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Domain Reduction Strategy for Stability Analysis of a NACA0012 airfoil

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Abstract

A new reduction domain strategy is applied in the framework of stability analysis for industrial applications. The aim of this work is to investigate if the geometrical approach proposed could be used as alternative or in combination with the numerical techniques that are currently employed for alleviating the prohibitive computational costs associated to the study of the stability of complex aerodynamic flows.

Global linear stability analysis has been applied to a NACA0012 airfoil to study the transient buffet phenomenon. The leading disturbance associated to buffeting remains located in a inner region of the computational domain, making this problem a perfect test case for the domain reduction strategy presented here. The base flow has been obtained using DLR-TAU code on a first fine mesh (100 chords) while different, smaller meshes have been used to perform the stability analysis. The eigenvalue problem associated to stability has been solved using a Krylov-Schur projection method with direct LU preconditioning. Results show that the reduction in the computational domain is an useful technique to recover disturbances bounded in a particular region of the aforementioned domain.

Keywords: Stability Analysis, Domain reduction, Buffet phenomenon.

1 Introduction

Flow unsteadiness are associated to undesirable load variations that could represent a serious problem for engineering applications. The flow instability analysis represents a powerful method for predicting the onset of flow bifurcations leading to this instabilities. Linear stability analysis, in the modal framework [2][3], is based on the decomposition of the flow variables as a sum of a steady base state and a small amplitude perturbation. When this decomposition is fed into the Navier-Stokes equations it results in a generalized eigenvalue problem (EVP). In the usual approach, the real part of the complex eigenvalue from the EVP represents the grow rate of the, initially small, disturbance while the imaginary part is responsible for the frequency. This technique could be successfully employed for problem with low degrees of freedom, which arise from simple flow configurations at relatives Reynolds numbers, such as the lid driven cavity proposed in [3][4]. However, the high memory demanded for solving EVP with high degrees of freedoms, makes these methods inefficient and even intractable for large 3D problems. Over the last decades, many efforts have been focus on detection of lower-memory demanding approaches, but the problem still remains open. Another possible and/or complementary approach is the domain reduction. In this case only part of the domain is considered, so the leading dimension of the EVP is reduced. Aims of this work is to investigate the applicability of this approach. Starting from a reference known fluid configuration, a smaller region of the domain will be studied in order to check if the stability analysis on the reduced domain is able to recover the stability proprieties of the flow in the region of interest. The transonic buffet phenomenon on the NACA0012 represents the perfect test case for this analysis. The Buffet phenomenon is a strong interaction between the boundary layer and the shock wave that appears on the airfoil during the transonic condition. As a consequence strong vibrations, called buffeting, appear affecting the structure of the wing. This phenomenon is strictly localized on the top of the airfoil, so the area of interest for domain reduction can be easily detected. An exhaustive stability analysis for this test case is available in [1].
Successful results on recovering the stability features in a reduced domain for this test case represent an important starting point for performing stability analysis of more complex geometries, currently not

affordable.

2 Mathematical definition of the Buffet Phenomenon

2.1 Transonic Buffet Phenomenon

The buffet phenomenon is defined as a strong shock boundary layer interaction during the transonic condition on an airfoil, see [5]. The phenomenon appears when the angle of attack α overcomes a specific critical value α_{cr} that depends on Mach number and Reynolds number, leading to a globally unstable flow configuration. More in detail, this cyclic and auto-feeded phenomenon starts after that the critical condition is reached, when the separation bubble starts to grow reducing the effective camber of the airfoil. As a consequence, the shock is shifted upstream, where the pre-shock Mach number is reduced, losing strength permitting the reattachment of the flow. This sudden flow motion is then followed by a shock wave downstream and the wave gains strength again. At this point the feedback loop is finished and the cycle restarts. The overall phenomenon appears as an oscillating shock wave that could damage the airfoil and is usually perceived by the pilot as a vibration in the controls.

2.2 The Mathematical model

In this work, the transonic buffet phenomenon is described by means of compressible Reynolds averaged Navier- Stokes equations (RANS). The turbulence model employed is the Spalart-Allmaras [6] with compressibility correction [7]. The set of equations can be written in compact form, as:

$$\frac{\partial}{\partial t}\int_{\Omega}\mathbf{q}d\Omega = -\int_{\partial\Omega}\bar{\bar{\mathbf{F}}}\cdot\mathbf{n}dS \tag{1}$$

whit $\mathbf{q}$ vector of the unknown flow variables:

$$\mathbf{q} = (\rho, \rho u, \rho v, \rho w, \rho E, \rho\nu)^T \tag{2}$$

Where Ω is the whole fluid domain, with $\partial\Omega$ as its boundary and $\mathbf{n}$ as outer normal.
In equation (2), ρ represents the fluid density, $(u, v, w)$are the velocity field components, ν is the turbulent kinematic viscosity calculated using the $S\acute{U}A$ turbulence model [6] and $E = C_vT + (u^2 + v^2 + w^2)/2$ is the total specific energy, where C_v is the specific heat capacity of the gas at constant volume and T is the temperature.
The spatial discretization of Ω domain is performed using a finite volume approach. More in detail,the whole domain Ω is divided into a finite number of subdomains Ω_i, each with N faces, and then a complementary dual mesh is defined over these subdomains. For each of these control volumes Ω_i, fixed in time and space, it is possible to express the variation of the fluid variables in time as follow:

$$\frac{\partial\mathbf{q}_i}{\partial t} + \frac{1}{|\Omega_i|}\sum_{j=1}^{N}U_j^F = 0 \tag{3}$$

Where $\mathbf{U}_j^F$ are the fluxes over the N boundary faces of the control volume Ω_i.
In order to obtain a steady-state solution, the time integration is performed using a pseudo time stepping τ, instead of t, into a backward Euler implicit scheme. In this case, the linear solver used for solving the implicit scheme is a Lower-Upper Symmetric Gauss Siedel Methods (LU-SGS) [8]. In order to improve the convergence, appropriated numerical techniques could be adopted, such as residual smooting, multigrid methods and local time stepping.
Equation (3) is then re-written in terms of steady solutions and pseudo time stepping, becoming:

$$\Omega_i\frac{\partial\bar{\mathbf{q}}_i}{\partial\tau} + \mathbf{R}_i = 0, \qquad \mathbf{R}_i = \frac{1}{|\Omega_i|}\sum_{j=1}^{N}U_j^F \tag{4}$$

Where the $\bar{\mathbf{q}}_i$ is the steady solution on the subdomain Ω_i, and $\mathbf{R}_i$ is its residual.
According with equation (4), the finite volume approach can be also written in the following compact form:

$$\mathbf{B}\frac{\partial \mathbf{q}}{\partial t} = \mathbf{R}(\mathbf{q}) \tag{5}$$

where B is the diagonal matrix which elements represent the volumes associated to each of the N finite cells of the discretized domain.
The mathematical problem definition is then closed imposing the boundary conditions on the body surfaces:

$$u = v = w = 0 \qquad \nu = 0 \qquad \frac{\partial T}{\partial \mathbf{n}} = \frac{\partial \rho}{\partial \mathbf{n}} = 0 \tag{6}$$

Finally a farfield boundary condition is imposed for the external contour $\partial\Omega_i$ of the Ω_i domain. For this last boundary condition,the crossing fluxes between the faces of contiguous subdomains are obtained using the AUSM Riemann solver [14], while the flow conditions outside the boundary faces are calculated according with the Whitfield theory [10].

3 Global Stability Analysis

The basic concept of global linear stability theory, in the modal framework [2][3] , is the decomposition of independent flow variables in the equations of motion in two parts. All quantities are represented as the sum of a $O(1)$ steady base state $\overline{q}$ and a small-amplitude $O(\varepsilon)$ time-dependent perturbation $\hat{q}$.

The separability of temporal and spatial derivatives in the Navier Stokes equations permits the introduction of an explicit temporal dependence of the disturbance quantities. These assumptions lead to the Ansatz:

$$q(\mathbf{x}, t) = \overline{q}(\mathbf{x}) + \varepsilon\widetilde{q}(\mathbf{x})exp(-i\omega t) \tag{7}$$

with $\varepsilon << 1$, and ω is the complex eigenvalue $\omega = \omega_r + i\omega_i$. With this representation, the imaginary component ω_i represents a frequency while the real part ω_r is the amplification/damping rate of the disturbance. The flow decomposition expressed in equation(7) is introduced in the equation (5), and a linearization around the steady state flow is performed, leading to the definition of the generalized eigenvalue problem:

$$A\widetilde{q} = \omega B\widetilde{q} \tag{8}$$

where B is the same volume diagonal matrix of the equation (5) and $A = \left[\frac{\partial \mathbf{R}}{\partial q}\right]_{\overline{q}}$ is the Jacobian matrix of the system that contains the stability properties of the flow.
In the system (8) the B matrix can not be singular, so the eigenvalue problem could also be expressed in its standard formulation:

$$\hat{A}\widetilde{q} = \omega\widetilde{q} \tag{9}$$

with

$$\hat{A} = (AB^{-1}) \tag{10}$$

For industrial CFD applications, the resulting eigenvalue system is complex, non-symmetric, sparse and very large, and its solution is therefore exceedingly expensive to compute. In the last decades, several efforts have been focus on detecting efficient algorithms, but this issue is still considered an open problem and the computational resources limitation has been regarded as the main bottleneck for the application of linear stability analysis to fully three-dimensional ows and complex geometries.

3.1 Solving the EVP

Due to the large leading dimension of the matrices in the equation (8), direct methods, like QZ algorithm, for recovering the whole spectrum are infeasible.
The usual procedure, in CFD industrial applications, involves the use of projection algorithms, which

project the system into a subspace which contains only some specific dominant directions of the system itself, defining a reduced eigenvalue problem. The solution of this reduced system permits to obtain only a selected portion of the spectrum. Projection is performed as follows:

$$V^T \hat{A} V = H \tag{11}$$

Where V is the $N \times M$ orthogonal projection matrix, M is the dimension of the subspace used for the projection, and H the $M \times M$ matrix of the reduced system.
One of the most efficient and reliable projection method is the Arnoldi algorithm [11]. In this case, the V matrix is built using a $m-$dimensional Krylov Subspace k_m, which use successive powers of the $\hat{A}$ matrix for defining the orthogonal base:

$$V = k_m(\hat{A}, x_1) = span\{x_1, \hat{A}x_1, \hat{A}^2 x_1, ..., \hat{A}^{m-1} x_1\} \tag{12}$$

Where x_1 is an initial, usually random, vector. The Arnoldi projection leads to a reduced eigenvalue problem where the dominant directions are associated to the first m largest eigenvalues of $\hat{A}$. An additional constrain is that, in the stability analysis, the region of interest is located close to the imaginary axis. That means where, for some configurations of the flow, the eigenvalues are related to modes that are unstable or close to be unstable. For this reason, a spectral transformation called $shift - inverse$ is usually associated to the Arnoldi projection. This technique replaces the $\hat{A}$ matrix with its inverse into the equation (12), taking advance of the relation that links the eigenvalues of a matrix with the eigenvalues of its inverse. In this way, the Arnoldi algorithm is forced to converge to the subspace associated to the first m smallest eigenvalue of the $\hat{A}$.
The disadvantage of this technique is that it requires the inversion of the $\hat{A}$ matrix. This is usually computed performing an LU decomposition, that represents the most expensive phase of the overall procedure for solving the eigenvalue problem in terms of memory demand.

4 Domain Reduction Strategy

Overall, the domain reduction strategy performed in this work could be reassumed into five steps.

- The computational domain is discretized using a two-dimensional non-structured mesh (reference mesh). The corresponding solution is recovered using TAU-DLR code.
- The stability analysis of the two-dimensional base flow is performed and the leading disturbances identified.
- A second smaller mesh is generated. This mesh comprised the region where the disturbances of interest from the previous stability analysis are located. This region could be re-meshed if necessary.
- The solution in the full computational domain is interpolated into the reduced mesh and new boundary conditions defined.
- The stability analysis of the flow in the reduced domain is performed and compared.

In the work here proposed the original domain is cut around the body for obtaining the reduced domain. No extra mesh is needed and the boundary conditions for the base flow in the cut mesh are extracted directly from the solution obtained on reference mesh. Then the stability analysis is performed on the reduced domain. The reason for applying this variant procedure is due to the good quality of the reference mesh for this specific test case of the NACA0012 airfoil, that is already sufficiently fine for performing the stability analysis. If a refinement of the original mesh was requested, an interpolation phase will be also request, adding complexity and introducing unwanted sources of error.

In fact, the domain reduction strategy proposed, in its generic formulation, aims to be primarily applied for cases where a stability analysis could be infeasible on the whole domain, due to the memory costs associated, and only a small portion of the domain is considered as region of interest that justifies a further refinement.

The analysis on the transonic buffet on NACA0012 airfoil, object of this paper, does not belong to this category of cases and has to be intended as a first step for understanding the applicability of the domain reduction methodology. The choice to avoid the interpolation phase could give a more clear overview on the effects and possible errors associated to the pure domain reduction.

As told before, TAU-DLR code, and internal UPM codes for cutting and mesh generation will be used. Stability analysis will be performed with in-house tools.

5 Results

In this section, the results of the stability analysis on the reference mesh are shown. The obtained solution will be used as reference for analysing the results obtained performing the stability analysis on the several reduced domains.
The transonic buffet phenomenon on NACA0012 airfoil here analysed has been already presented in [1], where the choices regarding the flow parameters are widely explained. In agreement with those, the Reynolds number based on the chord is set equal to 10^7, the Mach number is equal to 0.76 and the angle of attack α is set to 3.2 degrees.

5.1 Steady state calculations.

As the first step of the procedure, the whole flow domain is discretized on a hybrid circular mesh with radius equal to 100 chords, the mesh has been refined in the region where the shock wave appears, as it is shown on the right picture in figure 1. This choice is legitimate by the results presented in the same [1], where the circular domain of radius equal to 100 chords has been considered as sufficient large for not being affected by the imposition of the farfield boundary conditions.

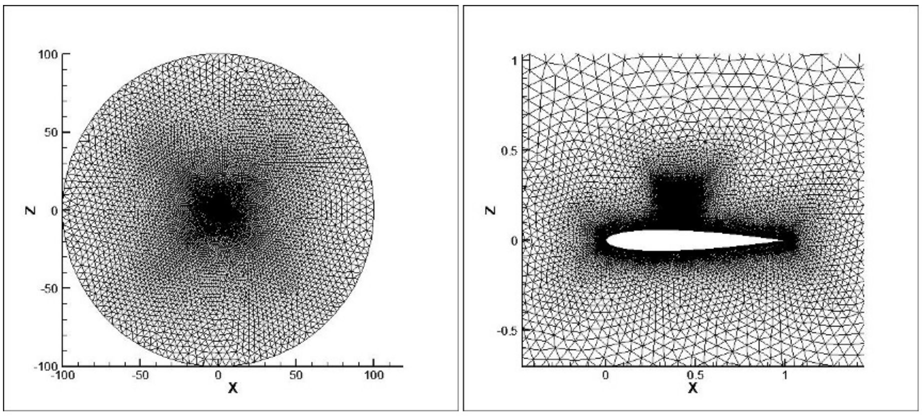

Figure 1: Hybrid mesh for the NACA0012 airfoil

To end up the first step of the procedure, the base flow solution for this NACA0012 airfoil configuration has been calculated using the aforementioned TAU-DLR code [8]. In pictures 2 and 4 the components of the steady base flow are depicted close to the airfoil surface. These results are, qualitatively and quantitative, in good agreement with those reported in [1].

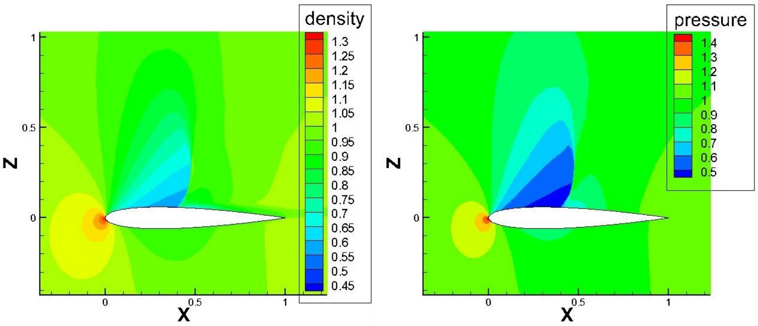

Figure 2: Base flow NACA0012: density(left), pressure(center).

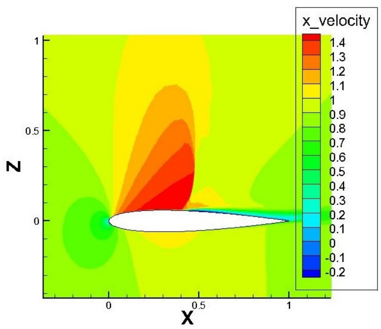

Figure 3: Base flow NACA0012:horizontal velocity(right).

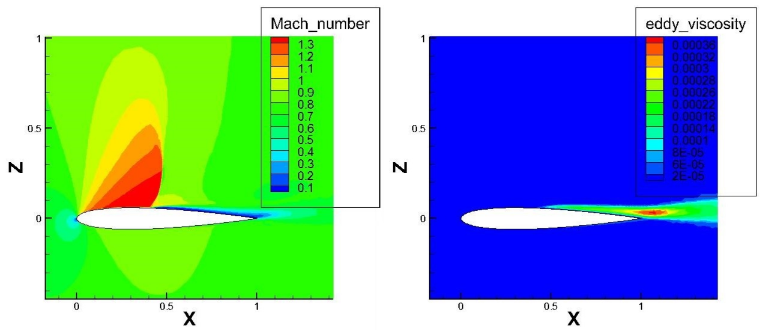

Figure 4: Base flow NACA0012: Mach number(left) eddy viscosity(right).

The reference mesh in figure 1 has been then cut for obtaining seven different reduced domains, with radius 90 chords, 80 chords, 70 chords, 60 chords, 50 chords, 40 chords and 30 chords respectively. Picture 5 shows the mesh for the particular domain of radius equal to 40 chords. It is important to remark, that the border of the domain is not regular. This is an intentional choice, since forcing the reduced domain into a regular, smooth contour could generate mesh elements of lower "quality" with respect to those of the original reference mesh. As told before the reference mesh is considered fine enough to do not require a further refinement around the airfoil, so the meshes for the domain reduction technique have been obtained simply cutting the reference mesh using a tool specifically developed for this work.

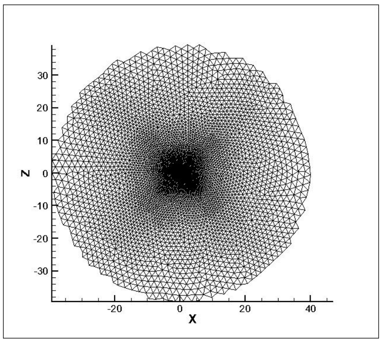

Figure 5: mesh for reduced domain of radius 40 chords.

Once the seven meshes for the reduced domains have been generated, the base flow solution for the NACA0012 airfoil has been calculated on them. Boundary condition for the farfield, in all the configurations, has been imposed using the corresponding value obtained in the original mesh. This particular treatment of farfield boundary conditions as Dirichlet boundary conditions, is possible because the base flow is, for definition, a steady flow solution. The external boundary values in the reduced domains will be the previously obtained in bigger domains.

5.2 Stability analysis on the reference mesh.

This subsection contains the results of the stability analysis performed on the whole fluid domain. The spectrum resulting from this analysis is shown on the left in figure 6. The eigenvalue associated to the buffet phenomenon, in this flow configuration, is the only one that lies in the positive region for the amplification/damping rate, in coordinates $[0.2365, 0.06157]$. The ρE component of its corresponding eigenvector, in the right side of figure 6, appears as a shock wave associated to the transonic phenomenon. It should be noticed that the structures related to the leading disturbances are located in a well resolved portion of the mesh. In agreement with the results in [1], the stability analysis shows that the flow configuration is actually unstable since not all the eigenvalues in the eigenvector belong to the negative part of the amplification/damping axis.

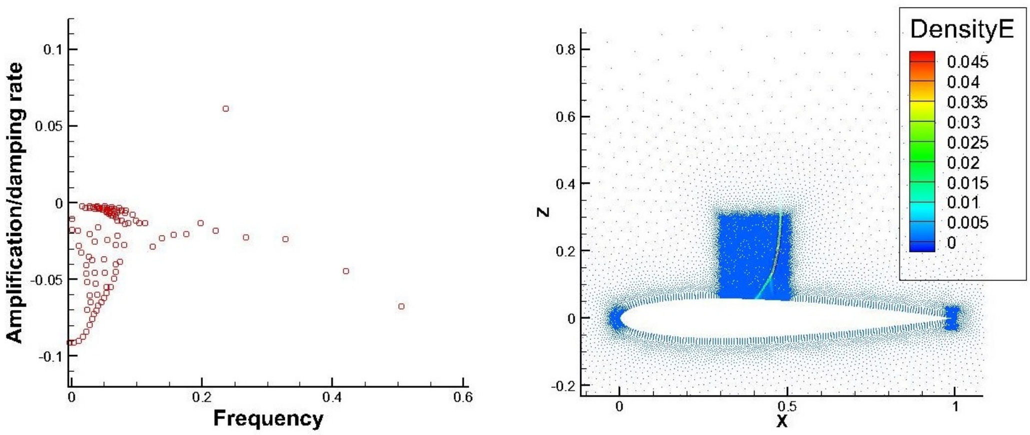

Figure 6: Spectrum solution and eigenvector of buffet transonic phenomenon on the NACA0012 airfoil. Reference mesh.

5.3 Stability analysis on the reduced domains.

For each reduced domain, the Jacobian has been extracted using TAU-DLR code from the base flow, and the stability analysis has been then performed. The results of all these stability analyses, summarized in pictures 7 and in table 1, shows that all the reduced domains identify the same eigenvalue associated to the buffet phenomenon. However it does not coincide exactly with the eigenvalue detected by the stability analysis on the reference mesh. The reason for this discrepancy comes from the use of different boundary conditions. A truly farfield condition is imposed in the outer boundary of the reference mesh but a Dirichlet-type boundary is used in the reduced domains.

In the same picture, another feature of the domain reduction technique is easily identified. While the buffet eigenvalue remains stabilized, the rest of the eigenspectrum is clearly affected by the reduction of the domain. This behaviour of the eigenspectrum is actually predictable, and it confirms that the eigenvalues associated to modes with structures acting in regions closed to the border of the computational domain are strongly affected by the domain reduction. Buffet phenomenon is associated to a mode whose structures are located and confined in the inner part of the mesh. This is also the reason why the reduction domain strategy should be applied only for detecting well isolated phenomenon inside the flow domain.

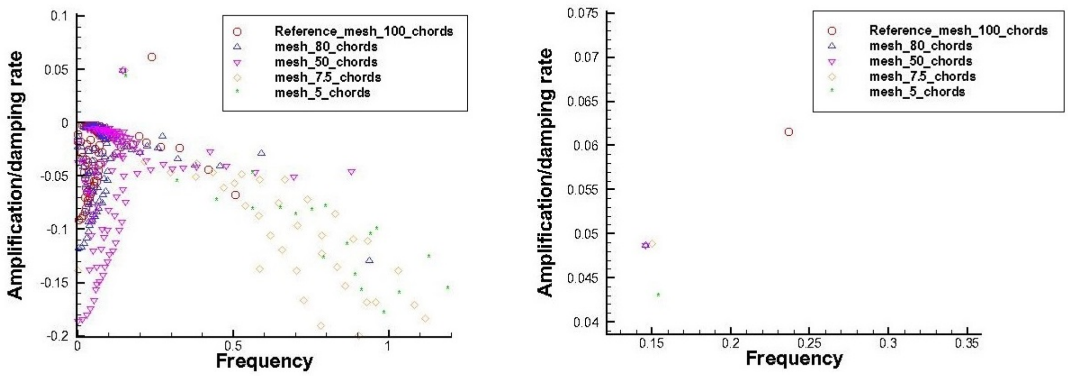

Figure 7: Spectrum solution of buffet transonic phenomenon on the NACA0012 airfoil.Comparison between reference and reduced meshes.

Buffet eigenvalue		
Mesh size (in chords)	Frequency	Damping rate
100	0.236497	6.15671(−2)
90	0.145751	4.86932(−2)
80	0.145751	4.86932(−2)
70	0.145751	4.86932(−2)
60	0.145752	4.86931(−2)
50	0.145748	4.86948(−2)
40	0.145765	4.86741(−2)
30	0.145738	4.88122(−2)
20	0.145527	4.81063(−2)
10	0.147597	5.03234(−2)
7.5	0.149906	4.89116(−2)
5	0.154082	4.29462(−2)
2	0.147835	−5.55713(−3)
1.5	0.121670	−5.21142(−2)

Table 1: Eigenvalue associated to the Buffet phenomenon, varying the dimension of the reduced domain

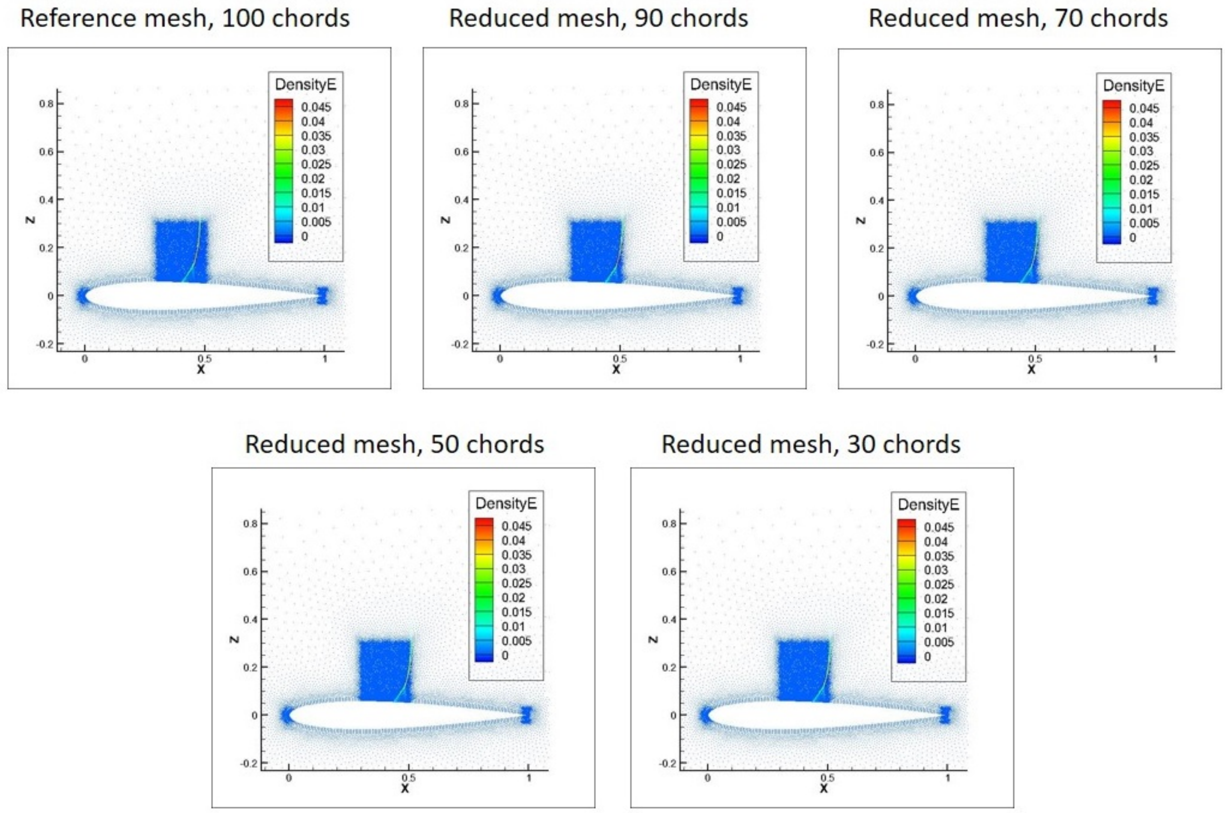

Figure 8: ρE eigenvector component of the buffet transonic phenomenon on the NACA0012 airfoil.Comparison between reference and reduced meshes.

The pictures in figure 8 show the ρE component of the eigenvector associated to the Buffet phenomenon, obtained by the stability analyses on the reference mesh and the reduced domains. Again, the results in all the reduced meshes are coherent, the structures are analogous not only for ρE but for all disturbance components (not shown). The previously commented discrepancy between the solution in the reference domain and those in the reduced domains manifests as a slight downstream shift of the shock wave on the reduced meshes with respect to the original location in the original mesh.

6 Conclusions

Two-dimensional global stability analysis has been performed for detecting the mode responsible of the transonic buffet phenomenon on the NACA0012 airfoil. To overcome the limitation of computational resources due to the huge memory requirements a domain reduction strategy has been implemented.
The results show that the stability analyses performed on the reduced domains are still able to predict the onset of instabilities and the amplification rate and frequency of the shock disturbance associated to the transonic phenomenon. Results regarding this leading mode are consistent for all the reduced domains presented in this work. This fact leads to a very first conclusion, domain reduction strategy is a powerful tool to analyse instabilities that remain confined in a specific region of the initial computational domain. However, and related to previous statement, stability analysis has shown quite sensitive to boundary conditions. The different treatment in the farfield condition between the original problem and the reduced ones translates in a modification of the position of the leading mode in the spectrum. This deviation does not affect the essential nature of the perturbation that remains unstable but provokes a slight change in the location of the structures along the airfoil.

As conclusion, the reduction domain strategy is here considered as a promising tool for overcoming the memory limitation on the applicability of the stability analysis in industrial problems. A deep study about the boundary condition treatment in the stability problem would bring more insight to the differences arisen in the analysis performed.

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