DOMINANCE MEASURING METHODS FOR THE SELECTION OF CLEANING SERVICES IN A EUROPEAN UNDERGROUND TRANSPORTATION COMPANY *

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Abstract. Dominance measuring methods are a recent approach for dealing with complex decision-making problems with imprecise, incomplete or partial information within multi-attribute value/utility theory. These methods compute pairwise dominance values and exploit the information included in the dominance matrix in different ways to derive measures of dominance intensity to rank the alternatives under consideration. We review dominance measuring methods proposed in the literature, describing how their possible drawbacks have been progressively overcome, and comparing their performance with other existing approaches, like surrogate weighting methods, the adaptation of classical decision rules to encompass an imprecise decision context, SMAA or Sarabando and Dias' method. An example of the selection of cleaning services in a European underground transportation company is used to illustrate dominance measuring methods in a real complex decision-making problem.

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1. Introduction

The additive model is widely used within multi-attribute value/utility theory (MAVT/MAUT) to rank alternatives in complex decision-making problems. It is considered a valid approach in many practical situations for the reasons described in [23, 29].

However, the information available in most real complex decision-making problems is not precise. Inputs are often described within prescribed bounds or as just satisfying certain relations. Different authors refer to this situation as *decision-making with imprecise information, incomplete information or partial information* [24, 25].

Several reasons are given in the literature that justify why a decision-maker (DM) may wish to provide imprecise information [28, 32]. For example, performances that reflect social or environmental impacts may be intangible or non-monetary, and performances may be taken from statistics or measurements. Besides, a DM might prefer not to reveal his/her preferences in public or could feel more comfortable providing a scale to represent attribute importance. Moreover, the decision could be taken in a group decision-making situation, where a negotiation process usually outputs imprecise information.

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Many papers on MAVT/MAUT have dealt with imprecise information. Sarabando and Dias [27] provided a brief overview of approaches proposed by different authors within the MAVT/MAUT framework to deal with imprecise information.

For instance, *surrogate weighting methods* (SWMs) deal with ranked attribute weights to output a best alternative and/or ranking of alternatives [4,31]. A weight vector is selected from a set of admissible weights to represent the set, which is then used to evaluate the alternatives by means of the multi-attribute value model. Commonly used SWMs are *rank sum weights* (*RS*), *rank reciprocal weights* (*RC*), *rank-order centroid weights* (*ROC*), and *equal weights* (*EW*).

The *stochastic multicriteria acceptability analysis* (*SMAA*) method was proposed to provide support when the weight information is missing [12]. The *SMAA-2* method [11] extends the analysis to the sets of weight vectors for any rank from best to worst for each decision alternative and can be used to identify good compromise alternatives, whereas *SMAA-O* [13] is a variant of SMAA for problems in which criteria are measured on ordinal scales.

Another option described in the literature for dealing with imprecision is based on the concepts of *pairwise* and *absolute dominance*.

The use of absolute dominance values is exemplified by the modification of four classical decision rules to encompass an imprecise decision context concerning weights and component values/utilities [22, 26]: the *maximax*, in which each alternative is rated on the basis of its maximum guaranteed value; the *maximin*, based on its minimum guaranteed value; the *minimax regret* rule, based on the maximum loss of value with respect to a better alternative; and the *central value* rule, based on the midpoint of the range of possible performances.

Regarding pairwise dominance, Eum *et al.* [5,14,21] provided linear programming characterizations of dominance and potential optimality for alternatives when information about values and/or weights is incomplete amd extended the approach to hierarchical structures [21]. Later, the concepts of *weak potential optimality* and *strong potential optimality* were developed in [14]. Besides, dominance and potential optimality with imprecision in weights, utilities and alternative performances is considered in [15].

A recent approach for dealing with imprecise information is to compute different measures of dominance to derive a ranking of alternatives. They are known as *dominance measuring methods* (*DMMs*). *DMMs* are based on the computation of a *dominance matrix*, D, including pairwise dominance values, which are exploited in different ways to derive measures of dominance to rank the alternatives under consideration.

In Section 1, we give an overview of the *DMMs* proposed in the literature and analyze their performance against the other existing approaches described above. An example concerning the selection of cleaning services in a European underground transportation company illustrates DMMs in Section 2. Finally, some conclusions are provided in Section 3.

2. Dominance measuring methods

We consider a decision-making problem with m alternatives $\{A_1, \ldots, A_m\}$ and n attributes $\{X_1, \ldots, X_n\}$, then the functional form of the additive model is

$$v(A_i) = \sum_{j=1}^{n} w_j v_j(x_{ij}),$$

where x_{ij} is the performance over attribute X_j for alternative A_i, and v_j and w_j are the component value/utility function and the weight for attribute X_j, respectively. Note that $\sum_{j=1}^{n} w_j = 1$ and $w_j \geq 0$.

The alternative performances could be precise or described under uncertainty by means of uniformly distributed *intervals*, *i.e.*,

$$x_{ij}^L \leq x_{ij} \leq x_{ij}^U, i = 1, \ldots, m, j = 1, \ldots, n.$$

Besides, imprecision could be also considered regarding DM preferences, both when assessing DM single attribute utilities, which represent DM preferences concerning the possible alternative performances, and weights, which represent the relative importance of criteria.
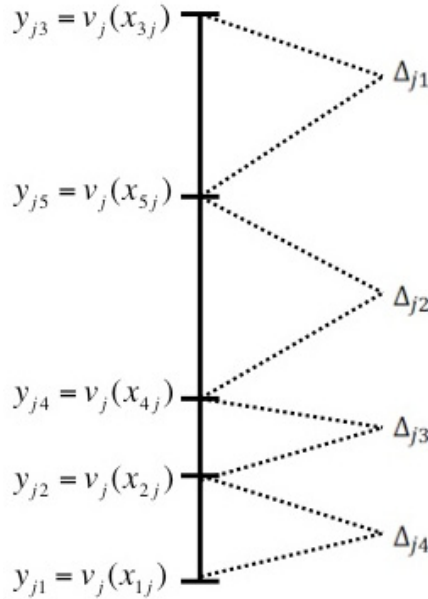
FIGURE 1. Ranking of alternatives and differences between consecutive alternatives for the attribute X_j.

In the first case, classes of component value/utility functions could be derived from the elicitation methods [6], and, consequently, the component value/utility associated with a specific performance would belong to an interval:

$$v_j^L(x_{ij}) \leq v_j(x_{ij}) \leq v_j^U(x_{ij}), i = 1, \ldots, m, \ \ j = 1, \ldots, n.$$

Alternatively, ordinal information about the component values/utilities of the alternatives could be considered [2, 28], *i.e.*, the DM provides a ranking of the alternatives in each attribute. Moreover, rankings of the difference between the values of consecutive alternatives could be also taken into account for each attribute. For instance, the DM might consider A_3 to be the best of five alternatives for attribute X_j, followed by A_5, A_4, A_2 and A_1 ($v_j(x_{3j}) \geq v_j(x_{5j}) \geq v_j(x_{4j}) \geq v_j(x_{2j}) \geq v_j(x_{1j})$). Also, the differences between consecutive alternatives might be ranked $\Delta_{j2} \geq \Delta_{j,1} \geq \Delta_{j,4} \geq \Delta_{j,3}$, with $\Delta_{j,2} = v_j(x_{5j}) - v_j(x_{4j})$, $\Delta_{j,1} = v_j(x_{3j}) - v_j(x_{5j})$, $\Delta_{j,4} = v_j(x_{2j}) - v_j(x_{1j})$ and $\Delta_{j,3} = v_j(x_{4j}) - v_j(x_{2j})$, as illustrated in Figure 1.

Imprecision concerning weights representing the relative importance of criteria could also be represented by intervals or by means of ordinal information. An example of an interval would be:

$$w_j^L \leq w_j \leq w_j^U, j = 1, \ldots, n,$$

If we consider *ordinal information about weights* [2, 27], the DM would provide an attribute importance ranking, arranged in descending order from the most to the least important attribute:

$$\mathbf{w} \in W = \{\mathbf{w} = (w_1, \ldots, w_n) | w_1 \geq w_2 \geq \ldots \geq w_n \geq 0\}, \text{ and } \sum_{j=1}^{n} w_i = 1.$$

As mentioned before, *dominance measuring methods* (*DMMs*) are based on the computation of a *dominance matrix*, D, including pairwise dominance values, which are exploited in different ways to derive measures of dominance to rank the alternatives under consideration.
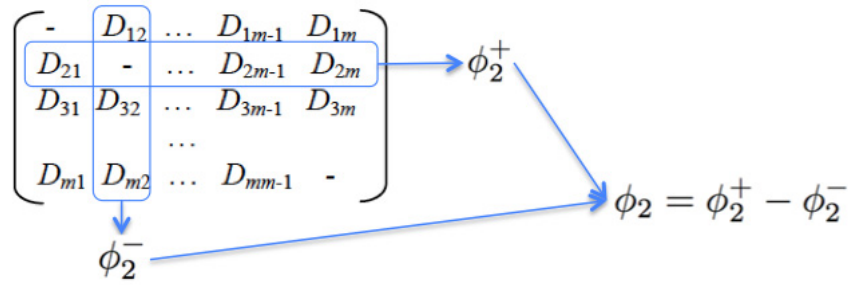
FIGURE 2. *AP1* and *AP2* methods.

A dominance matrix, D, can be defined as follows:

$$
D = \begin{pmatrix}
- & D_{12} & \cdots & D_{1(m-1)} & D_{1m} \\
D_{21} & - & \cdots & D_{2(m-1)} & D_{2m} \\
D_{31} & D_{32} & \cdots & D_{3(m-1)} & D_{3m} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
D_{m1} & D_{m2} & \cdots & D_{m(m-1)} & -
\end{pmatrix},
$$

where

$$
\begin{aligned}
D_{kl} = \quad & min\{\mathbf{w}\mathbf{v}_k - \mathbf{w}\mathbf{v}_l\} \\
s.t. \quad & \\
& \mathbf{v}_k = (v_{k1}, \dots, v_{kn}), \mathbf{v}_l = (v_{l1}, \dots, v_{ln}) \in V_{kl} \\
& \mathbf{w} = (w_1, \dots, w_n) \in W,
\end{aligned} \tag{2.1}
$$

where W and V_{kl} define the feasible region for weights and values associated with the alternatives A_k and A_l for each attribute, respectively. They represent imprecise information.

Note that given two alternatives A_k and A_l, alternative A_k dominates A_l if $D_{kl} \geq 0$, and there exists at least one $\mathbf{w}$, $\mathbf{v}_k$ and $\mathbf{v}_l$ such that the overall value of A_k is strictly greater than that of A_l. This concept of dominance is called *pairwise dominance*.

The optimization problems to be solved to derive pairwise dominance values are different depending on how the imprecision concerning DM preferences is represented, but always we have a linear or quadratic programming problem that can be solved using the *simplex method* or *Lemke's method*, respectively.

The first *DMM* was proposed by Ahn and Park [3]. It was applicable when the imprecision concerning the DM preferences was represented by both intervals or ordinal information.

Ahn and Park compute a dominating measure $\phi_k^+ = \sum\limits_{j=1,j\neq k}^{m} D_{kj}$ and a dominated measure $\phi_k^- = \sum\limits_{j=1,j\neq k}^{m} D_{jk}$ for each alternative A_k, and then derive a *net dominance* as $\phi_k = \phi_k^+ - \phi_k^-$.

Ahn and Park proposed two ranking methods for these measures: ranking the alternatives according to either ϕ_k^+ or ϕ_k values (denoted as the *AP1* and *AP2* methods, respectively, see Fig. 2).

Two new *DMMs* were proposed in [16,18]. The first one, *DIM1*, was based on the same idea as implemented by Ahn and Park. It also computed dominating and dominated measures but they were combined into a *dominance intensity* rather than a net dominance index, which was used as a measure of the strength of preference.

DIM1 was implemented as follows (see Fig. 3):

(1) Compute dominating indices for each alternative A_k:

$$
DI_{k+}^{row} = \sum_{l=1,l\neq k,D_{kl}>0}^{m} D_{kl}, \qquad DI_{k-}^{row} = \sum_{l=1,l\neq k,D_{kl}<0}^{m} D_{kl}.
$$

$$\begin{pmatrix} - & D_{12} & \dots & D_{1m\text{-}1} & D_{1m} \\ D_{21} & - & \dots & D_{2m\text{-}1} & D_{2m} \\ D_{31} & D_{32} & \dots & D_{3m\text{-}1} & D_{3m} \\ & & \dots & & \\ D_{m1} & D_{m2} & \dots & D_{mm\text{-}1} & - \end{pmatrix} \quad \begin{matrix} DI_{2+}^{row} \\ DI_{2-}^{row} \end{matrix} \rightarrow DI_{2}^{row}$$

$$DI_{2+}^{col} \ DI_{2-}^{col}$$
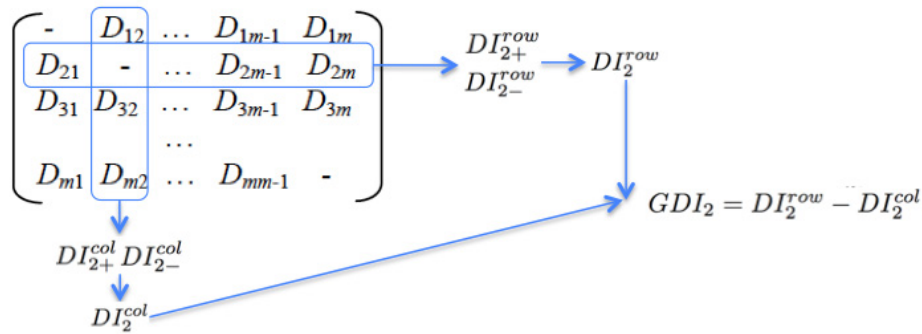$$DI_{2}^{col}$$

$$GDI_2 = DI_2^{row} - DI_2^{col}$$

FIGURE 3. *DIM1.*

(2) Compute the *dominating intensity* DI_k^{row} for each alternative A_k:

$$DI_k^{row} = \frac{DI_{k+}^{row}}{DI_{k+}^{row} - DI_{k-}^{row}}.$$

(3) Compute dominated indices for each alternative A_k:

$$DI_{k+}^{col} = \sum_{l=1, l\neq k, D_{lk}>0}^{m} D_{lk}, \qquad DI_{k-}^{col} = \sum_{l=1, l\neq k, D_{lk}<0}^{m} D_{lk}.$$

(4) Compute the *dominated intensity* DI_k^{col} for each alternative A_k:

$$DI_k^{col} = \frac{DI_{k+}^{col}}{DI_{k+}^{col} - DI_{k-}^{col}}.$$

(5) Calculate a *global dominance intensity* (GDI) for each alternative A_k, *i.e.*, $GDI_k = DI_k^{row} - DI_k^{col}, k = 1, \dots, m$, and rank the alternatives accordingly.

DIM1 improves *AP2* by reducing the duplicate information involved in the computations. However, *DIM1* has a drawback: the alternatives cannot be ranked if all the elements in D are negative because, since $DI_{k+}^{row} = 0$ and $DI_{k-}^{row}, \forall k$, their dominance intensity is zero in all cases. This drawback implies that *DIM1* is not independent of irrelevant alternatives.

The second method, *DIM2*, overcomes this drawback. It derives a *global dominance intensity index* to rank alternatives on the basis that

$$D_{kl} \leq \mathbf{w}^T (\mathbf{v}_k - \mathbf{v}_l) \leq -D_{lk}, \forall \mathbf{w} \in W, \mathbf{v}_k, \mathbf{v}_l \in V_{kl}.$$

DIM2 was implemented as follows (see Fig. 4):

(1) If $D_{kl} \geq 0$, then alternative A_k dominates A_l, and the dominance intensity of A_k over A_l (DI_{kl}) is 1, *i.e.*, $DI_{kl} = 1$.
 Else $(D_{kl} < 0)$:
 – If $D_{lk} \geq 0$, then alternative A_l dominates A_k, and $DI_{kl} = 0$.
 – Else $(D_{lk} < 0)$, $DI_{kl} = \frac{-D_{lk}}{-D_{lk} - D_{kl}}$.
(2) Calculate a *global dominance intensity* (GDI) for each alternative A_k, *i.e.* $GDI_k = \sum_{l=1, l\neq k}^{m} DI_{kl}$, and rank the alternatives accordingly.

Dominance Matrix   **Dominance Intensity Matrix**

$$\begin{pmatrix} - & D_{12} & \dots & D_{1m\text{-}1} & D_{1m} \\ D_{21} & - & \dots & D_{2m\text{-}1} & D_{2m} \\ D_{31} & D_{32} & \dots & D_{3m\text{-}1} & D_{3m} \\ & & \dots & & \\ D_{m1} & D_{m2} & \dots & D_{mm\text{-}1} & - \end{pmatrix} \Rightarrow \begin{pmatrix} - & DI_{12} & \cdots & DI_{1m} \\ DI_{21} & - & \cdots & DI_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ DI_{m1} & DI_{m2} & \cdots & - \end{pmatrix} \rightarrow DI_k = \sum_{l=1}^{m} DI_{kl}$$
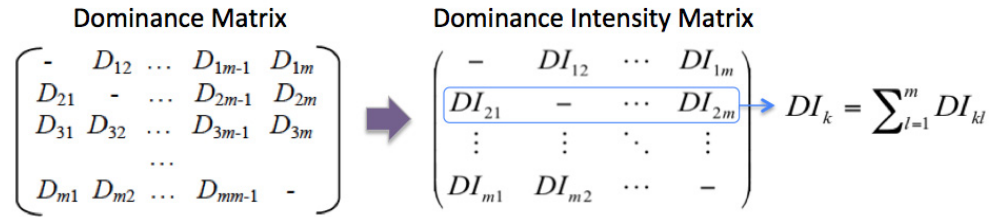
FIGURE 4. *DIM2.*

Simulation studies were carried out in [18] to compare the *DIM1* and *DIM2* methods with other existing approaches (the above modification of four classical decision rules, *SMAA* [12], *SMAA-2* [11] and the *AP1* and *AP2* methods) when the imprecision concerning the inputs is represented by value intervals.

    We use two measures of efficacy: the hit ratio and the rank-order correlation [3, 4]. The hit ratio is the proportion of all cases in which the method selects the same best alternative as in the TRUE ranking. Rank-order correlation indicates how similar the overall alternative ranking structures are in the TRUE ranking and in the ranking derived from the method. It is calculated using Kendall's τ [33]:

$$\tau = 1 - \frac{2 \times (\text{number of pairwise preference violations})}{\text{total number of pair preferences}} = \frac{S}{m(m-1)/2},$$

where S is the difference between the number of concordant (equally ordered) and discordant (differently ordered) pairs and m is the total number of alternatives.

    The results of simulation studies showed that *DIM2* performs better than the *AP1* method and the adaptation of classical decision rules. Although *SMAA-2* slightly outperforms *DIM2*, *DIM2* is applicable when incomplete information about weights is expressed not just as weight intervals but also as weights satisfying linear or non-linear constraints, weights represented by fuzzy numbers or weights fitting normal probability distributions.

    The performance of *DIM1* and *DIM2* was also compared in [20] with other existing approaches (surrogate weighting methods, modified classical decision rules and the *AP1* and *AP2* methods) when ordinal information represents imprecision concerning weights. As regards average hit ratios, *DIM2* and *ROC* outperform the other methods and, according to the *paired-samples t-test*, there is no significant difference between the two. However, *ROC* can be only applied when ordinal relations regarding attribute weights are provided.

    Despite its good performance, we later identified a drawback associated with *DIM2*. It did not take into account the size of the intervals $[D_{kl}, -D_{lk}]$ and how far they were from 0. Figure 5 illustrates the above situations in which different intervals lead to the same dominance intensities, D_{kl}. A new *DMM*, *DIM3*, was proposed in [17, 19] aimed at overcoming these drawbacks. It was based on *DIM2* and, specifically, on the fact that $\mathbf{w}^T (\mathbf{v}_k - \mathbf{v}_l) \in [D_{kl}, -D_{lk}], \forall \mathbf{w} \in W, \mathbf{v}_k, \mathbf{v}_l \in V_{kl}$, but *DIM3* incorporates the distance from the interval $[D_{kl}, -D_{lk}]$ to 0 to derive a *dominance intensity measure* to rank the alternatives under consideration:

(1) If $D_{kl} \geq 0$, then alternative A_k dominates A_l, and the dominance intensity of A_k over A_l is $DI_{kl} = d([D_{kl}, -D_{lk}], 0)$.
    Else ($D_{kl} < 0$):
    – If $D_{lk} \geq 0$, then A_l dominates A_k, and $DI_{kl} = -d([D_{kl}, -D_{lk}], 0)$.
    – Else ($D_{lk} < 0$),
$$DI_{kl} = \left[\frac{-D_{lk}}{-D_{lk} - D_{kl}} - \frac{-D_{kl}}{-D_{lk} - D_{kl}} \right] \times d([D_{kl}, -D_{lk}], 0).$$

(2) Calculate a global dominance intensity (GDI) for each alternative A_k, *i.e.*, $GDI_k = \sum_{l=1, \; l \neq k}^{m} DI_{kl}$, and rank the alternatives accordingly.
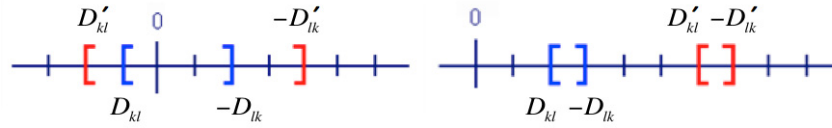
FIGURE 5. Drawbacks of *DIM2*.

New extensions of *DIM2* and *DIM3* were proposed in [1]. The *first extension* weights the dominance intensities derived in *DIM2* and *DIM3* according to the distance between the central weight vector (w_j^c) and the weight vector (w_j^{*lk}) associated with the optimal D_{lk} to solve the corresponding optimization problem. Note that the central weight vector is composed of the midpoints of the different weight intervals in the case of interval weights.

The aim of these extensions is to attach more importance to weight vectors closer to the central weight vector. To do this, the *DIM2* and *DIM3* methods must be applied, and the derived dominance intensities are then weighted using the following expression:

$$DI_{kl}^* = \frac{DI_{kl}}{d(w_j^c, w_j^{*lk})}.$$

The *second extension* is similar to the above in that weight vectors close to the central weight vector are given more importance. However, the weighting is now applied not to the dominance intensity values but to the pairwise dominance values in D. *DIM2* and *DIM3* are then applied as described before.

More recently, the same idea was extended concerning imprecision in DM preferences regarding both weights and component values/utilities. Instead of computing the pairwise dominance values (D_{kl}), the method calculates:

$$v_{kl} = \sum_{j=1}^{n} w_j^c v_{kj}^c - \sum_{j=1}^{n} w_j^c v_{lj}^c,$$

where $(w_1^c, \ldots, w_n^c)$ is the centroid or center of gravity of the polytope representing the weight space, and $(v_{k1}^c, v_{l1}^c), \ldots, (v_{kn}^c, v_{ln}^c)$ are the centroids or centers of gravity of the polytopes in the n attributes delimited by the constraints accounting for alternatives A_k and A_l. Note that the centroid is considered as the most representative point that verifies the constraints delimiting the polytope. Moreover, $D_{kl} \leq v_{kl} \leq -D_{lk}$.

The centroid of the polytope associated with constraints on component values in attribute X_j for the alternatives A_k and A_l is

$$\mathbf{v}_j^c = (v_{kj}^c, v_{lj}^c) = \frac{\int\limits_{[0,1]^2} V_j^{kl} dv}{\int\limits_{[0,1]^2} dv},$$

where V_j^{kl} is the set of constraints concerning component values in attribute X_j for alternatives A_k and A_l. Note that $V_j^{kl} \subset V_j$, which includes the constraints concerning component values in attribute X_j for all the alternatives.

A technique for finding all the endpoints from a polytope delimited by constraints representing ordinal information is proposed in [2]. These endpoints can then be averaged to derive the centroid of the polytope.

As the representation of a constraint set as just a point would be an oversimplification, an interval centered on the central value is built as follows:

$$I_{kl} = [I_{kl}^L, I_{kl}^U],$$

where $I_{kl}^L = v_{kl}^c - m_{kl}$ and $I_{kl}^U = v_{kl}^c + m_{kl}$, and

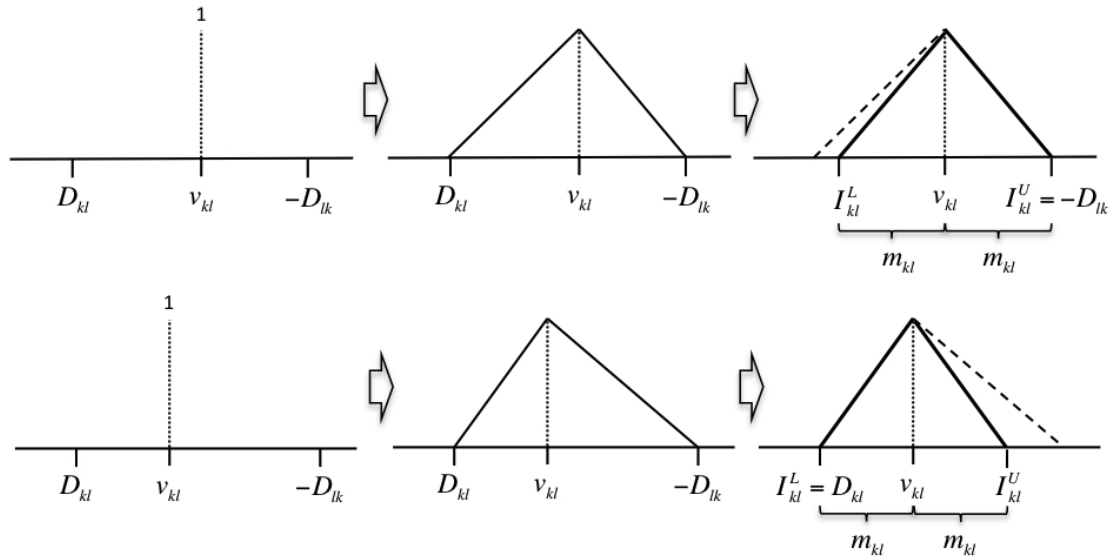$$m_{kl} = \min\{(-D_{lk} - v_{kl}^c), (v_{kl}^c - D_{kl})\}.$$

FIGURE 6. Building $\widetilde{I}_{kl}$.

Then the intervals $I_{kl} = [I_{kl}^L, I_{kl}^U]$ rather than $[D_{kl}, -D_{lk}]$ are used to compute the dominance intensities in *DIM3*.

In [2] the method is applied to deal with ordinal information in both weights and component values. The DM provides an attribute importance ranking. Besides, the method takes into account a ranking of the alternatives in each attribute and also a ranking of the difference of values between consecutive alternatives, as in Figure 1. Rather than intervals, however, triangular fuzzy numbers are built:

$$\widetilde{I}_{kl} = (I_{kl}^L, v_{kl}^c, I_{kl}^U),$$

with the membership function (see Fig. 6).

$$\mu_{\widetilde{I}_{kl}}(x) = \begin{cases} \dfrac{x - I_{kl}^L}{v_{kl}^c - I_{kl}^L}, & \text{if } I_{kl}^L \le x \le v_{kl}^c \\ 1, & \text{if } x = v_{kl}^c \\ \dfrac{x - I_{kl}^U}{v_{kl}^c - I_{kl}^U}, & \text{if } v_{kl}^c \le x \le I_{kl}^U \\ 0, & \text{otherwise.} \end{cases}$$

Then the *dominance intensities* are computed as follows:

- If $v_{kl}^c \ge 0$, then $DI_{kl} = d(\widetilde{I}_{kl}, 0, f)$, where d refers to Tran and Duckstein's distance [30], and f is a weight function for differentiating risk-averse, risk-neutral or risk-prone DMs, as explained later.
- Else ($v_{kl}^c < 0$), $DI_{kl} = -d(\widetilde{I}_{kl}, 0, f)$.

We compute a *dominance intensity measure* for each alternative A_k, $DIM_k = \sum_{l=1, l \ne k}^m DI_{kl}$, and rank alternatives according to DIM_k values.

As mentioned above, the method is based on Tran and Duckstein's distance between fuzzy numbers, which can be expressed for the particular case of the distance from a triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3)$ to a

constant (specifically 0) by:

- If $f(\alpha) = \alpha$ (neutral), then

$$d^2(\widetilde{a}, 0, f) = a_2^2 + \frac{1}{3}a_2(a_3 + a_1) + \frac{1}{18}[(a_3 - a_2)^2 + (a_2 - a_1)^2] - \frac{1}{18}[(a_2 - a_1)(a_3 - a_2)].$$

- If $f(\alpha) = 1$ (risk-prone), then

$$d^2(\widetilde{a}, 0, f) = a_2^2 + \frac{1}{2}a_2(a_3 + a_1) + \frac{1}{9}[(a_3 - a_2)^2 + (a_2 - a_1)^2] - \frac{1}{9}[(a_2 - a_1)(a_3 - a_2)].$$

- If $f(\alpha) = \alpha^2$ (risk-averse), then

$$d^2(\widetilde{a}, 0, f) = a_2^2 + \frac{1}{4}a_2(a_3 + a_1) + \frac{1}{144}[(a_3 - a_2)^2 + (a_2 - a_1)^2] - \frac{1}{96}[(a_2 - a_1)(a_3 - a_2)].$$

Note that $f(\alpha)$, which serves as a weight function, is a positive continuous function in $[0, 1]$, and the distance is computed as a weighted sum of distances between two intervals along all of the α-cuts from 0 to 1.

Thanks to the presence of function f, DM participation is flexible. For example, when the DM is risk-neutral, $f(\alpha) = \alpha$ is a reasonable assumption. A risk-averse DM might want to attach more weight to information at a higher α level by using other functions, such as $f(\alpha) = \alpha^2$ or a higher power of α. A constant ($f(\alpha) = 1$), or even a decreasing f function, can be utilized for a risk-prone DM.

The results of Monte Carlo simulation techniques [2] demonstrate that the proposed method is clearly better than the *DMMs* described above. Its performance is very similar to the method proposed by Sarabando and Dias [27], which was likewise developed to deal with ordinal information about DMs' preferences. Beside, Sarabando and Dias' method is less computationally demanding. However, it accounts only for ordinal information in both weights and component values, whereas *DMMs* are applicable when imprecision concerning the DM's preferences and alternative performances are simultaneously represented in different ways, as demonstrated in the illustrative example introduced in the next section. Moreover, in the last extension of *DIM* the DM attitude toward risk can be incorporated to the analysis.

3. An illustrative example

We consider the selection of a supplier for cleaning services in a European public underground transportation company, where several conflicting criteria must be taken into account simultaneously, such as improving service levels and reducing total service costs.

The GMAA [6, 7, 9] decision support system was used in [8] to deal with this selection problem based on decision analysis methodology. However, most of the attributes under consideration were subjective, and the DMs found it difficult to appraise the offers on their basis. Moreover, DMs also found it difficult to elicit the weights representing the relative importance of criteria by means of the trade-off weighting method [7, 10]. However, they stated that they would feel more comfortable if ordinal information were allowed about the relative importance of criteria and about the appraisal of offers with respect to some attributes.

In this example, we allow DMs to use ordinal information both for attribute weights and for the appraisal of some offers with respect to some attributes. Moreover, rankings of the difference between the values of consecutive offers can be also taken into account for the different attributes.

First we built an objective hierarchy including all the relevant aspects related to the problem under consideration, aimed at improving DM understanding of the decision, see Figure 7.

There are five main top-level objectives: *Delivery conditions and human resources* (Delivery Con), which accounts for the level of consistency and coherence of the human resources allocated to the services; *Technical merit and resources* (Tech Resourc), which is an important efficiency factor leading to a significant reduction in labor cost; *Price* (Price), which represents the lowest price offered by suppliers; *Quality control procedures*
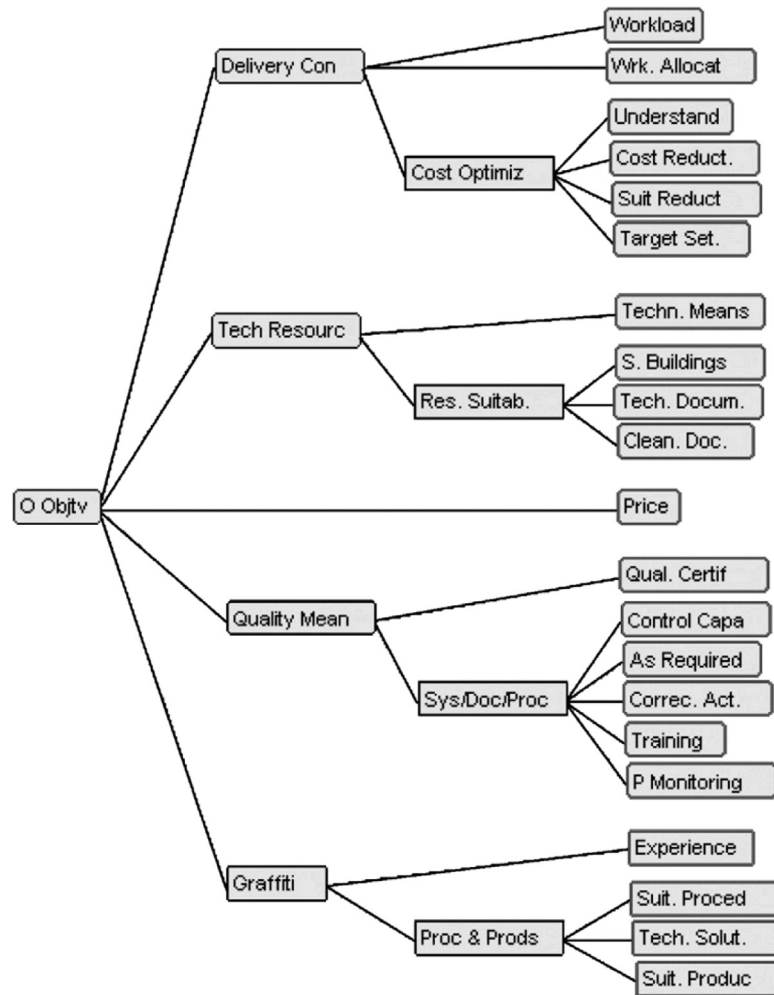
FIGURE 7. Objective hierarchy.

(Quality Mean), which accounts for accredited quality certifications and how quality systems and procedures are deployed; and *Graffiti prevention and cleanup* (Graffiti), which is one of the most common incidents detracting from the appearance of the underground buildings.

The *Delivery conditions and human resources* objective is split into three sub-objectives, *Workload*, *Workload allocation* and *Cost optimization program*. *Workload* (Workload) quantifies how much effort is deployed in terms of resources to achieve service performance objectives. *Workload allocation* (Wrk. Allocat) measures the coherence and consistency of the allocation of the resources throughout the underground facilities and buildings. *Cost optimization program* (Cost optimiz) measures the suitability of supplier cost reduction initiatives using two additional sub-objectives: *Suitability of cost reduction initiatives* (Suit Reduct), which quantifies how applicable and realistic the cost reduction initiatives are as things now stand and how they will ensure significant cost savings; and *Cost reduction target setting and performance monitoring* (Target Set), in which suppliers will be rewarded inasmuch as their offers include procedures and measures to successfully budget for and track cost savings.

Technical merit and resources (Techn. Resources) is also split into two sub-objectives: *Quantity of technical means* and *Technical resources suitability*. Suppliers will be rewarded inasmuch as their offers include large

TABLE 1. Attribute with continuous scale.

	Unit	Range
A_1: Workload	Absolute Percentage Error (APE)	[0,50]
A_2: Wrk. Allocat	Mean Absolute Percentage Error ($MAPE$)	[0,100]
A_5: Techn. Means	Number of pieces of equipments	[0,90]
A_8: Price	Monetary Units	[1500,3000]

amounts and extensive use of technical resources for *Quantity of technical means* (Techn. Means). *Technical resources suitability* (Res. Suitab.) is measured taking into account two additional sub-objectives: *Equipment suitability to client's buildings* (S. Buildings), which quantifies their appropriateness for underground facilities and possible adverse environmental impacts; and *Cleaning product technical documentation* (Clean. Doc.), which accounts for how well technical documentation describes the cleaning products.

Quality control procedures is measured in terms of two sub-objectives: *Number of quality certifications* (Qual. Certif) and *Quality measuring systems and procedures* (Sys/Doc/Proc). For the first, suppliers will be rewarded for having quality certifications accredited by national or European quality committees and institutions, whereas the second accounts for how quality systems and procedures are deployed.

This sub-objective is split into three sub-objectives: *Supplier quality control capabilities* (Control Capa.), which quantifies current supplier capabilities in terms of the supplier's quality assurance organization, its structure, number of resources, current skills and attested competencies; *Poor performance corrective actions* (Correc. Act.), which measures the proposed procedures and action plans to be put in place to solve poor performance and incidents when delivering cleaning services; and *Quality performance monitoring* (P. Monitoring), which accounts for the measuring system to be implemented, *i.e.*, what will be inspected, how the inspection will be carried out and the measurement criteria to be used during inspection are evaluated to verify that they guarantee greater objectivity, completeness and high-quality services.

Finally, *Graffiti prevention and cleanup* takes into account two sub-objectives, past experiences and graffiti cleanup procedures and products. *Past experiences in graffiti prevention and cleanup* (Experience) verifies supplier competence, assessing details on how the supplier has successfully implemented these specific services for other clients. *Graffiti cleanup procedures and product features* (Proc and Prods) is measured by taking into account three additional sub-objectives: *Suitability of graffiti cleanup procedures* (Suit. Proced), which measures their appropriateness for underground facilities and possible adverse environmental impacts, *Technical solution for prevention and cleanup* (Tech. Solut) and *Suitability of graffiti cleaning products* (Suit. Produc), which quantify how well the technical documentation describes the equipment, leading to a clear identification and understanding of the equipment and product performance, respectively.

Next, attributes were established for the lowest-level objectives to indicate to what extent they were achieved by the respective offers. Table 1 shows the attribute names, units and ranges for four out of the 16 attributes under consideration measured on a continuous scale. Ordinal information about the component utilities of the offers was considered for the remaining attributes, as described later.

The units for A_1: Workload and A_2: Wrk. Allocat, *i.e.*, APE and $MAPE$, represent the following:

- If the number of offers is odd and greater than 3, they are ordered from the highest to the lowest value, and a mean (M) is assessed discarding the highest and lowest values.
- If the number of offers is even and greater than 2, the mean is assessed considering all values. Then, we discard the offer whose difference with respect to the mean is the highest, positive or negative. A new mean (M) is now assessed taking the remaining values.
- If the number of offers is 2 or 3, the mean (M) is assessed considering all values.

TABLE 2. Offer performances.

	Offer 1	Offer 2	Offer 3	Offer 4	Offer 5	Offer 6
A_1: Workload	5.62	11.85	14.61	29.21	33.31	36.19
A_2: Wrk. Allocat	13.63	30.66	19	5.78	16.27	26.6
A_5: Techn. Means	23	19	63	31	80	24
A_8: Price	1808.8	1932.9	1708.3	1799.4	1754.9	2199.3

TABLE 3. Ordinal information concerning performances.

	Ordinal information
A_3: Suit Reduct	Offer3>Offer5>Offer2>Offer1>Offer4>Offer6
A_4: Target Set	Offer3>Offer1>Offer4>Offer2>Offer6>Offer5
A_6: S. Buildings	Offer3>Offer2>Offer4>Offer6>Offer1>Offer5
A_7: Clean Doc.	Offer4>Offer5>Offer1>Offer6>Offer2>Offer3
A_9: Qual. Certif	Offer1=Offer2=Offer3=Offer4>Offer5>Offer6
A_{10}: Control Capa	Offer3>Offer4>Offer2>Offer1>Offer5>Offer6
A_{11}: Correct. Act.	Offer4>Offer5>Offer3=Offer1>Offer2>Offer6
A_{12}: P. Monitoring	Offer3=Offer4>Offer1>Offer2>Offer5>Offer6
A_{13}: Experience	Offer3=Offer4=Offer5=Offer6>Offer2>Offer1
A_{14}: Suit. Proced	Offer3>Offer6>Offer4>Offer5>Offer2>Offer1
A_{15}: Tech. Solut	Offer3>Offer4>Offer6>Offer5>Offer2>Offer1
A_{16}: Suit. Produc	Offer5>Offer6>Offer4>Offer3>Offer2>Offer1

Then, the absolute percentage error and the mean absolute percentage error for the jth offer, APE_j and $MAPE_j$, are:

$$APE_j = \frac{ABS(M - value_j)}{M}, \quad MAPE_j = \frac{1}{N} \times \sum_{i=1}^{N} \frac{(M - value_j) \times 100}{M},$$

where N is the number of offers. Consequently, the best value for the attributes is 0, while the worst values are 50 and 100, respectively.

Six offers were considered for analysis. Table 2 shows the scores or performances of the different attributes using a continuous scale. Note that although the table reports precise performances, percentage deviations accounted for uncertainty about some performances. Specifically, a 20% and a 15% *Workload* deviation was introduced for Offers 5 and 6, and for Offer 4, respectively; a 10% and a 20% *Wrk. Allocat* deviation was introduced for Offers 3 and 6, and for Offer 2, respectively; and a 30% and a 15% *Techn. Means* deviation was introduced for Offers 3 and 5, respectively.

Besides, Table 3 shows ordinal information concerning the appraisal of the six offers for the remaining attributes.

Moreover, rankings of the difference between the values of some consecutive offers were also taken into account by DMs for some attributes, see Table 4.

For instance, for A_{16}: Suit. Produc, we have $\Delta_{16,5} \geq \Delta_{16,4} \geq \{\Delta_{16,1}, \Delta_{16,2}, \Delta_{16,3}\}$, where

$$\Delta_{16,1} = v_{16}(\text{Offer5}) - v_{16}(\text{Offer6}), \quad \Delta_{16,2} = v_{16}(\text{Offer6}) - v_{16}(\text{Offer4}),$$
$$\Delta_{16,3} = v_{16}(\text{Offer4}) - v_{16}(\text{Offer3}), \quad \Delta_{16,4} = v_{16}(\text{Offer3}) - v_{16}(\text{Offer2}),$$
$$\Delta_{16,5} = v_{16}(\text{Offer2}) - v_{16}(\text{Offer1}).$$

This means that the difference in the appraisal of offers 2 and 1 is greater than the difference in the appraisal of offers 3 and 2, which is greater than the remaining differences between offers for that attribute.

TABLE 4. Ordinal information about the difference between the values of consecutive offers.

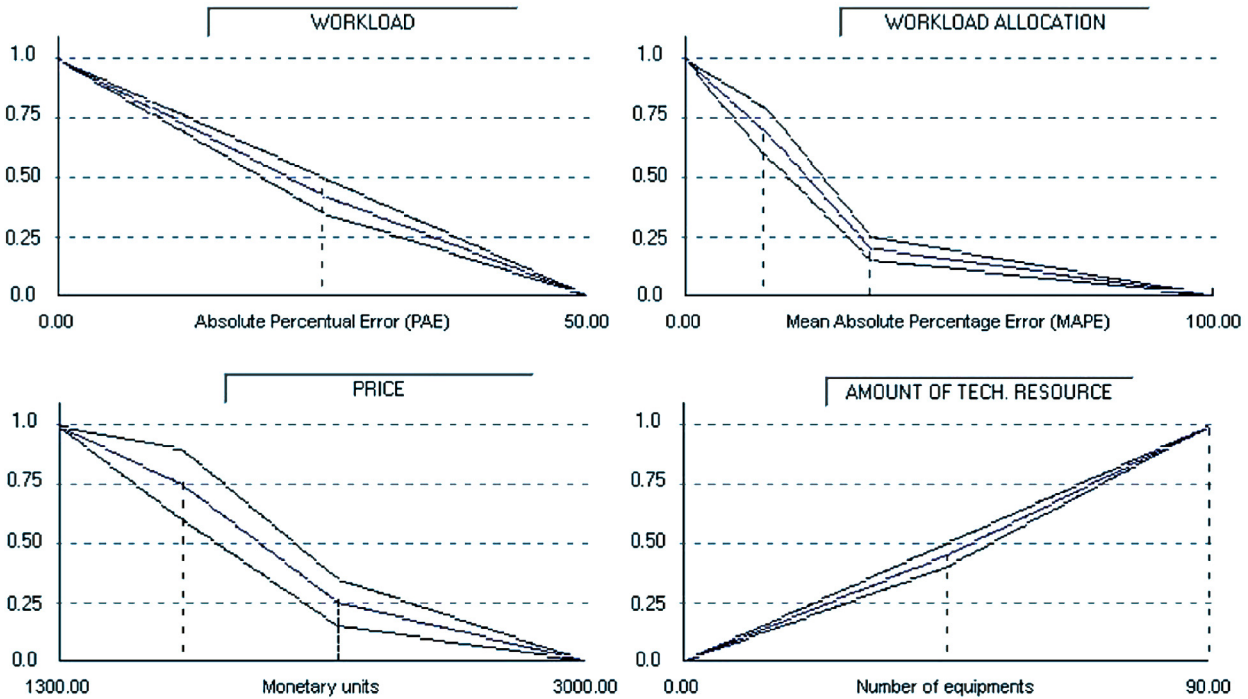|	Ordinal information
A_{10}: Control Capa	$\Delta_{10,5} \geq \{\Delta_{10,1}\ \Delta_{10,2}, \Delta_{10,3}, \Delta_{10,4}\}$
A_{11}: Correct. Act.	$\Delta_{11,4} \geq \{\Delta_{11,1}, \Delta_{11,2}, \Delta_{11,3}, \Delta_{11,5}\}$
A_{12}: P. Monitoring	$\Delta_{12,2} \geq \{\Delta_{12,3}, \Delta_{12,4}, \Delta_{12,5}\}$
A_{13}: Experience	$\Delta_{13,4} \geq \Delta_{13,5}$
A_{14}: Suit. Proced	$\Delta_{14,5} \geq \{\Delta_{14,1}, \Delta_{14,2}, \Delta_{14,3}, \Delta_{14,4}\}$
A_{16}: Suit. Produc	$\Delta_{16,5} \geq \Delta_{16,4} \geq \{\Delta_{16,1}, \Delta_{16,2}, \Delta_{16,3}\}$

FIGURE 8. Imprecise component utility functions.

Regarding the quantification of the DMs preferences, Figure 8 shows the imprecise component utility functions assigned by the DMs to those attributes for which no ordinal information about the offer performances were provided. Note that utility functions are decreasing for A_1: Workload, A_2: Wrk. Allocat and A_{11}: Price, and increasing for A_7: Tech. Means.

Looking at the appearance of the component utility functions we realize that the DM's attitude toward risk is prone for the four attributes. Thus, we will consider this situation in Tran and Duckstein's distance when applying the extension of *DIM3* with triangular fuzzy numbers to rank the offers.

As mentioned before, ordinal information was also used by DMs to represent the relative importance of the attributes as follows:

$$w_8 > 3w_1 > w_5 > w_9 > w_{13} > w_2 > \{w_3, w_4, w_6, w_7, w_{10}, w_{11}, w_{12}, w_{14}, w_{15}, w_{16}\},$$

TABLE 5. Offer ranking.

	1st	2nd	3th	4th	5th
Dominance intensity	0.4921	0.0216	−0.0997	−0.1351	−0.2789
Ranking	Offer1	Offer3	Offer4	Offer5	Offer2

i.e., the most important attribute for the decision is *Price* (w_8), which is at least three times more important than *Workload* (w_1). The next most important attributes are *Tech. Means* (w_5), *Qual. certif* (w_9), *Experience* (w_{13}) and *Wrk. Allocat* (w_2). Finally, *Wrk. Allocat* is more important than the remaining attributes.

Taking into account all the above information, the simplex method was used to solve the corresponding linear optimization problems to compute pairwise dominance values to derive the following *dominance matrix*:

$$D = \begin{pmatrix} - & -0.0372 & -0.1373 & -0.0983 & -0.1566 & 0.1857 \\ -0.4249 & - & -0.3816 & -0.3141 & -0.2769 & 0.0125 \\ -0.3439 & -0.2244 & - & -0.2434 & -0.1986 & 0.0475 \\ -0.3741 & -0.2308 & -0.2603 & - & -0.3091 & 0.0392 \\ -0.4428 & -0.2532 & -0.3299 & -0.2866 & - & 0.0002 \\ -0.5763 & -0.4608 & -0.5322 & -0.5077 & -0.4447 & - \end{pmatrix}.$$

Note that Offer 6 is dominated since all values in the last column are positive. Thus, it can be discarded for further analysis.

Table 5 shows the offer ranking derived from the last extension of *DIM3* with triangular fuzzy numbers. Dominance measuring methods are the only methods that can be used for the incomplete information under consideration, *i.e.*, performance intervals, ordinal information about the component values of some offers and rankings of the difference between the values of some consecutive offers, and ordinal information about weights. Note that the extension of *DIM3* with triangular fuzzy numbers accounts for DM's attitude toward risk.

Note that Offer 1 is best ranked, followed by offers 3, 4, 5 and 2. The three best ranked offers are the same than in the ranking reported in [8], in which the uncertainty about the offer performances and the imprecision concerning weights were both represented by intervals.

4. CONCLUSION

Dominance measuring methods (*DMMs*) have been shown to be a valid approach for dealing with complex decision-making problems with imprecise, incomplete or partial information within multi-attribute value/utility theory.

We have reviewed different *DMMs* for dealing with imprecise information about decision-maker preferences, identifying drawbacks associated with the original *DMMs* and describing other *DMMs* aimed at overcoming these shortcomings. The performance of these methods has been compared in the literature with other existing approaches on the basis of two efficiency measures (the hit ratio and the rank-order correlation.

The results of the applied Monte Carlo simulation techniques demonstrate that *DIM2* outperforms *AP1*, *AP2* and the adapted classical decision rules when both intervals and ordinal information about weights are considered. Besides, *SMAA-2* slightly outperformed *DIM2* when intervals were considered, and there was no significant difference between *DIM2* and the *ROC* method when ordinal information was considered (*DIM2* outperformed the other surrogate weighting methods).

However, *ROC* and *SMAA-2* are only applicable when ordinal information about weights is considered and *DMMs* are more general since other linear restrictions could be considered to derive the dominance matrix.

Besides, the last extension of *DIM3* with triangular fuzzy numbers outperforms previous *DMMs*, and there is no significant difference between *DIM3* and the method proposed by Sarabando and Dias when dealing with ordinal information concerning both weights and component values.

However, Sarabando and Dias account only for ordinal information regarding both weights and component values, whereas *DMM* is applicable when imprecision concerning DM preferences and alternative performances are simultaneously represented in different ways and the DM's attitude toward risk can be incorporated to the analysis, as demonstrated in the illustrative example.

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