A Simplified Capacitive Model for center-tapped Multi-windings Transformers

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***Abstract*— A simplified capacitive model for transformers with center-tapped windings has been developed. A Finite Element Analysis (FEA) tool is used to compute the electric energy in the windings of the transformer in order to obtain the required parameters of the model. Due to its reduced number of elements it can be easily used to model the capacitive effects in symmetrical (center-tapped) multi-winding magnetic components such as transformers for Push-Pull (PP), Half-Bridge (HB) and Full-Bridge (FB) applications. Some experimental results are compared with simulations.**

***Index Terms*— Magnetic components, transformers, capacitive model, capacitances.**

I. INTRODUCTION

In the design of magnetic components for power electronics applications accurate models are needed in order to obtain reliable simulations which are meant to be used in many stages of the system design such as the analysis of the dynamic response of the circuit and Electromagnetic Compatibility (EMC) of the system. In this sense the accurate modeling of capacitive effects in magnetic components, such as transformer for power converters, is very important in order to analyze the electric behavior and performance of the system in which the component will be placed [1-3]. Actual capacitive models for high frequency transformers have many parameters in the form of lumped capacitors in the equivalent electric circuit [3-5] and the number of required elements to model the capacitive effects rapidly increases as the number of windings of the transformer grows, as it is shown in Figure 1, or if a complex model is needed to take into account all the desired capacitive effects [5]. As a consequence, the capacitive modeling of the component can be a very time consuming task and the simulation of the magnetic component (in circuit simulators), due to the big number of distributed lumped capacitors, might have convergence issues.

Taking advantage of the accuracy of FEA simulations, which include all the 2D effects present in the modeled component, a simplified capacitive model for center-tapped multi-winding transformers, such as Push-Pull and Half/Full Bridge type transformers, is proposed in this paper. Due to its reduced number of elements the model can be easily obtained from a few FEA simulations and the results are comparable with more complex capacitive models.

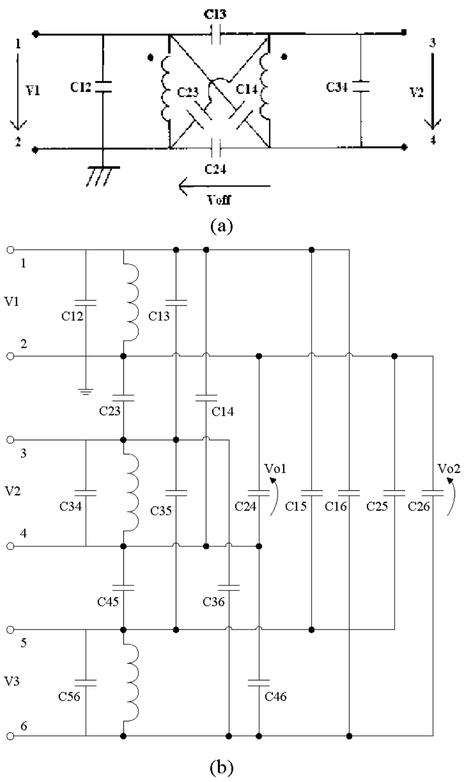


Figure 1. Classical capacitive model for two and three windings transformer (a and b respectively).

II. THE PROPOSED SYMMETRIC CAPACITIVE MODEL

The proposed capacitive model, which is suitable for HB/FB and Push-Pull type transformers, can be seen in Figure 2 (a and b respectively). Since the windings have one terminal shorted (center tap), the degrees of freedom [5], in both models, can be reduced to three and all the involved parameters, in form of lumped capacitors, can be calculated with the same number of FEA simulations. Following the procedure described in [5], the electric energy in the volume of the magnetic component can be calculated by means of equation (1). As both models have three degrees of freedom, equation (1) yields equation (2).

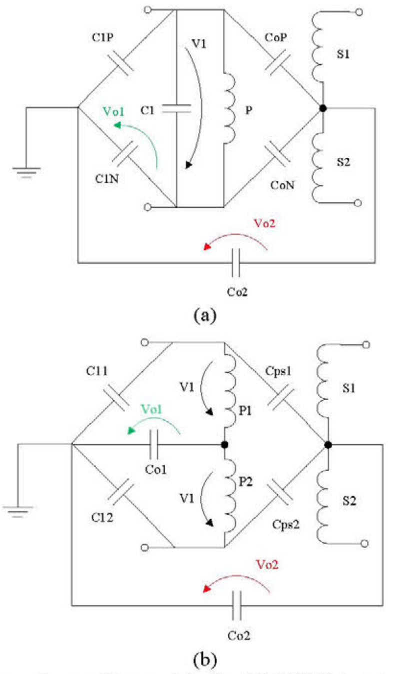


Figure 2. Proposed capacitive models for: (a) HB/FB type transformers and (b) Push-Pull type transformers.

$$\varepsilon = \frac{1}{2}\iiint \vec{D}.\vec{E}\, dV \qquad (1)$$

$$\varepsilon = \frac{1}{2}\iiint \left(\overrightarrow{D_1} + \overrightarrow{D_{01}} + \overrightarrow{D_{02}}\right).\left(\overrightarrow{E_1} + \overrightarrow{E_{01}} + \overrightarrow{E_{02}}\right) dV = \frac{1}{2}\iiint \left(\overrightarrow{D_1}.\overrightarrow{E_1} + \overrightarrow{D_1}.\overrightarrow{E_{01}} + \overrightarrow{D_1}.\overrightarrow{E_{02}} + \overrightarrow{D_{01}}.\overrightarrow{E_1} + \overrightarrow{D_{01}}.\overrightarrow{E_{01}} + \overrightarrow{D_{01}}.\overrightarrow{E_{02}} + \overrightarrow{D_{02}}.\overrightarrow{E_1} + \overrightarrow{D_{02}}.\overrightarrow{E_{01}} + \overrightarrow{D_{02}}.\overrightarrow{E_{02}}\right) dV = \frac{1}{2}\iiint \left[\left(\overrightarrow{D_1}.\overrightarrow{E_1}\right) + \left(\overrightarrow{D_{01}}.\overrightarrow{E_{01}}\right) + \left(\overrightarrow{D_{02}}.\overrightarrow{E_{02}}\right) + \left(\overrightarrow{D_1}.\overrightarrow{E_{01}} + \overrightarrow{D_{01}}.\overrightarrow{E_1}\right) + \left(\overrightarrow{D_1}.\overrightarrow{E_{02}} + \overrightarrow{D_{02}}.\overrightarrow{E_1}\right) + \left(\overrightarrow{D_{01}}.\overrightarrow{E_{02}} + \overrightarrow{D_{02}}.\overrightarrow{E_{01}}\right)\right] dV = \varepsilon_1 + \varepsilon_{01} + \varepsilon_{02} + \varepsilon_{101} + \varepsilon_{102} + \varepsilon_{012} \qquad (2)$$

A. *HB/FB transformers*

For half and full bridge type transformers, with one primary winding and two secondary windings (with a center tap), the three variables considered are the voltage applied to the primary winding and the offset voltages between one of the primary's terminals to ground and the secondary's center tap to ground (as it is shown in Figure 2a). The total electric energy associated to the circuit can be calculated with (3). For this model the three needed FEA simulations, which are represented in Figure 3, are as follows:

1- The first analysis consists in applying the voltage V1 in the primary winding. The secondary voltage is defined by the turns ratio, *tr*, (V1 ≠ 0, Vo1 = 0 and Vo2 = 0).
2- In the second analysis, a constant voltage is applied to all the turns of the primary winding while both windings of the secondary are set to 0V (V1 = 0, Vo1 ≠ 0 and Vo2 = 0).
3- Finally, in the last analysis all the turns of the primary winding are set to 0V while a constant voltage is applied to all the turns in the secondary side (V1 = 0, Vo1 = 0 and Vo2 ≠ 0).

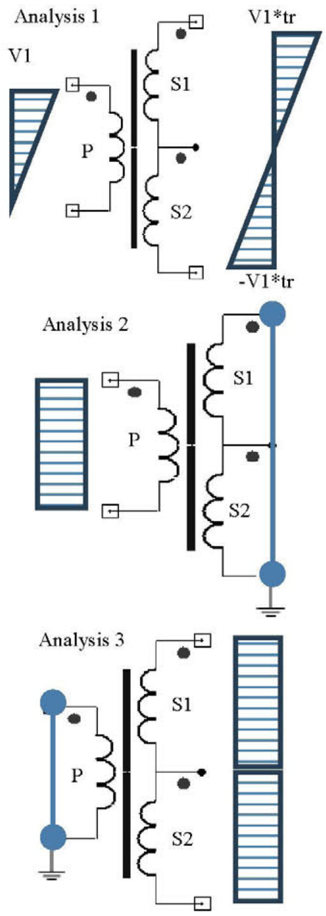


Figure 3. Representation of FEA simulations for HB/FB type transformers.

$$\varepsilon = \frac{1}{2}\left[V_1^2 C_1 + V_{01}^2 C_{1N} + V_{02}^2 C_{02} + (V_1 + V_{01})^2 C_{1P} + (V_{01} - V_{02})^2 C_{ON} + (V_1 + V_{01} - V_{02})^2 C_{OP}\right] = \frac{1}{2}V_1^2(C_1 + C_{1P} + C_{OP}) + \frac{1}{2}V_{01}^2(C_{1N} + C_{1P} + C_{OP} + C_{ON}) + \frac{1}{2}V_{02}^2(C_{02} + C_{OP} + C_{ON}) + V_1 V_{01}(C_{1P} + C_{OP}) + V_1 V_{02}(-C_{OP}) + V_{01}V_{02}(-C_{OP} - C_{ON}) = \varepsilon_1 + \varepsilon_{01} + \varepsilon_{02} + \varepsilon_{101} + \varepsilon_{102} + \varepsilon_{012} \qquad (3)$$

B. *Push-Pull type transformers*

In the case of Push-Pull type transformers, that consist in two primary windings and two secondary windings (both sides with center tap), the three variables considered are the voltage applied to both primary windings and the offset voltages between the primary and the secondary center tap to ground (Figure 2b). The electric energy associated to the proposed circuit can be computed by means of (4) and the required FEA simulations are represented in Figure 4.

$$\varepsilon = \frac{1}{2}\left[V_{01}^2 C_{01} + V_{02}^2 C_{02} + (V_1 + V_{01})^2 C_{11} + (V_{01} - V_1)^2 C_{12} + (V_1 + V_{01} - V_{02})^2 C_{ps1} + (-V_1 + V_{01} - V_{02})^2 C_{ps2}\right] = \frac{1}{2}V_1^2\left(C_{11} + C_{12} + C_{ps1} + C_{ps2}\right) + \frac{1}{2}V_{01}^2\left(C_{01} + C_{11} + C_{12} + C_{ps1} + C_{ps2}\right) + \frac{1}{2}V_{02}^2\left(C_{02} + C_{ps2} + C_{ps2}\right) + V_1V_{01}\left(C_{11} - C_{12} + C_{ps1} - C_{ps2}\right) + V_1V_{02}\left(-C_{ps1} + C_{ps2}\right) + V_{01}V_{02}\left(-C_{ps1} - C_{ps2}\right) \tag{4}$$

Then, using the superposition theorem, all the capacitors in the models can be extracted from FEA simulation results matching (2) with (3) or (4).

C. Calculating self and mutual capacitances

The differential and common mode equivalent capacitances (self and mutual capacitances respectively) of the models can be calculated using circuit analysis.

The equivalent self and mutual capacitance for the HB/FB type transformer model, which are the capacitance seen from the primary winding with the secondary open and the capacitance between primary and secondary with their respective terminals shorted, can be calculated applying Kirchhoff's laws to the circuits of the Figure 5 which are the representation of the lumped capacitors (without the windings for both self and mutual capacitances) of the model in the Figure 2a.

The self-capacitance can be calculated by means of the admittance matrix of the circuit 4a (placing a current source, i, in the terminals of the circuit) that is given by (5). The expression for the mutual capacitance can be found by simple inspection to the circuit 4b and it is given in (8).

$$Y = (jw)\begin{bmatrix} C_1 + C_{oP} + C_{1P} & -C_{oP} & -C_{1P} \\ -C_{oP} & C_{oP} + C_{oN} + C_{o2} & -C_{o2} \\ -C_{1P} & -C_{o2} & C_{o2} + C_{1N} + C_{1P} \end{bmatrix} \tag{5}$$

$$Z_{11} = \frac{1}{jwC_{self}} \tag{6}$$

$$C_{self} = \{[(C_{oP} + C_1 + C_{1P})C_{1N} + (C_{oN} + C_1)C_{1P} + (C_{oP} + C_1)C_{oN} + C_1C_{oP}]C_{o2} + [(C_{oN} + C_{oP})C_{1P} + (C_{oP} + C_1)C_{oN} + C_1C_{oP}]C_{1N} + [(C_{oP} + C_1)C_{oN} + C_1C_{oP}]C_{1P}\} \;/\; [(C_{1N} + C_{1P} + C_{oN} + C_{oP})C_{o2} + (C_{1N} + C_{1P})(C_{oN} + C_{oP})] \tag{7}$$

$$C_{mutual} = C_{o2} + \frac{(C_{1N}+C_{1P})(C_{oN}+C_{oP})}{C_{1N}+C_{1P}+C_{oN}+C_{oP}} \tag{8}$$

In terms of self and mutual capacitances, the lumped capacitances C_{1P}, C_{1N} and C_{o2} in the model of the Figure 2a, as they are terminal-to-ground capacitances, can be considered as "outside of the transformer" and they can be neglected in the self and mutual capacitances calculations. As a result, equations (7) and (8) can be simplified as follows.

$$C_{self} = C_1 + \frac{C_{oN}*C_{oP}}{C_{oN}+C_{oP}} \tag{9}$$

$$C_{mutual} = C_{oP} + C_{oN} \tag{10}$$

A similar analysis for the model of the PP type transformer results in the expressions (11), (12) and (13) for the self capacitance of primary (P1 or P2), the self capacitance between the positive terminal of P1 and the negative terminal of P2 and the mutual capacitance respectively.

$$C_{selfP1,P2} = \left[\left[C_{11} + \left(\frac{1}{C_{ps1}} + \frac{1}{C_{ps2}} + \frac{1}{C_{12}}\right)^{-1}\right]^{-1} + \frac{1}{C_{01}}\right]^{-1} \tag{11}$$

$$C_{selfP1P2} = \frac{C_{11}*C_{12}}{C_{11}+C_{12}} + \frac{C_{ps1}*C_{ps2}}{C_{ps1}+C_{ps2}} \tag{12}$$

$$C_{mutual} = C_{ps1} + C_{ps2} \tag{13}$$

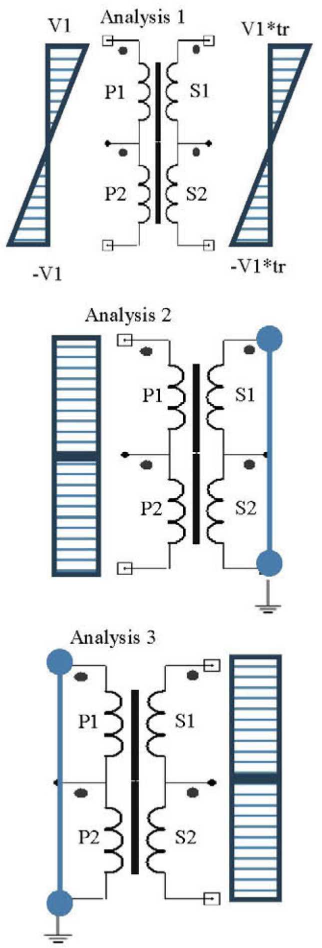


Figure 4. Representation of FEA simulations for Push-Pull type transformers.

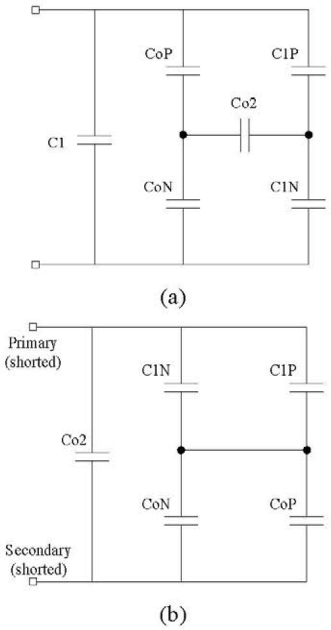


Figure 5. Equivalent capacitive circuit of the HB/FB type. (a) Seen from the primary winding with secondary open and (b) winding-to-winding capacitance with the corresponding (primary and secondary) terminals shorted.

III. RESULTS

Several magnetic components have been tested to assess the validity of the simplified model. The measured components are a Push-Pull and Half/Full bridge type planar transformers, whose specifications can be seen in Figure 7 in appendix II. The components have been modeled in the FEA tool MAXWELL16, where only nominal values in the properties of the materials were used (the relative permittivity of the non-conductive layers was set according to nominal values of FR4 specifications and the thickness of all layers, conductive and non-conductive, do not include tolerances in the fabrication).

The equivalent capacitances of the components were measured with the impedance analyzer Agilent 4294A. The calculated lumped capacitors for both models are shown in table 3 and the equivalent self and mutual capacitances (calculated and measured) are compared in table 4 (see nomenclature in table 2).

For comparison purposes, the classical capacitive model in Figure 1a is used for the calculations of mutual and self capacitances of the built components. After all the required lumped capacitors are calculated (see appendix II), a circuit simulator is used in order to set the connections in the central tap and determine the self and mutual equivalent capacitances (by means of the corresponding simulation, open or short circuit, for the respective capacitance). The calculated values, by means of both classical and simplified methods, of the equivalent self and mutual capacitances of both components are compared in table 1.

Table 1. Comparison between classical and proposed model calculated results.

Component	Description	Equivalent Capacitance (pF)		Relative error (%)
		Classical Model	Proposed Model	
HB/FB	C_{Self}	273.03	263.73	3.41
	C_{Mutual}	847.77	817.33	3.59
PP	C_{SelfP1}	323.42	319.35	1.27
	C_{Mutual}	463.97	460.37	0.78

IV. CONCLUSIONS

A New simple model that deals with the capacitive effects in center-tapped multi-winding magnetic components based on FEA computations has been presented. Since all the required parameters of the model are obtained from FEA calculations, the equivalent calculated capacitances include all the geometrical and physical effects (which is characteristic in FEA) resulting in a very accurate result. Due to its reduced number of elements, the convergence, when the component is analyzed in circuit simulators, is improved. The model can be properly used in Push-Pull, Half-Bridge and Full-Bridge type transformers.

As the calculated capacitive effect is very sensitive to the permittivity and insulator thickness, the overall accuracy of the results will be conditioned to the accuracy of the used values and, since possible variations from the specifications (such as dimensional tolerances and electric properties of the materials used in the manufactured components) were not reflected in the FEA model and calculations, along with the applied geometry transformation, in order to simplify the modeling in the FEA tool, differences from the measured values might be of importance. Even so, the measured values are very close to calculations and simple comparison with the results obtained using a more complex capacitive model show the accuracy of the proposed reduced model with respect to classical approaches.

Table 2. Description of measured equivalent capacitances.

Model	Equivalent Capacitance	Description
PP	C_{P1}	Self-capacitance of primary 1
	C_{P2}	Self-capacitance of primary 2
	C_{P1S1}	Mutual capacitance between primary 1 and secondary 1
	C_{P1S2}	Mutual capacitance between primary 1 and secondary 2
	C_{P2S1}	Mutual capacitance between primary 2 and secondary 1
	C_{P2S2}	Mutual capacitance between primary 2 and secondary 2
HB/FB	C_P	Self-capacitance of primary
	C_{PS1}	Mutual capacitance between primary and secondary 1
	C_{PS2}	Mutual capacitance between primary and secondary 2

Table 3. Calculated lumped capacitors.

Model/ Parameter		Calculated lumped Capacitance (pF)
PP	C11	-70.51
	C12	-70.51
	Cps1	230.15
	Cps2	230.21
	Co1	141.01
	Co2	0
HB/FB	C1p	-3.00E-12
	C1n	84.99E-12
	Cop	420.74
	Con	396.59
	C1	59.58
	Co2	87.99E-12

Table 4. Calculated and measured equivalent capacitances.

Model	Equivalent Capacitance	Calculated Capacitance (pF)	Measured Capacitance (pF)
PP	C_{P1}	319.35	332.20
	C_{P2}	319.35	330.50
	C_{P1S1}	460.37	557.30
	C_{P1S2}	460.37	566.90
	C_{P2S1}	460.37	546.60
	C_{P2S2}	460.37	542.80
HB/FB	C_P	263.73	276.85
	C_{PS1}	817.33	1004.10
	C_{PS2}	817.33	1007.30

APPENDIX I

CLASSICAL CAPACITIVE MODEL OF A TRANSFORMER WITH THREE AND FOUR WINDINS. LUMPED CAPACITORS.

For a three windings transformer, and five degrees of freedom, (1) yields equation (14) for the circuit of the Figure 1b.

$$\varepsilon = \frac{1}{2}\iiint\left(\overrightarrow{D_1}+\overrightarrow{D_2}+\overrightarrow{D_3}+\overrightarrow{D_{01}}+\overrightarrow{D_{02}}\right).\left(\overrightarrow{E_1}+\overrightarrow{E_2}+\overrightarrow{E_3}+\overrightarrow{E_{01}}+\overrightarrow{E_{02}}\right)dV = \varepsilon_1+\varepsilon_2+\varepsilon_3+\varepsilon_{01}+\varepsilon_{02}+\varepsilon_{12}+\varepsilon_{13}+\varepsilon_{101}+\varepsilon_{102}+\varepsilon_{23}+\varepsilon_{201}+\varepsilon_{202}+\varepsilon_{301}+\varepsilon_{302}+\varepsilon_{012} \quad (14)$$

From the circuit in the figure, the electric energy stored in the lumped capacitors can be calculated as follows:

$$\varepsilon = \frac{1}{2}\left[V_1^2C_{12}+V_2^2C_{34}+V_3^2C_{56}+V_{01}^2C_{24}+V_{02}^2C_{26}+{V_{13}}^2C_{13}+{V_{14}}^2C_{14}+{V_{15}}^2C_{15}+{V_{16}}^2C_{16}+{V_{23}}^2C_{23}+{V_{25}}^2C_{25}+{V_{35}}^2C_{35}+{V_{36}}^2C_{36}+{V_{45}}^2C_{45}+{V_{46}}^2C_{46}\right] \quad (15)$$

Where

$$V_{13} = V_1 - V_2 - V_{01} \quad (16)$$
$$V_{14} = V_1 - V_{01} \quad (17)$$
$$V_{15} = V_1 - V_3 - V_{02} \quad (18)$$
$$V_{16} = V_1 - V_{02} \quad (19)$$
$$V_{23} = V_2 + V_{01} \quad (20)$$
$$V_{25} = V_3 + V_{02} \quad (21)$$
$$V_{35} = (V_2 + V_{01}) - (V_3 + V_{02}) \quad (22)$$
$$V_{36} = V_2 + V_{01} - V_{02} \quad (23)$$
$$V_{45} = V_3 + V_{02} - V_{01} \quad (24)$$
$$V_{46} = V_{01} - V_{02} \quad (25)$$

Matching (14) with (15)

$$\varepsilon_1 = \frac{1}{2}V_1^2(C_{12}+C_{13}+C_{14}+C_{15}+C_{16}) \quad (26)$$
$$\varepsilon_2 = \frac{1}{2}V_2^2(C_{13}+C_{23}+C_{34}+C_{35}+C_{36}) \quad (27)$$
$$\varepsilon_3 = \frac{1}{2}V_3^2(C_{15}+C_{25}+C_{35}+C_{45}+C_{56}) \quad (28)$$
$$\varepsilon_{01} = \frac{1}{2}V_{01}^2(C_{13}+C_{14}+C_{23}+C_{24}+C_{35}+C_{36}+C_{45}+C_{46}) \quad (29)$$

$$\varepsilon_{02} = \frac{1}{2}V_{02}^2(C_{15} + C_{16} + C_{25} + C_{26} + C_{35} + C_{36} + C_{45} + C_{46}) \quad (30)$$

$$\varepsilon_{12} = -V_1V_2(C_{13}) \quad (31)$$

$$\varepsilon_{13} = -V_1V_3(C_{15}) \quad (32)$$

$$\varepsilon_{101} = -V_1V_{01}(C_{13} + C_{14}) \quad (33)$$

$$\varepsilon_{102} = -V_1V_{01}(C_{15} + C_{16}) \quad (34)$$

$$\varepsilon_{23} = -V_2V_3(C_{35}) \quad (35)$$

$$\varepsilon_{201} = V_2V_{01}(C_{13} + C_{23} + C_{35} + C_{36}) \quad (36)$$

$$\varepsilon_{202} = -V_1V_{02}(C_{35} + C_{36}) \quad (37)$$

$$\varepsilon_{301} = -V_3V_{01}(C_{35} + C_{45}) \quad (38)$$

$$\varepsilon_{302} = V_3V_{02}(C_{15} + C_{25} + C_{35} + C_{45}) \quad (39)$$

$$\varepsilon_{012} = -V_{01}V_{02}(C_{35} + C_{36} + C_{45} + C_{46}) \quad (40)$$

For a transformer with four windings, the model of Figure 1a becomes like the circuit shown in Figure 6. As there are seven degrees of freedom, equation (1) becomes (41). The total electric energy stored in the capacitors of the circuit of the Figure 6 is given by expression (42). Then, and all the lumped elements can be found matching (41) with (42).

$$\varepsilon = \frac{1}{2}\iiint \left(\overrightarrow{D_1} + \overrightarrow{D_2} + \overrightarrow{D_3} + \overrightarrow{D_4} + \overrightarrow{D_{01}} + \overrightarrow{D_{02} + D_{03}}\right).\left(\overrightarrow{E_1} + \overrightarrow{E_2} + \overrightarrow{E_3} + \overrightarrow{E_4} + \overrightarrow{E_{01}} + \overrightarrow{E_{02}} + \overrightarrow{E_{03}}\right)dV = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4 + \varepsilon_{01} + \varepsilon_{02} + \varepsilon_{03} + \varepsilon_{12} + \varepsilon_{13} + \varepsilon_{14} + \varepsilon_{101} + \varepsilon_{102} + \varepsilon_{103} + \varepsilon_{23} + \varepsilon_{24} + \varepsilon_{201} + \varepsilon_{202} + \varepsilon_{203} + \varepsilon_{34} + \varepsilon_{301} + \varepsilon_{302} + \varepsilon_{303} + +\varepsilon_{401} + \varepsilon_{402} + \varepsilon_{403} + \varepsilon_{012} + \varepsilon_{013} + \varepsilon_{023} \quad (41)$$

$$\varepsilon = \frac{1}{2}\left[V_1^2C_{12} + V_2^2C_{34} + V_3^2C_{56} + V_{01}^2C_{24} + V_{02}^2C_{26} + V_{03}^2C_{28} + {V_{13}}^2C_{13} + {V_{14}}^2C_{14} + {V_{15}}^2C_{15} + {V_{16}}^2C_{16} + {V_{17}}^2C_{17} + {V_{18}}^2C_{18} + {V_{23}}^2C_{23} + {V_{25}}^2C_{25} + {V_{27}}^2C_{27} + {V_{35}}^2C_{35} + {V_{36}}^2C_{36} + {V_{37}}^2C_{37} + {V_{38}}^2C_{38} + {V_{45}}^2C_{45} + {V_{46}}^2C_{46} + {V_{47}}^2C_{47} + {V_{48}}^2C_{48} + {V_{57}}^2C_{57} + {V_{58}}^2C_{58} + {V_{67}}^2C_{67} + {V_{68}}^2C_{68}\right] \quad (42)$$

Where

$$V_{13} = V_1 - (V_2 + V_{01}) \quad (43)$$

$$V_{14} = V_1 - V_{01} \quad (44)$$

$$V_{15} = V_1 - (V_3 + V_{02}) \quad (45)$$

$$V_{16} = V_1 - V_{02} \quad (46)$$

$$V_{17} = V_1 - (V_4 + V_{03}) \quad (47)$$

$$V_{18} = V_1 - V_{03} \quad (48)$$

$$V_{23} = V_2 + V_{01} \quad (49)$$

$$V_{25} = V_3 + V_{02} \quad (50)$$

$$V_{27} = V_4 + V_{03} \quad (51)$$

$$V_{35} = (V_2 + V_{01}) - (V_3 + V_{02}) \quad (52)$$

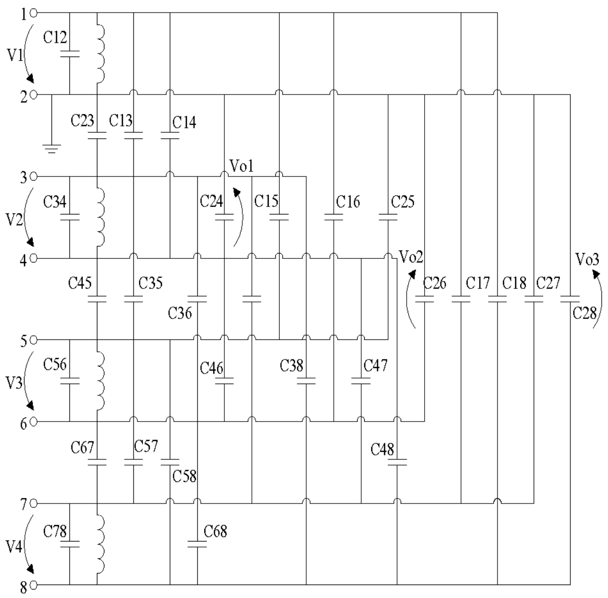


Figure 6. Classical capacitive model for a four windings transformer.

$$V_{36} = V_2 + V_{01} - V_{02} \quad (53)$$

$$V_{37} = (V_2 + V_{01}) - (V_4 + V_{03}) \quad (54)$$

$$V_{38} = V_2 + V_{01} - V_{03} \quad (55)$$

$$V_{45} = V_3 + V_{02} - V_{01} \quad (56)$$

$$V_{46} = V_{01} - V_{02} \quad (57)$$

$$V_{47} = V_{01} - (V_4 + V_{03}) \quad (58)$$

$$V_{48} = V_{01} - V_{03} \quad (59)$$

$$V_{57} = (V_3 + V_{02}) - (V_4 + V_{03}) \quad (60)$$

$$V_{58} = V_3 + V_{02} - V_{03} \quad (61)$$

$$V_{67} = V_4 + V_{03} - V_{02} \quad (62)$$

$$V_{68} = V_{02} - V_{03} \quad (63)$$

APPENDIX II

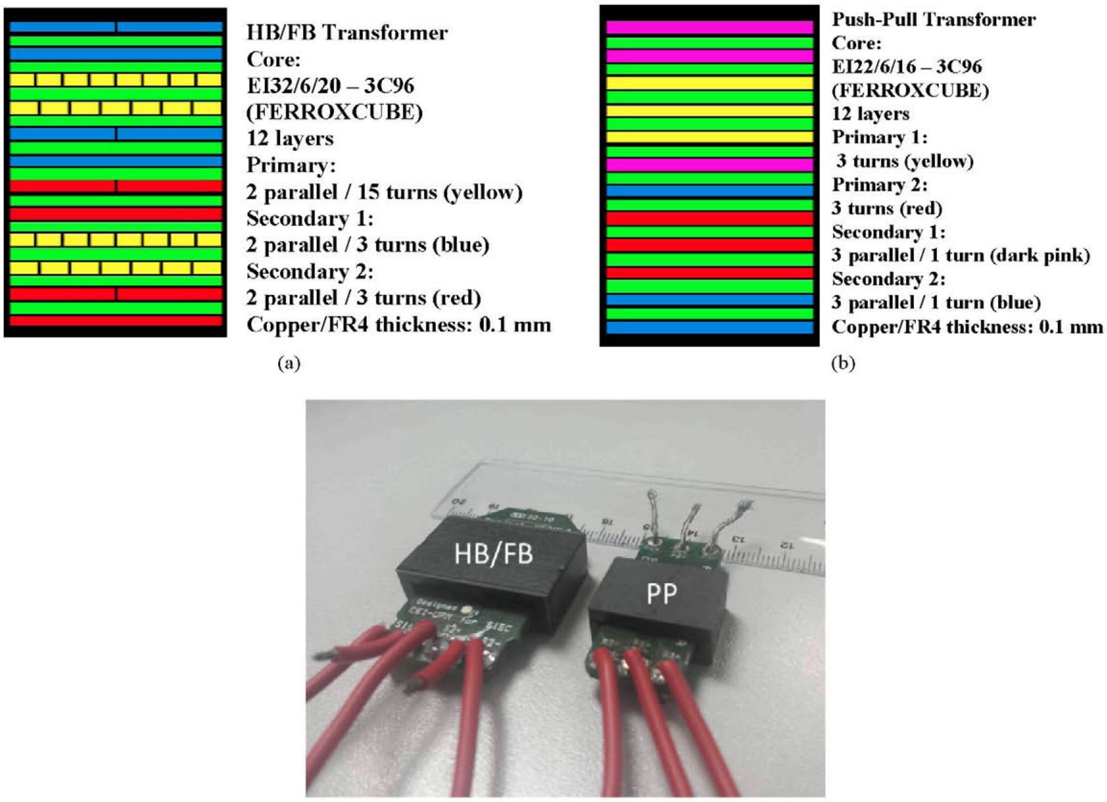
FEA ANALYSIS. BOUNDARIES, MATERIAL'S PROPERTIES AND RESULTS.

The built components, as well as their winding structure and specifications, are shown in Figure 7. For simplicity, 2D analysis is used for FEA simulations. The components were modeled in the FEA tool using the structure transformation described in [6].

The axis-symmetric representation of the components is shown in Figure 8. In the figure, the blue area is the solution region whose borders have been defined as far as two times the distance of any edge of the core from the origin of coordinates.

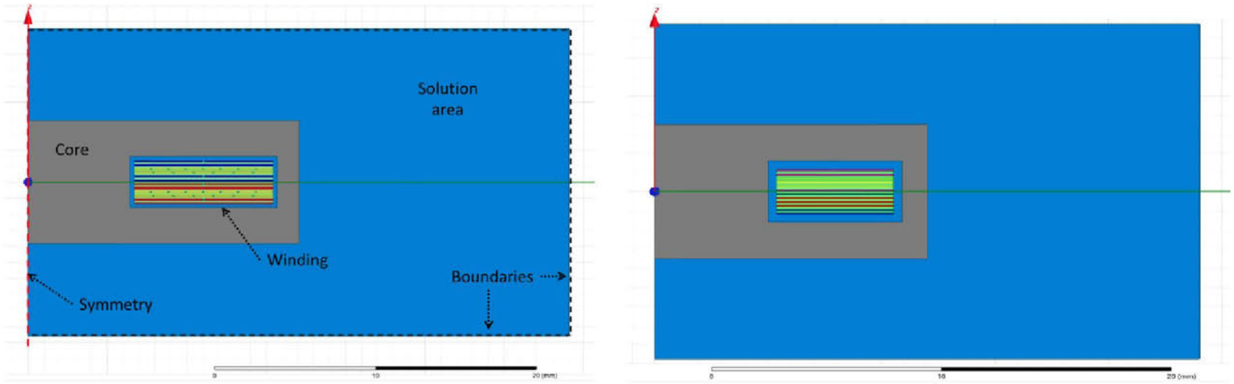
The relative permittivity used for the non-conductive material is 4.5 and the charge in the core has been set to zero. The necessary analyses for the calculations of the energy of the reduced model have already been illustrated in section II and the analyses that are necessary for the calculations of the energy in the classical model for a component with three and four windings can be deduced from [5]. The voltages across the windings have been distributed according to the required analysis with a value of 1 V.

The results of both components, in terms of energies, are summarized in table 5 for the simplified and classical models respectively.



(c)

Figure 7. Measured transformers. Sectional view of the PCB and specifications of the HB/FB and Push-Pull transformers in (a) and (b) respectively and (c) image of the built components (HB/FB and PP in left and right sides respectively).



(a) (b)

Figure 8. Axis-symmetric representation of the FEA modeled transformers. (a) HB/FB transformer and (b) PP transformer.

Table 5. FEA calculated energies of simplified and classical models of built components.

Component	Results (J/V^2)				
	Simplified model	Classical model			
PP	ε_1 = 159.6774E-12	ε_1 = 54.2882E-12	ε_{12} = -2.9371E-14	ε_{24} = -114.8140E-12	ε_{303} = -117.6968E-12
	ε_{01} = 230.1828E-12	ε_2 = 54.2882E-12	ε_{13} = -114.7690E-12	ε_{201} = 115.1620E-12	ε_{401} = -229.6047E-12
	ε_{02} = 230.18286E-12	ε_3 = 173.9030E-12	ε_{14} = -3.3244E-13	ε_{202} = -2.9013E-13	ε_{402} = -117.6968E-12
	ε_{101} = -6.1659E-14	ε_4 = 173.9763E-12	ε_{101} = -6.1099E-14	ε_{203} = -114.8140E-12	ε_{403} = 347.9527E-12
	ε_{102} = 6.1659E-14	ε_{01} = 115.1578E-12	ε_{102} = -114.7690E-12	ε_{34} = -117.6968E-12	ε_{012} = -5.9225E-13
	ε_{012} = -460.3655E-12	ε_{02} = 173.9030E-12	ε_{103} = -3.3244E-13	ε_{301} = -5.9225E-13	ε_{013} = -229.6047E-12
		ε_{03} = 173.9763E-12	ε_{23} = -2.9013E-13	ε_{302} = 347.8061E-12	ε_{023} = -117.6968E-12
HB/FB	ε_1 = 240.1573E-12	ε_1 = 285.2944E-12	ε_{12} = -67.8315E-12	ε_{201} = 282.8749E-12	
	ε_{01} = 408.6626E-12	ε_2 = 163.3885E-12	ε_{13} = -67.3784E-12	ε_{202} = -19.9977E-12	
	ε_{02} = 408.6626E-12	ε_3 = 225.6847E-12	ε_{101} = -259.8119E-12	ε_{301} = -189.8879E-12	
	ε_{101} = 420.7363E-14	ε_{01} = 406.4051E-12	ε_{102} = -260.1249E-12	ε_{302} = 420.6841E-12	
	ε_{102} = -420.7363E-14	ε_{02} = 405.7725E-12	ε_{23} = -32.5514E-12	ε_{012} = -276.7957E-12	
	ε_{012} = -817.3251E-12				

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