Monte Carlo Study on Distortion of the Space-Dimension in COBE Monopole Data

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ABSTRACT

A concise explanation of studies on distortion of space-time dimension is briefly introduced. Second we obtain the limits (i.e., bounded values) of the dimensionless chemical potential μ, the Sunyaev–Zeldovich (SZ) effect y and distortion of the space-dimension ε by Monte Carlo (MC) analysis of the parameter set (T, $d = 3 + \varepsilon$, μ, and y) in cosmic microwave data assuming that the SZ effect is positive ($y > 0$). In this analysis, the magnitude of the space-dimension d with distortion of the space-dimension ε is defined by $d = 3 + \varepsilon$. The limits of μ and y are determined as $|\mu| < 9 \times 10^{-5}$ (2σ) ($\mu = (-3.9 \pm 2.6) \times 10^{-5}$ (σ)), $|y| < 5 \times 10^{-6}$ (2σ) ($y = (2.0 \pm 1.4) \times 10^{-6}$ (σ)), while the distortion of the space-dimension is $|\varepsilon| < 6 \times 10^{-5}$ (2σ) ($\varepsilon = (-0.78 \pm 2.50) \times 10^{-5}$ (σ)). The magnitudes of these three estimated limits are ordered as $|\mu| \gtrsim |\varepsilon| > |y|$. The estimated limit of $|y| < 5 \times 10^{-6}$ appears to be related to re-ionization processes occurring at redshift $z_{ri} \sim 10$. We also present data analysis assuming a relativistic SZ effect.

Subject headings: cosmic background radiation; cosmological parameters; space-dimension

1. Introduction

First, we briefly introduce the essential background to topic of this paper. About one century ago, Ehrenfest (1917) published an interesting paper titled “In What Way Does It Become Manifest in the Fundamental Law of Physics that Space Has Three Dimensions?”. This paper remains actively discussed even today (see a paper “dimensionality” by Barrow (1983)). About 40 years ago, in quantum field theory, a method of dimensional regularization, permitting a distortion of space-time dimension $D - 4 = \varepsilon$, was proposed by 't Hooft and Veltman (1972), Bollini and Giambiagi (1972) and Ashmore (1972). This method is equivalent to assigning an infinitesimal fictitious mass λ_{min} to a photon or introducing a large momentum cut-off Λ in the momentum integration of the Feynman diagram ($2/\varepsilon$ corresponds to $\log(m_e/\lambda_{\text{min}})^2$ or $\log(\Lambda/m_e)^2$, where m_e is the mass of an electron). These methods are important in calculations of physical quantities such as the self-energy and the anomalous magnetic moment of electron (and muon), suggesting that non-integer values of $D = 3 + \varepsilon + 1$ may have a feasible physical interpretation.

Shortly after 't Hooft and Veltman, and their contemporaries proposed their method in 1972, Mandelbrot (1977) conceptualized the fractal dimension of space-time. Inspired by his book, several researchers have investigated fractal dimensions in the natural sciences (Zeilinger and Svozil 1985; Jarlskog and Yndurain 1985; Müller and Schäfer 1986; Grassi et al. 1986; Torres and Herrejón 1988). A range of estimated physical quantities is summarized in Table 1. Among these, the phase space of fractal d_H-dimension (named the Hausdorff dimension) is given by Mandelbrot (1977) based on Hausdorff (1918)

$$\gamma(d_H) = \frac{[\Gamma(1/2)]^{d_H}}{\Gamma(1 + d_H/2)}, \tag{1}$$

where Γ is the gamma function, and Eq. (1) can be multiplied by a factor $(\nu/c)^d$ in actual calculations. (See Eq. (2) below.)

Table 1: Various estimates of distortion parameters in the space-time dimension ($D = d + 1$ or $d = 3 + \varepsilon$).

Authors (Refs.)	Measured (or estimation) quantities	$\|D - 4\|$ or $\|d - 3\| = \|\varepsilon\|$
Zeilinger and Svozil (1985)	Anomalous $(g - 2)$ electron factor / QED with Hausdorff dimension d_H	$\|d_H - 4\| = (5.3 \pm 2.5) \times 10^{-7}$
Jarlskog and Yndurain (1985)	Perihelion of planet Mercury	$\|\varepsilon\|_M < 1.7 \times 10^{-9}$
	Binary PSR 1913+16	$\|\varepsilon\|_{PSR} < 1.7 \times 10^{-9}$
Müller and Schäfer (1986)	Lamb shift of hydrogen	$\|\varepsilon\| \approx 10^{-11}$
Grassi et al. (1986)	Eq. (2)	$\|\varepsilon\| < 2 \times 10^{-2}$
Torres and Herrejón (1988)	Eq. (2) and Stefan-Boltzmann constant including ε	$\|\varepsilon\| < 10^{-3}$

Here, we focus on a paper by Grassi et al. (1986). The Planck distribution, i.e., the black-body radiation law in $d = 3+\varepsilon$ dimensions (where d and ε denote the space-dimension and its distortion based on Eq. (1), respectively) is given by

$$U(T,\ \nu,\ \varepsilon) = N_c(d)\frac{(\nu/c)^d}{e^x - 1}, \tag{2}$$

where $x = h\nu/k_BT$, $N_c(d) = h\gamma(d)\cdot d(d-1) = 2hc\,\pi^{d/2}(d-1)/[c\Gamma(d/2)]$. The quantities h, ν, k_B, c, and T are Planck's constant, the light frequency, Boltzmann's constant, speed of light, and temperature, respectively. Grassi et al. (1986) suggested that upper limit may be placed on the distortion of space-dimension, such that $|d-3| < 0.02$ (Note that their investigation was published eight years earlier than the NASA COBE data (Mather et al. 1994)).

Measurements by NASA's COBE satellite revealed that our universe is almost homogenously filled with cosmic microwave radiation at 2.725 K. Caruso and Oguri (2009) analyzed the COBE/FIRAS (2005) data (where FIRAS denotes Far Infrared Absolute Spectrophotometer) in terms of the following formula:

$$U(T,\ \nu,\ \varepsilon) = N_c\frac{4\pi}{c}\frac{(\nu/c)^d}{e^x - 1}, \tag{3}$$

where N_c is assumed constant. The results of their analysis are summarized in Table 2. Our recent analyses of the same data using Eq. (2) (Biyajima and Mizoguchi 2012) are also depicted in Table 2. As evident from the table, the analyses yield significantly different results, including the signs of ε. These differences are contradicting in spite fact that both studies used the same data, a similar formulation and the CERN MINUIT program. After recalculating our results (listed in the third row of Table 2), we are able to attribute these discrepancies to differences in the numerical values of hc/k_B (Mohr et al. 2012; Biyajima and Mizoguchi 2012).

Second, we note that the COBE data contain two effects, namely, the chemical potential μ, and the Sunyaev–Zeldovich (SZ) effect y (Zeldovich and Sunyaev 1969; Sunyaev and Zeldovich 1970) in the black-body radiation spectra. As a next step, it is worthwhile to analyze the COBE data in terms of these effects, in addition to the distortion of the space-dimension ε.

As is well known, the COBE monopole data contain the following residual spectrum, given by

$$\begin{aligned} &[\text{COBE residual spectrum}] \\ &\equiv\ [\text{monopole data}]\ -[U_{\text{Planck}}\ (T=2.725\ \text{K})] \end{aligned} \tag{4}$$

The monopole spectrum is analyzed by means of the Bose–Einstein distribution, which involves the dimensionless chemical potential μ, (Fixsen and Mather 2002)

$$U_{\text{BE}}(T,\ \nu,\ \mu) = C_B \frac{(\nu/c)^3}{e^{(x+\mu)}-1}, \tag{5}$$

where $C_B = 8\pi h$. Moreover, it is necessary to include for the effect of inverse Compton scattering $e^- + \gamma \to e^- + \gamma$, described by

$$U_{\text{SZ}}(T,\ x,\ y) = C_B \frac{(\nu/c)^3 yxe^x}{(e^x-1)^2}\left(x\coth\frac{x}{2}-4\right). \tag{6}$$

Here, the SZ effect y (Zeldovich and Sunyaev 1969; Sunyaev and Zeldovich 1970) is the parameter for inverse Compton scattering, defined by

$$y \equiv \int dln_e\sigma_T \frac{k_B T_e}{m_e c^2}, \tag{7}$$

where l, n_e, σ_T, and T_e denote the size of the high-temperature region in the Universe, number density of electrons, the cross section of Thomson scattering, and the temperature of electrons, respectively. In this paper, we focus on the positivity of the SZ effect y.

To account for the distortion of the spatial dimension in the COBE monopole data, we adopt the following formula, in which $C_B\,(\nu/c)^3$ in the above equation is replaced by

$N_c(d)\,(\nu/c)^d$,

$$U(T,\,\nu,\,\varepsilon,\,\mu,\,y) = N_c(d)\,\left(\frac{\nu}{c}\right)^d \times \left[\frac{1}{e^{(x+\mu)}-1} + \frac{yxe^x}{(e^x-1)^2}\left(x\coth\frac{x}{2}-4\right)\right] \quad (8)$$

Equation (8) forms the basis of our investigation on the distortion of the space-dimension in the COBE monopole data.

This paper is divided into several sections. In Section 2, we apply Eq. (8), including and excluding ε, to the COBE monopole data and estimate the physical quantities (T, ε, μ, y) and χ^2. Assuming these estimates, the COBE data are analyzed at fixed temperatures by the Monte Carlo (MC) method. Average parameter values are estimated for allowed combinations of parameter ensembles. Section 3 is denoted to the COBE residual spectrum for these combinations of parameter ensembles. Concluding remarks and discussion, including our analyses of relativistic formula of Eq. (6) in Itoh et al. (1998); Challinor and Lasenby (1998) and an analysis of the dipole spectrum derived from Eq. (2), are presented in Section 4.

2. Analyses of COBE Monopole Data in Terms of the Chemical Potential μ and the SZ Effect y

2.1. Analysis of Monopole Data by Equation (8)

Applying Eq. (8) to the COBE monopole data (Fixsen and Mather 2002; COBE/FIRAS 2005), we obtain the results listed in the upper sections of Table 3. From these results, we calculate two parameter sets ($\varepsilon = 0$, μ, y) and ($\varepsilon \neq 0$, μ, y) at fixed temperatures. The χ^2 widths ($\Delta\chi^2$) of the allowed parameter sets are then estimated from the center-column parameters (listed in the central columns of Table 3) and Fig. 1. From the widths $\Delta\chi^2$ and the standard deviations in the parameters ($\delta\varepsilon$, $\delta\mu$, δy), we can estimate the distributions of

Table 2: Analysis of COBE monopole data using Eqs. (2) and (3)

	N_c or $2hc$ (MJy cm^3 sr^{-1})	T (K)	ε ($\times 10^{-6}$)	χ^2	inputs for hc/k_B (cm K^{-1})
Eq. (2)	39.7289	2.7250 ± 0.0001	2.9 ± 24.6	45.1	1.43878[a]
Eq. (3)	39.73 ± 0.002	$2.726 \pm 3 \times 10^{-5}$	-9.6 ± 0.1	45.0	1.43939[b]
Our recalculation of Eq. (3) using $hc/k_B = 1.43878$ and $N_c =$ const.					
Eq. (3)	39.731 ± 0.003	2.7251 ± 0.0001	-53 ± 106	44.8	1.43878[a]

[a] $h = 6.6260696 \times 10^{-27}$ [erg s], $c = 2.9979246 \times 10^{10}$ [cm s^{-1}], and $k_B = 1.3806488 \times 10^{-16}$ [erg K^{-1}] (Mohr et al. 2012) are used (Biyajima and Mizoguchi 2012).

[b] Communication with Caruso and Oguri.

Note. — The difference between the results from Eqs. (2) and (3) is attributed to different numerical values of hc/k_B. In the third row, Eq. (3) is recalculated using $hc/k_B = 1.43878$ (cm K^{-1}), assuming $N_c = constant$.

parameters (ε, μ, y) at fixed temperatures using the MC method. More details are provided in the following sub-section.

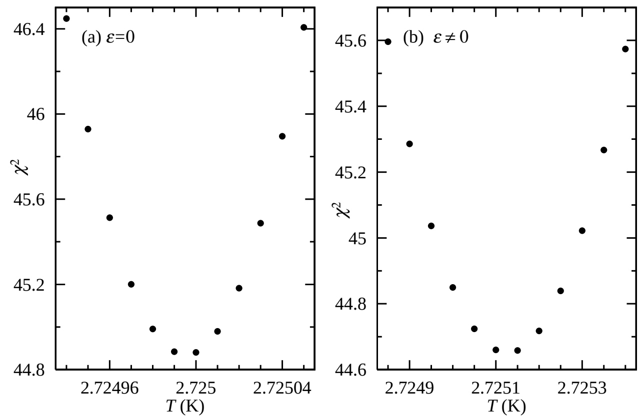

Fig. 1.— χ^2 as a function of fixed temperature T. (a) $\varepsilon = 0$ and (b) $\varepsilon \neq 0$

2.2. Application of MC Method to the Parameter-Set (T, μ, y) (Case $\varepsilon = 0$)

The random variables are introduced as follows:

$$\begin{aligned} T &: T \pm \delta T \to T + \delta T \times U(-1,\ 1) \\ \mu &: \mu \pm \delta\mu \to \mu + \delta\mu \times U(-1,\ 1) \\ y &: y \pm \delta y \to y + \delta y \times U(-1,\ 1) \\ \chi^2 &: \chi^2 < \chi^2_{\rm min} + \Delta\chi^2 \\ \Delta\chi^2 &: \text{depending on the temperature interval } T \pm \delta T. \end{aligned} \tag{9}$$

where $U(-1,\ 1)$ are uniform random numbers in $(-1,\ 1)$ (c.f. Mizoguchi and Biyajima (2001)). The resulting χ^2 values are presented in Fig. 1. The parameter distributions obtained from our analysis are shown in Fig. 2. Assuming that the SZ effect y is positive (as defined in the caption of Fig. 2), we can determine the intervals of the temperature T and the chemical potential μ. The results are presented in Table 4.

Table 3: Analysis of COBE monopole data by means of Equation (8)

T (K)	ε ($\times 10^{-5}$)	μ ($\times 10^{-5}$)	y ($\times 10^{-6}$)	χ^2
2.72501 ± 0.00002	—	-1.1 ± 3.2	—	45.0
2.72504 ± 0.00008	-3.1 ± 6.2	-4.9 ± 8.3	—	44.7
2.72500 ± 0.00004	—	-2.6 ± 5.6	1.6 ± 4.8	44.9
2.72513 ± 0.00023	-7.6 ± 13.4	-6.9 ± 9.5	-3.7 ± 10.4	44.7
T = fixed				
2.72495	—	-8.3 ± 1.0	5.8 ± 2.6	45.93
2.72498	—	-4.6 ± 1.0	3.0 ± 2.6	44.99
2.72500	—	-2.1 ± 1.0	1.2 ± 2.6	44.88
2.72502	—	0.4 ± 1.0	-0.6 ± 2.6	45.18
2.72504	—	2.9 ± 1.0	-2.5 ± 2.6	45.90
T = fixed				
2.72490	5.2 ± 2.5	-0.4 ± 6.9	5.8 ± 3.4	45.29
2.72500	-0.4 ± 2.5	-3.2 ± 6.9	1.6 ± 3.4	44.85
2.72510	-6.1 ± 2.5	-6.2 ± 7.0	-2.6 ± 3.4	44.66
2.72520	-11.8 ± 2.5	-9.0 ± 6.9	-6.8 ± 3.4	44.72
2.72535	-20.3 ± 2.5	-13.4 ± 6.9	-13.2 ± 3.4	45.27

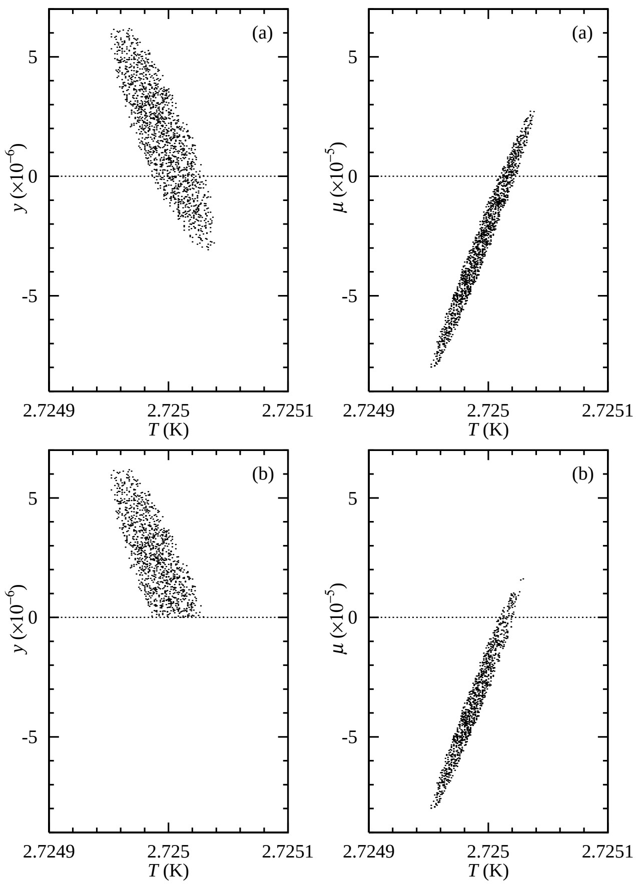

Fig. 2.— Parameter ensemble (T, μ, y) with $\chi^2 < \chi^2_{\rm min}(44.87) + \Delta\chi^2(1.00)$. Number of events generated is 3×10^4 (30k). (a) $N = 1529$. (b) $N_{y>0} = 1142$.

2.3. Application of MC to the Parameter Set (T, ε, μ, and y) (Case $\varepsilon \neq 0$)

Inserting the space distortion $d = 3 + \varepsilon$ into the above calculations yields the distributions shown in Fig. 3. Numbers of events satisfying $y > 0$ for the two calculations (assuming isotropic and distorted space) are listed in Table 4.

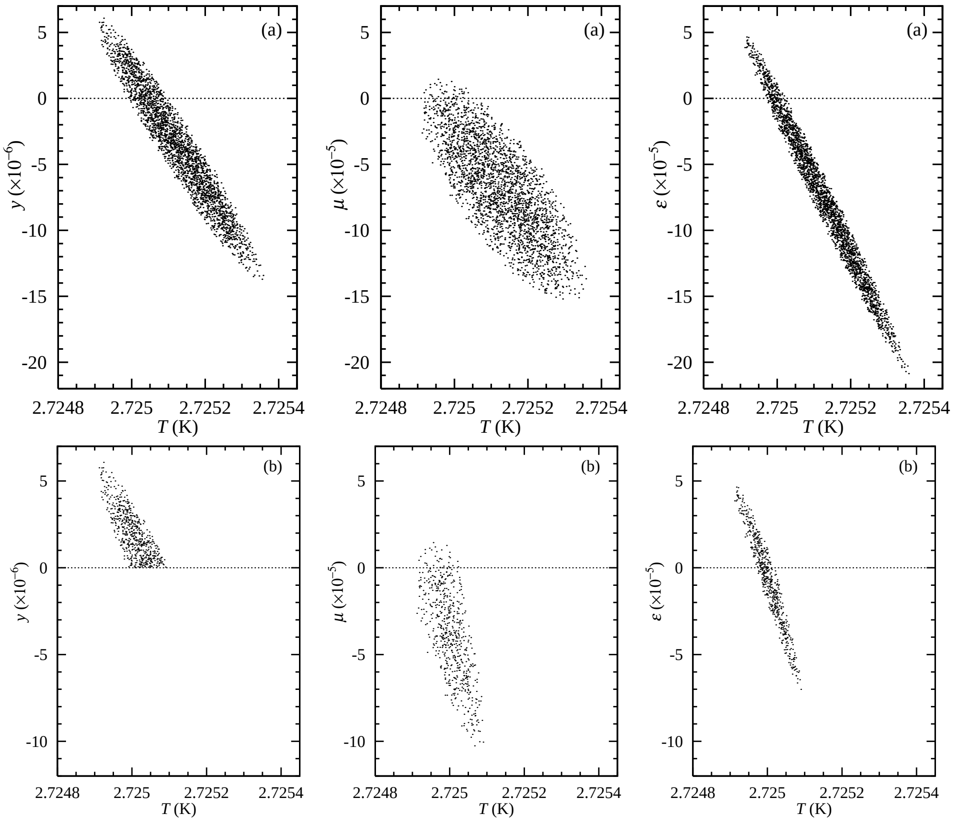

Fig. 3.— Parameter ensemble $(T,\ \varepsilon,\ \mu,\ y)$ with $\chi^2 < \chi^2_{\rm min}(44.65) + \Delta\chi^2(0.67)$. Number of events generated is 3×10^6 (3M). (a) $N = 2699$. (b) $N_{y>0} = 597$.

3. Analyses of Residual Spectrum in terms of Allowed Parameter Combination Ensembles

If $|\mu| \ll 1$, Eq. (8) is simplified, and the residual spectrum becomes

$$
\begin{aligned}
& U^{(\rm residual\ spectrum)} \\
=& N_c(d) \cdot \left(\frac{\nu}{c}\right)^d \\
& \times \left[\frac{1}{e^{(x+\mu)}-1} - \frac{1}{e^x - 1} + \frac{yxe^x}{(e^x-1)^2}\left(x\coth\frac{x}{2} - 4\right)\right] \\
\cong & N_c(d) \cdot \left(\frac{\nu}{c}\right)^d
\end{aligned}
$$

$$\times \left[-\mu \frac{e^x}{(e^x - 1)^2} + \frac{yxe^x}{(e^x - 1)^2} \left(x \coth \frac{x}{2} - 4 \right) \right]. \tag{10}$$

Computing the residual spectrum by means of Eq. (10) yields Fig. 4, in which the allowed combinations of parameters ensembles are evaluated for $\varepsilon = 0$ and $\varepsilon \neq 0$. This figure reveals slight difference between the residual spectra for $\varepsilon = 0$ and $\varepsilon = -0.755 \times 10^{-5}$.

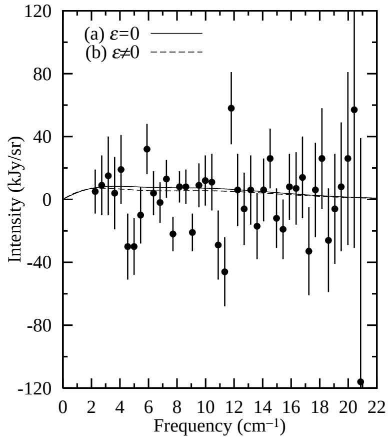

Fig. 4.— Analysis of COBE residual spectrum for allowed combinations of parameter ensembles (ε, T, μ, $y > 0$). (a) $T = 2.72499$ K, $\mu = -3.5879 \times 10^{-5}$ and $y = 2.627 \times 10^{-6}$ (see Table 4), and $\chi^2 = 50.2$. (b) $T = 2.72500$ K, $\varepsilon = -0.775 \times 10^{-5}$, $\mu = -3.885 \times 10^{-5}$ and $y = 2.001 \times 10^{-6}$ (see Table 4), and $\chi^2 = 47.8$.

4. Concluding Remarks and Discussions

(C1) We have computed the distortion of the space-dimension ε in the NASA COBE monopole data (Fixsen and Mather 2002; COBE/FIRAS 2005) using Eqs. (3) and (2). Difference between the two results is attributed to numerical difference in the hc/k_B values. The magnitude of our estimation of ε is $(3 \pm 25) \times 10^{-6}$, should be compared with that of Caruso and Oguri (2009) (namely, $-(9.6 \pm 0.1) \times 10^{-6} \approx -1 \times 10^{-5}$). As seen in third row of Table 2, when we adopt the same numerical value for hc/k_B as Biyajima and Mizoguchi

(2012) in Eq. (3), we obtained $\varepsilon = (-53 \pm 106) \times 10^{-6}$ (σ) (and $|\varepsilon| \leq 1.74 \times 10^{-4}$ (2σ)). It seems to be inadequate to tune the physical constant factor hc/k_B as Caruso and Oguri (2009).

(C2) To extend our investigation of the distortion of the space-dimension in the COBE data, we propose that chemical potential μ and the SZ effect y, in addition to ε, be included in Eq. (8). Applying Eq. (8) to the COBE monopole data and the MC method to parameters sets (T, ε, μ, and y) and (T, $\varepsilon = 0$, μ, and y), we have obtained the allowed parameter ensembles, assuming the positivity of the SZ effect y. The following limits of the parameters have been obtained at $T = 2.72500 \pm 0.00004$ K;

$$\begin{aligned} &|\varepsilon| < 6 \times 10^{-5}\ (2\sigma)\ (\varepsilon = (-0.78 \pm 2.50) \times 10^{-5}\ (\sigma)), \\ &|\mu| < 9 \times 10^{-5}\ (2\sigma)\ (\mu = (-3.9 \pm 2.6) \times 10^{-5}\ (\sigma)), \\ &|y| < 5 \times 10^{-6}\ (2\sigma)\ (y = (2.0 \pm 1.4) \times 10^{-6}\ (\sigma)). \end{aligned} \tag{11}$$

Assuming no distortion of the space-dimension, i.e., $\varepsilon = 0$, the following limits are obtained at $T = 2.72499 \pm 0.00002$ K,

$$\begin{aligned} &|\mu| < 8 \times 10^{-5}\ (2\sigma), \\ &|y| < 6 \times 10^{-6}\ (2\sigma). \end{aligned} \tag{12}$$

The above limit of $|\mu|$ is almost identical to that reported by Fixsen and Mather (2002) ($|\mu| < 9 \times 10^{-5}$). However, our estimated limit of $|y|$ is smaller than that of these authors ($|y| < 1.2 \times 10^{-5}$), probably because SZ effect $|y|$ was assumed as positive in our study. Moreover, it is interesting that limits $|\mu|$ and $|y|$ slightly depend on the magnitude of distortion $|\varepsilon|$. Allowing for the distortion of the space-dimension, among three limits we observe the following order:

$$|\mu| \gtrsim |\varepsilon| > |y|. \tag{13}$$

(C3) According to Durrer (2008), as the Universe is re-ionized at some redshift z_{ri}, during the ionization process, the electrons gain kinetic energy (estimated at around 10 eV). Denoting the electron temperature at the re-ionization by T_{ri}, we obtain the following expression for the SZ effect:

$$\begin{aligned} y &= \frac{\sigma_T T_{ri} n_e(t_0)}{m_e H_0 \sqrt{\Omega_m}(Z_{ri}+1)^2} \int_0^{z_{ri}} dz (z+1)^{5/2} \\ &= 5 \times 10^{-7} \frac{\Omega_b h^2}{\sqrt{\Omega_m h^2}} (z_{ri}+1)^{3/2} \left(\frac{T_{ri}}{10 \text{ eV}} \right), \end{aligned} \tag{14}$$

where Ω_m, Ω_b, and h are the matter density, the baryon density, and the scale parameter, respectively. Given $\Omega_m h^2 \cong 0.13$, $\Omega_b h^2 \cong 0.022$, the re-ionization redshift for the limit $y < 10^{-5}$ is $z_{ri} < 50(T_{ri}/10 \text{ eV})^{-2/3}$. The limit $y < 10^{-6}$ implies that

$$z_{ri} < 10 \left(\frac{T_{ri}}{10 \text{ eV}} \right)^{-2/3}. \tag{15}$$

This set of parameters ($T_{ri} \cong 10$ eV, and $z_{ri} \sim 10$) yields $y \sim 10^{-6}$, which is consistent with the limits in Eqs. (11) and (12).

(D1) For the sake of reference, we analyze the same data by means of the relativistic formula denoted by $U_{\text{SZ}(*)}$ (Itoh et al. 1998; Challinor and Lasenby 1998)

$$\begin{aligned} & U_{\text{SZ}(*)}(T,\ x,\ y_*,\ \theta_e) \\ & = C_B \frac{y_* \theta_e x e^x}{(e^x-1)^2} \left[Y_0 + \theta_e Y_1 + \theta_e^2 Y_2 + \theta_e^3 Y_3 + \theta_e^4 Y_4 \right], \end{aligned} \tag{16}$$

where $y_* = \int dl n_e \sigma_T$, $\theta_e = k_B T_e / m_e c^2$, $Y_0 = x \coth(x/2) - 4$, and

$$\begin{aligned} Y_1 = & -10 + \frac{47}{2} x \coth\frac{x}{2} - \frac{42}{5} x^2 \coth^2\frac{x}{2} + \frac{7}{10} x^3 \coth^3\frac{x}{2} \\ & + \frac{x^2}{\sinh^2(x/2)} \left(-\frac{21}{5} + \frac{7}{5} x \coth\frac{x}{2} \right). \end{aligned}$$

Y_2, Y_3 and Y_4 are explicitly given in Itoh et al. (1998). According to descriptions on temperature of electron T_e's in Weinberg (2008); Naselsky et al. (2006), the temperatures of the ionized plasma in cluster of galaxies is approximately on $(6.5 \sim 7) \times 10^6$ K. Our results are presented in Table 5. It is shown that estimated values are almost the same in Table 3. (Notice that "$y_* \theta_e$" in Table 5 corresponds to "y" in Table 3.)

(D2) The dipole spectrum can be analyzed by the derivative of the Planck distribution (Fixsen et al. 1996; Henry et al. 1968; Sugiyama 2001), given as:

$$\begin{aligned} \frac{\partial U_{\text{Planck}}}{\partial T}\delta T &= C_B \cdot \left(\frac{\nu}{c}\right)^3 \frac{xe^x}{(e^x-1)^2}\frac{\delta T}{T}, \\ &\xrightarrow{\delta T \to T_{\text{amp}}} N_c(d) \cdot \left(\frac{\nu}{c}\right)^d \frac{xe^x}{(e^x-1)^2}\frac{T_{\text{amp}}}{T}. \end{aligned} \tag{17}$$

The results of this analysis are provided in Table 6 and Fig. 5. As seen in Table 6, the χ^2 is improved by imposing a finite $\varepsilon = d - 3$ on the system. From the dipole amplitude $T_{\text{amp}} = 3.597 \pm 0.044$ mK, the velocity of our solar system (with the Universe fixed) is estimated as $v = 396$ km s^{-1}, about 6.6 % higher than the estimate of Fixsen et al. (1996). The anomalously large value of ε obtained in this analysis requires further investigation.

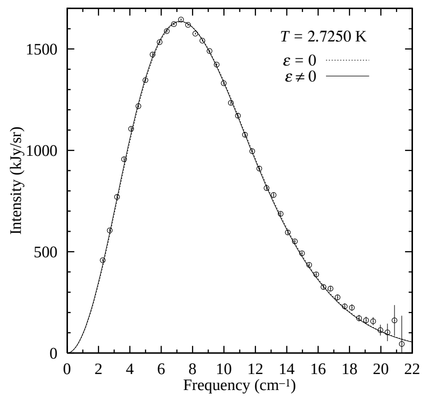

Fig. 5.— Analysis of COBE dipole spectrum at fixed T (2.7250 K), computed from Eq. (17).

(D3) From Eq. (2), a modified Stefan-Boltzmann law is derived as,

$$\int_0^\infty \frac{x^d}{e^x - 1}dx \approx \Gamma(d+1)\left[\zeta(4) - \varepsilon\sum_{n=1}^{\infty}\frac{\ln n}{n^4}\right], \tag{18}$$

Table 4: Averages of parameter ensembles ($N(y$ all) and $N_{y>0}$ events) assuming $\chi^2 < \chi^2_{\rm min} + \Delta\chi^2$ in Figs. 2 and 3

Calc.	y(all and > 0) ($\times 10^{-6}$)	T (K)	μ ($\times 10^{-5}$)	ε ($\times 10^{-5}$)	$\langle\chi^2\rangle$
I (N =1529)	1.682 ± 2.138	2.72499 ± 0.00002	-2.707 ± 2.477	—	45.47
I ($N_{y>0}$ =1142)	2.627 ± 1.551	2.72499 ± 0.00002	-3.5879 ± 2.098	—	45.45
II (N =2699)	-3.674 ± 4.178	2.72513 ± 0.00009	-6.883 ± 3.563	-7.578 ± 5.462	45.09
II ($N_{y>0}$ =597)	2.001 ± 1.367	2.72500 ± 0.00004	-3.885 ± 2.585	-0.775 ± 2.497	45.14

Table 5: Analysis of COBE monopole spectrum by means of Eq. (16) and $\theta_e = 0.6$, 0.8 and 1.0 keV (Fixed)

T_e (keV)	T (K)	μ ($\times 10^{-5}$)	y_* ($\times 10^{-3}$)	$y_*\theta_e$ ($\times 10^{-6}$)	ε ($\times 10^{-5}$)	χ^2
0.6	2.72500 ± 0.00004	-2.6 ± 5.6	1.4 ± 4.1	1.6 ± 4.8	—	44.9
0.8	2.72500 ± 0.00004	-2.6 ± 5.6	1.1 ± 3.1	1.7 ± 4.9	—	44.9
1.0	2.72500 ± 0.00004	-2.6 ± 5.6	0.9 ± 2.5	1.7 ± 4.9	—	44.9
0.6	2.72512 ± 0.00028	-6.9 ± 10.7	-3.1 ± 10.7	-3.7 ± 12.5	-7.5 ± 16.3	44.7
0.8	2.72512 ± 0.00029	-6.9 ± 11.1	-2.4 ± 8.3	-3.7 ± 13.0	-7.5 ± 17.0	44.7
1.0	2.72513 ± 0.00028	-6.9 ± 10.7	-1.9 ± 6.5	-3.8 ± 12.6	-7.6 ± 16.3	44.7

Table 6: Analysis of COBE dipole spectrum by Eq. (17) at $T = 2.7250$ K

T (K)	$T_{\rm amp}$ (mK)	ε ($\times 10^{-2}$)	χ^2
2.7250 (fixed)	3.380 ± 0.004	—	118.2
2.7250 (fixed)	3.597 ± 0.044	-2.0 ± 0.4	92.5

where $\zeta(4)$ is the Riemann's zeta-function and the second term $\sum_{n=1}^{\infty} \ln n/n^4 = 9.89113 \times 10^{-2}$. Through Eq. (18), we are able to discuss the problem of the distortion of the space-dimension in a future in more detail, provided that the error bar of ε becomes smaller.

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