Improvements in the MoM Analysis of 2-D Planar Multilayered Periodic Structures

Rafael Florencio , Rafael R. Boix , and José A. Encinar

***Abstract*—In this paper the authors apply the Method of Moments (MoM) to the analysis of the scattering of a multilayered periodic strip grating by a plane wave with oblique incidence and arbitrary polarization. It is shown that the use of an improved MoM spatial domain approach is one order of magnitude faster than the use of the classical MoM spectral domain approach when basis functions that account for edge singularities are used in the modeling of the current density on the metallizations.**

***Index Terms*—periodic strip gratings, multilayered media, integral equations**

I. Introduction

Periodic strip gratings have been traditionally used as frequency selective surfaces and polarization selective surfaces [1]. Also, they have found an application as twist reflectors for reflector antennas [2], and as hard/soft surfaces [2] for horn antennas. The analysis of the scattering of periodic strip gratings by plane waves has been usually carried out by means of the MoM in the spectral domain both in free-space [1] and in the case of gratings printed on one dielectric layer [2]. However, the application of the MoM in the spectral domain leads to the computation of infinite summations with slow convergence. In this paper we carry out the analysis of the scattering of periodic strip gratings embedded in multilayered substrates by means of an improved MPIE formulation of the MoM in the spatial domain. It is shown that the improved spatial domain version of the MoM can be up to one order of magnitude faster than the spectral domain version.

II. Theory and Numerical Results

Figs. 1(a) and (b) show a strip grating embedded in a multilayered substrate. A plane wave impinges on the strip grating with oblique incidence and arbitrary polarization. The state of polarization is characterized by the angle γ defined in [1]. Assuming the strips are perfect electric conductors (PEC), the current density on the strips $\mathbf{J}(x,y)$ can be obtained as the solution of the following electric field integral equation (EFIE)

$$\hat{\mathbf{z}} \times \left[\mathbf{E}_{\text{exc}}(x,y,z=-h_P) + \sum_{m=-\infty}^{+\infty} \int_{b_1+ma}^{b_1+ma+w} \int_{-\infty}^{+\infty} \right.$$

$$\left. \overline{\mathbf{G}}^{E}(x-x', y-y', z=-h_P | z'=-h_P) \cdot \mathbf{J}(x',y') dx' dy' \right]$$

$$= 0 \qquad b_1 < y < b_1 + w; \; -\infty < x < +\infty \qquad (1)$$

where $\overline{\mathbf{G}}^{E}(x-x', y-y', z|z')$ is the dyadic Green's function

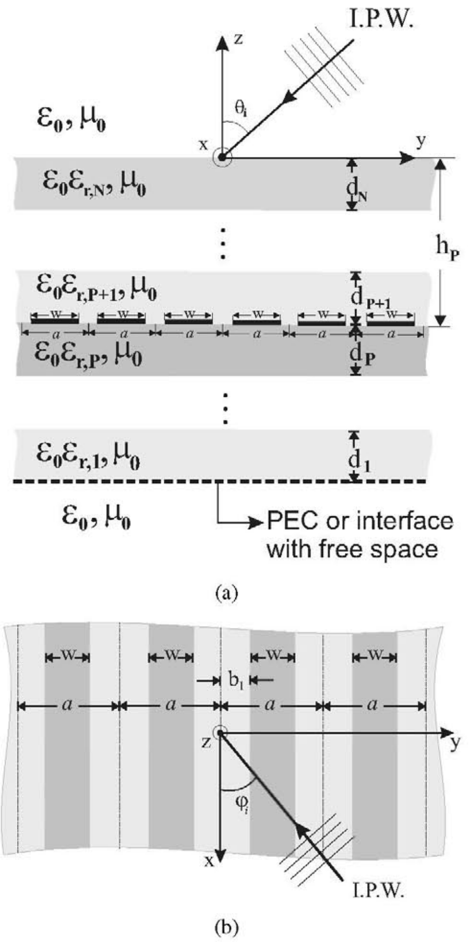

Fig. 1. Side view (a) and top view (b) of a periodic strip grating embedded in a multilayered substrate. A plane wave impinges on the periodic strip grating in a direction given by the spherical coordinate angles θ_i and φ_i.

of the multilayered substrate and $\mathbf{E}_{\text{exc}}(x,y,z)$ is the electric field created by the impinging wave in the absence of the periodic strips. Let us assume the current density on the strips is approximated in terms of basis functions as

$$\mathbf{J}(x,y) = \sum_{j=1}^{N} a_j \left(\mathbf{J}_j(y) e^{jk_0 \sin\theta_i \cos\varphi_i x} \right) \qquad (2)$$

Then, the unknown coefficients $a_j\,(j=1,\ldots,N)$ can be obtained by applying the Galerkin's version of the MoM to the EFIE (1). As a result of the application of the MoM, a system of linear equations is obtained for a_j. It can be shown that the elements of the coefficient matrix of this system of equations can be obtained in a simple way in the spectral domain as

$$\Gamma_{ij} = a \sum_{m=-\infty}^{+\infty} \left[\left(\tilde{\mathbf{J}}_i^{\mathrm{d}}(k_{ym}) \right)^* \right]^t \cdot \tilde{\overline{\mathbf{G}}}^{E,\mathrm{c}}(k_x = k_0 \sin\theta_i \cos\varphi_i,$$

$$k_y = k_{ym}, z = -h_P | z' = -h_P) \cdot \tilde{\mathbf{J}}_j^{\mathrm{d}}(k_{ym}) \quad (i,j=1,\ldots,N) \quad (3)$$

where $k_{ym} = k_0 \sin\theta_i \cos\varphi_i + 2\pi m/a$, $\tilde{\mathbf{J}}_i^{\mathrm{d}}(k_{ym})$ is the discrete Fourier transform of $\mathbf{J}_i(y)$ and $\tilde{\overline{\mathbf{G}}}^{E,\mathrm{c}}(k_x,k_y,z|z')$ is the continuous 2-D Fourier transform of $\overline{\mathbf{G}}^E(x-x',y-y',z|z')$ ($\tilde{\overline{\mathbf{G}}}^{E,\mathrm{c}}$ can be obtained in closed-form for a multilayered medium). The problem with (3) is that the infinite summations involved in the computation of Γ_{ij} are slowly convergent.

In the frame of this paper, we have been able to prove that the coefficients Γ_{ij} can be expressed as

$$\Gamma_{ij} = \left(-j\omega S_{ij}^A - \frac{1}{j\omega} S_{ij}^\phi \right) \quad (4)$$

where S_{ij}^A and S_{ij}^ϕ are given by the integrals

$$S_{ij}^A = \int_{-w}^{+w} G_{xx}^{A,\mathrm{P}}(y, z=-h_P|z'=-h_P) f_{ij}^A(y)\, dy \quad (5)$$

$$S_{ij}^\phi = \int_{-w}^{+w} G^{\phi,\mathrm{P}}(y, z=-h_P|z'=-h_P) f_{ij}^\phi(y)\, dy \quad (6)$$

In Eqns. (5) and (6) $G_{xx}^{A,\mathrm{P}}(y,z|z')$ and $G^{\phi,\mathrm{P}}(y,z|z')$ stand for 2-D periodic multilayered Green's function for the x component of the vector potential and the scalar potential respectively (MPIE formulation [3]). Also, $f_{ij}^A(y)$ stand for correlations between the basis functions for the current density, and $f_{ij}^\phi(y)$ stand for correlations between the basis functions for the charge density on the strips (which can be obtained in terms of the basis functions for the current density by invoking the continuity equation). Although $G_{xx}^{A,\mathrm{P}}$ and $G^{\phi,\mathrm{P}}$ can only be expressed in terms of slowly convergent series, in this paper we have substantially accelerated the convergence of these series by using Kummer's transformation, exponential approximations and Ewald's method as in [4]. Also, $G_{xx}^{A,\mathrm{P}}$ and $G^{\phi,\mathrm{P}}$ have been interpolated in terms of y after regularization as in [5]. Finally, we have been able to obtain closed-form expressions of $f_{ij}^A(y)$ for sinusoidal basis functions [6] and for edge singularity basis functions weighted by Chebyshev polynomials [6]. All these improvements have made it possible that the computation of Γ_{ij} by means of Eqns. (4) to (6) can be carried out within a short CPU time.

Table I shows results for the phase of the reflection coefficient of a strip grating on a grounded four layered substrate. These results have been obtained with both the spectral domain approach (Eqn. (3)) and the spatial domain approach (Eqn. (4)). Also, the results have been obtained for the two type of basis functions aforementioned: sinusoidal basis functions (SBF) and edge singularity basis functions (ESBF). The CPU times provided are for a a laptop computer with a 32 bits Intel Core Duo 2.66 GHz processor and 4 GB RAM. Note that whereas convergence in the value of $\angle R_0$ within 6 significant figures is obtained with 7 ESBF, only convergence within 2 significant figures is obtained with 50 SBF. This means that ESBF provide a much faster convergence than SBF, which is a well known fact. Also, note that whereas the CPU times required by both the spectral and the spatial approach are similar for SBF, the CPU time required by the spectral approach is typically one order of magnitud larger than that required by the spatial approach for ESBF. This is due to the fact that the infinite summations of (3) take much longer to converge for ESBF than for SBF.

TABLE I

PHASE OF THE REFLECTION COEFFICIENT $\angle R_0$ OF A PERIODIC GRATING ON A GROUNDED FOUR-LAYERED SUBSTRATE. THE RESULTS SHOWN HAVE BEEN OBTAINED WITH SINUSOIDAL BASIS FUNCTIONS (SBF) AND EDGE-SINGULARITY BASIS FUNCTIONS (ESBF). THE CPU TIMES OBTAINED WITH (3) AND (4) ARE ALSO SHOWN. PARAMETERS: PEC AT THE BOTTOM, $P=N=4$, $d_4=0.7\,\mathrm{mm}$, $d_3=0.3\,\mathrm{mm}$, $d_2=0.5\,\mathrm{mm}$, $d_1=0.3\,\mathrm{mm}$, ϵ_{r4}=2.1, ϵ_{r3}=12.5, ϵ_{r2}=9.8, ϵ_{r1}=8.6, $f=10\,\mathrm{GHz}$, $a=0.5\lambda_0$, $w=0.2a$, $\theta_i=0^\circ$, $\varphi_i=90^\circ$, $\gamma=90^\circ$.

N	$\angle R_0$	CPUT (s) with (3)	CPUT (s) with (4)
49 (SBF)	-57.5140°	0.343	0.359
59 (SBF)	-57.5350°	0.530	1.170
7 (ESBF)	-57.6191°	0.215	0.021
9 (ESBF)	-57.6191°	0.260	0.023

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