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Mixed Control Charts Using EWMA Statistics

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ABSTRACT In this paper, two mixed control charts are designed for process monitoring when the quality characteristic of interest follows a normal distribution. The mixed control chart starts with monitoring the number of non-conforming items but switches to monitoring using exponentially weighted moving average (EWMA) statistic or hybrid EWMA statistic when the decision is indeterminate with the attribute data. The average run lengths are calculated to evaluate the performance of the proposed control charts according to the mean shift. The performance of both control charts is compared with each other and with the existing control chart. Simulation study is given to demonstrate the efficiency of the proposed control charts.

INDEX TERMS Exponentially weighted moving average, hybrid exponentially weighted moving average, normal distribution, average run length, mixed chart.

I. INTRODUCTION

One of the main aims of industries is to enhance the quality of their product. The high quality of product reflects good reputation of the industry in the global market. The use of control charts helps industries manage the high quality product. The control chart works on two control limits called the upper control limit (UCL) and the lower control limit (LCL). The process is said to be in control if the quality characteristic lies within these limits, but it is declared as out-of-control, otherwise. It is desired that the process runs on the target line so that products can be manufactured according to the specification. Practically, it is not possible to manufacture the whole products at the target line because of some uncontrollable factors. Alternately, the process may be shifted from the target line and goes beyond the specified specifications limits. A quick indication is necessary in this situation to bring it back to the target line. A quick action minimizes the non-conforming product. So, a well-designed control chart provides quick indication about the shift in the process.

Two types of control charts are popularly used in the industry for the process monitoring. One is a variable control chart such as X-bar control chart which is used when the quality characteristic of interest is measurable. On the other hand, an attribute control chart such as *np* control chart is used when data is classified as good or bad. The *np* control chart is used to monitor the non-conforming product in the industry. Many authors worked on the designing of attribute control charts for various situations including, for example, Epprecht and Costa [1], Wu et al [2], Luo and Wu [3], Epprecht et al [4], Chan et al [5], Wu et al [6], Wu and Wang [7], Wu et al [8]., De Araújo Rodrigues et al [9] and Haridy et al. [10]. Similarly, several authors focused on the development of variable control charts including, for example, Al-Oraini and Rahim [11], He and Grigoryan [12], Sulek et al [13], Montgomery [14], Schoonhoven and Does [15] and Aslam et al [16].

The Shewhart control charts are unable to detect small shifts in the process. To overcome this issue, Roberts [17] introduced a control chart using the exponential weighted moving average (EWMA) statistic, which is more efficient to detect a small shift in the process. This type of control charts uses the current information as well as the previous information to make the final decision about the state of process. Many authors designed control charts using EWMA statistic including, for example, Wetherill and Brown [18], Yashchin [19], Hawkins and Olwell [20], Abbas et al. [21] and Aslam et al [22]. Recently, Haq [23] designed the control chart using a new EWMA statistic called the hybrid exponentially weighted moving average (HEWMA). Haq [23] showed that the control chart based on HEWMA statistic is more efficient than the control chart based on EWMA statistic. Azam et al [24] designed the HEWMA control chart using repetitive sampling and proved the efficiency of their proposed chart over Haq [23] in terms of the average run length. More details about EWMA control charts can be seen in Capizzi and Masarotto [25], Jiang et al [26], Zarandi et al [27], Abbasi and Miller [28], Aslam et al [22] and Zhang[29].

Attribute inspection is easy while the variable inspection is more informative. Sometimes, a mixed inspection using attribute as well as variable data is necessary in the industry [(Li et al [30]]. The combined use of attribute and variable data can be applied to a control chart, which is called a mixed control chart. Mixed control charts have combined advantages of variable and attribute control charts. Recently, Aslam et al [31] proposed a mixed control chart using attributes and variables data for monitoring the process mean or the fraction nonconforming. They proved the efficiency of the proposed mixed control chart over the existing control charts. More details about control charts can be seen in Ahmad et al [32], Azam et al [24] and Zaman et al [33].

As mentioned earlier, the control charts using EWMA and HEWMA statistics are more efficient in detecting small shift in the process. In the industry, sometime, it may be necessary to use attribute inspection and variable inspection at the same time. The mixed chart may save time to make decision about the state of process. No attention has paid to design mixed control chart using EWMA and HEWMA statistics. In this paper, therefore, we will present a mixed control chart using EWMA statistic and HEWMA statistic by assuming that the quality of interest follows the normal distribution. The average run length properties are analyzed for the shifted process. Simulation study is given and the proposed chart is compared with the existing charts.

II. A MIXED CHART USING EWMA (np-EWMA CHART)

In this section, we will present a mixed chart using EWMA. We will also derive the average run length formulas. It is assumed that the quality of interest follows a normal distribution with mean m and variance σ^2 when the process is in control and that an item will be defective (or non-conforming) when the quality is beyond the upper specification limit (USL) [Aslam et al [16]]. The proposed mixed control chart called np-EWMA chart is based on three pairs of control limits and proceeds as follows:

Step 1: Take a sample of size n at each subgroup. Calculate the number of non-conforming items (denoted by D) from the sample.

Step 2: Declare the process as out-of-control if $D > UCL_1$ or $D < LCL_1$. Declare as in-control if $LCL_2 \le D \le UCL_2$. Otherwise, go to Step 3

Step 3: For the sample in Step 1, measure variable data X's and calculate $\bar{X} = (\frac{1}{n})\sum_{i=1}^{n} X_i$. The EWMA statistic at the i-th subgroup with a specified smoothing constant λ is calculated as

$$M_i = \lambda\bar{X} + (1-\lambda)M_{i-1}$$

Step 4: Declare the process as out-of-control if $M_i > UCL_3$ or $M_i < LCL_3$. Declare as in-control if $LCL_3 \le M_i \le UCL_3$.

The proposed np-EWMA chart is the extension of some existing charts. When $UCL_1 = UCL_2$ and $LCL_1 = LCL_2$, the proposed chart reduces to an np chart. When $UCL_1 = \infty$, $LCL_1 = -\infty$ and $UCL_2 = LCL_2 = 0$, the proposed chart reduces to an EWMA chart. The proposed chart also reduces to the chart by Aslam et al [31] when $\lambda = 1$.

The approximate distribution of M_i for a large value of i when the process is in control is given as

$$M_i \sim N\left(m, \frac{\lambda}{(2-\lambda)}\frac{\sigma^2}{n}\right)$$

Let p_0 be the fraction nonconforming when the process is in control. Then, the number of nonconforming items from a sample of size n has a binomial distribution with parameters n and p_0. In fact, there is a relation between p_0 and m. That is, the fraction nonconforming of p_0 is obtained by

$$p_0 = P(X > USL|m) = 1 - \Phi(\frac{USL-m}{\sigma}) \tag{1}$$

Therefore, three pairs of control limits of the proposed chart are given as follows:

$$UCL_1 = np_0 + k_1\sqrt{np_0(1-p_0)} \tag{2a}$$
$$LCL_1 = \max[0, np_0 - k_1\sqrt{np_0(1-p_0)}] \tag{2b}$$
$$UCL_2 = np_0 + k_2\sqrt{np_0(1-p_0)} \tag{3a}$$
$$LCL_2 = \max[0, np_0 - k_2\sqrt{np_0(1-p_0)}] \tag{3b}$$
$$UCL_3 = m + k_3\sigma\sqrt{\frac{\lambda}{n(2-\lambda)}} \tag{4a}$$
$$LCL_3 = m - k_3\sigma\sqrt{\frac{\lambda}{n(2-\lambda)}} \tag{4b}$$

where k_1, k_2 and k_3 are control coefficients to be determined.

The probability that the process is declared to be in control for the proposed control chart is given as follows when the process is actually in control [Aslam et al [16]].

$$P_{in,0} = A_{10} + A_{20}A_{30} \tag{5}$$

where

$$A_{10} = P(LCL_2 \le D \le UCL_2|p_0) = \sum_{d=\lfloor LCL_2\rfloor+1}^{\lfloor UCL_2\rfloor}\binom{n}{d}p_0^d(1-p_0)^{n-d}$$
$$A_{20} = P(UCL_2 \le D \le UCL_1|p_0) + P(LCL_1 \le D \le LCL_2|p_0) = \sum_{d=\lfloor UCL_2\rfloor+1}^{\lfloor UCL_1\rfloor}\binom{n}{d}p_0^d(1-p_0)^{n-d} + \sum_{d=\lfloor LCL_1\rfloor+1}^{\lfloor LCL_2\rfloor}\binom{n}{d}p_0^d(1-p_0)^{n-d}$$
$$A_{30} = P(LCL_3 \le M_i \le UCL_3|m) = 2\Phi(k_3) - 1$$

The performance of the proposed control chart will be evaluated using the average run length (ARL) which is used to indicate when the process is declared to be out of control. Let ARL_0 denote the in-control ARL. Then,

$$ARL_0 = \frac{1}{1 - P_{in,0}} = \frac{1}{1-(A_{10} + A_{20}A_{30})} \tag{6}$$

To derive ARL for the shifted process, we suppose that the process has shifted to $p = p_1$ and $m + c\sigma = \mu_1$ due to some extraneous factors.

As before, we should not consider p_1 and μ_1 (shifted mean) separately because they are related to each other. By following Aslam et al [31] the fraction nonconforming p_1 when the process is out-of-control is given as follows:

$$p_1 = P(X > USL|m + c\sigma) = 1 - \Phi\left(\frac{USL - m - c\sigma}{\sigma}\right) = 1 - \Phi\left(\frac{USL - m}{\sigma} - c\right) \tag{7}$$

Hence, the probability that the process is declared to be in control for the shifted process is given as follows:

$$P_{in,1} = A_{11} + A_{21}A_{31} \tag{8}$$

where

$$A_{11} = P(LCL_2 \le D \le UCL_2|p_1) = \sum_{d=\lfloor LCL_2\rfloor+1}^{\lfloor UCL_2\rfloor} \binom{n}{d} p_1^d (1 - p_1)^{n-d}$$

$$A_{21} = P(UCL_2 \le D \le UCL_1|p_1) + P(LCL_1 \le D \le LCL_2|p_1) = \sum_{d=\lfloor UCL_2\rfloor+1}^{\lfloor UCL_1\rfloor} \binom{n}{d} p_1^d (1-p_1)^{n-d} + \sum_{d=\lfloor LCL_1\rfloor+1}^{\lfloor LCL_2\rfloor} \binom{n}{d} p_1^d (1 - p_1)^{n-d}$$

$$A_{31} = P(LCL_3 \le M_i \le UCL_3|\mu_1) = \Phi\left(k_3 - c\sqrt{\frac{n(2-\lambda)}{\lambda}}\right) - \Phi\left(-k_3 - c\sqrt{\frac{n(2-\lambda)}{\lambda}}\right)$$

Therefore, the out-of-control ARL for the shifted process is given by

$$ARL_1 = \frac{1}{1 - (A_{11} + A_{21}A_{31})} \tag{9}$$

Let r_0 be the target in-control ARL. The following algorithm is used to determine the control coefficients and the values of ARL_1 for various shift constant c.

1) The value of p_0 is specified first. Then, obtain $\frac{USL-m}{\sigma}$ from Eq. (1).
2) Specify the target value r_0 and sample size n.
3) Determine k_1, k_2 and k_3 subject to $ARL_0 \ge r_0$.
4) Calculate the values of ARL_1 from Eq.(9) for various values of the shift constant c. The value of p_1 is obtained from Eq.(7).

The values of ARL_1 for various sample size and smoothing constant are placed in Tables 1-3 according to shift constants when $p_0 = 0.1$ and $r_0 = 370$. From these tables, we note that the proposed control chart is efficient to detect a shift in the process even for smaller c. For example, when $n = 20$ and $\lambda = 0.1$, the value of ARL_1 drops to 70 at c= 0.05 and further to 13.6 at c= 0.1.

TABLE 1. Average run lengths of the np-EWMA chart when $n = 20$.

c	$k_1 = 3.8934;\ k_2 = 0.8556;\ k_3 = 2.6121$					
	λ					
	0.1	0.2	0.3	0.4	0.5	1
0	370.01	370.01	370.01	370.01	370.01	370.01
0.005	354.74	360.36	362.27	363.24	363.82	364.98
0.01	321.67	341.33	348.36	351.98	354.18	358.66
0.02	234.77	285.69	307.20	319.07	326.59	342.62
0.03	158.24	224.35	257.98	278.31	291.91	322.95
0.04	104.73	170.40	209.91	236.11	254.73	300.84
0.05	69.99	127.78	167.80	196.72	218.51	277.39
0.07	33.46	72.16	105.34	132.98	156.09	230.22
0.09	17.82	42.20	66.59	89.22	109.79	186.80
0.1	13.55	32.84	53.37	73.29	92.05	167.34
0.2	3.15	5.32	8.82	13.29	18.54	53.12
0.3	2.25	2.44	3.10	4.17	5.62	18.43
0.4	1.80	1.81	1.89	2.15	2.58	7.55
0.5	1.50	1.50	1.50	1.54	1.66	3.72
0.6	1.30	1.30	1.30	1.30	1.32	2.20
0.7	1.17	1.17	1.17	1.17	1.17	1.54
0.8	1.09	1.09	1.09	1.09	1.09	1.23
0.9	1.04	1.04	1.04	1.04	1.04	1.09
1	1.02	1.02	1.02	1.02	1.02	1.03

III. A MIXED CHART USING HEWMA (np-HEWMA CHART)

In this section, we will present a mixed chart using HEWMA statistic under the same assumptions described in the previous section.

We define a new sequence $\{HE_1, HE_2, \ldots\}$ as follows:

$$HE_t = \lambda_2 M_t + (1 - \lambda_2) HE_{t-1} \tag{10}$$

where M_t is a usual EWMA defined in Section 2 and λ_2 is another smoothing constant between 0 and 1. HE_t is called a hybrid EWMA (HEWMA).

Based on it, the proposed mixed chart called np-HEWMA chart is stated as follows:

Step 1: Take a sample of size n at each subgroup. Calculate the number of non-conforming items (denoted by D) from the sample.

Step2: Declare the process as out-of-control if $D > UCL_1$ or $D < LCL_1$. Declare as in-control if $LCL_2 \le D \le UCL_2$. Otherwise, go to Step3

Step 3: For the sample in Step 1, measure variable data X's and calculate $\bar{X} = (\frac{1}{n})\sum_{i=1}^{n} X_i$ from the sample data. The EWMA statistic at subgroup i is given as

$$M_i = \lambda_1 \bar{X} + (1 - \lambda_1) M_{i-1}$$

TABLE 2. Average run lengths of the np-EWMA chart when $n = 30$.

c	k_1=3.4764; k_2=0.7169; k_2=3.1036					
	λ					
	0.1	0.2	0.3	0.4	0.5	1
0	370.00	370.00	370.00	370.00	370.00	370.00
0.005	346.96	350.27	351.38	351.94	352.27	352.94
0.01	314.73	326.31	330.28	332.28	333.48	335.91
0.02	238.77	272.18	284.44	290.79	294.67	302.59
0.03	166.90	217.79	238.50	249.63	256.55	270.98
0.04	110.70	168.92	195.97	211.21	220.92	241.62
0.05	71.59	128.08	158.56	176.75	188.67	214.78
0.07	30.27	70.87	100.55	120.90	135.23	168.75
0.09	14.13	38.88	62.36	81.10	95.48	132.12
0.1	10.17	29.07	49.03	66.18	79.96	116.86
0.2	2.20	3.57	6.25	9.92	14.24	35.25
0.3	1.70	1.76	2.11	2.81	3.83	11.89
0.4	1.41	1.41	1.43	1.54	1.79	4.80
0.5	1.22	1.22	1.22	1.23	1.27	2.42
0.6	1.11	1.11	1.11	1.11	1.12	1.54
0.7	1.05	1.05	1.05	1.05	1.05	1.19
0.8	1.02	1.02	1.02	1.02	1.02	1.06
0.9	1.01	1.01	1.01	1.01	1.01	1.01
1	1.00	1.00	1.00	1.00	1.00	1.00

TABLE 3. Average run lengths of the np-EWMA chart when $n = 40$.

c	k_1=3.2542; k_2=0.6595; k_3=2.9799					
	λ					
	0.1	0.2	0.3	0.4	0.5	1
0	370.00	370.00	370.00	370.00	370.00	370.00
0.005	341.61	348.86	351.33	352.57	353.32	354.83
0.01	293.84	317.49	326.02	330.42	333.10	338.56
0.02	188.79	242.78	265.94	278.76	286.90	304.26
0.03	110.67	173.55	206.37	226.25	239.53	269.70
0.04	63.61	119.97	155.54	179.21	195.93	236.65
0.05	37.23	81.93	115.45	139.91	158.21	206.17
0.07	14.43	38.72	62.83	83.74	101.23	154.48
0.09	6.85	19.47	34.78	50.13	64.39	114.86
0.1	5.10	14.26	26.24	39.03	51.48	98.99
0.2	1.79	2.23	3.37	5.11	7.39	23.82
0.3	1.46	1.46	1.55	1.81	2.25	7.16
0.4	1.24	1.24	1.24	1.26	1.33	2.91
0.5	1.11	1.11	1.11	1.11	1.12	1.62
0.6	1.05	1.05	1.05	1.05	1.05	1.19
0.7	1.02	1.02	1.02	1.02	1.02	1.05
0.8	1.00	1.00	1.00	1.00	1.00	1.01
0.9	1.00	1.00	1.00	1.00	1.00	1.00
1	1.00	1.00	1.00	1.00	1.00	1.00

and the HEWMA as

$$HE_i = \lambda_2 M_i + (1-\lambda_2)\, HE_{i-1}$$

where $\lambda_1 \in [0, 1]$ and $\lambda_2 \in [0, 1]$ are smoothing constants.

Step 4: Declare the process as out-of-control if $HE_i > UCL_3$ or $HE_i < LCL_3$ Declare as in-control if $LCL_3 \le HE_i \le UCL_3$.

The proposed np-HEWMA chart is an extension of the proposed np-EWMA chart in Section 2 when $\lambda_2 = 1$. The proposed chart reduces to the one in Aslam et al.[31] when $\lambda_1 = \lambda_2 = 1$.

According to [Haq [23]] HE_i has a normal distribution as follows when the process is in control:

$$HE_i \sim N\left(m, \frac{\lambda_2^2\lambda_1}{(2-\lambda_1)}\left[\frac{1-(1-\lambda_2)^{2i}}{\lambda_2(2-\lambda_2)} - \frac{(1-\lambda_1)^2\left\{(1-\lambda_1)^{2i}-(1-\lambda_2)^{2i}\right\}}{(1-\lambda_1)^2-(1-\lambda_2)^2}\right]\frac{\sigma^2}{n}\right)$$

For a large value of i, HE_t follows

$$HE_i \sim N\left(m, \frac{\lambda_1\lambda_2\sigma^2}{(2-\lambda_2)(2-\lambda_1)}\right) \quad (11)$$

Therefore, the control limits of the proposed chart based on HEWMA statistic are given as follows

$$UCL_1 = np_0 + k_1\sqrt{np_0(1-p_0)} \quad (12a)$$
$$LCL_1 = \max[0, np_0 - k_1\sqrt{np_0(1-p_0)} \quad (12b)$$
$$UCL_2 = np_0 + k_2\sqrt{np_0(1-p_0)} \quad (13a)$$
$$LCL_2 = \max[0, np_0 - k_2\sqrt{np_0(1-p_0)}] \quad (13b)$$
$$LCL_3 = m - k_3\sqrt{\frac{\lambda_1\lambda_2}{(2-\lambda_2)(2-\lambda_1)}\frac{\sigma^2}{n}} \quad (14a)$$
$$UCL_3 = m + k_3\sqrt{\frac{\lambda_1\lambda_2}{(2-\lambda_2)(2-\lambda_1)}\frac{\sigma^2}{n}} \quad (14b)$$

The in-control ARL for this proposed chart can be determined similarly to the proposed chart using EWMA as described in Section 2. Suppose now that the process mean is shifted from m to $m + c\sigma = \mu_1$. Then, the probability of being declared as in control for the shifted process is given by

$$P_{in,1} = B_{11} + B_{21}B_{31} \quad (15)$$

where

$$B_{11} = P(LCL_2 \le D \le UCL_2 | p_1) = \sum_{d=\lfloor LCL_2\rfloor+1}^{\lfloor UCL_2\rfloor}\binom{n}{d} p_1^d(1-p_1)^{n-d}$$

$$B_{21} = P(UCL_2 \le D \le UCL_1 | p_1) + P(LCL_1 \le D \le LCL_2 | p_1) = \sum_{d=\lfloor UCL_2\rfloor+1}^{\lfloor UCL_1\rfloor}\binom{n}{d} p_1^d(1-p_1)^{n-d} + \sum_{d=\lfloor LCL_1\rfloor+1}^{\lfloor LCL_2\rfloor}\binom{n}{d} p_1^d(1-p_1)^{n-d}$$

$$B_{31} = P(LCL_3 \le HE_i \le UCL_3 | \mu_1) = \Phi\left(k_3 - c\sqrt{\frac{n(2-\lambda_2)(2-\lambda_1)}{\lambda_1\lambda_2}}\right) - \Phi\left(-k_3 - c\sqrt{\frac{n(2-\lambda_2)(2-\lambda_1)}{\lambda_1\lambda_2}}\right)$$

Hence, the ARL for the shifted process is given as follows

$$\mathrm{ARL}_1 = \frac{1}{1-(\mathrm{B}_{11}+\mathrm{B}_{21}\mathrm{B}_{31})} \tag{16}$$

The values of ARL_1 for various values of λ_1 and λ_2 according to various shift constants are placed in Tables 4-6 when $p_0 = 0.1$ and $r_0 = 370$.

From tables 4-6, we note that the proposed control chart is efficient to detect a shift in the process even for smaller c. For example, when $\lambda_1 = \lambda_2 = 0.10$ and $n = 30$, the value of $\mathrm{ARL}_1 = 17.3$ at c= 0.02. It is seen that for a fixed value of λ_2, the values of ARLs increases as λ_1 increases.

TABLE 4. Average run lengths of the np-HEWMA chart when λ_2= 0.10 and n= 20.

c	k_1=3.304276; k_2=0.64375; k_3=3.103591					
	λ_1					
	0.1	0.2	0.3	0.4	0.5	1
0	370.00	370.00	370.00	370.00	370.00	370.00
0.005	253.24	300.90	319.16	328.78	334.72	346.96
0.01	107.61	187.12	230.59	257.11	274.81	314.73
0.02	17.33	51.02	87.55	120.83	149.46	238.77
0.03	5.37	15.55	31.08	49.95	70.46	166.90
0.04	3.18	6.63	12.95	21.93	33.28	110.69
0.05	2.75	3.93	6.70	11.04	17.03	71.59
0.07	2.64	2.72	3.26	4.41	6.23	30.27
0.09	2.57	2.58	2.63	2.91	3.50	14.13
0.1	2.54	2.54	2.55	2.66	2.98	10.17
0.2	2.10	2.10	2.10	2.10	2.10	2.20
0.3	1.70	1.70	1.70	1.70	1.70	1.70
0.4	1.41	1.41	1.41	1.41	1.41	1.41
0.5	1.22	1.22	1.22	1.22	1.22	1.22
0.6	1.11	1.11	1.11	1.11	1.11	1.11
0.7	1.05	1.05	1.05	1.05	1.05	1.05
0.8	1.02	1.02	1.02	1.02	1.02	1.02
0.9	1.01	1.01	1.01	1.01	1.01	1.01
1	1.00	1.00	1.00	1.00	1.00	1.00

IV. ADVANTAGES OF PROPOSED CHARTS

In this section, the efficiency of the proposed charts will be discussed in terms of ARLs. We will compare the proposed np-EWMA chart with the existing mixed chart by Aslam et al [31] as well as with the proposed np-HEWMA chart.

A. PROPOSED np-EWMA CHART VS EXISTING MIXED CHART

The proposed np- EWMA chart is a generalization of the mixed control chart by Aslam et al [31] because the np-EWMA chart reduces to Aslam et al [31] chart when $\lambda = 1$. The values of ARLs of the np-EWMA chart when $\lambda = 1$ are also reported in Tables 1-3. From Tables 1-3, it can be seen that the proposed chart provides smaller values of ARLs for all shift constants. Figure 1 shows the ARLs of the proposed np-EWMA chart with $\lambda = 0.1$ and the existing control chart according to the shift constants when $r_0 = 370$ and $p_0 = 0.10$. From Figure 1, it can be seen that the curve of ARLs for the existing control chart is much higher than the proposed np-EWMA chart at all levels of shift constant. So, the proposed np-EWMA chart performs better than the existing control chart in detecting early shift in the manufacturing process.

TABLE 5. Average run lengths of the np-HEWMA chart when $\lambda_2 = 0.20$ and $n = 30$.

c	k_1=3.580099; k_2=0.760283; k_3=3.103607					
	λ_1					
	0.1	0.2	0.3	0.4	0.5	1
0	370.01	370.01	370.01	370.01	370.01	370.01
0.005	300.91	327.25	336.62	341.43	344.35	350.27
0.01	187.12	252.69	280.71	296.10	305.80	326.31
0.02	51.02	114.54	160.34	192.35	215.42	272.19
0.03	15.55	46.00	79.43	110.02	136.33	217.79
0.04	6.63	19.94	38.77	60.03	81.60	168.92
0.05	3.93	10.04	20.12	33.22	48.32	128.09
0.07	2.72	4.13	7.24	11.98	18.38	70.87
0.09	2.58	2.83	3.86	5.72	8.46	38.88
0.1	2.54	2.62	3.20	4.38	6.21	29.07
0.2	2.10	2.10	2.10	2.10	2.11	3.57
0.3	1.70	1.70	1.70	1.70	1.70	1.76
0.4	1.41	1.41	1.41	1.41	1.41	1.41
0.5	1.22	1.22	1.22	1.22	1.22	1.22
0.6	1.11	1.11	1.11	1.11	1.11	1.11
0.7	1.05	1.05	1.05	1.05	1.05	1.05
0.8	1.02	1.02	1.02	1.02	1.02	1.02
0.9	1.01	1.01	1.01	1.01	1.01	1.01
1	1.00	1.00	1.00	1.00	1.00	1.00

TABLE 6. Average run lengths of the np-HEWMA chart when $\lambda_2 = 0.50$ and n=30.

c	k_1=3.47307; k_2=0.668984; k_3=3.103594					
	λ_1					
	0.1	0.2	0.3	0.4	0.5	1
0	370.00	370.00	370.00	370.00	370.00	370.00
0.005	334.72	344.34	347.62	349.27	350.27	352.27
0.01	274.82	305.80	317.01	322.79	326.31	333.48
0.02	149.46	215.41	245.06	261.64	272.18	294.67
0.03	70.46	136.32	175.84	200.83	217.78	256.55
0.04	33.28	81.60	120.04	148.17	168.92	220.92
0.05	17.03	48.32	79.78	106.47	128.08	188.67
0.07	6.23	18.38	35.08	53.22	70.87	135.23
0.09	3.50	8.46	16.63	27.09	38.88	95.48
0.1	2.98	6.21	11.96	19.76	29.07	79.96
0.2	2.10	2.11	2.28	2.75	3.57	14.24
0.3	1.70	1.70	1.70	1.71	1.76	3.83
0.4	1.41	1.41	1.41	1.41	1.41	1.79
0.5	1.22	1.22	1.22	1.22	1.22	1.27
0.6	1.11	1.11	1.11	1.11	1.11	1.12
0.7	1.05	1.05	1.05	1.05	1.05	1.05
0.8	1.02	1.02	1.02	1.02	1.02	1.02
0.9	1.01	1.01	1.01	1.01	1.01	1.01
1	1.00	1.00	1.00	1.00	1.00	1.00

B. PROPOSED np-HEWMA CHART VS PROPOSED np-EWMA CHART

Now, we compare the efficiency of the proposed np-EWMA chart and the proposed np-HEWMA chart. The proposed

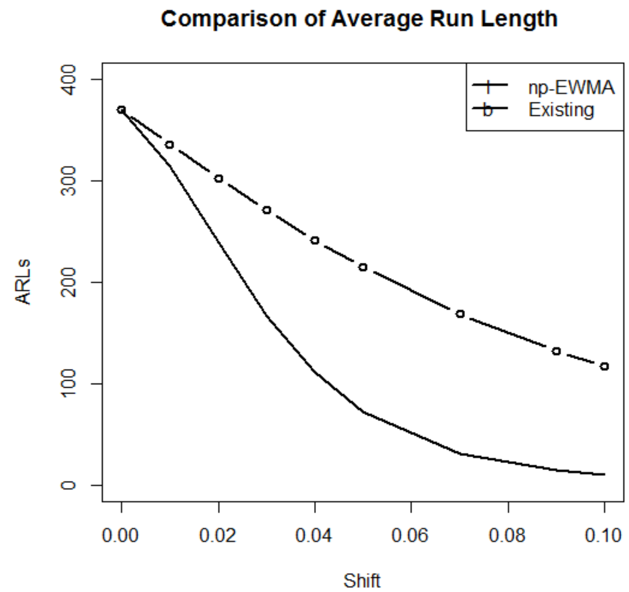

FIGURE 1. Comparisons of np-chart and existing control chart in ARLs.

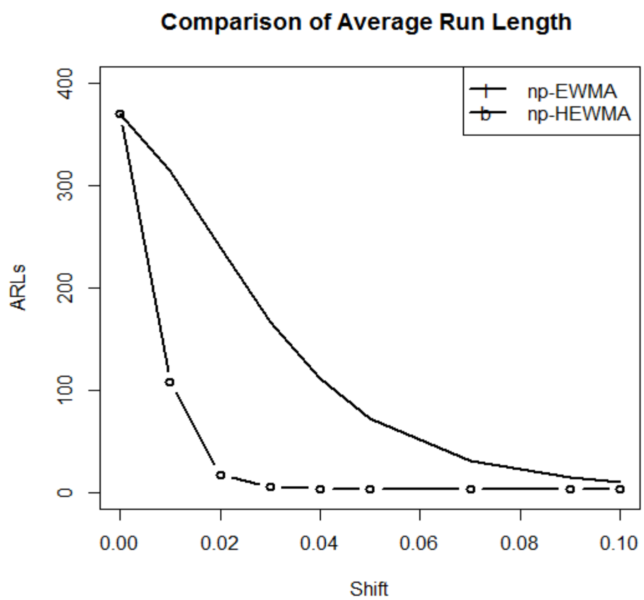

FIGURE 2. Comparisons of two proposed control chart in ARLs.

np-HEWMA chart is the extension of the proposed np-EWMA chart. Figure 2 shows the ARLs of the proposed np-EWMA chart with $\lambda = 0.1$ and the proposed np-HEWMA chart with $\lambda_1 = \lambda_2 = 0.1$ according to the shift constants when $r_0 = 370$ and $p_0 = 0.10$. From Figure 2, we note that the proposed np-HEWMA chart performs better than proposed np-EWMA chart at every shift in the manufacturing process.

C. SIMULATION STUDY

In this section, we will discuss the application and the efficiency of the proposed charts with the help of simulated data. For simulation study, first 30 observations are generated from a in control process following a normal distribution with $m = 0$ and $\sigma = 1$. Let $\lambda_1 = \lambda_2 = 0.1$, $n = 30$, $r_0 = 370$ and $p_0 = 0.10$. Next 23 observations are generated when the process has shifted to a normal distribution with $m = 0.02$ and $\sigma = 1$. The USL is calculated from Eq. (7). Declare the item as defective if $X > USL$.

The subgroup of size 30 is used for counting the number of defectives which are reported as follows

3, 3, 3, 1, 2, 1, 1, 4, 4, 2, 5, 4, 4, 4, 4, 5, 3, 2, 1, 7, 3, 1, 2, 3, 4, 0, 5, 4, 1, 4, 3, 7, 6, 1, 9, 4, 6, 7, 8, 1, 6, 2, 3, 3, 7, 2, 2, 5, 5, 7, 4, 2, 8, 6, 5

The proposed np-HEWMA chart with $\lambda_1 = \lambda_2 = 0.1$, $n = 30$, $r_0 = 370$ and $p_0 = 0.10$ is shown in Figure 3.

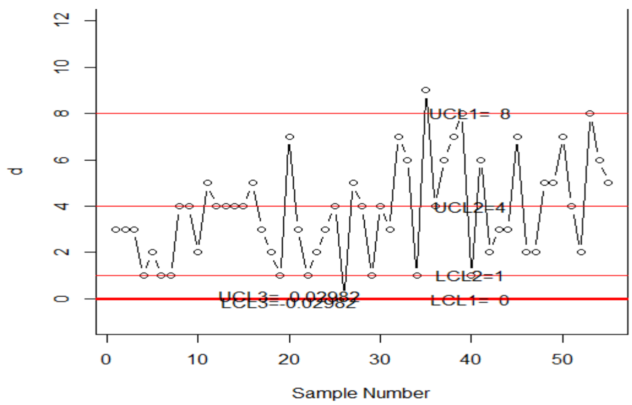

FIGURE 3. The proposed HEWMA chart for simulated data.

The data of defectives items are plotted on Figure 3. From Figure 3, it can be seen that the proposed chart detects the shift at 37th observations. So, the proposed chart has ability to detect the shift in the process quickly. The numbers of defectives are also plotted on the proposed np-EWMA and the existing chart by Aslam et al [31] in Figure 4 and Figure 5, respectively.

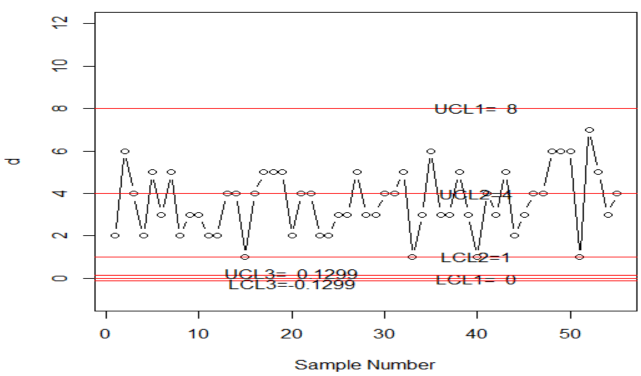

FIGURE 4. The proposed np-EWMA chart for simulated data.

From these figures, we can see that both control charts declare that the process is in control although the manufacturing process has been shifted after 30th observation. So, these control charts are unable to detect a shift in the

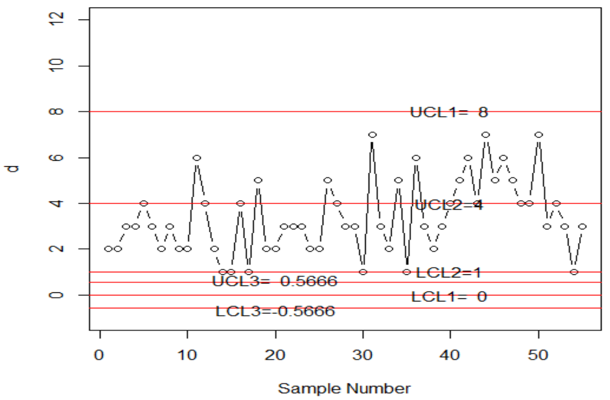

FIGURE 5. The existing chart for simulated data.

manufacturing process. Only the proposed np-HEWMA chart has this ability to detect a small shift in the manufacturing process.

D. EXAMPLE

In this section, we discuss the application of the proposed control HEWMA chart to a real process where frozen orange juice concentrate is packed in cans. The inspection department is interested to observe that it is possibly leak on the side or near the joint. If a leak found, then it is considered as a defective item. The similar data is also reported in Montgomery [14]. Let $n = 20$, $r_0 = 370$ and $\lambda_1 = \lambda_2 = 0.1$ for this example. The numbers of defective are as follows

1, 1, 4, 1, 1, 0, 3, 1, 0, 1, 5, 4, 1, 2, 1, 3, 3, 4, 1, 1, 4, 2, 3, 3, 3, 5, 4, 0, 3, 1, 1, 2, 1, 5, 1, 1, 3, 4, 1, 3, 2, 2, 2, 1, 2, 1, 1, 5, 4, 1, 1, 3, 3, 2, 1

The control chart coefficients are as: $k_1 = 3.8934$, $k_2 = 0.8556$ and $k_3 = 2.6121$. The control limits are shown in Figure 6.

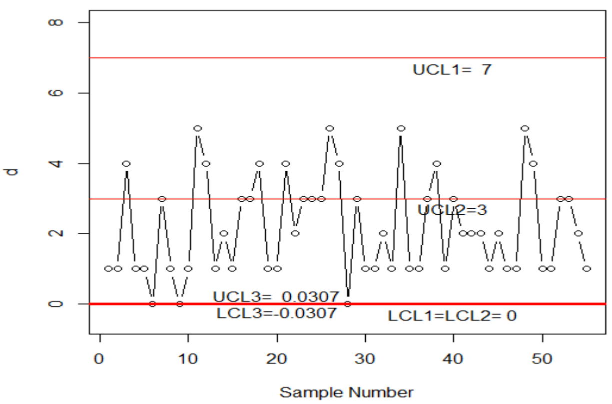

FIGURE 6. The proposed np-HEWMA chart for real data.

It can be seen from Figure 6 that some points are near the control limits which clearly indicates that there may be some issues in process. So, some causes of variation should be removed.

V. CONCLUSION AND RECOMMENDATIONS

Two mixed control charts are proposed in this manuscript. The structures of both mixed control charts are presented to evaluate the ARLs. Some tables are provided for practical use. The advantages of the proposed chart over the existing chart are discussed. The application of both charts is discussed with the help of simulated data as well as a real example. It is concluded that the proposed HEWMA chart is more efficient than the proposed EWMA chart and an existing chart. It has the ability to detect a small shift in the manufacturing process. The proposed chart under the time truncated life test under a non-normal distribution can be studied as future research.

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