)	7.34	10(2)	17.99	15(0)	10.51	10(0)	5.78	10(1)
	0.05	1.5	94.78	115(20)	107.41	110(32)	129.59	130(55)	62.04	80(7)	54.65	65(14)	54.13	60(23)
		2.0	30.56	40(5)	32.57	40(9)	37.05	45(16)	26.23	40(2)	17.25	30(4)	19.82	25(7)
		2.5	17.22	25(2)	20.27	30(5)	23.91	30(9)	21.08	30(1)	10.51	20(2)	10.80	15(3)
		3.0	15.33	20(1)	11.04	15(2)	15.66	20(5)	21.08	20(0)	10.51	15(1)	10.21	15(2)
		3.5	15.33	20(1)	11.04	15(2)	10.80	15(3)	21.08	20(0)	10.51	10(0)	5.78	10(1)
		4.0	15.33	20(1)	10.28	15(1)	10.21	15(3)	21.08	20(0)	10.51	10(0)	5.78	10(1)
	0.01	1.5	139.71	155(26)	164.15	165(46)	184.91	185(76)	80.67	110(9)	73.64	90(18)	88.05	90(33)
		2.0	43.04	60(7)	46.61	60(13)	54.93	65(22)	32.60	50(2)	22.81	40(5)	22.50	30(8)
		2.5	24.43	40(3)	25.08	35(6)	27.10	35(10)	30.27	40(1)	15.98	25(2)	13.75	20(4)
		3.0	21.32	35(2)	16.31	25(3)	17.99	25(6)	30.27	30(0)	15.11	20(1)	10.26	15(2)
		3.5	20.10	25(1)	15.21	20(2)	13.75	20(4)	30.27	30(0)	15.11	20(1)	10.26	15(2)
		4.0	20.10	25(1)	15.21	15(1)	11.70	20(3)	30.27	30(0)	15.11	15(0)	10.04	15(1)

second random sample of size 8 units will be drawn from the lot and will again be subjected to the test for 700 hours. Finally, the lot will be accepted if at most 7 failures are observed from both the samples, otherwise, the lot will be rejected. In this case, ASN required to make a decision about the submitted lot is 25.84. Next, suppose that an experimenter wishes to implement a single acceptance sampling plan under the same specifications. Then from Table 3, a sample of size 30 units are required and a maximum of 7 failures can be tolerated to accept the lot. It can be observed from the Table 3 that ASN using the double acceptance sampling plan is 25.84 which is less than the sample size 30 obtained using the single acceptance sampling plan. Hence, in this case a double acceptance sampling plan is suggested.

Further, if an experimenter wishes to implement a two-stage group acceptance sampling plan by considering 3 units in each group with other specifications being the same. Then from Table 5, we conclude that design parameters are $(g_1, g_2, c_1, c_2) = (7, 3, 3, 7)$ and ASN is 26.41. This suggests that a random sample of 21 units should be drawn and 3 units will be allocated to each 7 groups. If during an experiment total number of failures exceed 7 then that lot will be rejected, however, if the total number of failures are at most 3 then that lot will be accepted. If the number of observed failures is between 4 and 7, then a second sample of size 9 units should be drawn and 3 groups will be formed by allocating 3 units to each group. During next 700 hours, if total number of failures from the two samples exceeds 7 then the lot will be rejected, otherwise, it will be accepted. In this case ASN required to make a decision is 26.41. In addition, the group acceptance sampling plan requires a sample size of 30 units with acceptance number 7. As a consequence, the two-stage group acceptance sampling plan is suggested over the group acceptance sampling plan. Furthermore in this case double acceptance sampling plan provide the minimum sample size among all the proposed sampling plans.

4. IMPLEMENTATION OF SAMPLING PLANS

In this section, we analyze a real data set to illustrate the implementation of sampling plans in practical situations. This data set is given in Lawless (2003) and represents the number of millions revolutions to failure for 23 ball bearings. The corresponding 23 failure times are

17.88, 28.92, 33.0, 41.52, 42.12, 45.60,
48.40, 51.84, 51.96, 54.12, 55.56, 67.80,
68.64, 68.64, 68.88, 84.12, 93.12, 98.64,
105.12, 105.84, 127.92, 128.04, 173.40

Krishna and Kumar (2013) have shown that GIE distribution fits the data set reasonably well compared to some other well-known distributions. For the sake of completeness, we also fit the data through various criteria using different models. For comparison purpose, apart from GIE distribution, we take into consideration generalized exponential (GE) distribution, Weibull (W) distribution, inverted exponential (IE) distribution and exponential (E) distribution. We applied different goodness-of-fit tests which include chi-squared test statistic (χ^2), log-likelihood criterion (log-L), Akaike's information criterion (AIC), Bayesian information criterion (BIC) and Kolmogorov-Smirnov (K-S) test statistic. The observed and the expected frequencies are reported in Table 7 for different models and the corresponding values of the goodness-of-fit tests are presented in Table 8. Based on these criteria, it is concluded that GIE distribution fits the data set reasonably good. One may also refer to Abouammoh and

Table 7. The observed and expected frequencies

Intervals	Observed frequencies	Expected frequencies				
		GIE	GE	W	IE	E
0-35	3	2.82	2.93	3.55	4.77	8.83
35-55	7	6.56	5.70	4.54	3.68	3.43
55-80	5	6.43	6.55	6.04	3.10	3.14
80-100	3	2.93	3.39	3.85	1.70	1.84
100-	5	4.25	4.41	5.02	9.74	5.76

Table 8. Goodness-of-fit tests for different distributions

Distribution	PDF	$\hat{\lambda}$	$\hat{\gamma}$	log-L	AIC	BIC	K-S	χ^2
GIE	$\gamma\lambda t^{-2}e^{-\frac{\lambda}{t}}\left(1-e^{-\frac{\lambda}{t}}\right)^{\gamma-1}$	129.996	5.3076	113.549	231.098	233.369	0.0916	0.49
GE	$\frac{\gamma}{\lambda}e^{-\frac{t}{\lambda}}\left(1-e^{-\frac{t}{\lambda}}\right)^{\gamma-1}$	0.0323	5.2783	112.9778	229.9557	232.2267	0.1056	0.79
W	$\gamma\lambda(\lambda t)^{\gamma-1}e^{-(\lambda t)^{\gamma}}$	81.8745	2.1018	113.692	231.3839	233.6549	0.1510	1.79
IE	$\lambda t^{-2}e^{-\frac{\lambda}{t}}$	55.0550		121.7259	245.4519	246.5874	0.3060	8.09
E	$\frac{1}{\lambda}e^{-\frac{t}{\lambda}}$	0.0138		121.4338	244.8675	246.003	0.3068	9.51

Alshingiti (2009), Dey and Kundu (2009) and Dey and Pradhan (2013) for various other applications of this data set.

Now suppose a producer proposes a lot of ball bearing and claims that the specified median life of the ball bearings is 70 million revolutions. Further consider that consumer's risk is 25% and producer's risk is 5% where r_2 is given by 1.5, 2.0, 2.5 and 3.0. Now assume that an experimenter would like to run the experiment for 49 million revolutions and is interested to implement a sampling plan for the proposed lot. Since the shape parameter is unknown and so the experimenter can consider the given data as a historical data set for which corresponding inferences may be available. These statistical inferences about the given data set can be used to implement a sampling plan for the proposed lot. So for the proposed lot, value of γ can be selected close to the estimated value of the shape parameter of the historical data. In consequence, we take the shape parameter as $\gamma = 5.3$ and then report the corresponding sampling plans in Table 9.

These plans may be used to make adequate inference about the proposed lot. Furthermore, if an experimenter is interested to choose a plan having minimum sample size then double acceptance sampling plans may be suggested among all the proposed sampling plans. It is to be noticed that shape parameter γ for the proposed lot is unknown and we considered the shape parameter from the historical data set. So, it would be further interesting to see the effect of mis-specification on the lot acceptance probabilities for producer's risk p_α and consumer's risk p_β. Therefore let us assume that the true value of the shape parameter of the proposed lot is γ_0 and so p can be re-written as $p_0 = 1 - \left[1 - \left(1 - 0.5^{\frac{1}{\gamma}}\right)^{\frac{m}{am_0}}\right]^{\gamma_0}$. Now if design parameters of sampling plans, obtained using γ, still satisfies the inequalities (6) and (7) under p_0 then the choice of the shape parameter γ would be considered as reasonably good. Next we obtain the acceptance probabilities using the sampling plans as discussed in Table 9 for different arbitrary choices of γ_0 like 4.3, 4.8, 5.8, 6.3. These probabilities are reported in Table 10.

Observe that up to $\gamma_0 = 5.8$, p_β still satisfies the consumer's risk except for the cases r = 1, 3 where r_2 = 1.5. Similarly p_α satisfies the producer's risk except for the case $\gamma_0 = 4.3$ when r = 1, 3, 5 and r_2 = 1.5. In other words, higher valued shape parameters may satisfy producer's risk while smaller values may satisfy consumer's risk.

Recall that, in Table 8 we observed that the GIE distribution fits the data set reasonably well followed by GE, W, IE and E distributions. We now compare the behaviour of acceptance sampling plans obtained using these distributions. Notice that sampling plans for GE, W and E distributions have been discussed in literature before. The choices of shape parameters for GE and W distributions are taken exactly same as the corresponding estimated values for the given data set, whereas, for the IE and E distributions p turns out to be independent of the shape parameter. For the sake of convenience, we have

Table 9. Acceptance sampling plans for the ball bearing lot

		$r=1$							$r=3$							$r=5$						
β	r_2	n_1	n_2	c_1	c_2	ASN	p_α	$n(c)$	g_1	g_2	c_1	c_2	ASN	p_α	$n(c)$	g_1	g_2	c_1	c_2	ASN	p_α	$n(c)$
0.25	1.5	13	9	1	3	17.29	0.9700	21(3)	5	2	1	3	17.45	0.9713	21(3)	3	2	1	3	19.09	0.9541	30(4)
	2.0	7	5	0	1	8.63	0.9911	11(1)	3	1	0	1	9.73	0.9900	12(1)	2	1	0	1	11.03	0.9853	15(1)
	2.5	7	5	0	1	8.63	0.9995	6(0)	3	1	0	1	9.73	0.9994	6(0)	2	1	0	1	11.03	0.9991	10(0)
	3.0	7	5	0	1	8.63	0.9999	6(0)	3	1	0	1	9.73	0.9999	6(0)	2	1	0	1	11.03	0.9999	10(0)

Table 10. Acceptance probabilities under mis-specification of shape parameter γ for GIE distribution

r	r_2	$\gamma_0 = 4.3$		$\gamma_0 = 4.8$		$\gamma = 5.3$		$\gamma_0 = 5.8$		$\gamma_0 = 6.3$	
		p_β	p_α	p_β	p_α	p_β	p_α	p_β	p_α	p_β	p_α
1	1.5	0.1970	0.9436	0.2214	0.9592	0.2448	0.9700	0.2673	0.9775	0.2888	0.9829
	2.0	0.1998	0.9829	0.2174	0.9878	0.2341	0.9911	0.2499	0.9933	0.2649	0.9949
	2.5	0.1998	0.9987	0.2174	0.9992	0.2341	0.9995	0.2499	0.9996	0.2649	0.9997
	3.0	0.1998	0.9999	0.2174	0.9999	0.2341	0.9999	0.2499	0.9999	0.2649	0.9999
3	1.5	0.1928	0.9458	0.2178	0.9609	0.2418	0.9713	0.2649	0.9785	0.2869	0.9837
	2.0	0.1629	0.9809	0.1796	0.9864	0.1954	0.9900	0.2106	0.9925	0.2250	0.9943
	2.5	0.1629	0.9986	0.1796	0.9991	0.1954	0.9994	0.2106	0.9996	0.2250	0.9997
	3.0	0.1629	0.9999	0.1796	0.9999	0.1954	0.9999	0.2106	0.9999	0.2250	0.9999
5	1.5	0.1228	0.9163	0.1418	0.9385	0.1605	0.9541	0.1789	0.9653	0.1969	0.9733
	2.0	0.0956	0.9721	0.1078	0.9800	0.1198	0.9853	0.1314	0.9890	0.1428	0.9915
	2.5	0.0956	0.9979	0.1078	0.9987	0.1198	0.9992	0.1314	0.9994	0.1428	0.9996
	3.0	0.0956	0.9998	0.1078	0.9999	0.1198	0.9999	0.1314	0.9999	0.1428	0.9999

presented various sampling plans in Table 11, by taking the same values of β, α, r_2 and a as considered above. The bold scripts sample sizes indicate the cases where distributions other than GIE have smaller sample size. However, the corresponding acceptance probabilities at producer's risk p_α are smaller than that of GIE distribution. Therefore it is suggested that plans obtained using better models require smaller sample size. As a consequence, we conclude that plans obtained using GIE distribution have smaller sample size followed by GE, W, IE and E distributions.

Further it is also seen that the estimated shape parameter of the GE distribution is approximately same as that of GIE distribution (see Table 8) and in some cases, GE distribution provide smaller sample size as compared to the GIE distribution (see Table 11). With this view we further study the case of mis-specification of shape parameter of the GE distribution and then also compare the behaviour of corresponding acceptance probabilities with the acceptance probabilities of GIE distribution reported in Table 10. The choices of γ_0 are considered same as in case of GIE distribution. Based on the plans as reported in Table 11, the corresponding acceptance probabilities for GE distribution are reported in Table 12.

Tabulated values suggest that GE distribution is more sensitive to the mis-specification of shape parameters compared to the GIE distribution. As a consequence, the corresponding acceptance probabilities at the consumer' risk p_β are higher for GE distribution. It is also observed that associated acceptance probabilities at pro-

Table 11. Comparison between acceptance sampling plans of GIE distribution and other selected distributions

		$r=1$							$r=3$							$r=5$						
r_2		n_1	n_2	c_1	c_2	ASN	p_α	$n(c)$	g_1	g_2	c_1	c_2	ASN	p_α	$n(c)$	g_1	g_2	c_1	c_2	ASN	p_α	$n(c)$
1.5	GIE	13	9	1	3	17.30	0.9701	21(3)	5	2	1	3	17.46	0.9714	21(3)	3	2	1	3	19.10	0.9543	30(4)
	GE	17	7	1	4	20.66	0.9566	24(4)	6	2	1	4	20.87	0.9563	24(4)	3	2	1	4	21.05	0.9514	30(5)
	W	26	12	4	8	33.24	0.9514	42(9)	10	4	5	9	36.80	0.9624	42(9)	7	2	7	9	37.72	0.9557	50(11)
	IE	28	27	7	17	51.49	0.9506	54(17)	12	6	9	17	50.99	0.9543	54(17)	8	4	10	19	57.19	0.9630	60(19)
	E	54	47	12	35	100.59	0.9518	101(35)	20	15	18	36	99.96	0.9509	114(39)	13	8	20	36	99.90	0.9514	110(38)
2	GIE	7	5	0	1	8.63	0.9911	11(1)	3	1	0	1	9.73	0.9901	12(1)	2	1	0	1	11.03	0.9854	15(1)
	GE	6	6	0	1	**8.13**	0.9579	**10(1)**	2	2	0	1	**8.13**	0.9579	15(2)	2	1	0	2	12.34	0.9894	**10(1)**
	W	12	6	1	3	14.68	0.9639	18(3)	4	2	1	3	14.67	0.9639	18(3)	2	2	1	3	15.19	0.9592	25(4)
	IE	14	6	3	5	16.40	0.9607	19(5)	6	2	4	6	19.98	0.9675	24(6)	3	1	3	5	16.81	0.9570	20(5)
	E	24	14	5	12	36.02	0.9512	38(12)	10	4	8	13	37.74	0.9524	45(14)	7	2	10	14	39.94	0.9598	45(14)
2.5	GIE	7	5	0	1	8.63	0.9995	6(0)	3	1	0	1	9.73	0.9994	6(0)	2	1	0	1	11.03	0.9992	10(0)
	GE	6	6	0	1	**8.13**	0.9911	10(1)	2	2	0	1	**8.13**	0.9911	12(1)	2	1	0	1	**10.93**	0.9840	10(1)
	W	7	7	0	2	11.15	0.9771	14(2)	3	2	0	2	11.80	0.9709	15(2)	2	1	1	2	11.28	0.9763	15(2)
	IE	7	4	1	2	**8.14**	0.9555	10(2)	3	2	1	3	11.75	0.9718	15(3)	1	1	0	2	**8.16**	0.9578	10(2)
	E	17	7	3	7	21.40	0.9528	24(7)	6	2	3	7	21.44	0.9524	24(7)	5	1	7	8	25.67	0.9544	30(9)
3	GIE	7	5	0	1	8.63	0.9999	6(0)	3	1	0	1	9.73	0.9999	6(0)	2	1	0	1	11.03	0.9999	10(0)
	GE	6	6	0	1	**8.13**	0.9978	**5(0)**	2	2	0	1	**8.13**	0.9978	6(0)	2	1	0	1	**10.93**	0.9960	**5(0)**
	W	6	4	0	1	**7.30**	0.9659	9(1)	2	2	0	1	**7.95**	0.9569	9(1)	2	1	1	2	11.28	0.9913	10(1)
	IE	4	3	0	1	**5.11**	0.9597	7(1)	2	2	0	2	**9.25**	0.9808	12(2)	1	1	0	2	**8.16**	0.9886	10(2)
	E	14	5	3	5	15.92	0.9533	18(5)	4	2	0	5	16.21	0.9589	18(5)	4	1	5	6	20.70	0.9537	25(7)

Note: $n = g \times r$, bold scripts represents the smaller sample size as compare to GIE distribution.

Table 12. Acceptance probabilities under mis-specification of shape parameter γ for GE distribution

		$\gamma_0=4.3$		$\gamma_0=4.8$		$\gamma=5.3$		$\gamma_0=5.8$		$\gamma_0=6.3$	
r	r_2	p_β	p_α	p_β	p_α	p_β	p_α	p_β	p_α	p_β	p_α
1	1.5	0.1909	0.9083	0.2203	0.9375	0.2494	0.9573	0.2781	0.9708	0.3060	0.9801
	2.0	0.2054	0.9209	0.2236	0.9429	0.2411	0.9586	0.2581	0.9698	0.2744	0.9778
	2.5	0.2054	0.9777	0.2236	0.9862	0.2411	0.9913	0.2581	0.9945	0.2744	0.9965
	3.0	0.2054	0.993	0.2236	0.9962	0.2411	0.9979	0.2581	0.9988	0.2744	0.9993
3	1.5	0.1892	0.9078	0.2186	0.9371	0.2476	0.9571	0.2762	0.9706	0.3041	0.9799
	2.0	0.2054	0.9209	0.2236	0.9429	0.2411	0.9586	0.2581	0.9698	0.2744	0.9778
	2.5	0.2054	0.9777	0.2236	0.9862	0.2411	0.9913	0.2581	0.9945	0.2744	0.9965
	3.0	0.2054	0.9930	0.2236	0.9962	0.2411	0.9979	0.2581	0.9988	0.2744	0.9993
5	1.5	0.1740	0.8986	0.2018	0.9303	0.2295	0.9522	0.2568	0.9671	0.2838	0.9774
	2.0	0.1958	0.9727	0.2190	0.9833	0.2417	0.9897	0.2637	0.9936	0.2850	0.9960
	2.5	0.0778	0.9604	0.0892	0.9752	0.1007	0.9843	0.1123	0.9901	0.1238	0.9936
	3.0	0.0778	0.9872	0.0892	0.9930	0.1007	0.9961	0.1123	0.9978	0.1238	0.9988

ducer' risk p_α tend to be smaller compared to the corresponding probabilities of plans obtained using GIE distribution. This suggests that sampling plans using GIE distribution are practically more useful than that of using GE and other considered distributions.

5. SAMPLING PLANS FOR OTHER PERCENTILE POINTS

In previous sections we obtained various acceptance sampling plans under the assumption that the quality of the test units is measured with respect to their median lifetime. This section deals with establishing sampling plans when quality is measured with respect to other percentile points θ_p. Assume that for a proposed plan an experiment is performed for $\tilde{t}_0$ units of time where $\tilde{t}_0$ may be different from the t_0. Here $\tilde{t}_0$ can be considered as a multiple of specified lifetime θ_0 and so we can take $\tilde{t}_0 = \tilde{a}\theta_0$ for some positive constant $\tilde{a}$. Now let the probability of accepting a lot under the given sampling plan be $L_a(\tilde{p})$ where $\tilde{p}$ denotes the probability that a unit fails before the time $\tilde{t}_0$. Observe that $\tilde{p} = F(\tilde{t}_0; \gamma, \lambda)$ and using the value of λ from expression (2) it is rewritten as

$$\tilde{p} = 1 - \left[1 - \left(1 - (1-p)^{\frac{1}{\gamma}}\right)^{\frac{\theta_p}{a\theta_0}}\right]^{\gamma} \tag{9}$$

Now design parameters having minimum sample size and satisfying both the consumer's risk and producer's risk can be obtained by solving the following optimization problem:

Minimize $ASN(\tilde{p}_1)$

Subject to $L_\alpha\left(\tilde{p}_1 \middle| \frac{\theta_p}{\theta_0} = 1\right) \le \beta$

$L_\alpha\left(\tilde{p}_1 \middle| \frac{\theta_p}{\theta_0} = \tilde{r}_2\right) \ge 1 - \alpha.$

By comparing Eq. (5) and Eq. (9), it is observed that for given values of β, α and $\tilde{r}_2$, sampling plans for other percentiles can be obtained using the tables based on the median provided

$r_2 = \tilde{r}_2$ and $\frac{\theta_p}{\theta_0} = \frac{m}{m_0}$ or $r_2 = \tilde{r}_2$ and

$$a = \frac{\tilde{a}\log\left(1 - 0.5^{\frac{1}{\gamma}}\right)}{\log\left(1 - (1-p)^{\frac{1}{\gamma}}\right)}. \tag{10}$$

For instance consider Example 1 where an experimenter wishes to obtain sampling plan for 75th percentile point instead of median lifetime and the corresponding experiment is performed for 565 hours. Further assume that values of β, α and $r_2 = \tilde{r}_2$ remain the same. So, we have $\tilde{a} = 0.565$ and using Eq. (10) we find that $a = 1.0$. Therefore in practice sampling plan based on median lifetime corresponding to $a = 1.0$ can be used to obtain sampling plans based on 75th percentile lifetime.

6. CONCLUSION

In this paper, we have proposed a two-stage group acceptance sampling plan for the generalized inverted exponential distribution based on the truncated life test. We considered the median lifetime of the product as a quality parameter and then reported design parameters satisfying the consumer's risk and producer's risk simultaneously. We have further established that the plans based on median lifetimes can also be used for other percentile points as well. Although the group acceptance, double acceptance and single acceptance sampling plans become the particular case of the proposed plans, we have also presented a comparison among them. Our simulation study reveals that the two-stage group and the double acceptance sampling plans respectively require smaller sample size than the group and the single acceptance sampling plans. In fact we noticed that a different group size or different sampling plan may be selected in order to achieve the minimum sample size. Therefore design of an algorithm which can suggest sampling plan (with group size) with minimum sample size may be a considered as a future direction. Further, we have illustrated practical implementations of the proposed plans through a real data set. The case of mis-specification of shape parameter has also been discussed. We also compared the proposed sampling plans obtained under generalized inverted exponential distribution with some other well known distributions. In the process we observed that distribution which fits the data set good provide smaller sample size compared to the distributions with moderate fitting. Finally, we mention that the plans reported in this paper can also be used for the inverted exponential distribution.

ACKNOWLEDGEMENTS

Authors are thankful to the Editor, Associate Editor and anonymous referees for their constructive suggestions that led to much improvement on an earlier version of this manuscript. The first author thanks the Council of Scientific and Industrial Research (CSIR) India for funding this research work.

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