

# Límite cuántico de la fotodetección

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- 2- Estadística de Poisson
- 3- BER
- 4- Sensibilidad

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## 1- Introducción

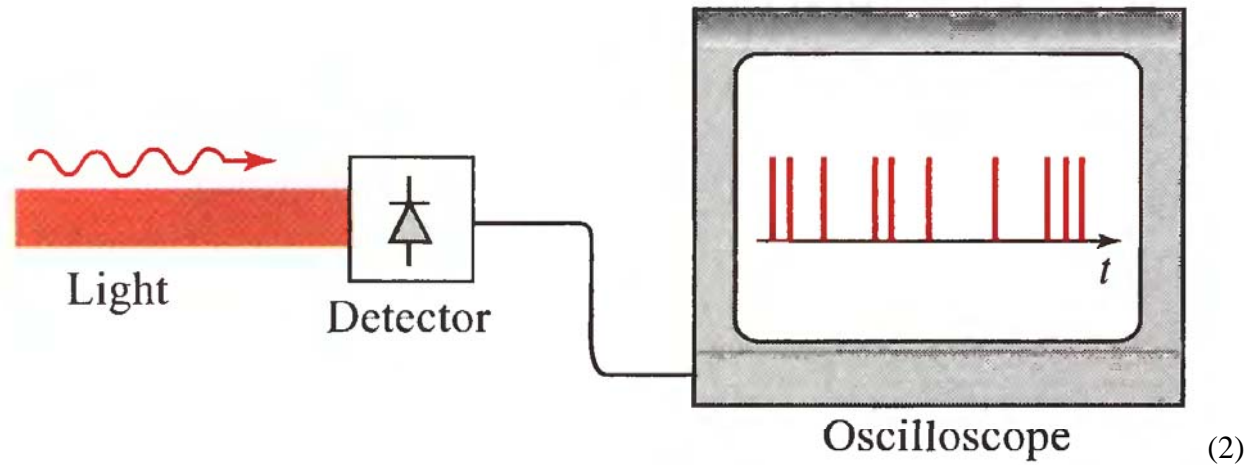
Para potencias muy bajas, aparecen los efectos cuánticos

Energía de un cuanto de energía electromagnética (fotón)

$$E_{\text{fotón}} = \underbrace{h}_{\text{Js}} \underbrace{\nu}_{1/s} \text{ (J)}$$

Potencia de un grupo de fotones

$$\boxed{\underbrace{P}_{\text{Wattios}} = \underbrace{N}_{\text{Fotones/seg}} \underbrace{h\nu}_{\text{Julios}}}$$



$$\text{Flujo de fotones} \rightarrow \frac{P}{h\nu} \left[ \frac{\text{Fotones}}{\text{seg}} \right]$$

Número medio de fotones detectados en un tiempo  $T$

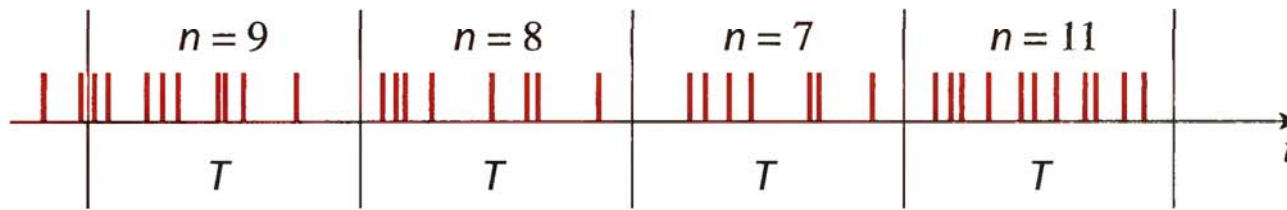
$$\bar{n} = \frac{P}{h\nu} T \left[ \text{Fotones} \right]$$

Mean photon-flux density for various sources of light.

Source	Mean Photon-Flux Density (photons/s-cm <sup>2</sup> )
Starlight	10 <sup>6</sup>
Moonlight	10 <sup>8</sup>
Twilight	10 <sup>10</sup>
Indoor light	10 <sup>12</sup>
Sunlight	10 <sup>14</sup>
Laser light <sup>a</sup>	10 <sup>22</sup>

<sup>a</sup> A 10-mW He–Ne laser beam at  $\lambda_o = 633$  nm focused to a 20- $\mu$ m-diameter spot.

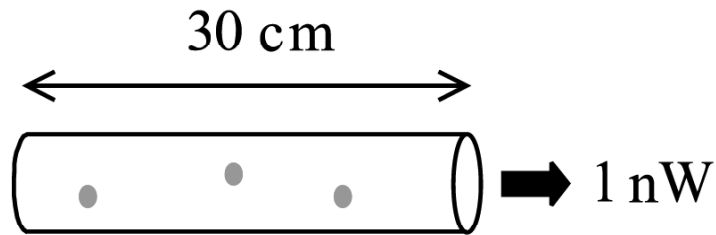
(2)



Random arrival of photons in a light beam of constant power  $P$  within during intervals of duration  $T$ . Although the optical power is constant, the number  $n$  of photons arriving within each interval is random.

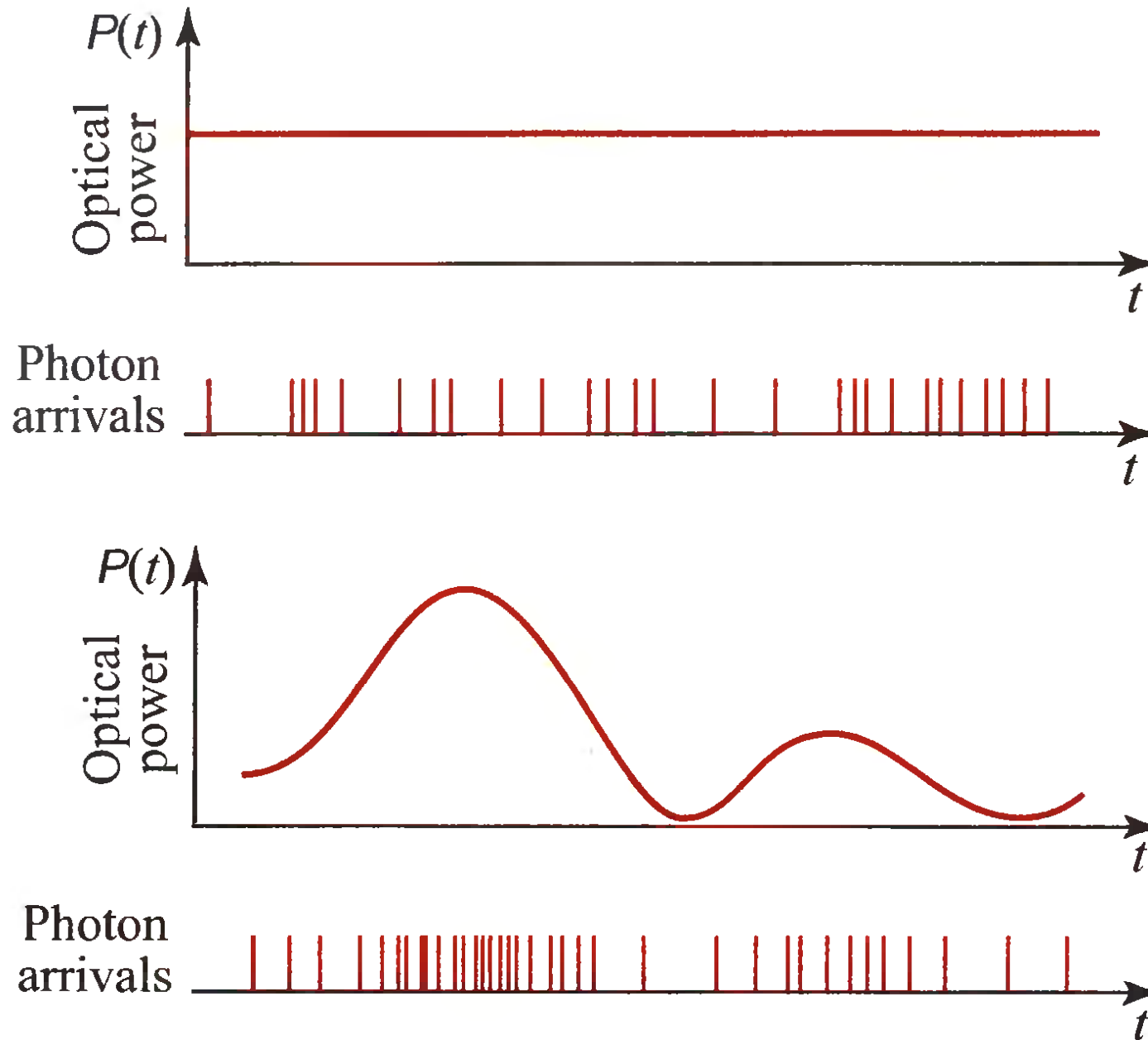
(2)

Ejemplo:



A 30 cm section of a beam light at 633 nm with a power of 1 nW contains three photons on average. (3)

$$\left. \begin{array}{l} \bar{n} = \frac{PT}{h\nu} \\ T = \frac{L}{c} \end{array} \right\} \bar{n} = \frac{PL\lambda}{hc^2} = \frac{(10^{-9})(30 \cdot 10^{-2})(0,633 \cdot 10^{-6})}{(6,62 \cdot 10^{-34})(3 \cdot 10^8)^2} = 3,18$$



(2)

## 2- Estadística de Poisson

Para potencias muy bajas → El ruido shot no se puede caracterizar con estadística gaussiana

→ Estadística de Poisson

→  $P(n)$  → Probabilidad de recibir  $n$  fotones

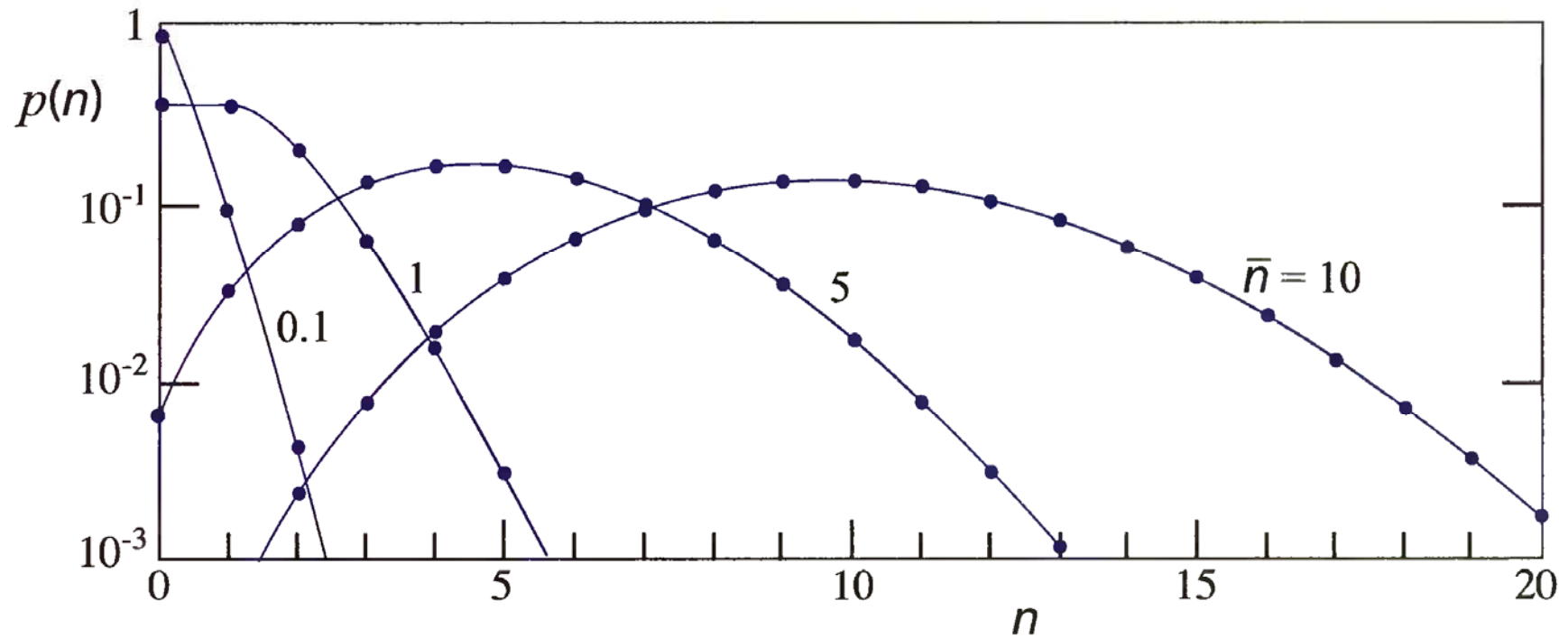
(siendo  $\bar{n}$  el número de fotones promedio esperados)

$$\text{Distribución de Poisson} \rightarrow \boxed{P(n) = \frac{e^{-\bar{n}} \bar{n}^n}{n!}} \quad [n = 0, 1, 2, \dots]$$

Limite Clásico → Distribución Gaussiana

$$P(n) = \frac{e^{-\bar{n}} \bar{n}^n}{n!} \quad [n \gg \bar{n}] \quad \rightarrow \quad P(n) = \frac{1}{\sqrt{2\pi\bar{n}}} e^{-\frac{(n-\bar{n})^2}{2\bar{n}}}$$





Poisson distribution  $p(n)$  of the photon number  $n$ .

(2)

### 3- BER

$$P(n) = \frac{e^{-\bar{n}} \bar{n}^n}{n!} \quad [n = 0, 1, 2, \dots]$$

Probabilidad de recibir  $\underbrace{1}_n$  fotón, cuando  $\bar{n} = 0 \rightarrow \underbrace{P(1/0)}_{\substack{\text{Probabilidad de} \\ \text{decidir 1,} \\ \text{cuando es 0}}} = 0$

Probabilidad de recibir  $\underbrace{0}_n$  fotones, cuando  $\bar{n} \neq 0 \rightarrow \underbrace{P(0/1)}_{\substack{\text{Probabilidad de} \\ \text{decidir 0,} \\ \text{cuando es 1}}} = e^{-\bar{n}}$

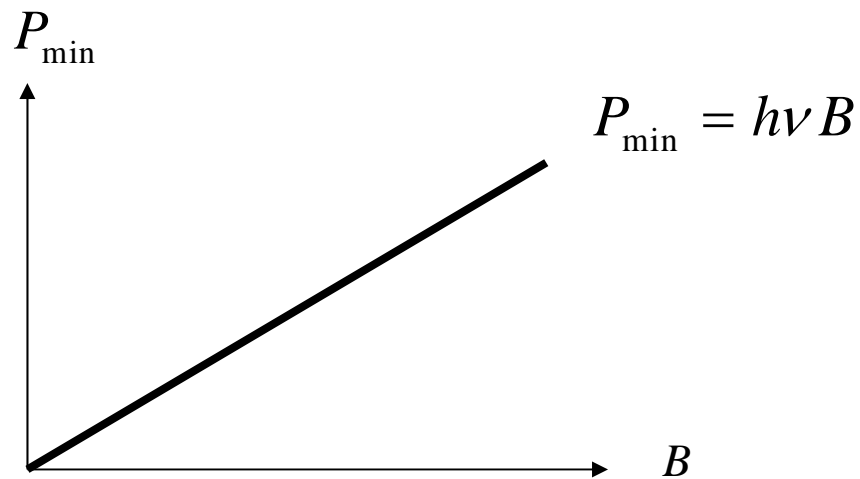
$$BER = \frac{P(0/1) + P(1/0)}{2} \rightarrow \boxed{BER = \frac{1}{2} e^{-\bar{n}}}$$

Ejemplo:  $BER = 10^{-9} \rightarrow \bar{n} = 20,7 \rightarrow 21$

## 4- Sensibilidad

### Limite Ideal

$$\left. \begin{aligned} P &= \frac{E}{t} \rightarrow P_{\min} = \frac{E_{\min}}{T_B} = \frac{h\nu}{T_B} \\ \frac{1}{T_B} &= B \end{aligned} \right\} \boxed{P_{\min} = h\nu B}$$



## Límite cuántico

Número promedio de fotones contenidos en un bit "1"  $\rightarrow \bar{n} \left[ \frac{\text{fotones}}{\text{bit "1"}} \right]$

Número promedio de fotones contenidos en un bit ("1" o "0")  $\rightarrow \bar{N}_{ph} \left[ \frac{\text{fotones}}{\text{bit}} \right]$

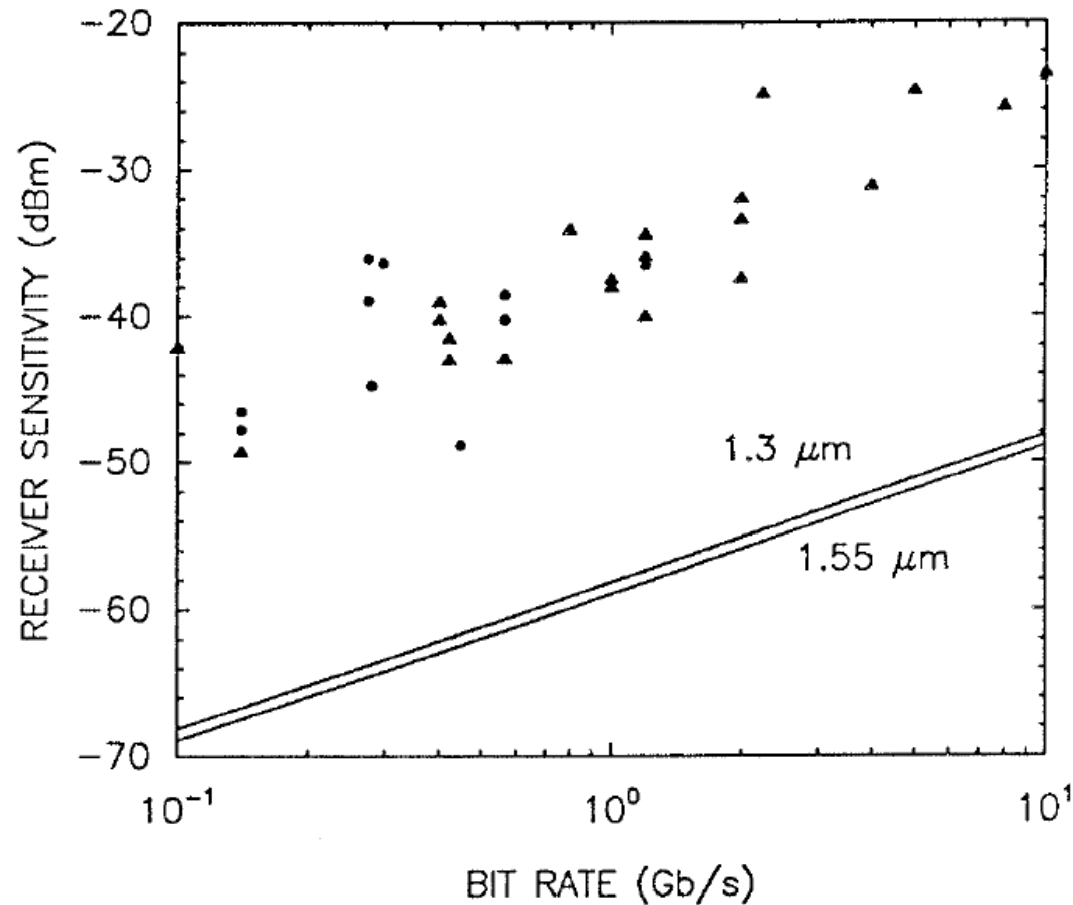
$$\bar{N}_{ph} \rightarrow \frac{"1" + "0"}{2} \rightarrow \frac{\bar{n} + 0}{2} \rightarrow \frac{\bar{n}}{2}$$

$$\left. \begin{array}{l} \bar{P}_{rec} = \frac{P_1 + P_0}{2} = \frac{P_1}{2} \\ P_1 = \bar{n} h \nu \underbrace{B}_{\frac{1}{T}} \end{array} \right\} \rightarrow \boxed{\bar{P}_{rec} = \frac{1}{2} \bar{n} h \nu B = \bar{N}_{ph} h \nu B}$$

Ejemplo:  $BER = 10^{-9} \rightarrow \bar{n} = 20,7 \rightarrow \bar{N}_{ph} = \frac{20,7}{2} \approx 10$

$$\lambda = 1,55 \mu m, B = 10 Gb / s \rightarrow \bar{P}_{rec} \approx 10 h \nu B = 13 nW \rightarrow (-48,9 dBm)$$

En la práctica este límite se excede  $\sim 20dB \rightarrow \bar{N}_p = 10 \cdot 100 = 1000$



**Figure 4.23:** Measured receiver sensitivities versus the bit rate for *p-i-n* (circles) and APD (triangles) receivers in transmission experiments near 1.3- and 1.55- $\mu\text{m}$  wavelengths. The quantum limit of receiver sensitivity is also shown for comparison (solid lines).

(1)

**Typical Sensitivities (Mean Number of Photons per Bit)  
of Some Optical Receivers Operating at Bit Rates  
in the Range 1 Mb / s to 2.5 Gb / s**

Receiver	Receiver Sensitivity (photons/bit)
Photon-limited ideal detector	10
Si APD	125
Er-doped silica-fiber preamplifier/ InGaAs <i>p-i-n</i> photodiode	215
InGaAs APD	500
<i>p-i-n</i> photodiode	6000

(2)

$$P_1 = \bar{n}h\nu B$$

$$\left. \begin{array}{l} \text{NRZ} \rightarrow \Delta f \approx \frac{B}{2} \rightarrow P_1 = 2\bar{n}h\nu\Delta f \\ \text{Recordando que } SNR = \frac{\eta_R P_1}{2h\nu\Delta f} \end{array} \right\} \rightarrow \underline{SNR = \eta_R \bar{n}} \xrightarrow{SNR=Q^2} Q = \sqrt{\eta_R \bar{n}}$$

$$\left. \begin{array}{l} \text{RZ} \rightarrow \Delta f \approx B \rightarrow P_{in} = \bar{n}h\nu\Delta f \\ \text{Recordando que } SNR = \frac{\eta_R P_1}{2h\nu\Delta f} \end{array} \right\} \rightarrow \underline{SNR = \frac{1}{2} \eta_R \bar{n}} \xrightarrow{SNR=Q^2} Q = \sqrt{\frac{1}{2} \eta_R \bar{n}}$$

$$\left. \begin{array}{l} \text{Ejemplo} \rightarrow \lambda = 1,55 \mu m \\ B = 10 Gb/s \end{array} \right\} \left. \begin{array}{l} SNR \approx 20 dB \rightarrow \bar{n} \approx 100 \\ P_1 \approx 130 nW \end{array} \right\}$$

(1) Agrawal, "Fiber-Optic Communication Systems", 3<sup>rd</sup> Ed., Wiley, 2002

(2) Saleh and Teich, "Fundamentals of Photonics", 1<sup>st</sup> Ed., Wiley, 2007

(3) Fox, "Quantum Optics", Oxford University Press, 2006