BLUE CHEESE COSMOLOGY: LENSING BY COSMIC STRINGS

by

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Abstract

The light bending effects around cosmic strings in universes with varying rates of expansion are investigated. A relationship between the angular deflection and the expansion rate is found. This is made possible by the Blue Cheese model, which is a generalization to a cylindrical realm of the Swiss Cheese model.

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1 Introduction

First theorized by Tom Kibble, cosmic strings are one-dimensional topological defects that are believed to have been formed in the early universe during phase transitions [6]. They are also believed to be on the order of a femtometer in diameter, which is why we, in this model, treat the cosmic string as a line source with zero thickness. The cosmic string is particularly interesting in spacetime because it behaves like flat space locally, and has a conical shape globally [10]. As I will show later, this conical metric looks almost identical to Minkowski spacetime, except that it has a range on the azimuthal coordinate that is not from 0 to 2π. This scaled down range is what creates the conical space and gives the manifold a characteristic deficit angle. The discovery of cosmic strings would allow us to better understand the link between small scale particle physics and large scale cosmology. The discovery would give insight into numerous processes throughout the universe, in particular the physics of the early universe and galaxy formation [10].

Phase transitions can be typically seen in everyday thermodynamics. When water freezes, ice is formed and this is known as a phase transition. Phase transitions are also seen in cosmology, especially in the early universe. The universe rapidly cooling after the big bang is a phase transition, and from this cosmic strings may have formed. Cosmic strings are like cracks, or defects, that could have formed in the universe and as their name implies, they may span cosmological distances, perhaps even the size of the universe. The "great attractor" could also have some link to cosmic strings [9].

Cosmic strings may allow us to understand how galaxies are formed. The galaxies could have possibly been formed after the formation of cosmic strings. Because of the immense density of these objects, they could have attracted the matter in the universe, and caused them to form along the cosmic string's axis.

In 2003, a galaxy image pair, Capodimonte-Sternberg-Lens Candidate number 1 or CSL-1, was considered to be a great candidate for a cosmic string. This image pair was initially observed by Mikhail Sazhin in the Capodimonte Deep Field. The twin galaxy images were identical in shape, spectrum, luminosity, and redshift. The images were first thought to have been produced by a straight cosmic string acting as a gravitational lens for the galaxy. Later in 2005, new data from high quality imaging arose from the Hubble Space Telescope asserting that these images were actually two separate galaxies and not a double image produced by a cosmic string lens [1].

In addition to their unique lensing effects, cosmic strings may be observed in another manner. Cosmic strings, as I've stated before, are incredibly dense and thus could produce observable gravitational wave signals. These strings are not the straight, static strings I will use in my simulation, however they contain "wiggles" or loops and move around in spacetime. Due to the recent discovery of gravitational waves from a binary black hole system by the Laser Interferometer Gravitational Wave Observatory, a new field of astronomy is rising, and cosmic strings may be one of the more exotic objects to be detected in this manner [9].

Cosmic strings are incredibly dense, and thus could be the source of gravitational waves, however this paper focuses on another method of observation: gravitational lensing. Gravitational lensing occurs when a massive object is between a source of light and the light's observer, and the object is able to bend the light significantly as it passes the object. The lensing effects, as we will explore, are also thought to produce double images of the lensed object.

The “Swiss Cheese” Model is a model used to investigate the lensing effects of Black Holes. The general idea is to carve a vacuum sphere out of a dust-filled Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime, which is an exact solution of the Einstein equation, describing a homogenous and isotropic universe and place a Schwarzschild black hole inside this region [5]. In a spacetime of this type, there are three regions: the interior Schwarzschild, the spherical vacuum, and FLRW exterior. I will pursue a modified version of this method for the case of a cosmic string, in which a long cylindrical region is carved out of a FLRW spacetime with a cosmic string along its axis.

Later in Chapter 2, I will outline the formalism and units used, and describe more efforts that have been previously made to characterize cosmic strings that are relevant to this work. In Chapter 3, I will characterize the cosmic string’s gravitational effects, and derive its effect on spacetime, i.e., its conical deficit angle. I will also derive the angular separation between the double images. After that in Chapter 4, I will derive the boundary conditions of the system to ensure continuity, and smoothness of the light ray’s trajectory. Chapter 5 shows the details of the simulation used to model this system, and Chapter 6 brings this simulation into the expanding spacetime realm.

2 Formalism and the model

Before I describe the nitty-gritty details, I'll set out the formalism to be used in this work. I use the standard –,+,+,+ spacetime metric signature. For most of the calculations cylindrical coordinates (t, r, ϕ, z) are used. Generally, the z coordinate is ignored in calculations, and the problem can effectively be seen as a $(2+1)$–dimensional polar coordinate space problem. I do this because I can model the cosmic string as infinitely along the z-axis, and thus has no effect on the motion in the polar plane. If the z axis is completely uniform, all effects concerning this axis are equivalent, and, thus can be ignored.

The units in this paper are not the fundamental units $(c = G = 1)$, however, we use

$$3 \times 10^{8}\ \frac{\mathrm{m}}{\mathrm{s}} \tag{1}$$

to be the speed of light. This is done, so that we may find results in space to be in the "mks" units, and not spacetime units. Newton's constant is the standard

$$G = 6.67 \times 10^{-11}\ \frac{\mathrm{N} \times \mathrm{m}^2}{\mathrm{kg}^2}. \tag{2}$$

Astronomical units of distance are used because much of the literature, historically, uses these units, however, the units are convenient because one can easily relate the measurements to typical cosmological scales. I frequently use the Megaparsec,

$$1\ \mathrm{Mpc} = 3.1 \times 10^{24}\ \mathrm{cm}. \tag{3}$$

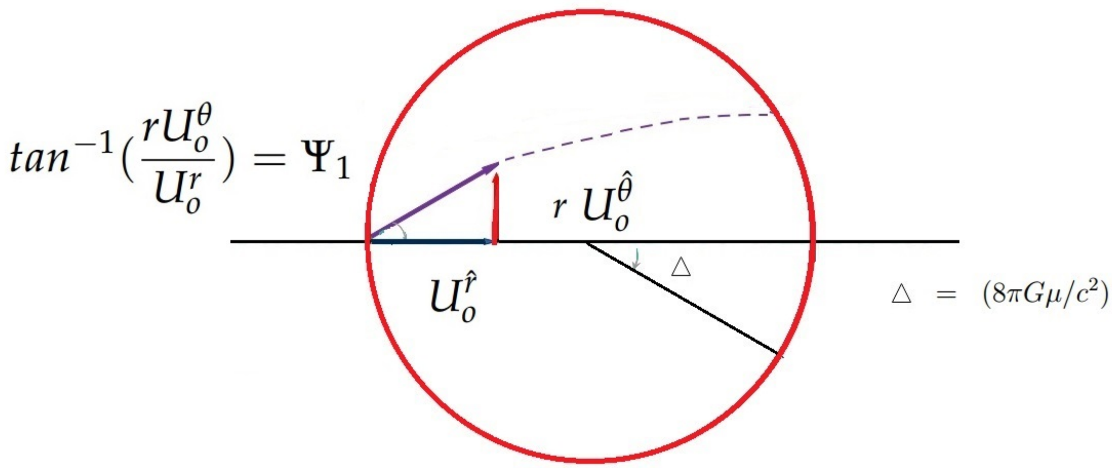


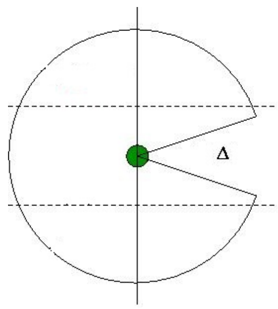
Figure 1: This figure shows the system in which I will perform my simulations. The light ray starts on the boundary, simulating a ray that has arrived from infinity. It then propagates through the cosmic string region where the string is located at the origin. The light ray exits the region and the deflection is investigated. Ψ_2 is defined with the exiting vectors in the same way as Ψ_1 is defined by the entering vectors

This problem is shown in figure 1, which describes the model. Comoving coordinates are used in the FLRW region, however the metric of the cosmic string is described more carefully in the next chapter.

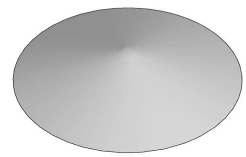
To continue with this situation, we must establish some mathematical way to propagate our light rays through the cosmic string region, while exhibiting the light bending effects from the cosmic string. We can borrow the same line of thinking expressed in the Swiss Cheese model outlined by Kantowski, Chen, and Dai, used to understand the light bending effects around Schwarzschild black holes [10]. The Swiss Cheese model allowed for a spherically symmetric Schwarzschild metric to be embedded into external FLRW spacetime. A spherical cavity was taken out of the external FLRW spacetime, and a Schwarzschild black hole was placed in the cavity. Unlike the Swiss Cheese model, I'm going to carve out a cylindrical space in FLRW spacetime and place the cylindrical metric of a cosmic string inside this cavity. In Chapter 4, the matching conditions between the two regions will be considered.

The cosmic string is static, straight and contains no loops and has an upper bound on its linear mass density $\mu = 6.73 \times 10^{27}$ g cm^{-1} [4]. This upper bound is due to the matching conditions of the string's spacetime and the exterior flat spacetime. Gott shows that the string's linear mass density approaches an asymptotic limit for which the spacetimes cannot be matched [4].

In the case of a cosmic string, this gives 3 regions: the cosmic string conical metric, a cylinder vacuum characterized by the cosmic string, and the exterior FLRW spacetime. In our scenario, we will later characterize the cosmic string as a line source, so we're really left with two regions: the exterior FLRW, and the interior cylindrical metric characterized by the cosmic string. This model allows us to characterize this phenomenon and accurately represent the trajectory of light. The next chapter will be primarily focused with matching of the boundaries so that we have an accurate depiction of how the curve behaves at these boundaries. I'm then going to shoot light rays starting on the boundary between FLRW and the conical region. Each ray will leave the source with some angle, Ψ_1, as designated in figure 1 and propagate through the expanding spacetime. In this region careful consideration of the system is required, and as stated before, these boundary matching is looked at in more detail in the next chapter. In this region, we choose to use non comoving coordinates.



(a) This figure shows the conical metric of the cosmic string, with conical deficit angle Δ. When the angle Δ is cut out, we're left with a cone.



(b) The conical metric created with the deficit angle is cut out. This metric, as we will later see, is simply the Minkowski metric with a redefined azimuthal coordinate.

This is done because the spacetime metric for a cosmic string is static, unlike the FLRW spacetime metric. Comoving coordinates are commonly used in cosmology, and represent coordinates assigned to observers that move with the Hubble flow, i.e., the expansion of the universe. This spacetime is not expanding, however the external spacetime still is. The result is that the boundary expands a bit. The light ray will, then, hit the expanded boundary at Ψ_2. The deflection will be determined based on the difference between these angles. We can then see the dependence upon the scale factor, and Ψ_1, the deflection has.

3 Cosmic String Gravity

This chapter quantitatively describes the gravitational effects of the cosmic string, and the derivation of the conical deficit angle. Double images are more closely examined.

The conical deficit angle can be visualized as a small portion cut out from a disk surrounding the cosmic string, so the light ray's deflection is caused by this rescaling of the azimuthal ϕ coordinate. This can be seen in figure 2.

Following Gott's work [4], we have the metric

$$\mathrm{d}s^2 = -(c\mathrm{d}t)^2 + \mathrm{d}\rho^2 + \left(1 - \frac{4G\mu}{c^2}\right)^{-2} (\rho^2 \mathrm{d}\phi^2 + \mathrm{d}z^2). \tag{4}$$

This line element only applies for solutions for which $0 < \mu < \frac{c^2}{4G}$, and contains the characteristic deficit angle $\Delta = \frac{8\pi\mu G}{c^2}$. This can be seen if we define a new azimuthal coordinate

$$\phi^{'} = \left(1 - \frac{4G\mu}{c^2}\right)\phi. \tag{5}$$

Taking the derivative

$$\mathrm{d}\phi^{'} = \left(1 - \frac{4G\mu}{c^2}\right)\mathrm{d}\phi, \tag{6}$$

rearranging the terms and squaring, we get

$$(1 - \frac{4G\mu}{c^2})^{-2}(\mathrm{d}\phi^{'})^2 = \mathrm{d}\phi^2. \tag{7}$$

Substituting this into the line element we see that

$$\begin{aligned} \mathrm{d}s^2 &= -(c\mathrm{d}t)^2 + \mathrm{d}r^2 + \frac{(1-4G\mu/c^2)^2}{(1-4G\mu/c^2)^2}\rho^2(\mathrm{d}\phi^{'})^2 + \mathrm{d}z^2 && (8)\\ &= -(c\mathrm{d}t)^2 + \mathrm{d}\rho^2 + r^2(\mathrm{d}\phi^{'})^2 + \mathrm{d}z^2. && (9)\end{aligned}$$

This is really just Minkowski spacetime with a redefined azimuthal coordinate: $0 < \phi^{'} < 2\pi(1 - 4G\mu/c^2)$.

I will now derive the relationship between the deficit angle of the cosmic string and the cosmic string's mean density. Looking at the metrics of the region in the two different coordinate systems:

$$\mathrm{d}s^2 = \mathrm{d}r^2 + r^2\mathrm{d}\phi^2 + \mathrm{d}z^2, \tag{10}$$

where $\phi \in [0, 2\pi - \Delta]$

$$\mathrm{d}s^2 = \mathrm{d}\rho^2 + (1 - \frac{4G\mu}{c^2})^{-2}[\rho^2\mathrm{d}\theta^2 + \mathrm{d}z^2], \tag{11}$$

where $\theta \in [0, 2\pi]$.

Looking at the circumferences of these regions at a distance R, which is the radial distance where the regions meet, we see that:

$$\begin{aligned} \int_0^{2\pi-\Delta} r\mathrm{d}\phi &= R(2\pi - \Delta) && (12)\\ \int_0^{2\pi} \frac{1}{1-4G\mu/c^2} R\,\mathrm{d}\theta &= \frac{R}{1-4G\mu/c^2}2\pi && (13)\end{aligned}$$

Setting these equal to each other

$$
\begin{aligned}
2\pi - \Delta &= \frac{2\pi}{1 - 4G\mu/c^2} && (14)\\
\Delta &= \left(1 - \frac{1}{1 - 4G\mu/c^2}\right) 2\pi && (15)\\
&= \left(2\pi - \frac{2\pi}{1 - 4G\mu/c^2}\right) && (16)\\
&= \left(\frac{2\pi(1 - 4G\mu/c^2) - 2\pi}{1 - 4G\mu/c^2}\right) && (17)\\
&= \left(\frac{8\pi G\mu/c^2}{1 - 4G\mu/c^2}\right) && (18)\\
&= (8\pi G\mu/c^2) && (19)
\end{aligned}
$$

Where in the last step, the $1 \gg 4G\mu/c^2$

4 Boundary matching

The purpose of this chapter is to correctly model our system of the cosmic string, cylindrical vacuum, and exterior FLRW metric. This matching has been somewhat controversial in the past. Dyer, Oattes, and Starkman, showed that these surfaces do not match smoothly, and Lake later, agreed [2]. In 1991, Unruh explained that it is in fact possible to match these surfaces together, which I will do in this section [9].

Before I set out to match the sections of spacetime, I should first look at the metric for a cosmic string. As Linet [8] states in his paper, using cylindrical coordinates (t, ρ, ϕ, z), where ρ is the radial coordinate, the general solution for the stress-energy tensor of the form

$$T_t^t = T_z^z = \mu \frac{\delta(\rho)}{\rho} \tag{20}$$

gives the general solution line element:

$$\mathrm{d}s^2 = -(c\rho^C \mathrm{d}t)^2 + A^2\rho^{2C^2-2C}(\rho^{2-2C}\mathrm{d}\phi^2 + \mathrm{d}z^2) + \mathrm{d}\rho^2. \tag{21}$$

Linet shows that by taking $C = 0$, the line element correctly models a cosmic string, which is taken to be a line source at $\rho = 0$, as

$$\mathrm{d}s^2 = -(c\,\mathrm{d}t)^2 + \left(1 - \frac{4G\mu}{c^2}\right)^{-2} (\rho^2\mathrm{d}\phi^2 + \mathrm{d}z^2) + \mathrm{d}\rho^2. \tag{22}$$

From here, we can now do the matching of the cosmic string, characterized by the linear mass density, to FLRW. This will give us an idea of how the deficit angle is related to the

FRLW geometry,

$$\mathrm{d}s^2 = -(c\,\mathrm{d}t)^2 + (\beta t^\alpha)^2(\mathrm{d}r^2 + r^2\mathrm{d}\theta + \mathrm{d}z^2), \tag{23}$$

where β is a constant. If I set the line elements in equations 19 and 21 equal, we have the following conditions:

The time components give:

$$1 = 1, \tag{24}$$

the angular components give

$$(\beta t^\alpha)r = \rho^2 \left(1 - \frac{4G\mu}{c^2}\right)^{-1}, \tag{25}$$

the z components give:

$$(\beta t^\alpha) = \left(1 - \frac{4G\mu}{c^2}\right)^{-1}, \tag{26}$$

We must also derive the coordinate transformations between the two regions. Taking our basis to be

$$e_t = \frac{\partial}{\partial t} \tag{27}$$

$$e_r = \frac{\partial}{\partial r} \tag{28}$$

$$e_\theta = \frac{\partial}{\partial \theta} \tag{29}$$

We will want to establish a correspondence between tangent vectors defined in the FLRW region and those defined in the cylindrical vacuum region. One way to do this is to find orthonormal frames for vectors in both regions with a natural correspondence between basis elements in the two frames, so we start by using the basis e_μ in the FLRW region to construct an orthonormal basis $e_{\hat{\mu}}$ We impose

$$U = U^\mu e_\mu = U^{\hat{\mu}} e_{\hat{\mu}} \tag{30}$$

Expanding this for the radial coordinate:

$$U^r \frac{\partial}{\partial r} = U^{\hat{r}} \frac{1}{\beta t^\alpha} \frac{\partial}{\partial r} \tag{31}$$

It is obvious to see

$$\begin{aligned} U^r &= U^{\hat{r}} \frac{1}{\beta t^\alpha} &(32)\\ \beta t^\alpha U^r &= U^{\hat{r}} &(33) \end{aligned}$$

The same is true for the azimuthal coordinate

$$\begin{aligned} U^\theta \frac{\partial}{\partial \theta} &= U^{\hat{\theta}} \frac{1}{\beta t^\alpha} \frac{\partial}{\partial \theta} &(34)\\ U^\theta &= \frac{U^{\hat{\theta}}}{\beta t^\alpha} &(35)\\ U^{\hat{\theta}} &= \beta t^\alpha U^\theta &(36) \end{aligned}$$

We can take these normalized vectors in the orthonormal basis and use this basis to transform to the new set coordinates inside the region. Looking at the inside metric

$$\mathrm{d}s^2 = -(c\,\mathrm{d}t)^2 + \left(1 - \frac{4G\mu}{c^2}\right)^{-2} (\rho^2 \mathrm{d}\phi^2 + \mathrm{d}z^2) + \mathrm{d}\rho^2 \tag{37}$$

So the orthonormal basis here is

$$\begin{aligned} \frac{\partial}{\partial t} &= e_{\hat{t}} && (38) \\ \frac{\partial}{\partial \rho} &= e_{\hat{\rho}} && (39) \\ \left(1 - \frac{4G\mu}{c^2}\right) \frac{1}{\rho} \frac{\partial}{\partial \phi} &= e_{\hat{\phi}} && (40) \end{aligned}$$

On the boundary between the regions, we identify the two orthonormal bases

$$\begin{aligned} U^{\hat{t}} &= U^{\hat{t}} && (41) \\ U^{\hat{r}} &= U^{\hat{\rho}} && (42) \\ U^{\hat{\theta}} &= U^{\hat{\phi}} && (43) \end{aligned}$$

So the complete transformation between vector components is

$$\begin{aligned} U_i^t &= U_o^t && (44) \\ U^\rho &= \beta t^\alpha U^r && (45) \\ U^\phi &= \left(1 - \frac{4G\mu}{c^2}\right) U^\theta && (46) \end{aligned}$$

The region of the cosmic string also expands as time increases

$$\rho = \beta t^{\alpha} r_{\text{boundary}} \tag{47}$$

Now that we have successfully characterized the model, we can simulate the results of the light deflection.

5 Algorithm

We will begin the light ray with these initial conditions:

$$\begin{aligned}
t_0 &= 3.1688 \times 10^9 \text{ years} && (48)\\
U^t &= \mathrm{d}t/\mathrm{d}s = \sqrt{\frac{g_{rr}(U^r)^2 + g_{\phi\phi}(U^\phi)^2}{g_{tt}}} \text{ s/step} && (49)\\
r_0 &= \text{radius of the boundary Mpc} && (50)\\
U^r &= \mathrm{d}r/\mathrm{d}s = .1 \text{ Mpc/step} && (51)\\
\theta_0 &= \pi \text{ radians} && (52)\\
U^\theta &= \mathrm{d}\phi/\mathrm{d}s = .1 \text{ radians/step} && (53)\\
z_0 &= 0 \text{ Mpc} && (54)\\
U^z &= \mathrm{d}z/\mathrm{d}s = 0. \text{ Mpc/step} && (55)
\end{aligned}$$

Where U^μ is the tangent vector to the null geodesic representing the light ray. Notice U^t is calculated, which is done so that the null condition is satisfied.

$$g_{\mu\nu}U^\mu U^\nu = 0. \tag{56}$$

We can come to this relation using the fact that for null paths the norm of the vector should be zero, i.e., the dot product of U^μ with itself is zero. This is met as the U^t is calculated based on the specified values for U^r and U^ϕ that are given. I will use these initial conditions to propagate the light ray, while α will vary from 0.5 to 0.66.

To solve these equations (second order, non-linear, coupled ODE's), I used the Fourth

Order Runge-Kutta method. Fortunately, Matlab has a built-in function, ODE45, which uses the Fourth Order Runge-Kutta method to solve differential equations. Passing the initial conditions, and the equations to ODE45 gives the solution to the equation.

The geodesic equations must be set up now. We can find the Christoffel symbols for the line element of the cosmic string line source

$$\mathrm{d}s^2 = -(c\,\mathrm{d}t)^2 + \frac{1}{(1-4G\mu/c^2)^2}(\rho^2\mathrm{d}\phi^2 + \mathrm{d}z^2) + \mathrm{d}\rho^2, \tag{57}$$

and we find that the only non-vanishing, relevant ones are

$$\Gamma^r_{\phi\phi} = -r\left(1-\frac{4G\mu}{c^2}\right)^2 \tag{58}$$

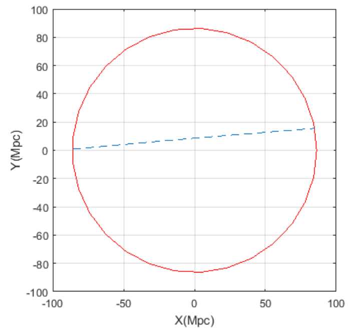
$$\Gamma^\phi_{r\phi} = \frac{1}{r}. \tag{59}$$

Thus, the geodesic equation is

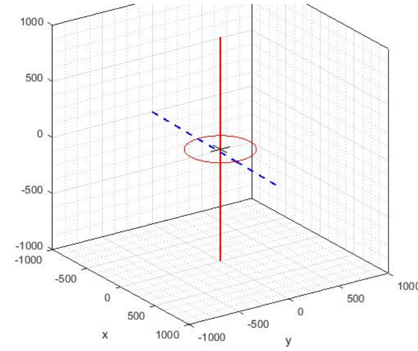
$$\frac{\mathrm{d}^2 r}{\mathrm{d}s^2} = r\left(1-\frac{4G\mu}{c^2}\right)^2 (U^\phi)^2 \tag{60}$$

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}s^2} = -\frac{2U^rU^\phi}{r}. \tag{61}$$

When μ goes to zero, the Christoffel symbols, and thus the geodesic equation become identical to the Minkowski case.



(a) This figure shows the bending around a cosmic string of a light ray starting on the boundary of the region. The dotted line shows the trajectory of the light ray, while the circle represents the region of the cosmic string.



(b) This figure shows the bending around a cosmic string. Notice that the bending only occurs in the cylindrical metric. The dotted line shows the trajectory of the light ray, while the circle represents the region of the cosmic string.

Using those initial conditions listed in equations 50 – 57, allowing our algorithm to iterate through the geodesic equations, providing the trajectory of light. Figure 4 shows the result of this calculation using $\alpha = 0.5$. The calculated deficit angle, using $\Delta = \frac{8\pi G\mu}{c^2}$, is 0.0186 radians or 1.07^o.

The line element $\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + R(t)^2\mathrm{d}\sigma^2$ where $\mathrm{d}\sigma^2 = \frac{\mathrm{d}r^2}{1-kr^2} + r^2(\mathrm{d}\phi^2 + \mathrm{d}z^2)$, and k is the curvature of space is examined. For the $k = 0$ or flat case, we're left with $\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + R(t)^2(\mathrm{d}r^2 + r^2(\mathrm{d}\phi^2 + \mathrm{d}z^2)$. We can write $R(t) = \beta t^\alpha$, where α determines the composition of the universe obtained by the Friedmann equation. The table below shows the values for α

α	Universe
$2/3$	Matter Dominated
$1/2$	Radiation Dominated

Passing the initial conditions from equations 50 –57 to the solver, I can vary the intitial U^ϕ from 0 to larger values, until the light reaches the arrival point on the other side of the string. I do this for every α from 0.5 to 0.66.

I cannot just place a cosmic string at the center because unlike Minkowski space, FLRW contains some matter spread uniformly throughout the universe. The area of the cosmic string I must carve out must contain an equal amount of mass as the cosmic string. The universe is fairly close to the critical density, so I will choose this to be the density used in the calculation. The value for the critical density will change depending on α. The following calculation shows how large of a cylindrical region should be carved out. To keep the density of the entire spacetime to uniform, we must carve out some region, whose size is determined by the linear mass density of the cosmic string. If we have a cosmic string with a linear mass density $\mu = 10^{24}$ kg/cm, which is a typical value for a cosmic string's linear mass density [4], being placed in a carved out cylindrical region of FLRW, we must be sure that the amount of mass cut out of FLRW be equivalent to the mass of the cosmic string, so that the density of the entire space, cosmic string region and FLRW, have the uniform density of FLRW. We are close to the critical density, so we will use that in our calculation. The critical density is

$$\rho_{crit} = \frac{3H^2}{8\pi G}. \tag{62}$$

Where $H = \dot{R}/R$. Thus the critical density depends on the scale factor, and thus α. Using $R = \beta t^\alpha$,

$$\frac{\dot{R}}{R} = \frac{\alpha}{t}, \tag{63}$$

We can then use the critical density to determine how much of FLRW we should carve out.

$$\rho_{critical} = \frac{M}{V}. \tag{64}$$

For a cylinder, the density can be written as

$$\rho_{critical} = \frac{M}{\pi R^2 h}. \tag{65}$$

If the linear mass density is defined as $\mu = M/h$, we can rewrite the critical density as:

$$\rho_{critical} = \frac{\mu}{\pi R^2}. \tag{66}$$

Setting $\mu = 10^{24}$ kg/m, and solving for the values of R, we get the values in the table below.

α	Radius(Mpc)
.50	86.16
.51	84.47
.52	82.85
.53	81.29
.54	79.78
.55	78.33
.56	76.93
.57	75.58
.58	74.28
.59	73.02
.60	71.80
.61	70.63
.62	69.49
.63	68.38
.64	67.31
.65	66.28
.66	65.27

Using these values as the size of the region for the cosmic string and the initial conditions, I simulate this system and make a plot a of $\Delta\Psi$ versus α, keeping Ψ_1 fixed at 0.1 radians. I also make a plot of Ψ_2 versus Ψ_1 keeping α fixed to 0.50.

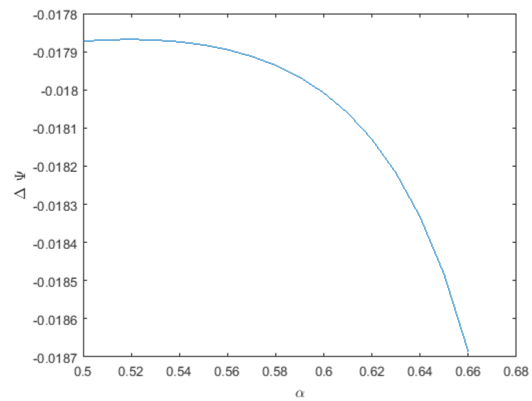


Figure 4: This figure shows the relationship between the “deflection” , $\Delta\Psi = \Psi_2 - \Psi_1$, and α for Ψ_1 fixed at 0.1 radians.

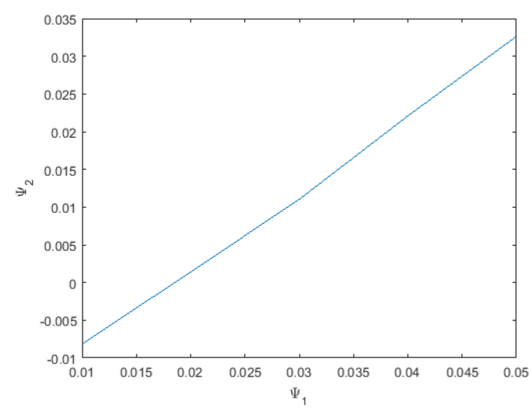


Figure 5: This figure shows the linear relationship between Ψ_1 and Ψ_2 having a slope of 1 and offset 0.018.

6 Conclusions and Future Work

Figure 5 shows the interesting result that the deflection does not depend on Ψ_1, which means that no matter where along the boundary the light ray enters the region, $\Delta\Psi$ is always the same. This is seen in the linear relationship between Ψ_1 and Ψ_2.

The deflection does however depend on α. The curve shows a decrease from – 0.0179 to – 0.0187. The negative value is due to Ψ_1 always being greater than Ψ_2, when making Ψ_1 positive. The values are of the same order as the calculated deficit angle. These results are promising for trying to determine the value of α as Ψ_1 is one less parameter to be determined before α can be found.

The next step of this project is to determine precisely the distance to the cosmic string, and be able to determine the linear mass density of the string. If these values are known, a curve like the one in figure 5 can be produced. From this curve, the type of universe at this time can be discovered.

The code should be further optimized, in addition to decreasing run time, such that rounding errors, and other numerical errors are minimized. The code should also be expanded to include the string's linear mass density chaning with time. An analytical solution should be pursed as well. This solution, if it exists, should be matched to the numerical solutions in this work.

Not only would the discovery of cosmic strings be monumental, but it is promising to learn that cosmic strings could allow us to determine the type of universe that exists today.

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