Editorial on "Multiplex networks: Structure, dynamics and applications"

There is a wide range of systems in the real world where components cannot function independently, so that these components interact with others through different channels of connectivity and dependencies. Complex Networks theory is, in fact, the formal tool for describing and analyzing fields as disparate as sociology (social networks, acquaintances or collaborations between individuals), biology (metabolic and protein networks, neural networks) or technology (phone call networks, computers in telecommunication networks) [1,3,5,7–9].

Since the publication of the two seminal and classic papers, by D.J. Watts and S.H. Strogatz on *small world networks*, [10], and by A.L. Barabási and R. Albert on *scale-free networks*, [2], there is growing interest for complex networks reflected by the large number of papers that have been published on this topic, and the new issues and research lines which have been tackled.

The traditional complex network approach to nature has generally focused on the case in which each constituent system (or elementary unit) is represented by a network node, and nodes are connected to each other by a unique type of steady edge. Obviously, this assumption is almost invariably a gross oversimplification, as different kinds of connections exist in a system and cannot be lumped together and treated uniformly.

Multilayer networks [4,6] explicitly incorporate multiple channels of connectivity in a system and constitute the natural mathematical environment to describe systems whose units are interconnected through different categories and kinds of connections: each channel (relationship, category, activity) is represented by a layer, and the same node or entity may have different kinds of interactions (coworker-ship, friendship, vicinity, etc.) with the rest of the nodes or entities.

As a particular case of multilayer networks, multiplex networks provide a natural description for systems in which the same elementary units may interact among them through many kinds of liaisons or links. Each type of liaison or relationship (e.g., a task, an activity, or a category) is represented by a different layer. Thus, multiplex networks provide a natural description for systems in which entities have the same set of neighbors in each layer [4].

Multilayer and multiplex networks generalize the "*traditional*" network theory, developing a solid foundation and providing the consequent new associated tools to study multilayer and multicomponent systems in a comprehensive fashion.

The study and development of some of these tools, some theoretical results related to multiplex networks and some other aspects and applications related to multiplex networks feature among the core objectives of this special issue.

The studies contained in this special issue cover a wide range of topics related not only to the topological properties of multiplex networks but also to their dynamics and the relationships between them.

The issue opens with a review by *S. Havlin, H.E. Stanley, A. Bashan, J. Gao and D.Y. Kenett* on the mathematical framework for the percolation of Networks (NONs), resulting from the connection of several interdependent random networks. Recently, methods of statistical physics have successfully been applied to shed the light on the problem from many different perspectives, significantly advancing our understanding of percolation in NONs. The percolation properties of a NON greatly differ from those of single isolated networks, as the constituent networks of a NON are connected by node dependencies, and a NON is subject to cascading failure. Although networks with broader degree distributions, such as scale-free networks, are more robust when analyzed as single networks, they become more vulnerable in a NON. The effect of space embedding on network vulnerability it is also reviewed. It is shown that for spatially embedded networks any finite fraction of dependency nodes will lead to abrupt transitions.

The special issue continues with a study by *L. Li, W. Li, J. Kurths, Q. Luo, Y. Yang and S. Li* on pinning adaptive synchronization for complex dynamic networks with multi-links. Several new synchronization criteria for this kind of networks are given, which provide local or global stable synchronized states in the presence of uncertainty and deterioration.

The third paper by *D. Higham and A. Alsayed* presents a generalization of the betweenness centrality measure to the case of time-dependent networks, such as those arising in telecommunication and on-line social media networks. This measure, called temporal betweenness, quantifies the importance of a node in terms of its propensity to act as an intermediary when messages are passed around through the network. Its effectiveness is illustrated with both synthetic examples and real data sets on voice calls, emails and Twitter messages.

The following four papers deal with different aspects related to the structure and the topological and dynamical properties of multiplex networks.

B. Min, S. Lee, K.-M. Lee and K.-I. Goh discuss some aspects of multiplex networks with special emphasis on the role played by link overlaps. Empirical evidence points to the importance of such overlaps, i.e., of the fact that if two nodes are connected in one layer of the network, this increases the probability of their connection in other layers as well. Using a dynamics that sequentially first increases and then decreases attachments, the authors discover rich phase diagrams, with first-order transitions and high latencies, that demonstrate the dramatic effects that multiplex percolation may give rise to.

M. Salehia, P. Siyari, M. Magnani and D. Montesi, the authors propose a new concept of multidimensional epidemic threshold characterizing diffusion processes on interdependent networks, allowing different diffusion rates on the different networks and arbitrary degree distributions. The authors obtain a set of critical values of transmission rates and present an algorithm to numerically evaluate these values.

L. Rossi and M. Magnani discuss visualization strategies for multiplex networks, proposing that the development of specific visualization methods for multiplex networks will be one of the main drivers pushing current research results into daily practice.

M. Romance, L. Solá, J. Flores, E. García, A. García del Amo and R. Criado present some mathematical results on the Perron vector for multiplex networks. The conclusions they present result from the relationships between the irreducibility of some nonnegative block matrix associated with a multiplex network and the irreducibility of the corresponding matrices to each layer as well as the irreducibility of the adjacency matrix of the projection network. The authors also present the precise (non linear) relations that allow to express the Perron eigenvector of the multiplex network in terms of the Perron eigenvectors of its layers.

Some important applications related to multiplex networks have received special attention in the last two papers of this special issue.

The first, by *J. Borondo, A.J. Morales, R.M. Benito and J.C. Losada*, analyzes the structural properties of Twitter considered as a (three-layer) multiplex network. After a review of the main findings obtained in previous papers of the same authors, some new results about the relationships among the structure of the Follower Layer, the Mention Layer and the Retweet Layer are presented. In particular, the emergence of multiple leaders at different layers is studied, by analyzing the influence of two different elite collectives (politicians and mass media).

Finally, *J.A. Capitan, J. Aguirre and S. Manrubia* analyze the structure of genotype networks by means of the dynamics of a population of replicators evolving on it. It is shown that populations are organized according to a non-trivial hierarchical structure. Remarkably, current algorithms measuring (topological) modularity are unable to find the structure that conditions population dynamics. The problem of community detection is then tackled by analyzing the dynamical properties both in the stationary and in the transient regimes. The authors devote attention to the characterization of this latter scenario, by considering different (and meaningful) initial conditions. Finally, the results are compared to those obtained through static methods (typical in the community detection literature), revealing that no full agreement exists between the typical community detection algorithm and the results obtained from the *"intrinsic"* dynamics of neutral network.

We are extremely grateful to the international journal *Chaos, Solitons* & *Fractals*, and specially to E. Presani, Y. Gao, Editors in Chief and Publishers for having hosted this theme issue. We would also like to express our most sincere thanks and great appreciation to all the contributors of the different manuscripts for their work and support. It has been a great honor for us to count on the above leaders on Complex Networks to elaborate this Special Issue. Finally, it is almost an obligation to thank all those colleagues who helped us in the realization of this special issue, and especially those colleagues who assisted in the reviewing process of the papers.

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