11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
E. Oñate, J. Oliver and A. Huerta (Eds)

AERODYNAMIC SHAPE OPTIMIZATION OF A 3D WING VIA VOLUMETRIC B-SPLINES

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Key Words: *Aerodynamic shape optimization, geometry parameterization, NURBS, Control Box, B-Splines, Gradient-based Optimization.*

Abstract. This paper shows a gradient-based aerodynamic shape optimization of a three-dimensional wing using volumetric B-Splines. The wing is enclosed in a volumetric parallelepiped, commonly referred as *control box*, which can be seen as a rubber box that deforms the enclosed space. The deformation of the wing is provided by the manipulation of the control points, where the gradients are calculated using the continuous adjoint solution. This technique can be applied to arbitrary three-dimensional complex designs and provides some advantages over other traditional geometry parameterization methods, such as deformation locality and the ability to handle some geometric constraints.

1 INTRODUCTION

Gradient-based methods for aerodynamic shape optimization [1] aim to minimize a suitable cost or objective function, as aerodynamic drag, by means of deformations on a selected geometry parameterization. The selection of the geometry parameterization is a crucial step in any optimization process in order to obtain viable geometries. Some parameterizations are directly linked with engineer definitions, such as the PARSEC method [2], which uses eleven geometric parameters to manipulate the shape of two-dimensional airfoils. More recently, the class/shape function transformation (CST) method [3] is a formulation that describes aircraft components surfaces as the product of a class function and a shape function.

In a previous work of the authors, Non-Uniform Rational B-Splines (NURBS) [4] and Free Form Deformation (FFD) [5] have been employed. The use of NURBS does not provide a direct physical meaning of the geometry, so that the designer could understand the effect of modifying the position of one particular control point in an intuitive way, but has the ability to explore the design space for unconventional shapes and provides some level of local refinement for irregular shapes. However, surface NURBS parameterizations face some practical problems to be implemented on complex three-dimensional geometries. On the other hand, FFD seems more suitable to be implemented on complex three-dimensional configurations, although lacks locality, so the displacement of one control point affects the whole design.

In this paper, the selected design parameters are the control points of a volumetric B-Spline, which encloses the geometry in a parallelepiped, commonly referred as *control box*. This can

* ISDEFE subcontracted

be seen as a transparent rubber box that deforms the space by the manipulation of control points. This technique overcomes some practical problems; avoids the need to build a suitable NURBS surface that matches the original geometry, while the control box provides deformation locality. However, while in the FFD technique, the parametric coordinates are obtained directly from the Cartesian coordinates by a mathematical property of the Bernstein polynomials, the control box approach employs a generalized polynomial basis and requires an additional effort to calculate these parametric coordinates.

This paper is organized as follows: First, an introduction on gradient-based methods for aerodynamic shape optimization is given in section 1. Then, the mathematical background of the proposed geometry parameterization is detailed in section 2. Section 3 details the test case to be used, which is the DPW-W1 three-dimensional wing from the 3rd AIAA CFD Drag Prediction Workshop [6], and the validation of the gradients calculated with the adjoint in comparison with those computed by finite differences. Meanwhile, section 4 shows the numerical results of an optimization obtained with this parameterization and a gradient-based algorithm. The design parameters are the vertical displacement of the control points, as is shown in Figure 1. Finally, section 5 draws the conclusions and suggests future activities.

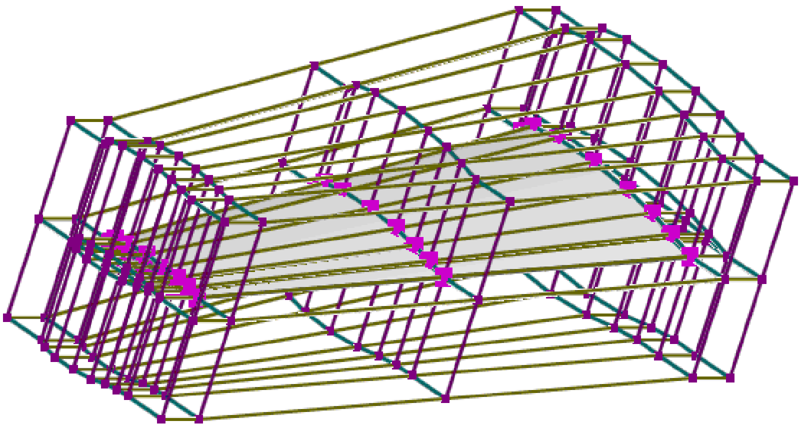

Figure 1: Selected test case, DPW-W1 wing and control box parameterization.

2 MATHEMATICAL BACKGROUND

The mathematical definition of the control box is given by:

$$N(u,v,w) = \sum_i \sum_j \sum_k B_i(u) B_j(v) B_k(w) C_{i,j,k} \tag{1}$$

where u, v, and w are the parametric coordinates, B_i is the basis function defined for each three parametric directions *i, j, k,* and C are the control points spatial coordinates. The basis functions are calculated using the following recursive expression:

$$B_{i,0}(t) = \begin{cases} 1 & \text{if } i \le t < i+1 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

$$B_{i,k}(t) = \frac{(t-i)B_{i,k-1} + (i+k-i)B_{i+1,k-1}}{k-1}$$

$$t \cdot (N - p) \in [0,1)$$

where p is the order of the basis, k is the order each iteration step, and N is the number of control points in a row. The same expression will be obtained from the conventional NURBS surface expression by using an uniform knot vector, no weights and the control points are placed to enforce one parametric coordinate is constant. On the other hand, if the order is $p = N\text{-}1$, the Bernstein polynomials are obtained, which are the most common mathematical expression employed by free form deformation techniques. So, in a sense, the control box can be seen as a generalized form of B-Splines.

The parametric coordinates u, v, and w of each vertex of the computational grid are initially calculated from its spatial coordinates x, y, and z. This calculation is only required once, at the beginning of any optimization process, because the parametric coordinates remain constant upon deformations due to displacement of the control points. The algorithm developed to solve the so-called *inversion point problem* consists on running a Newton-Raphson algorithm once provided an appropriate estimate.

The estimation is based on the property that, under some conditions, the parametric coordinates can be obtained directly through a linear transformation of the sub-box, formed by 8 control points, that encloses the vertex.

Let's consider a sub-box formed by eight adjacent control points *C000, C100, C010, C001, C101, C011, C110,* and *C111*. There is a transformation matrix M that accomplishes:

$$M \cdot C \rightarrow \begin{cases} C100 - C000 = \{1,0,0\} \\ C010 - C000 = \{0,1,0\} \\ C001 - C000 = \{0,0,1\} \end{cases} \tag{3}$$

If the transformation matrix M is applied to all eight control points creates a rectangular cube and the order of the control box is 1 or $N\text{-}1$, being N the number of control points (so the basis functions are the Bernstein polynomials), then the parametric coordinates $U=\{u,v,w\}$ can be directly obtained from the Cartesian coordinates $X=\{x,y,z\}$ as:

$$U \approx (X - C000) \cdot M \tag{4}$$

In any case, the exact solution is refined with a Newton-Raphson algorithm:

$$U^{n+1} = U^{n} + \left(X - B(u,v,w)\right)\begin{bmatrix} \dfrac{\partial B(u,v,w)}{\partial u} & \dfrac{\partial B(u,v,w)}{\partial v} & \dfrac{\partial B(u,v,w)}{\partial w} \end{bmatrix}^{-1} \tag{5}$$

The basis employed in this work are monotonic positive; although discontinuities may appear, for example, if several control points are located close together, which may affect the convergence of the algorithm.

3 TEST CASE AND GEOMETRIC CONSTRAINTS

The test case selected is the DPW-W1 three-dimensional wing, from the 3rd AIAA Workshop on Drag Prediction [6] and the geometric restrictions considered are beam constraints at 20% and 75% of the chord and the leading edge curvature. The parameterization is a control box with B-Splines basis functions, which are first order in the vertical and wing span directions, and third order in the wing section direction. The wing is divided into 3 sections along the span direction at the root, mid-span, and tip, as shown in Figure 1, where there are 12 control points for each section to control the shape of the profile, as shown in Figure 2, and three layers to perform independent deformations of the upper and lower wing.

The design variables are the vertical displacements of the 36 control points.

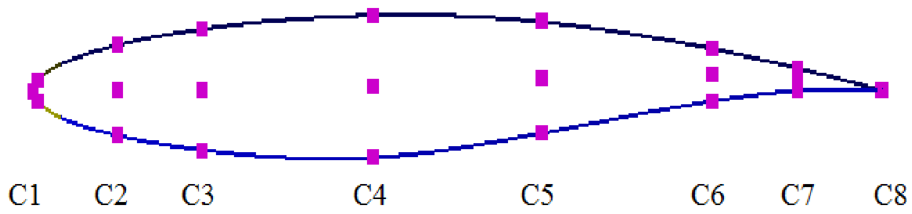

Figure 2: Section of the DPW1 wing, where the initial location of the control points is shown.

In order to handle the geometric constraints, certain control points are initially located on the aerodynamic surface. Some of them are strategically located at 20% and 75% of the chord, and three control points very close together at the leading edge to manipulate the leading edge curvature. In this way, it is possible to track the displacement of the geometry. The process is similar to track a vertex of the computational grid that, luckily, could be located at that position. The basis functions indicate how the geometry is deformed upon movements of the control points. For this particular case, the basis functions in the direction of the profile are shown in Figure 3.

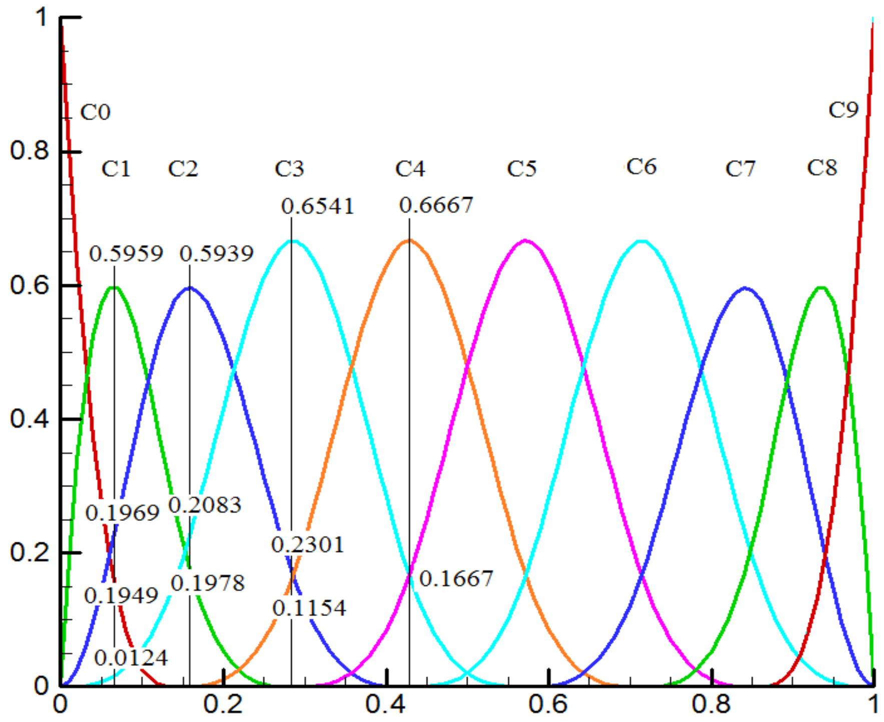

Figure 3: Third order B-Spline basis functions, where the exact influence of each control point is indicated at each control point initial position.

The control points C3 and C7 are located at 20% and 75% of the chord respectively. Upon vertical movements of the control points, the thickness T of the profile is modified as:

$$\Delta T_{20} = 0.0004 \cdot \Delta C_1 + 0.2301 \cdot \Delta C_2 + 0.6541 \cdot \Delta C_3 + 0.1154 \cdot \Delta C_4$$
$$\Delta T_{75} = 0.0005 \cdot \Delta C_5 + 0.1978 \cdot \Delta C_6 + 0.5939 \cdot \Delta C_7 + 0.0.2083 \cdot \Delta C_8 \quad (6)$$

On the other hand, the curvature at the leading edge is the inverse of the radius, which can be approximately calculated as:

$$r = \frac{abc}{4K} \tag{7}$$

Where *abc* is the product of the segments and *K* is the area of the triangle formed by the three control points at the leading edge, as it is illustrated in Figure 4.

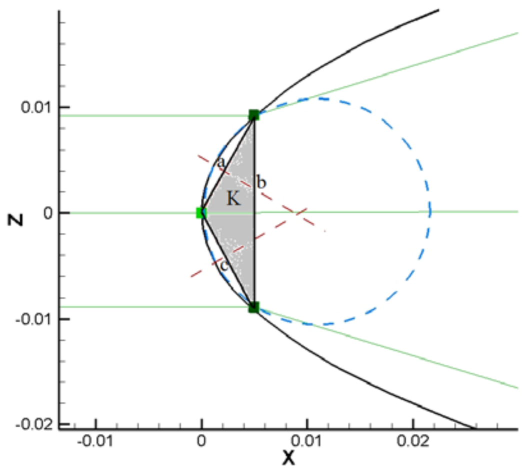

Figure 4: Calculation of the leading edge curvature as an approximation through three points closely located.

4 NUMERICAL RESULTS: GRADIENT-BASED OPTIMIZATION

The DPW wing is optimized at Mach 0.76 and angle of attack 0.5, employing Euler numerical solution. Gradients are calculated from the adjoint solution, which provides the surface sensitivity of a cost function, such as aerodynamic drag and lift. The gradients to design variables δJ are obtained as an integration over the surface *S* as follows:

$$\delta J = \int_S \delta\overline{x} \cdot \left(\delta j \cdot \overline{n}\right) \tag{8}$$

Where δx are the so called geometric sensitivities, δj is the adjoint solution provided by the numerical simulation, and *n* is the surface normal. In this case, the design variables are vertical displacement of the control points and the geometric sensitivities are exactly the basis function. This formulation does not take into account tangential deformations and therefore it could be inaccurate at numerical level at the leading and trailing edge, that are kept fixed in order to maintain the angle of attack.

For validation purposes, the adjoint gradients are compared with those obtained by finite differences in the baseline configuration, as it is shown in Figure 5.

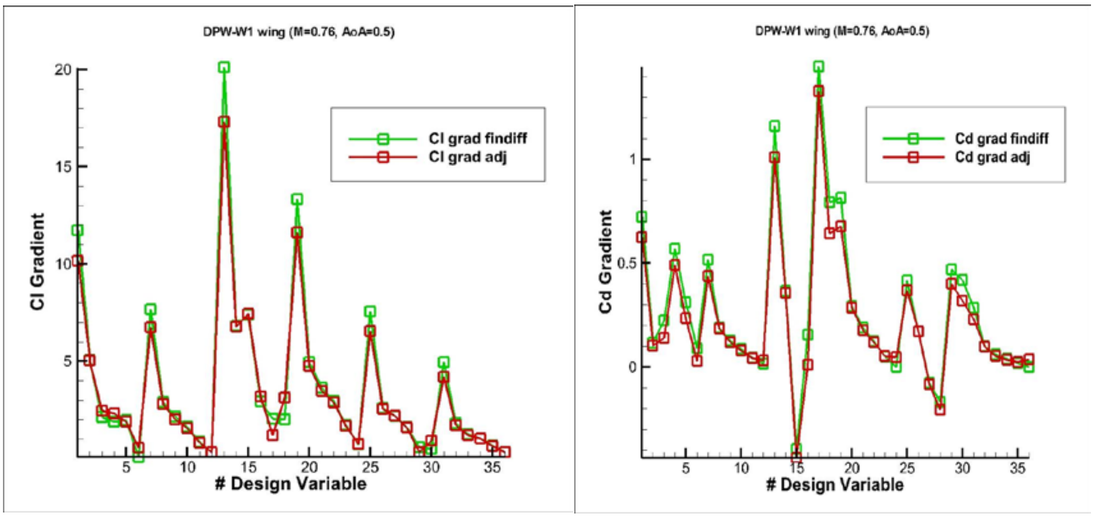

Figure 5: Comparison of the gradients obtained from the adjoint with those computed by finite differences.

At a first glance, there is a good agreement between the gradients obtained from the adjoint solution and by finite differences, at least for optimization purposes. Small discrepancies can be explained by an insufficient grid resolution, inaccurate calculation of the surface normal and a degradation of the adjoint solution near the leading edge.

The objective function selected for the optimization is to minimize the inverse of efficiency; there is no lift restraint. The optimization method is a simple steepest descend, which proves to be very robust to inaccurate adjoint solutions and easy to implement. The main difficulty relays on selecting the step size. One suggestion is to run two optimization steps with a very small stepping and then, to infer a convenient stepping considering that the convergence roughly follows an exponential. Figure 6 shows the convergence process.

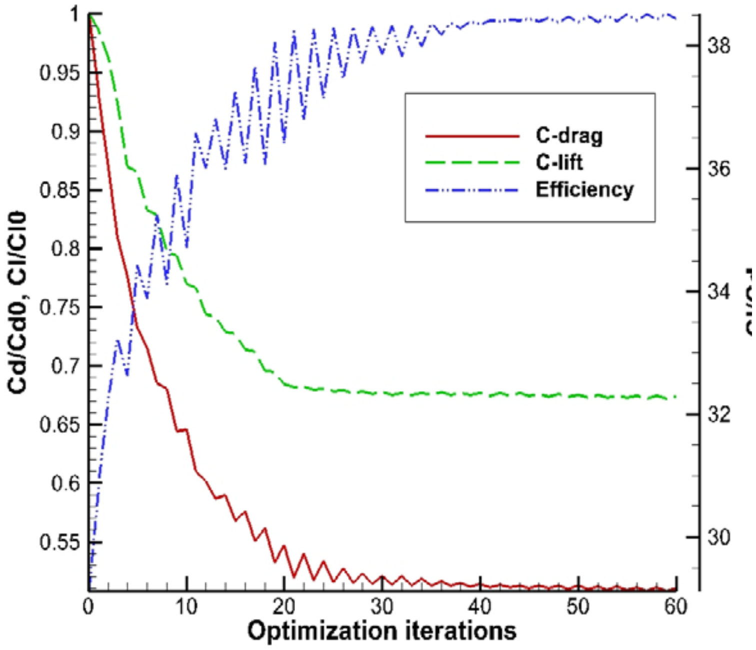

Figure 6: Evolution of aerodynamic coefficients through the optimization iterations of the DPW wing at M= 0.76 and $\alpha = 0.5$

The aerodynamic efficiency obtained has been increased roughly 25% compared to the original, at the expense of drastically reducing the lift, which was not constrained. The final shapes and pressure distribution obtained are shown in Figure 7, while Figure 8 shows the original and final Mach distribution. As can be observed in the mentioned figures, the shock wave has been considerably reduced, although it has not been completely disappeared. One of the main problems with gradient based methods is that the optimization falls into a local minimum, where the gradients are zero, so the optimizer cannot achieve further improvements and it is more likely to happen with high density of design variables. This is an open issue that can be tackled with a multilevel deformation, one coarse level that envelops a finer one, and global search algorithms. But these improvements fall out of the scope of this work, which only aims to demonstrate the feasibility of this kind of geometry parameterization.

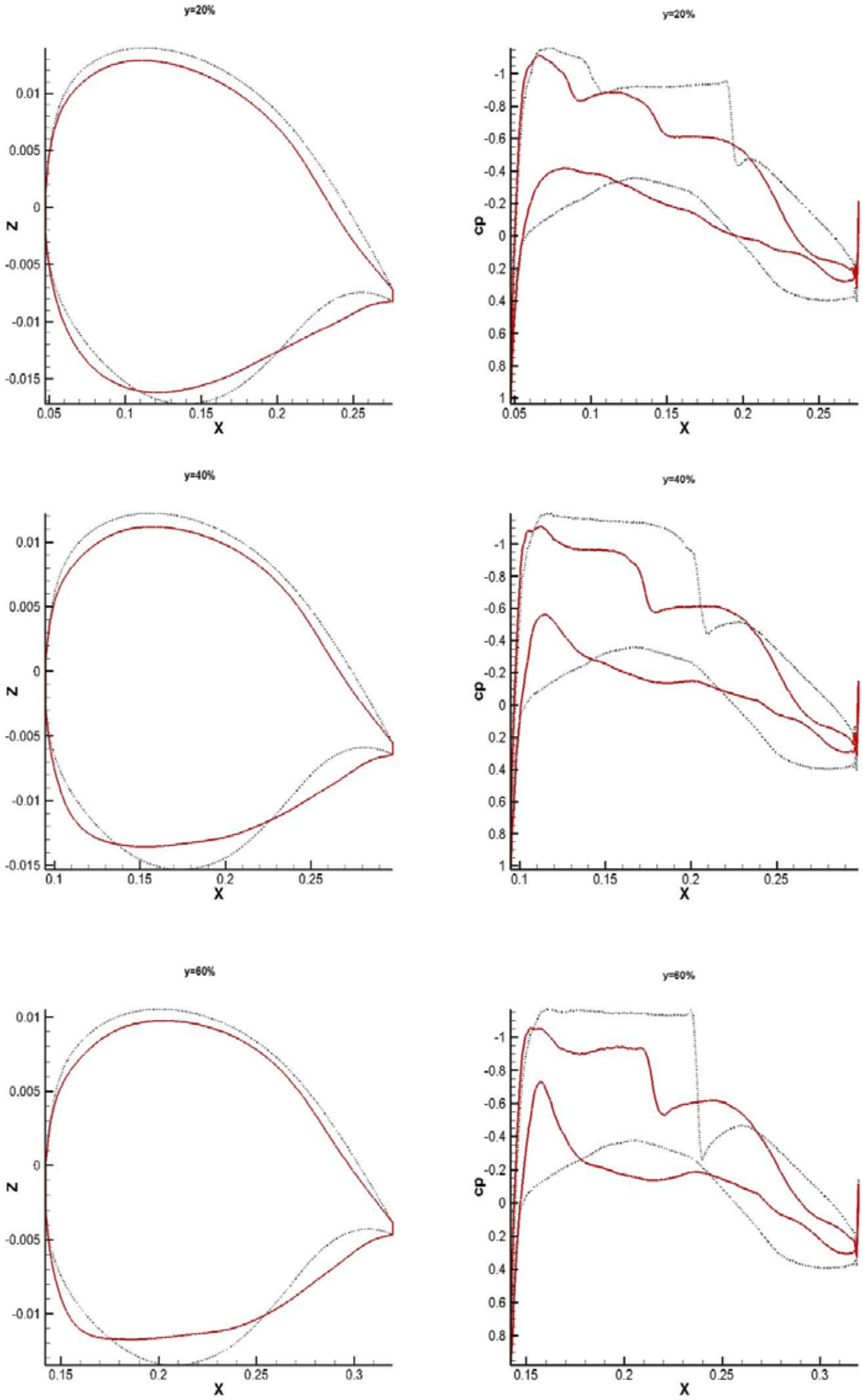

Figure 7: Baseline and optimized profiles (left) and pressure distribution (right)

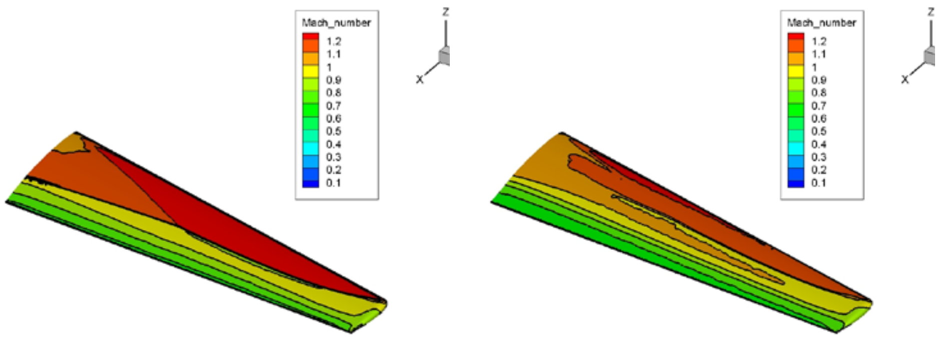

Figure 8: Mach distribution; original (left), optimized (right).

5 CONCLUSIONS AND FUTURE WORK

The use of volumetric B-Splines for aircraft design is a suitable alternative that provides the same good properties of NURBS surfaces, such as locality and control of the smoothness, while the implementation for complex geometries is much easier. The design is encapsulated inside a volumetric control box, in a similar way as the free form deformation technique. Actually, the control box technique can be seen as a generalized form of these two techniques. A clever deployment of the control points, for example on the aerodynamic surface, allows handling some common geometric restrictions, such as thickness and curvature. Also, some engineering design variables, such as twist and wing span can be easily mapped. On contrast, it requires some additional effort to calculate the parametric coordinates of the computational grid. In this work, an efficient algorithm has been developed for this purpose, and its computational cost is not relevant, as it has to be executed only once at the beginning of the optimization process. This technique has been applied to the optimization of a three dimensional wing.

There is still further potential to be exploited, such as considering the inclusion of other constraints, i.e. volume; considering multi-level deformations with different levels of detailed parameterizations; additionally, the control box can be employed with the CAD geometry, which is primarily composed by surface NURBS, in order to maintain the underlying CAD during the optimization; on the other hand, there is potential to link the computational grid to engineer design variables through the control box concept, which can also be investigated.

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