

# A MCDA Framework for the Remediation of Zapadnoe Uranium Mill Tailings: A Fuzzy Approach

ANTONIO JIMÉNEZ-MARTÍN<sup>1,\*</sup>, MIGUEL MARTÍN<sup>1</sup>,  
DANYL PÉREZ-SÁNCHEZ<sup>2</sup>, ALFONSO MATEOS<sup>1</sup>  
AND ALLA DVORZHAK<sup>2</sup>

<sup>1</sup>*Decision Analysis and Statistics Group, Universidad Politécnica de Madrid, SPAIN*

<sup>2</sup>*Departamento de Medio Ambiente, CIEMAT, SPAIN*

We propose a theoretical framework based on MCDA and fuzzy logic to analyze remediation alternatives for the Zapadnoe uranium mill tailings (Ukraine). We account for potentially conflicting economic, social, radiological and environmental objectives, which are included in an objective hierarchy. Fuzzy rather than precise values are proposed for use to evaluate remediation alternatives against the different criteria and to quantify preferences, such as the weights representing the relative importance of criteria. Remediation alternatives are evaluated by means of a fuzzy additive multi-attribute utility function and ranked on the basis of the similarity of the respective trapezoidal fuzzy number representing their overall utility to the anti-ideal point.

## 1 INTRODUCTION

The Zapadnoe uranium mill tailings site is situated in the south-western part of the main industrial site of the former Pridneprovsky Chemical Plant (PChP), located at Dneprodzerzhinsk (Ukraine). The tailings site operated from 1949 until 1954. The total volume of waste was  $3.5 \times 10^5 m^3$  and the total activity was  $1.8 \times 10^{14}$  Bq [1].

\* Corresponding authors E-mail: antonio.jimenez@upm.es

Uranium mill tailings, disposed of using the hydraulic discharge method, account for most of the wastes. The tailings site was covered in 2000 by an engineered multi-layer soil cover. As a result, the wastes are covered by a layer of non-radioactive backfill, composed of construction and industrial wastes, sand, clayey loam soils, clinker, rubbish, etc., with a total thickness of 0.2 to 2.8 m. The southern part of the tailings have been covered by a 0.3 to 1.0 m thick layer of crushed stone and asphalt layer. The slopes of the tailings pile are covered by layers of clay loam and organic soil, with a combined thickness of 0.5-1.0 m.

The tailings are situated on the slope of a sequence of the terraces of the Dnieper River. The ground generally slopes from south to north. The tailings themselves are located within the second terrace. The first (lower) terrace is situated to the north of the tailings. The third and fourth (higher) terraces are situated to the south of the waste site. The tailings site was surrounded by dikes that were not surfaced with protective impermeable screens and are currently buried below the layers of backfilled soil. The surface of the tailing pile is equipped with a system for collecting runoff rainwater. This water runs into Konoplyanka River.

There are two aquifers at the Zapadnoe tailings site. The *technogenic aquifer* is a perched water horizon that is recharged by infiltration of atmospheric precipitation through the waste cover. The water from this aquifer infiltrates further down to the underlying aquifer in the alluvial deposits. The *regional aquifer* in the alluvial deposits is composed of alluvial sands, sandy loam and clay loam deposits. The alluvial deposits are overlain by loess deposits and underlain by the upper part of the fissured crystalline basement rocks. The groundwater in the alluvial aquifer flows north towards the Konoplyanka and Dnieper rivers.

A series of rainfall events led to the erosion of the surface and slopes of the protective dikes from 2002 to 2004. Remedial works were carried out in 2005. These works included backfilling the eroded areas with clayey soil, and reinforcing the slopes by a geotechnical polymer net material. The eroded surfaces were covered with an organic soil layer and planted with grasses. The surface run-off drainage system was also repaired.

The tailings site is surrounded by other industrial sites and technological communications lines that employ 2500 people. The surface of the tailings site is equipped with warning signs prohibiting the entry of unauthorized personnel, but the site is not fenced.

There are currently two main sources of data regarding the physical, chemical and radioactive characteristics of wastes disposed at the Zapadnoe tailings site. The first characterization studies were carried out in 2000 [2]. Six characterization boreholes were drilled and the core material was subjected to

various lithological, chemical and radiometric analyses. The second characterization was carried out in 2009 as part of the National PChP Remediation Program [3]. Information about radiation exposure due to soil, water and air contamination was collected for various radionuclides. Water samples were also analyzed to gather information on contamination by chemically toxic materials.

Discrepancies between the results of inventory studies carried out in 2000 and 2009 have been identified. In particular, the 2009 studies suggest that uranium and radium concentrations in the wastes are about a factor of two higher than previously estimated. The estimated mean Th-230 activity increased by a factor of about three, and discrepancies were also observed for Pb-210.

More recently, the context for a safety assessment of the Zapadnoe tailings site has been described in [1]. The safety assessment was carried out by Ecomonitor and Geo-EcoConsulting following the steps set out in the ENSURE II project (funded by the Swedish International Development Agency, SIDA, to provide assistance to Ukraine in the remediation of uranium-contaminated territories and facilities at the Dnieprodzerzhynsk industrial site).

Bugay *et al.* [1] includes information on the operational history of the tailings site, on its engineering features, as well as on the chemical, physical and radioactive characteristics of the waste materials in the tailings. Environmental conditions (such as the geology, geomorphology and hydrogeological setting) and climate are also described.

Safety assessment can be considered as the starting point for an analysis of remediation alternatives. It would be equivalent to the no action alternative. The selection of a preferred remediation alternative is a complex decision-making problem, which has to take into account factors other than the radiological and chemical toxicity impacts of the wastes. For example, the direct costs of the application and maintenance of remediation alternatives (manpower, consumables, equipment needed for application, management), the job creation effects and other indirect costs or benefits should be considered as economic criteria. Social impacts, as well as direct impacts on human health and safety, should also be considered. These impacts include community satisfaction, and the impact of remediation on the social characteristics of the neighborhood.

In the next section, we proposed a fuzzy MCDA framework for selecting remediation alternatives for the Zapadnoe uranium mill tailings on the basis of MCDA stages: problem structuring, elicitation of DMs' preferences and the fuzzy evaluation of remediation alternatives. Finally, some conclusions are provided in Section 3.

## 2 A FUZZY MCDA FRAMEWORK

The goal of multi-criteria decision analysis (MCDA) is to structure and simplify the task of making hard decisions insofar as the nature of the decision permits [4]. MCDA works on the assumption that the appeal of an alternative depends on the preferences concerning the possible consequences and the likelihood of their materializing.

What makes MCDA unique is the way in which these factors are quantified and formally incorporated into the problem analysis. Existing information, collected data, models and professional judgments are used to quantify the likelihoods of ranges of consequences, whereas utility theory is used to quantify preferences.

The usual or traditional approach to MCDA calls for single or precise values for the different model inputs, i.e., for the weights as well as for the performances of the alternatives in terms of the identified criteria. However, we adopt a less demanding approach for the decision-maker (DM), who is able to provide fuzzy numbers instead of single values.

Fuzzy logic (FL) introduced by Zadeh [5] is a mathematical modeling tool using vague or imprecise measurements particularly suited to for decision-making problems [6,7]. In FL, a linguistic scale is usually built to characterize model inputs [8]. Each linguistic term is associated with a triangular or trapezoidal fuzzy number (see Table 1 and Figure 1), and fuzzy arithmetic is used to compute model outputs.

As shown in Figure 1, we consider the set of normalized trapezoidal fuzzy numbers with support on  $[0,1]$ ,  $TF[0,1]$ , i.e., tuple  $(a_1, a_2, a_3, a_4)$  with  $0 \leq a_1 \leq a_2 \leq a_3 \leq a_4 \leq 1$ , together with a membership function,

$$\mu_{\tilde{A}} = \begin{cases} 0 & \text{if } x < a_1 \\ (x - a_1)/(a_2 - a_1) & \text{if } a_1 < x < a_2 \\ 1 & \text{if } a_2 < x < a_3 \\ (x - a_4)/(a_3 - a_4) & \text{if } a_3 < x < a_4 \\ 0 & \text{if } a_4 < x \end{cases}, \quad (1)$$

indicating the degree of membership of value  $x \in R$  to the fuzzy number  $\tilde{A}$ .

We use the arithmetic proposed in [9] in  $TF[0,1]$  for computations. Thus, if  $\tilde{A} = (a_1, a_2, a_3, a_4; w_{\tilde{A}})$  and  $\tilde{B} = (b_1, b_2, b_3, b_4; w_{\tilde{B}})$ , then

$$\begin{aligned} \tilde{A} \oplus \tilde{B} &= (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4; \min\{w_{\tilde{A}}, w_{\tilde{B}}\}), \\ \tilde{A} \otimes \tilde{B} &= (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3, a_4 \times b_4; \min\{w_{\tilde{A}}, w_{\tilde{B}}\}). \end{aligned} \quad (2)$$

The fuzzy number resulting from such a computation is usually translated into a linguistic term on the previously defined scale by means of a similarity

Term	Trapezoidal fuzzy number
Very Low (VL)	(0, 0, 0, 0.05)
Low (L)	(0, 0.075, 0.125, 0.275)
Medium-Low (M-L)	(0.125, 0.275, 0.325, 0.475)
Medium (M)	(0.325, 0.475, 0.525, 0.675)
Medium-High (M-H)	(0.525, 0.675, 0.725, 0.875)
High (H)	(0.725, 0.875, 0.925, 1)
Very High (VH)	(0.925, 1, 1, 1)

TABLE 1  
Trapezoidal fuzzy numbers corresponding to linguistic terms.

function [10]. Following the MCDA methodology, we build an objective hierarchy including all relevant criteria and then establish attributes for the bottom-level objectives of the hierarchy to indicate to the extent to which they are achieved.

The performance of each of the options in relation to each of the considered attributes has to be determined from the results of the safety assessment and other studies, and translated into a trapezoidal fuzzy number. Also, the relative importance of the attributes in the objective hierarchy has to be represented by means of trapezoidal fuzzy numbers. Finally, a fuzzified additive utility function can be used to derive a global utility value for each option, on the basis of which remediation alternatives can be ranked.

## 2.1 Problem structuring

To identify the criteria to be incorporated into the analysis, we consulted experts taking part in the ENSURE II project and reviewed the literature on MCDA applications for evaluating remediation alternatives [11], and especially applications to uranium mill tailings sites [12].

On this basis, we built an objective hierarchy applicable to remediation options for the Zapadne tailings site (Figure 2). There are four main top-level criteria for the appropriate management of the Zapadne tailings site (global objective): environmental impact, radiological impact, social impact and economic impact.

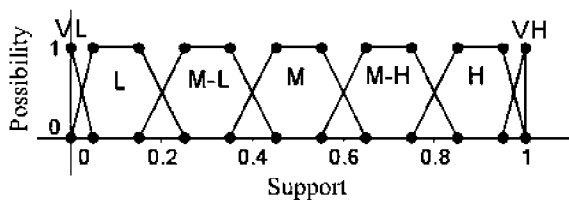


FIGURE 1  
A fuzzy linguistic scale.

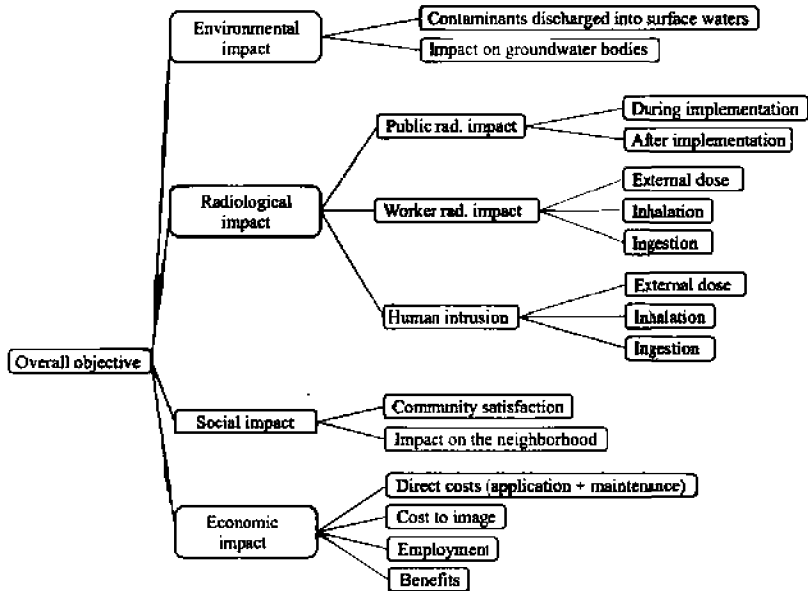


FIGURE 2  
Objective hierarchy.

The *environmental impact* is caused by contaminants discharged into surface waters, which can impair the functioning of aquatic biota, and by infiltration through the tailings to the underlying aquifer, which has an impact on groundwater bodies. Both radioactive and toxic chemical contamination is taken into account and measured in terms radiation dose or degree of chemical exposure. The doses and exposures derived from the safety assessment are adopted as reference values for the no action alternative and remediation options are evaluated with respect to these values.

The *radiological impact* is split into three subobjectives. *Public radiological impact* refers to the doses received by the population through external exposure, inhalation (concentration in the air) and ingestion (via drinking water, food, inadvertently). It differentiates the doses received by the population during and after the implementation of the remediation alternative, leading to two new subobjectives, respectively.

The *radiological impact on workers* refers to radiation doses received by workers as a consequence of the process of implementing a remediation alternative. This objective is split into three subobjectives accounting for the external dose (radiation exposure at the surface of the tailings site), and the doses received by inhalation and ingestion. To measure these objectives, the corresponding attributes take into account the number of workers needed to

implement the remediation alternative, the number of hours each worker is exposed to the radiation and the radiation doses per hour through exposure at the surface, inhalation and ingestion, respectively.

Finally, *human intrusion* refers to the radiation received by intruders at the Zapadnoe tailings site. The objective is again split into three subobjectives accounting for the external dose, and the doses received by inhalation and by ingestion. The corresponding attributes take into account an estimation of the number of intruders at the Zapadnoe tailings site per year on the basis of historical data, the average number of hours each intruder spent at the site by intrusion and the radiation doses per hour through exposure at the surface, inhalation and ingestion, respectively.

*Social impact* is split into community satisfaction and the impact on neighborhoods or regions. *Community satisfaction* refers to how a remediation alternative is perceived by individuals belonging to a critical group living in the area and the *impact on the neighborhood* accounts for the impact on the local community as a whole, including dust, light, noise, odor and vibration during the remediation works and associated with traffic, including week-day and weekend day- and nighttime operations. The fuzzy linguistic scale is used to quantify both social objectives.

Under *economic impact*, *direct costs* refer to the costs of the implementation and maintenance of a remediation alternative (manpower, consumables, equipment needed for implementation, management requirements). A monetary attribute is used for this aspect. *Cost to image* comprises indirect costs associated with a remediation alternative. It relates to public perceptions, e.g., a reluctance to purchase products from the area, even if uncontaminated, or a drop in tourism. Both the no action alternative and the various remediation options may have associated indirect costs.

*Employment* corresponds to job creation in the implementation of a remediation alternative and afterwards. Short- and long-term jobs are taken into account and the corresponding attribute is measured in person-months. Finally, *benefits* refers to direct economic benefits associated with the implementation of a remediation alternative (e.g., sale of waste materials for reuse). It is measured in monetary units.

## **2.2 Elicitation of preferences**

The GMAA decision support system provides two procedures for assessing component utilities [13,14]. They represent the experts' preferences concerning the possible alternative performances: by directly constructing a piecewise linear utility function from the best and the worst attribute values and up to three intermediate values with their respective imprecise utilities; or using indifference judgments between lotteries and sure amounts. In both cases, the system accepts value intervals specified as responses to the probability

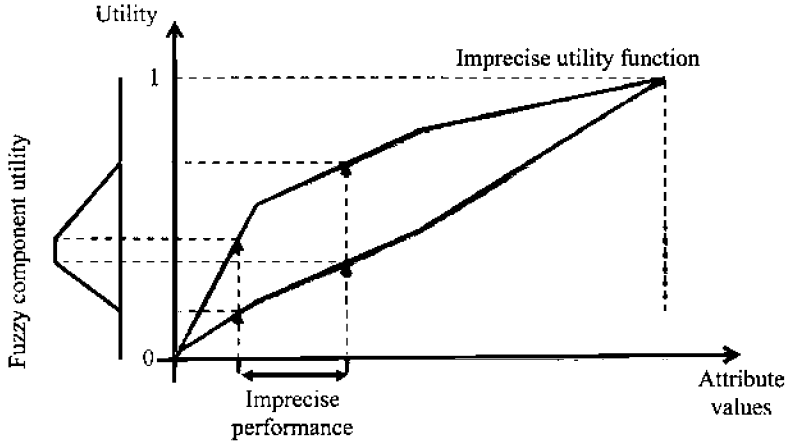


FIGURE 3  
Fuzzy component utilities.

questions that the expert is asked, which leads to classes of utility functions (see Figure 3).

As interval values represent imprecise performances, a fuzzy component utility will be derived from a particular interval through the class of utility functions (see Figure 3).

Weights representing the relative importance of criteria in the objective hierarchy have also to be elicited. We use a fuzzy adaptation of the procedure included in the GMAA system for eliciting weights based on tradeoffs [13,15]. In this procedure, the interviewed individual has to make indifference judgments between lotteries and multiple sure amounts, where value intervals are possible responses. Weight intervals (rectangular fuzzy numbers) or a fuzzy linguistic scale can also be used for direct assignment.

Once the relative importance of the objectives has been rated along the branches of the hierarchy (Figure 2), the attribute weight can be assessed by multiplying the respective weights (represented by trapezoidal fuzzy numbers) of the objectives in the path from the root (global objective) to each leaf (attribute).

### 2.3 Fuzzy evaluation of remediation alternatives

Once the preferences have been quantified, remediation alternatives (including the no action alternative) can be evaluated by means of a fuzzified additive multi-attribute utility function. The form of the function is

$$u(\widetilde{S}_j) = \oplus_{j=1}^n (\widetilde{k}_j \otimes u_j(\widetilde{x}_{ij})), \quad (3)$$



where  $\tilde{k}_j$  is the trapezoidal fuzzy number representing the  $i$ th attribute weight,  $x_{ij}$  is the performance for the remediation alternative in the  $i$ th attribute and  $u_j(\tilde{x}_{ij})$  is the fuzzy component utility associated with the above performance or the trapezoidal fuzzy number associated with the selected linguistic term. We use the  $\oplus$  and  $\otimes$  operators proposed in [9].

Note that if the linguistic scale is used to value remediation alternatives in respect of a particular attribute, then the respective trapezoidal fuzzy numbers (see Table 1 and Figure 1) are used as fuzzy component utilities.

Remediation alternatives are then ranked on the basis of the trapezoidal fuzzy numbers representing their overall utility. A significant number of ranking approaches have been proposed in the literature. A review and comparison of some can be found in [16-19]. The maximizing set and minimizing set method proposed in [20] has become one of the most popular approaches, which compares and ranks fuzzy numbers in terms of their left, right and total utilities, which are computed based on the introduced maximizing set and minimizing set.

More recently, Wang and Luo [21] presented an alternative ranking that overcomes a drawback associated with Chen's approach. They define two new alternative indices for comparing and ranking fuzzy numbers, defined in terms of a DM's attitude towards risks and the left and right areas between the fuzzy numbers and the two ideal points.

In this paper, we propose an alternative approach for ranking trapezoidal fuzzy numbers on the basis of a similarity function [10], in which the similarity of the fuzzy overall utility of each remediation alternative is computed regarding the anti-ideal point (0,0,0,0).

In the similarity function we consider three parameters consisting of the ratio between the common area and the joint area under the membership functions of trapezoidal fuzzy numbers, the geometric distance between them, and the distance between the centroid of both trapezoidal fuzzy numbers. Moreover, the difference between the heights of the generalized fuzzy numbers is also considered. Given  $\tilde{A} = (a_1, a_2, a_3, a_4)$  and  $\tilde{B} = (b_1, b_2, b_3, b_4)$ , the adaptation of the similarity function to normalized trapezoidal fuzzy numbers (height equal to 1) can be then defined as

- if  $\max\{(a_4 - a_1), (b_4 - b_1)\} \neq 0$ , then

$$S(\tilde{A}, \tilde{B}) = \left( 1 - (1 - \alpha - \beta) \times \left( 1 - \frac{\int_0^1 \mu_{\tilde{A} \cap \tilde{B}}(x) dx}{\int_0^1 \mu_{\tilde{A} \cup \tilde{B}}(x) dx} \right) - \alpha \frac{\sum |a_i - b_i|}{4} - \beta \frac{d[(X_{\tilde{A}}, Y_{\tilde{A}}), (X_{\tilde{B}}, Y_{\tilde{B}})]}{M} \right),$$

- otherwise,

$$S(\tilde{A}, \tilde{B}) = \left( 1 - \left( \frac{1 - \alpha - \beta}{2} + \alpha \right) \times \frac{\sum |a_i - b_i|}{4} - \left( \frac{1 - \alpha - \beta}{2} + \beta \right) \times \frac{d[(X_{\tilde{A}}, Y_{\tilde{A}}), (X_{\tilde{B}}, Y_{\tilde{B}})]}{M} \right),$$

where  $\alpha + \beta < 1$ ,  $\mu_{\tilde{\chi}}$  is the membership function of  $\tilde{\chi}$ ,

$$M = \max_{[0,1] \times [0,1/2]} \{d((x, y), (x', y'))\},$$

$$\mu_{\tilde{A} \cap \tilde{B}}(x) = \min_{0 \leq x \leq 1} \{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}, \mu_{\tilde{A} \cup \tilde{B}}(x) = \max_{0 \leq x \leq 1} \{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\},$$

$(X_{\tilde{A}}, Y_{\tilde{A}})$  and  $(X_{\tilde{B}}, Y_{\tilde{B}})$  are the centers of gravity, defined as

$$X_{\tilde{A}} = \frac{Y_{\tilde{A}}(a_3 + a_2) + (1 - Y_{\tilde{A}})(a_4 + a_1)}{2}$$

$$Y_{\tilde{A}} = \begin{cases} \frac{\left(\frac{a_3 - a_2}{a_4 - a_1} + 2\right)}{6}, & \text{if } a_4 - a_1 \neq 0 \\ 1/2, & \text{if } a_4 - a_1 = 0 \end{cases},$$

and analogously for  $(X_{\tilde{B}}, Y_{\tilde{B}})$ ; and  $d$  is a distance in  $R^2$ .

### 3 CONCLUSIONS

The evaluation of remediation alternatives for the Zapadnoe uranium mill tailings site is a complex decision-making problem involving environmental, radiological, social and economic criteria. The MCDA methodology provides a framework for structuring the problem incorporating individual or group preferences. Moreover, thanks to fuzzy logic, the inputs to the decision-support process may contain vague or imprecise information, which is less demanding for experts and makes the analysis suitable for group decision-making. We have set out a basis for such an evaluation. The actual evaluation is ongoing and will be described in a subsequent publication.

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## REFERENCES

- [1] Bugay, D. Voitsekhovitch, O., Lavrova, A., Skalskyy, A., Tkachenko, E. (2012). Definition of the assessment context and scenarios for the selected cases. Data analyses and data collection. Case A Zapadne tailings. Report on the Project Ensure-II: Work Package 2, Kiev: Ecomonitor.
- [2] STC-USR & DIIT. (2000). Progress report on comprehensive investigation of tailings to solve the problem of radioactive waste immobilization and isolation on sites of Dnieprodzerginsk Town. Stage 2. Physical, chemical, spectroscopic and radiospectrum analysis of radioactive waste. Kiev: STC "Cvetnie Metally" and Ukrainian Scientific Research and Design Institute for Industrial Technology.
- [3] UHMI. (2009). Analytical measurements of samples of material from tailings according to parameters of spectrum of priority alpha- and gamma-emitting radionuclides of U - Th series using methods of low-background semiconductor alpha- and gamma-spectroscopy. Final report on contract No.31 with SE Baryer, Kiev: UHMI Institute (in Ukrainian).
- [4] Belton, V. (1990). Multiple criteria decision analysis practically the only way to choose. L.C. Hendry and R.W. Englese (eds.), Operational Research Tutorial Papers 53-101, Birmingham: Operational Research Society.
- [5] Zadeh, L.A. (1965). Fuzzy sets. *Information and Control*, **8**, 338–353.
- [6] Carlsson, C., Fuller, R. (2001). Fuzzy reasoning in decision making and optimization. Studies in fuzziness and soft computing series. Berlin/Heidelberg: Springer.
- [7] Kahraman, C. (2008). Fuzzy multi-criteria decision making. Theory and applications with recent developments. Optimization and its application, 16. New York: Springer-Verlag.
- [8] Dursun, M. (2007). Evaluating solid waste management scenarios using fuzzy multi-criteria decision making approaches. Thesis of Master of Science. Istanbul: Galatasaray University.
- [9] Xu, Z., Shang, S., Qian, W., Shu, W. (2010). A method for fuzzy risk analysis based on the new similarity of trapezoidal fuzzy numbers. *Expert Systems with Applications*, **37**, 1920–1927.
- [10] Vicente, E., Mateos, A., Jiménez, A. (2013). A new similarity function for generalized trapezoidal fuzzy numbers. *Lecture Notes in Computer Science*, **7895**, 400–411.
- [11] Brinkhoff, P. (2011). Multi-criteria Analysis for Assessing Sustainability of Remedial Actions. Applications in Contaminated Land Development. Goteborg: Chalmers University of Technology.
- [12] Goldammer, W., Nüsser, A., Bütow, E., Lühr, H.-P. (1999). Integrated assessment of radiological and non-radiological risks at contaminated sites. *Mathematische Geologie*, **3**, 53–72.
- [13] Jiménez, A., Ríos-Insua, S., Mateos, A. (2003). A decision support system for multi-attribute utility evaluation based on imprecise assignments. *Decision Support Systems*, **36**, 65–79.
- [14] Jiménez, A., Rios-Insua, S., Mateos, A. (2006). A generic multi-attribute analysis system. *Computers & Operations Research*, **33**, 1081–1101.

- [15] Keeney, R.L., Raiffa H. (1976). *Decisions with Multiple Objectives: Preferences and Value-Tradeoffs*. New York: Wiley.
- [16] Bortolan, G., Degani, R. (1985). A review of some methods for ranking fuzzy subsets. *Fuzzy Sets and Systems*, **15**, 1–19.
- [17] Chen, S.J., Hwang, C.L. (1992). *Fuzzy Multiple Attribute Decision Making: Methods and Applications*. New York: Springer.
- [18] Wang, X., Kerre, E.E. (2001a). Reasonable properties for the ordering of fuzzy quantities (I). *Fuzzy Sets and Systems*, **118**, 375–385.
- [19] Wang, X., Kerre, E.E. (2001b). Reasonable properties for the ordering of fuzzy quantities (II). *Fuzzy Sets and Systems*, **118**, 387–405.
- [20] Chen, S.H. (1985). Ranking fuzzy numbers with maximizing set and minimizing set. *Fuzzy Sets and Systems*, **17**, 113–129.
- [21] Wang, Y.-M., Luo, Y. (2009). Area ranking of fuzzy numbers based on positive and negative ideal points. *Computers and Mathematics with Applications*, **58**, 1769–1779.