Enhanced MoM for the Analysis of Multilayered Periodic Structures Containing Dipoles with Application to the Design of Reflectarray Antennas

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***Abstract*—The spectral domain Method of Moments (MoM) is customarily used in the analysis of multilayered periodic structures. In this paper a hybrid MoM is introduced for the analysis of multilayered periodic structures containing sets of dipoles at two metallization levels. In the hybrid MoM the matrix entries involving basis functions of dipoles at different metallization levels are computed in the spectral domain. However, the matrix entries involving basis functions at the same metallization level are computed in the spatial domain. The implemented hybrid MoM is applied to the design of multilayered reflectarray antennas made of dipoles under the local periodicity assumption. Thanks to the use of interpolated expressions for the periodic spatial Green's functions in terms of both coordinates and incidence angles, the CPU time required by the hybrid MoM in the design of the antennas is around twenty times faster than that required by the spectral domain MoM.**

***Index Terms*—Moment methods, multilayered media, periodic structures, reflectarrays.**

I. INTRODUCTION

Reflectarray antennas are an interesting alternative to phased arrays and reflector antennas. The first single layer elements employed in reflectarray antennas suffered from several drawbacks such as narrow bandwidth, lack of linearity in the phase curves, and phase range lower than 360 degrees. However, all these problems were solved to a large extent by the introduction of multilayered multi-resonant elements, and by the use of optimization techniques that choose the adequate element dimensions to compensate for the spatial phase delay at several frequencies [1]. In particular, the use of an element based on three stacked rectangular patches in conjunction with optimization techniques made it possible to design a contoured beam reflectarray antenna for space applications with two independent coverages (European and North American) and orthogonal polarizations in each coverage [2]. Also, the same element was used to design a contoured beam transmit-receive antenna for DBS applications with dual frequency dual polarization capabilities [3]. The main problem with the element based on three stacked patches is that it requires a complex manufacturing process derived from the bonding of three layers with printed elements. This manufacturing problem is partially alleviated if the stacked patches are substituted by two orthogonal sets of parallel dipoles at two metallization levels since only one layer with printed elements (at both sides) is required in this latter case. A broadband dual polarization reflectarray antenna based on an element with two orthogonal sets of three parallel dipoles has been presented in [4]. And an element containing two orthogonal sets of four parallel dipoles that are shifted half a period has been used in [5] to design the dual frequency dual polarization reflectarray antenna with South American coverage that is reported in [3]. Apart from a simplification in the manufacturing process, the elements based on two sets of parallel dipoles have a better electrical performance than the element based on stacked patches since the former lead to lower cross-polarization [5].

When designing a reflectarray made of either stacked rectangular patches or two sets of parallel dipoles, it is necessary to choose the sizes of the patches/dipoles that lead to the appropriate reflection phase. In the determination of the sizes of the metallizations of a certain cell, it is common practice to assume that this cell is surrounded by an infinite periodic array of cells of the same dimensions. This simplification is known as the local periodicity assumption. The assumption is justified by the fact that it leads to theoretical antenna designs that show a good agreement with measurements [2]–[5]. When the local periodicity assumption is used in the design of a frequency optimized reflectarray made of patches or dipoles, the numerical analysis of the scattering of a plane wave obliquely incident on periodic multilayered structures has to be carried out a huge number of times. Therefore, very efficient numerical tools are required for the solution of this scattering problem. Although the numerical method traditionally employed for the characterization of reflectarray cells in multilayered periodic environments is the spectral domain MoM [6], [7], Florencio et al. very recently introduced a hybrid MoM for the analysis of multilayered periodic structures containing stacked patches [8]. In this hybrid MoM, whereas the matrix entries involving basis functions at different metallization levels are computed in the spectral domain as quickly convergent double infinite summations, the matrix entries involving basis functions at the same metallization level are determined in the spatial domain as double integrals that require low-order quadrature rules. The singularities of these integrals are handled by means of the singularity extraction technique and by the use of Ma-Rokhlin-

Wandzura (MRW) quadrature rules [9]. When the hybrid MoM has been applied to the design of a reflectarray antenna made of stacked patches, it has proven to be between one and two orders of magnitude faster than the spectral domain MoM [8]. In this paper, we extend the hybrid MoM introduced in [8] to deal with multilayered periodic structures containing two orthogonal sets of parallel dipoles at two metallization levels. Two novelties are introduced with respect to the formulation presented in [8]. First, new cross-correlations are derived for the basis functions of different coplanar dipoles, which can be expressed in terms of complete elliptic integrals of first, second and third kind [10]. Second, a 4-D interpolation of the spatial domain periodic Green's functions is carried out in terms of Chebyshev polynomials. This interpolation not only involves the spatial domain variables (as in [8]) but also the angles of incidence of the impinging wave, which makes it possible to considerably reduce the total number of Green's functions that has to be computed for the design of a complete reflectarray antenna. Thanks to this interpolation, the novel hybrid MoM has been applied to the design of the focused beam reflectarray antenna fabricated and measured in [4], and it has proven to be around twenty times faster than the standard spectral domain MoM.

II. BRIEF DESCRIPTION OF THE NUMERICAL PROCEDURE

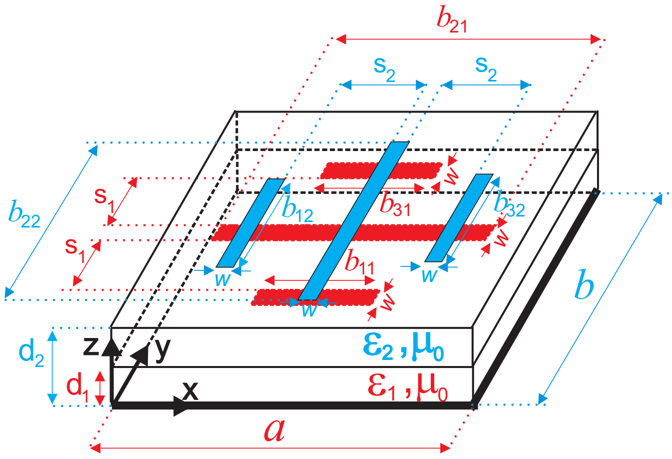


Fig. 1. Multilayered reflectarray cell containing two orthogonal sets of three parallel dipoles.

Fig. 1 shows the element used to design the reflectarray antennas studied in [4]. This element consists of two sets of three parallel dipoles allocated at the two dielectric interfaces of a two-layered substrate. The dimensions of one set of dipoles are used to control the phase-shift in one polarization, and the dimensions of the other set control the phase shift in the orthogonal polarization. Let us assume that this reflectarray element is placed in a periodic environment, and that a plane wave impinges on the multilayered periodic structure with oblique incidence direction characterized by the angular spherical coordinates θ_{inc} and φ_{inc}. In order to obtain the reflection matrix of the cell (see [5, eqn. 1]) that is required for the design of a reflectarray antenna under the local periodicity condition, we need to determine the surface current density on the dipoles. Let $\mathbf{J}_l^j(x,y)$ be the surface current density of the l-th dipole ($l = 1,2,3$) placed at the j-th dielectric interface ($j = 1,2$). If the dipoles are assumed to be PEC with negligible thickness, the six vector quantities $\mathbf{J}_l^j(x,y)$ can be obtained as the solution of the following set of two coupled electric field integral equations (EFIE)

$$\hat{\mathbf{z}} \times \left[\mathbf{E}^{\mathrm{ms}}(x,y,z=d_i) + \sum_{j=1}^{2} \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \sum_{l=1}^{3} \int_{S_{jl}^{mn}} \overline{\mathbf{G}}^{E}(x-x', y-y', z=d_i, z'=d_j) \cdot \mathbf{J}_l^j(x',y')\, dx'dy' \right] = \mathbf{0}$$

$$(x,y) \in S_{ik}^{00} \qquad (i = 1,2;\ k = 1,2,3) \qquad (1)$$

where S_{jl}^{mn} is the surface of the l-th dipole located at the j-th dielectric interface of the mn-th periodic cell, $\overline{\mathbf{G}}^{E}(x-x', y-y', z, z')$ is the dyadic Green's function of the multilayered substrate [8], and $\mathbf{E}^{\mathrm{ms}}(x,y,z)$ is the electric field created by the impinging wave in the absence of the patches. In order to solve the two coupled EFIE, we have expanded the vector quantities $\mathbf{J}_l^j(x,y)$ in terms of Chebyshev polynomials weighted by functions that account for the current density edge singularities as in [8, Eqns. (30) and (31)], which ensures a fast convergence of MoM with respect to the number of basis functions [8, Fig. 7]. Then, the unknown weighting coefficients of the basis functions have been determined by means of the hybrid Galerkin's version of MoM which is described in [8, Eqns. (4) to (15)]. In particular, the MoM matrix entries involving basis functions of dipoles located at different dielectric interfaces ($i \neq j$ in (1)) have been computed as double infinite summations with fast exponential convergence [8, Eqn. (7)]. However, the MoM matrix entries involving basis functions of dipoles located at the same dielectric interface ($i = j$ in (1)) have been computed as double finite integrals containing the correlations between basis functions times the periodic Green's functions for the scalar and vector potential [8, Eqns. (9) to (15)]. The cross-correlations between basis functions belonging to the same dipoles have been obtained in terms of complete elliptic integrals of the first and second kind [8, Eqn. (33)]. The cross-correlations between basis functions belonging to different dipoles were more difficult to obtain, but we have finally succeeded to express them in terms of complete elliptic integrals of first, second and third kind [10]. The complete elliptic integrals of third kind have been expressed in turn as complete and normal elliptic integrals of first and second kind [10], for which fast computation routines are available. The double integrals leading to the MoM matrix entries involving basis functions of different dipoles show logarithmic singularities that have been conveniently handled by means of Ma-Rokhlin-Wandzura (MRW) quadrature rules [9].

The efficient computation of the multilayered periodic Green's functions involved in the spatial domain MoM matrix entries is a crucial step in the application of the hybrid MoM to multilayered periodic structures. These Green's functions were accurately interpolated in 2-D [8] in terms of the spatial coordinates after extraction of the source singularity and the first quasidynamic images through the multilayered substrate [8, Eqns. (24) to (27) and (29)]. When the 2-D interpolation is used, a different set of Green's functions for the potentials have to be computed for each element of the antenna since the periodic Green's functions depend on the incidence angles of the impinging wave $\theta_{\rm inc}$ and $\varphi_{\rm inc}$, and these incidence angles vary from element to element. In order to reduce the total number of periodic Green's functions required for the design of the antenna, in this paper we have carried out an accurate 4-D interpolation of the periodic Green's functions, not only in terms of the spatial coordinates as in [8], but also in terms of the incidence angles. The 4-D interpolation has made it possible to reduce the computation of the total number of different periodic Green's functions that are required for the design of a whole reflectarray antenna. For instance, in the case of an antenna with roughly one thousand elements such as those studied in [4] and [5], the 4-D interpolation leads to a reduction of the number of required Green's function by a factor of twenty.

III. NUMERICAL RESULTS

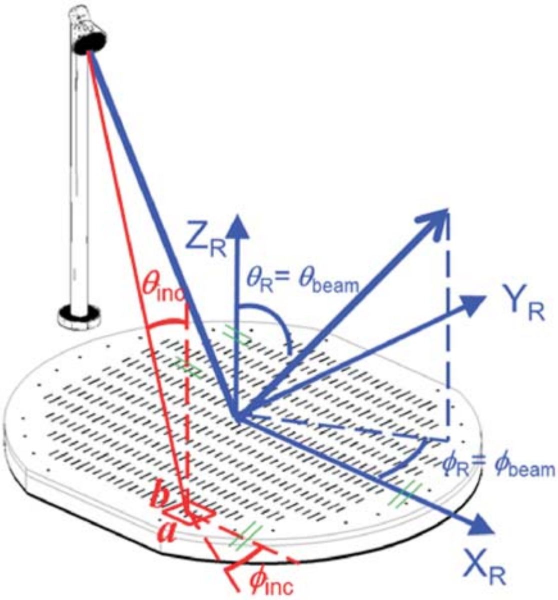
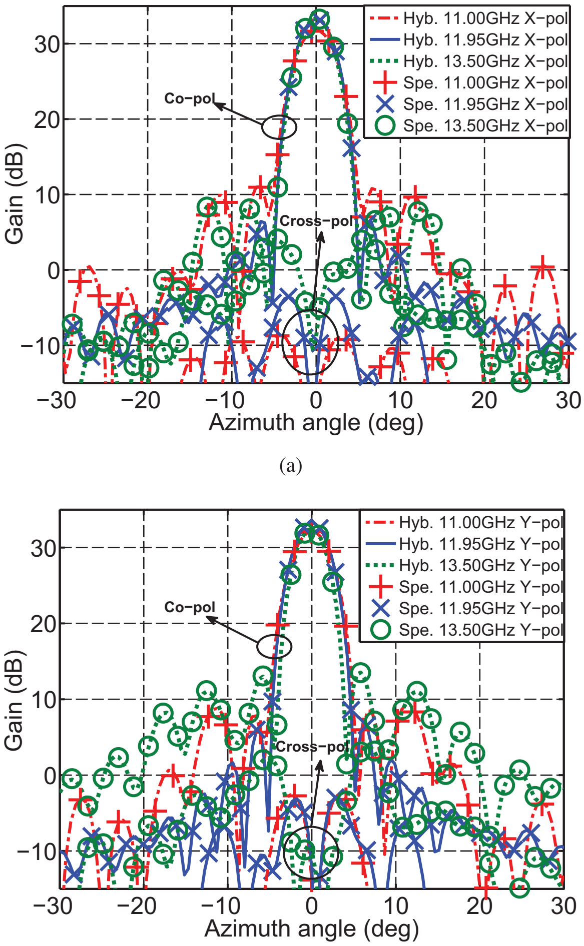


Fig. 2. Reflectarray made of elements with two orthogonal sets of three parallel dipoles.

A reflectarray antenna similar to that measured in [4, Figs. 6 and 7] has been designed under the local periodicity assumption with both the standard spectral domain MoM [6], [7] and the hybrid MoM of Section II. As shown in Fig. 2, the reflectarray is based on the element with two orthogonal sets of dipoles depicted in Fig. 1. The reflectarray has been chosen to generate a focused beam in the direction $\theta_{\rm beam} = 16.9^\circ$ and $\varphi_{\rm beam} = 0^\circ$ at 11.95 GHz for both linear polarizations. It has a circular shape and consists of 861 elements arranged in a 33×33 grid with cell size $a = 12$ mm $\times$ $b = 12$ mm (diameter of 396 mm). The reflectarray is fed by a circular corrugated horn with its phase centre located at $(x_{\rm R}, y_{\rm R}, z_{\rm R})$=$(-0.193, 0.0, 0.635)$ meters with respect to a coordinate system with origin at the reflectarray center (see Fig. 2). The feeding horn provides an illumination of -11 dB at the antenna edge. The substrate consists of two layers of Diclad® 880 ($\varepsilon_{r1} = \varepsilon_{r2} = 2.17$ and $\tan\delta_1 = \tan\delta_2 = 0.0009$) of thicknesses $d_1 = 3.175$ mm and $d_2 - d_1 = 0.127$ mm. For each element of the reflectarray, the lengths of the central dipoles b_{21} and b_{22} have been adjusted by a zero finding routine which iteratively calls the MoM software employed (spectral domain or hybrid) to achieve the required phase for X and Y polarizations at 11.95 GHz. While b_{21} and b_{22} are varied, we have kept $b_{11} = b_{31} = 0.63 b_{21}$, $b_{12} = b_{32} = 0.63 b_{22}$, $s_1 = s_2 = 3.5$ mm and $w = 0.5$ mm.

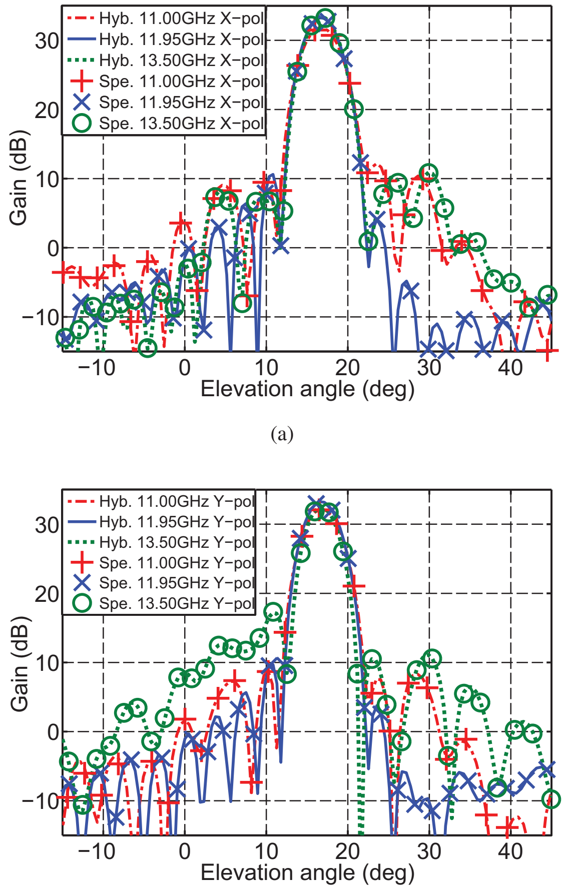


(a)

(b)

Fig. 3. Simulated radiation patterns in azimuth plane for X polarization (a) and Y polarization (b). Solid line, dotted line, and dots and dashes stand for the results obtained with the hybrid MoM. Symbols (×, ○ and +) stand for the results obtained with the spectral MoM.

Figs. 3 and 4 show the radiation patterns obtained in the design of the antenna of Fig. 2 with both the spectral domain



(a)

(b)

Fig. 4. Simulated radiation patterns in elevation plane for X polarization (a) and Y polarization (b). Solid line, dotted line, and dots and dashes stand for the results obtained with the hybrid MoM. Symbols (×, ○ and +) stand for the results obtained with the spectral MoM.

MoM and the hybrid MoM under the local periodicity condition. The numerical results indicate that the antenna shows a bandwidth of at least 21% for a gain variation smaller than 1 dB, cross-polar discrimination better than 33 dB and side lobe level smaller than 20 dB as can be checked with the measurements presented in [4, Figs. 6 and 7]. The results obtained with both the spectral domain MoM and the hybrid MoM are superimposed, which is an additional validation of both approaches. Table I shows the CPU time gain achieved with the hybrid MoM with respect to the spectral domain MoM when both 2-D and 4-D interpolations of the spatial Green's functions are carried out in the hybrid MoM. Note that whereas the hybrid MoM is roughly 10 times faster than the spectral domain MoM when 2-D interpolations are used, the hybrid MoM becomes more than 20 times faster when 4-D interpolations are used. As commented above, this is because the number of Green's functions that have to be computed per reflectarray element considerably diminishes when the 4-D interpolation is applied instead of the 2-D interpolation.

TABLE I

RATIOS BETWEEN THE CPU TIMES EMPLOYED BY THE SPECTRAL DOMAIN MOM AND THE HYBRID MOM FOR THE DESIGN OF THE REFLECTARRAY ANTENNA OF FIG. 2.

$T_{\mathrm{spectral}}/T_{\mathrm{hybrid}}^{2-\mathrm{D}}$	9.6
$T_{\mathrm{spectral}}/T_{\mathrm{hybrid}}^{4-\mathrm{D}}$	21.6

IV. CONCLUSION

In this paper the authors apply a hybrid spectral domain-spatial domain MoM to the analysis of multilayered periodic structures containing dipoles at two metallization levels. The hybrid MoM is used for the design of a dual polarization reflectarray antenna based on a cell with two orthogonal sets of three parallel dipoles. Thanks to the use of a 4-D interpolation of the spatial Green's functions, the hybrid MoM turns out to be around twenty times faster than the standard spectral domain MoM in the design of the antenna.

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