Deterministic coherence resonance in a ring of coupled chaotic oscillator

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ABSTRACT

We study synchronization in a ring of three unidirectional coupled chaotic coupled chaotic Rössler oscillator in the presence of a small mismatch between their natural frequencies $\omega_1 < \omega_2 < \omega_3$. The forward (1 → 2 → 3 → 1) and the backward (1← 2 ← 3 ← 1) coupling directions are considered. As the coupling strength increases, the common route to synchronization for both configurations is: intermittent phase synchronization →imperfect phase synchronization → perfect phase synchronization → lag or anticipated synchronization. The difference in synchronization scenario for the two configurations occurs only for small couplings in the regime of intermittent phase synchronization characterized by the time-averaged dominant frequency in the chaotic power spectrum and the slope of the time dependence of the difference between the oscillators' phases. Although phase synchronization is more easily achieved for the backward coupling configuration, the forward coupling results in significant coherence enhancement which occurs within a narrow range of the coupling strengths as soon as the oscillators synchronize their phases. In this regime all oscillators behave almost periodically.

Introduction

Synchronization is commonly understood as a collective state of coupled systems. Generally, synchronization means some relations between functions of different processes due to their interactions [1]. As a result of synchronization, coupled oscillatory systems adjust their individual frequencies in a certain relation.
The notion of synchronization has been extended to chaotic dynamics since the appearance of the work of Fujisaka and Yamada [2] who first demonstrated that two identical chaotic systems can change their individual behaviors from uncorrelated oscillations to completely identical oscillations as the coupling strength is increased.

Model

The Rössler oscillator is a prototypical system frequently used for studying synchronization of chaotic oscillators [6]. We consider two possible configurations shown in Fig.1.: (a) The forward (1 → 2 → 3 → 1) and (b) he backward (1← 2 ← 3 ← 1) coupling directions.

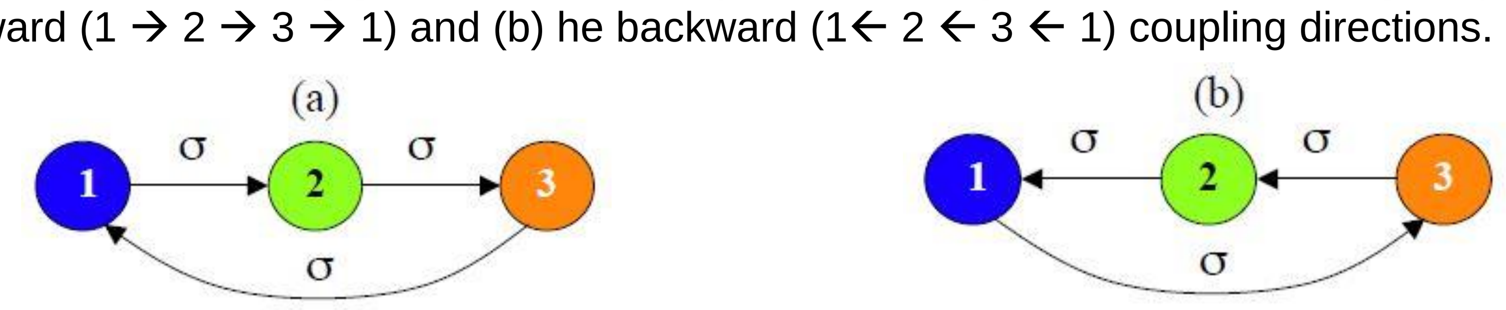


Figure 1: Ring configurations of three oscillators unidirectional coupled in (a) forward and (b) backward directions.

Any of them can be described by the following system of equations:

$$\dot{x}_i = -w_i \cdot y_i - z_i + \sigma \cdot (x_j - x_i)$$
$$\dot{y}_i = w_i \cdot x_i + a \cdot y_i$$
$$\dot{z}_i = b + z_i \cdot (x_i - c)$$

where *i*, *j* = 1, 2, 3 (*i* ≠ *j*) is the oscillator number, x_i, y_i, z_i, are the state variables of the *ith* oscillator, $\omega_1 = 0.95$, $\omega_2 = 0.97$ and $\omega_3 = 0.99$ are the oscillators' natural frequencies, and is the coupling strength. A slave oscillator *i* is coupled through variable x_j of a neighboring master oscillator *j*. Being uncoupled (σ = 0), the oscillators are chaotic for a = 0.165, b = 0.2, and c = 10. Starting from different initial conditions, they oscillate asynchronously, as seen from the time series in Fig. 2 (a)

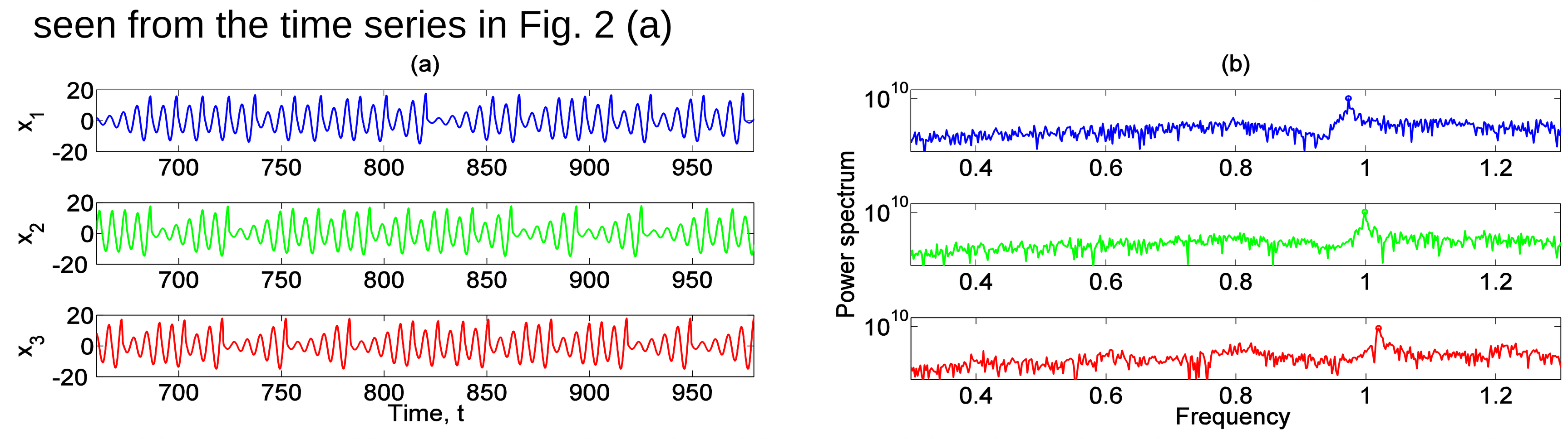


Figure 2: (a) Timer series and (b) power spectra of x variables of three uncoupled Rössler oscillators demonstrating asynchronous chaotic behavior with dominant frequencies, Ω_1^0, Ω_2^0 and Ω_3^0.

The chaotic power spectra of the system variables, as those shown in Fig. 2 (b), exhibit maxima at the dominant frequencies: $\Omega_1^0 \approx 0.975$, $\Omega_2^0 \approx 0.998$, $\Omega_3^0 \approx 1.02$. Due to the system nonlinearity, these frequencies are a little higher than the natural frequencies of the corresponding oscillators.
Now, we consider how synchronization emerges when the coupling strength increases. Quantitatively, phase synchronization between a pair of oscillators *I* and *j* can be characterized by the difference between their instantaneous phases $\theta_{ij} = \phi_i - \phi_j$ where $\phi_1 = arctan\left({}^{y_{i,j}}/_{x_{i,j}}\right)$ (ref). Since the oscillators have distinct natural frequencies, θ_{ij} of the uncoupled oscillators either increases or decreases monotonically in time (depending on a sign of the frequency mismatch). The oscillators begin to interact already for a very small coupling strength (σ > 5 x 10^{-3}) that manifests itself as the appearance of irregular windows of phase synchronization in the time series. This regime is referred to as *intermittent phase synchronization.*
This situation is illustrated in Fig. 3, where we plot the time dependences of θ_{21} situation where for three different coupling strengths. The horizontal parts of these dependences correspond to the windows of phase synchronization where the dominant frequency of the slave oscillator is locked by the corresponding master oscillator.

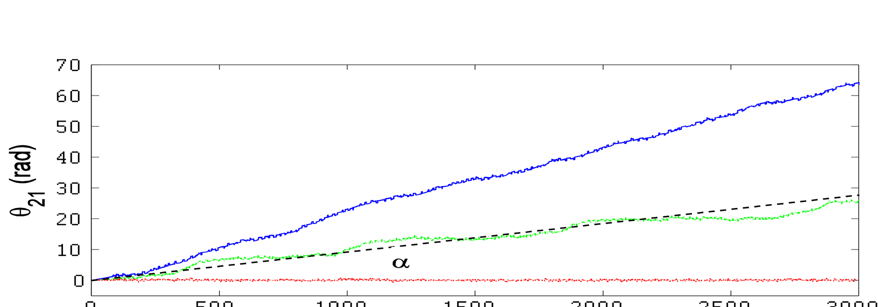


Figure 3: Phase difference θ_{21} between oscillators 2 and 1 as a function of time for σ = 6.6 x 10^{-3} (upper blue line), 26.4 x 10^{-3} (middle green line), and 46.2 x 10^{-3}(lower black line). The dashed line is a linear fit for the middle dependence with slope ***α***. The horizontal parts of these dependences indicate the regions of intermittent phase synchronization.

Intermittent Phase Synchronization and Deterministic Coherence Resonance

In the forward coupling configuration, the sequence of the closest spaced oscillators (1 → 2 → 3 → 1) is arranged so that the natural frequency of a slave oscillator is higher than the frequency of a corresponding master oscillator, i.e. ω1 < ω2 < ω3. Thus, oscillator 1 is master for oscillator 2, and oscillator 2 is master for oscillator 3, while oscillators 2 and 3 are slaves for 1 and 2, respectively. In the backward coupling configuration, the sequence of the most closely spaced oscillators (3→ 2 → 1) is arranged so that the natural frequency of a slave oscillator is lower than the frequency of the slave oscillator leads to phase synchronization. To characterize intermittent phase synchronization, we use the time-averaged dominant frequency $\bar{\Omega}$ and slope ***α***.

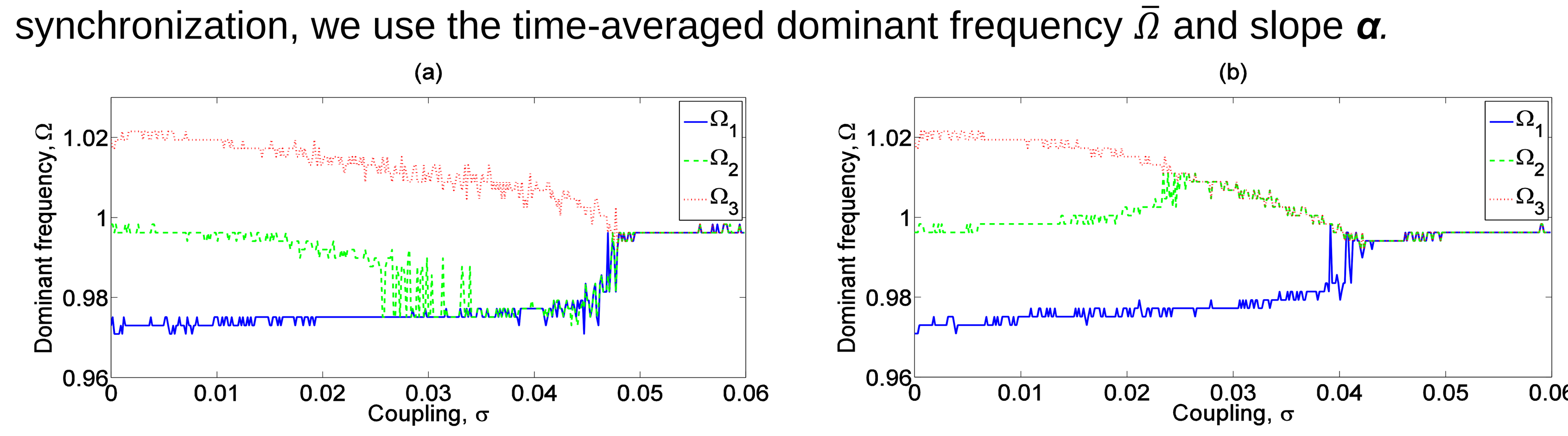


Figure 4: (a,b) Time-averaged dominant frequencies $\bar{\Omega}_i$ and slopes α_{ij} as a function of coupling strength for (a,c) forward and (b,d) backward directions.

In the region of phase synchronization, synchronization quality is characterized by comparing amplitudes of coupled oscillators. The commonly used measures of lag and anticipated synchronization are cross-correlation and similarity functions, C and S, defined respectively as [3, 4]. The higher maximum cross-correlation C_{max} and the lower minimum similarity S_{min} mean better synchronization. Figure 5 show how similarity vary with the coupling strength.

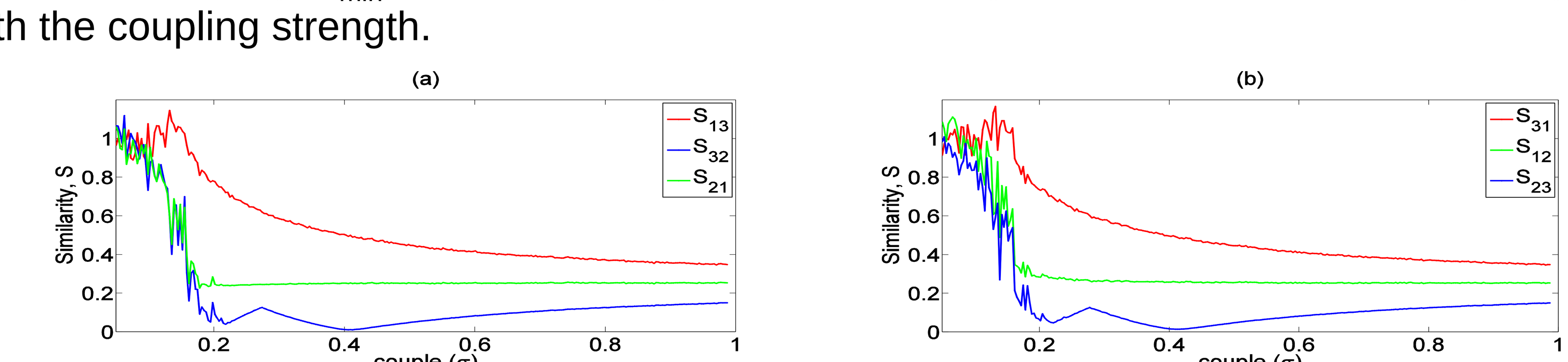


Figure 5: (a) minimum similarity as a function of coupling strength for every pair of oscillators for forward and (b) for backward directions of coupling.

For small coupling (σ < 0.048) in the region of intermittent phase synchronization, C_{max} is very low [Fig. 5(a,b)], while S_{min} [Fig. 5(c,d)] is very high. As the coupling increases from σ = 0.048 to σ = 0.180, imperfect phase synchronization becomes perfect thus resulting in slowly increasing C_{max} and slowly decreasing S_{min}. In this regime the phase difference θ fluctuates around its average value, but does not extend the modulation period, i.e. $\theta\epsilon[-\pi, \pi]$. As σ increases, the amplitude of these fluctuations decreases leading to perfect phase synchronization.

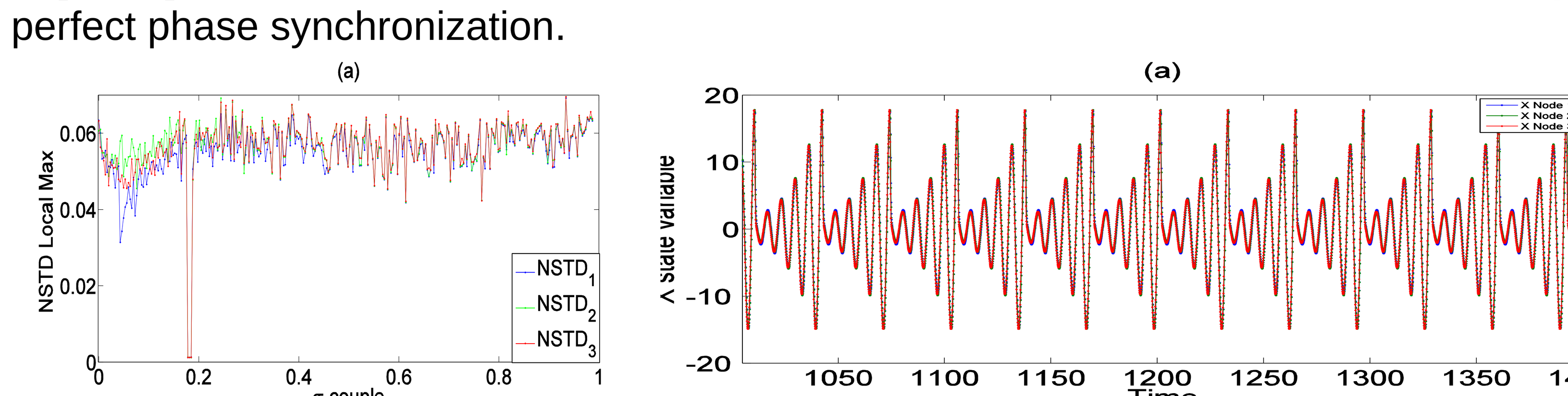
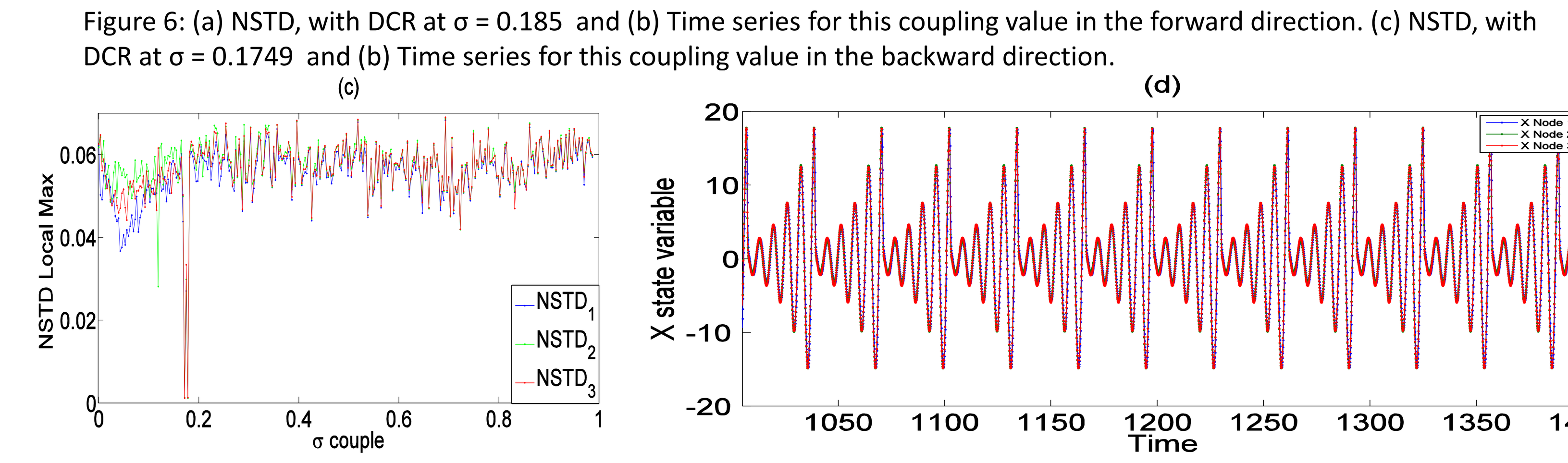


Figure 6: (a) NSTD, with DCR at σ = 0.185 and (b) Time series for this coupling value in the forward direction. (c) NSTD, with DCR at σ = 0.1749 and (b) Time series for this coupling value in the backward direction.



Conclusions

We have studied the route to synchronization in a ring of three unidirectional coupled Rössler oscillators with small mismatch between their natural frequencies $\omega_1 < \omega_2 < \omega_3$,in forward (1 → 2 → 3 → 1) and the backward (1← 2 ← 3 ← 1) coupling directions. As the coupling strength increases, the oscillators first synchronize their phases intermittently and then adjust their amplitudes. We quantitatively characterized intermittent phase synchronization by the time-averaged dominant frequency in the power spectrum of every oscillator and the linearly approximated slope of the time-dependent phase difference for each pair of the coupled oscillator. Then, we have observed that phase synchronization is more easily achieved when a master is faster than a slave.
Finally, We have analyzed not only forward but also backward direction and we found Deterministic Resonance Coherence for both scenarios, although there are some differences: each scenario has a specific couple value to reach Deterministic Resonance Coherence. The more sensitive to coupling is the backward direction.

References

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