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Train scheduling and rolling stock assignment in high speed trains

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Abstract

The Train Timetabling Problem (TTP) has been widely studied for freight and passenger rail systems. A lesser effort has been devoted to the study of high-speed rail systems. A modeling issue that has to be addressed is to model departure time choice of passengers on railway services. Passengers who use these systems attempt to travel at predetermined hours due to their daily life necessities (e.g., commuter trips). We incorporate all these features into TTP focusing on high-speed railway systems.

We propose a Rail Scheduling and Rolling Stock (RSch-RS) model for timetable planning of high-speed railway systems. This model is composed of two essential elements: i) an infrastructure model for representing the railway network: it includes capacity constraints of the rail network and the Rolling-Stock constraints; and ii) a demand model that defines how the passengers choose the departure time. The resulting model is a mixed-integer programming model which objective function attempts to maximize the profit for the rail operator.

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1. Introduction

Public transport planning can be decomposed in five successive stages, named: i) network design, ii) line planning, iii) timetable planning, iv) rolling stock assignment and v) crew planning and rostering. Sequential planning might provide solutions which are not global optima. Therefore, it has recently originated a high interest to

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study simultaneously several phases. (Cadarso & Marín, 2012) propose an integrated model which combines timetable planning and rolling stock assignment model.

This paper deals with the Train Timetabling Problem (TTP) in high speed railway (HSR) systems. Our main contributions are:

1. The modelling of the choice of trip departure time by passengers;
2. The integration of timetable planning with relevant aspects of rolling stock assignment;
3. The resolution of railway conflicts considering time as a continuous variable and the number of planned events of the system as a discrete finite set (i.e., each event corresponds to the departure/arrival of a train from/to a station).

The vast majority of the existing literature is focused on considering graphs with space-temporal nodes (i.e., from a macroscopic point of view). On the contrary, we model the network microscopically and considering the passenger demand. Both features have received simultaneously a lesser attention in literature. An approach in this direction is given in (Espinosa-Aranda & García-Ródenas, 2013) which propose a model focused on conflict resolution taking the demand into account.

2. Mathematical modeling

2.1. Problem definition

For a better understanding of the problem, some definitions are explained below:

- A *segment* is a sorted pair of consecutive stations, without any station between them. Segments are indexed by (a, b) being a and b stations. (a, b) indicates the riding direction of the trip from a to b , while (b, a) represents the opposite one.
- A *track* is the infrastructure linking the stations. In high speed systems there are some reasonable assumptions: i) a train can go along each track only in one riding direction; ii) each segment is only covered by one track (i.e., a pair of rails) although there could be more than one; and iii) the used security systems are based on the control of the position of the trains at each instant, not in control signal blocking in each track segment. These assumptions allow to identify and to simplify tracks with segments.
- A *line* is a sorted set of segments which represent a pattern of movements of a set of trains.
- A *service* is a determined route of a train between the end stations in a line. Each service is defined by its origin and departure stations and timetable.

Considering the features of the studied problem, we make some assumptions on the railway infrastructure and the rolling stock.

Assumptions on the railway infrastructure

A1: Double track lines are considered. Each track is defined by nodes which represent the stations and all the trains in the same track must travel in same direction.

A2: The route of each service is fixed and defined by a station sequence. It is not mandatory to stop in all stations.

A3: The speed of each train is defined by the characteristics of the track while there is not any conflict. In case of conflict, the successor train must reduce its speed maintaining the minimum dwell time defined by controllers.

A4: A train which does not stop in a station could overtake a train stopped in a station. Stations are the unique overtaking areas.

Figure 1 shows the characteristics of the problem. This network consists of 4 tracks, 3 lines and 3 stations. Note that lines 1 and 3 share same tracks but have different stops.

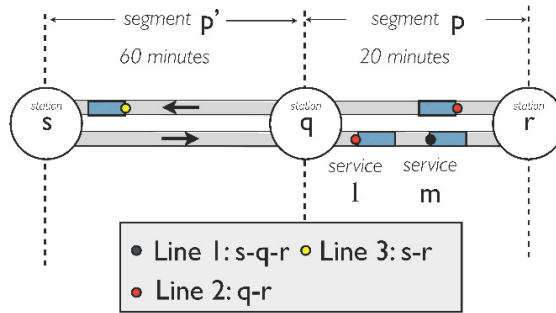


Figure 1: Problem illustration

Asumptions on the Rolling Stock.

A5: There are k trains assigned to each existent line. In the proposed example, train 1 is in line 1, train 2 in line 2 and train 1, again, in line 3. We also assume that the station where the trains are located in their departure, which is the same station where they end their trip, is known (schedule periodicity).

A6: Trains stop in stations during a determined dwell time.

A7: There is no priority between trains. First come first served policy.

A8: Each train k may do a potential number of services n_k . For example, if the planning period of the proposed example is 10 hours, the trains of lines 1 and 3 could make a maximum of $10 \cdot 60 / 80 = 7.5$ trips. Considering that the trains must end their planning period in the same station they started, the number of total services must be an even number. Therefore, the maximum number of services would be 6. On the other hand, trains of line 2 have a shorter route and could make $10 \cdot 60 / 10 = 30$ services. Finally, the total number of services that must be planned is $6 + 6 + 2 \cdot 30 = 72$.

The optimization model proposed will determine what of the n_k potential services will be effectively realized.

A9: We assume that each train k can be assigned a composition in C , which determines its capacity. In this case, we assume the existence of a dummy composition $0 \in C$ which represents a non-performed service. The operational cost of this service is 0.

A10: We assume the fleet size is big enough to operate the optimal solution given by the optimisation model.

Furthermore, we assume that the necessary time for shunting operations and their associated operational costs are 0.

2.2. Mathematical model

The notation defined for this problem is as follows:

2.2.1. Sets and indexes

- Q : set of stations indexed by q .
- C : set of composition types indexed by c .
- P : set of segments indexed by p
- L : set of rail services indexed by l ; each service is defined by an ordered sequence of tracks and a list of stations where the train stops.
- W : set of origin-destination pairs indexed by ω .

- W_l : set of origin-destination pairs served by line l .
- W_{lq}^+ : set of origin-destination pairs served by line l which origin is before station q and destination is after station q .
- Q_l : set of stations where service l stops.
- $L_p \subset L$: set of services traversing the track p .
- L_F : set of last service of each train.
- L_ω : set of services which serve the origin-destination pair ω .
- P_l : set of pairs of consecutive tracks (p', p) realized by service l .
- p_l^0 : first track traversed by service l .
- $p_{l\omega}^0$: first track traversed by service $l \in L_\omega$ considering demand ω .
- l^+ : next service (if it exists) realized by the train which has just done the service l .

2.2.2. Variables

- D_p^l : departure time of service l at track $p = (q, r)$.
- A_p^l : arrival time of service l at track $p = (q, r)$.
- $Z_p^{lm} = 1$: if service l departs earlier than service m at the track p ; 0 otherwise.
- $Y_i^{l\omega}$: auxiliary binary variable used for modelling the non-linear demand for origin-destination pair ω served by service $l \in L_\omega$. The index i represents the node where the demand is discretized.
- $T_i^{l\omega}$: auxiliary variable which represents the demand for pair ω served by service $l \in L_\omega$ in i th interval.
- G_ω^l : potential demand for origin-destination pair ω which should be served by service $l \in L_\omega$.
- \hat{G}_ω^l : demand of origin-destination pair ω served by services $m \in L_\omega$ and realized before service l .
- \tilde{G}_ω^l : accumulated demand of origin-destination pair ω served by all services until service l (included).
- F_ω^l : demand of origin-destination pair ω served by services $l \in L_\omega$.
- K_c^l : this variable equals to 1 if service l is realized with a composition c .

2.2.3. Parameters

- h_q : minimum headway time between two consecutive services arriving or departing at station q via the same track.
- \underline{t}_q^l : minimum dwelling time of service l (defined as 0 if q is not a stopping station for service l).
- \overline{t}_q^l : maximum dwelling time of service l at station q .
- \overline{t}_p^l : maximum transit time over track p of service l .
- \underline{t}_p^l : minimum transit time over track p of service l .
- κ_c^l : is the capacity of a service l if a composition c is defined.
- ρ_{ll^+} : minimum necessary time for a train, which realized service l , for being available for its next service l^+ .
- $\delta_i^\omega, \alpha_i^\omega, t_\omega^0$: parameters which define the accumulated demand function for origin-destination pair ω .
- π_l^c : operational cost of service l with a composition type c .

- ψ_{ω}^l : ticket price for origin-destination pair ω in service l .
- τ_{ω} : maximum time a user of origin-destination pair ω waits a service until he/she decides not making the trip.
- $(0, \tau)$: planning period.
- M : an enough large positive number.

2.2.4. Decision variables

The main objective is to determine the timetable of the HSR network and the composition of the trains. Variables D_p^l and A_p^l represent the departure and arrival time of service l in track P , respectively. These variables lead to the sequencing variables Z_p^{lm} defined as:

$$Z_p^{lm} := \begin{cases} 1 & \text{if } D_p^l \leq D_p^m \\ 0 & \text{otherwise} \end{cases}$$

The timetable is completely determined by the arrival and departure time at stations. Rolling stock variables are defined as:

$$K_c^l := \begin{cases} 1 & \text{if service } l \text{ is carried out using rolling stock type } c \\ 0 & \text{otherwise} \end{cases}$$

The previous definitions are illustrated on Figure 2.

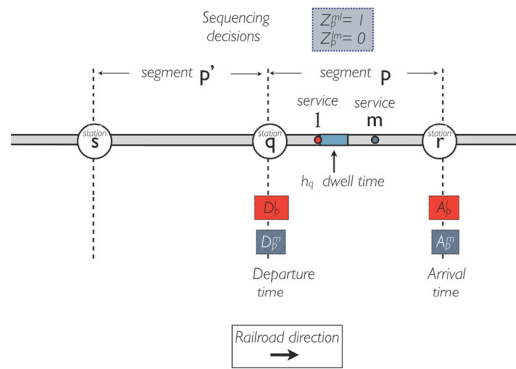


Figure 2: Notation illustration

2.2.5. Constraints

Network capacity. This set of constraints models the railway operations and the overtaking between trains. These constraints are linearly formulated and the set of feasible solutions provide the departure/arrival time of services, travel times in tracks (speed) and stopping times, obtaining a conflict free scheduling.

Train-dispatching sequencing. A departing sequence of services should be equal to their arrival sequence over the same track (it has only a track in which there is no overtaking).

$$Z_p^{lm} + Z_p^{ml} = 1; \quad \forall p \in P, \forall l, m \in L_p, l \neq m \quad (1)$$

Inter-segment headway safety. Two services l and m traversing track $p = (q, r)$ must maintain the minimum headway time.

$$MZ_p^{ml} + D_p^m - D_p^l \geq h_p; \quad \forall p \in P, \forall l, m \in L_p \text{ with } l \neq m \quad (2)$$

$$MZ_p^{ml} + A_p^m - A_p^l \geq h_p; \quad \forall p \in P, \forall l, m \in L_p \text{ with } l \neq m \quad (3)$$

Transit times at segment. For each track $p = (q, r)$ the transit time of each service is determined between its lower-and upper bounds:

$$A_p^l - D_p^l \leq \underline{t}_p^l; \quad \forall p \in P \text{ and } \forall l \in L_p \quad (4)$$

$$A_p^l - D_p^l \geq \underline{t}_p^l; \quad \forall p \in P \text{ and } \forall l \in L_p \quad (5)$$

Dwelling time requirements. Each train has to dwell for a positive predetermined time at station q . The dwelling time should be zero, otherwise. For each pair of consecutive tracks (p', p) incident to q , i.e. $p' = (s, q)$ and $p = (q, r)$, and for each service traversing p and p' , namely $\forall (p', p) \in P_l$, we have

$$D_p^l - A_{p'}^l \geq \underline{t}_q^l; \quad \forall l \in L, \forall (p', p) \in P_l, \forall q \in Q_l \quad (6)$$

$$D_p^l - A_{p'}^l \leq \underline{t}_q^l; \quad \forall l \in L, \forall (p', p) \in P_l, \forall q \in Q_l \quad (7)$$

$$D_p^l - A_{p'}^l = 0; \quad \forall l \in L, \forall (p', p) \in P_l, \forall q \notin Q_l. \quad (8)$$

Constraint (7) imposes a maximum stopping time per station. This constraint is important in high speed trains which travel time is lower compared to other types of train services.

Demand. In this paragraph the demand time dependencies are modelled. We assume that users of a origin-destination pair ω who want to travel during time period t are prepared to use any adequate railway service which stops during time interval $[t, t + \tau_\omega]$. This assumption is modelled considering that for each origin-destination pair ω there is a function $G_\omega(T)$ which provides the total number of users willing to realize their travel before time T . This function may be non linear. It is also worth noting that the number of intermediate stops may affect the scheduling process, and therefore, the captured demand.

Potential demand. For each $\omega = (q, r)$ and for each $l \in L_\omega$, the potential demand of service l is denoted as $G_\omega^l = G_\omega(D_q^l)$, that is, the number of users who want to make their trip before time D_q^l and could use service l .

Effective demand. Passengers G_ω^l may not travel using service l due to two reasons: i) they use a previous service because it is close to the time period at which they want to travel or ii) they reject the service due to the fact that it is realized too late. Parameter τ_ω is introduced for calculating this maximum temporal amplitude.

Denote \hat{G}_ω^l as the number of passengers of origin-destination pair ω attended before service l and assume $G_\omega^{\tau l} = G_\omega(D_q^l - \tau_\omega)$. Denote \hat{G}_ω^l as the accumulated demand attended by pair ω before service l . The following constraints are introduced to model the above considerations:

$$F_\omega^l \leq G_\omega^l - \hat{G}_\omega^l; \quad \forall \omega, \forall l \in L_\omega \quad (9)$$

$$F_\omega^l \leq G_\omega^l - G_\omega^{\tau l}; \quad \forall \omega, \forall l \in L_\omega \quad (10)$$

$$\tilde{G}_\omega^m \leq \hat{G}_\omega^l + Z_{p_{l\omega}^0}^{lm} M; \quad \forall \omega, \forall l, m \in L_\omega \text{ with } l \neq m \tag{11}$$

$$\tilde{G}_\omega^l = \hat{G}_\omega^l + F_\omega^l; \quad \forall \omega, \forall l \in L_\omega \tag{12}$$

$$\hat{G}_\omega^l \geq 0; \quad \forall \omega, \forall l \in L_\omega \tag{13}$$

$$G_\omega^l = G_\omega(D_q^l) \tag{14}$$

$$G_\omega^{\tau l} = G_\omega(D_q^l - \tau_\omega) \tag{15}$$

where $p_{l\omega}^0$ is the first traversed track by service l for fulfilling origin-destination pair ω .

The right hand side of constraint (9) indicates the number of non-attended passengers of origin-destination pair ω who want to travel before service l arrival. The right hand side of constraint (10) indicates the potential demand of service l which does not reject to travel because of the time inadequacy for the purpose of its trip.

The number of passengers of origin-destination pair ω served by l , F_ω^l , are potential non-attended users who have not rejected to travel. Therefore, the number of users F_ω^l must be lower than these two quantities as modeled through constraints (9)-(10).

Constraint (11) imposes that the attended demand before service l , \tilde{G}_ω^l , is bigger than the accumulated effective demand of services realized before l . Constraint (12) shows that the attended demand when service l is realized is the attended demand before its realization plus the demand which uses l .

The objective function shall force the inequalities transformation to equalities and the variables will take a value depending on their mean.

$G_\omega(T)$ linearisation. From a computational point of view it is desirable to obtain a mixed-integer linear programming model. Therefore, the non-linear function $G_\omega(T)$ is approximated using a piecewise-defined function (see Figure 3) where n is the number of points prefixed by user.

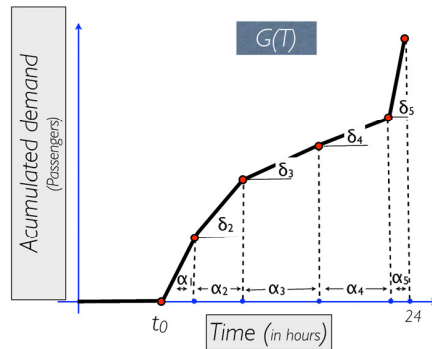


Figure 3: Accumulated linearised demand $G(T)$

For linearisation of equation (14) the following constraints are introduced:

$$G_\omega^l = \sum_{i=1}^n \delta_i^\omega T_i^{l\omega}; \quad \forall \omega \in W, \forall l \in L_\omega \tag{16}$$

$$D_{q_{l\omega}^0}^l = \tau K_0^l + t_0^\omega Y_0^{l\omega} + \sum_{i=1}^n T_i^{l\omega}; \quad \forall \omega \in W, \forall l \in L_\omega \tag{17}$$

$$\alpha_i^\omega Y_{i+1}^{l\omega} \leq T_i^{l\omega} \leq \alpha_i^\omega Y_i^{l\omega}; \quad \forall \omega \in W, \forall l \in L_\omega, i = 1, \dots, n - 1 \tag{18}$$

$$0 \leq T_n^{l\omega} \leq \alpha_n^\omega Y_n^{l\omega}; \quad \forall \omega \in W, \forall l \in L_\omega \tag{19}$$

$$Y_{i+1}^{l\omega} \leq Y_i^{l\omega}; \quad \forall \omega \in W, \forall l \in L_\omega, i = 0, \dots, n - 1 \tag{20}$$

$$Y_0^{l\omega} \in [0, 1], Y_i^{l\omega} \in \{0, 1\}, \quad \forall i = 1, \dots, n, \forall \omega, \forall l \in L_\omega \tag{21}$$

where $q_{i\omega}^0$ is the first track traversed by service l for fulfilling demand w and τ is the planning period, for example 24 hours. Constraint (17) imposes that if a service is assigned composition 0, i.e. without capacity, the departure time of the service will be fixed outside of the planning period, avoiding any possible railway conflict. Therefore, a subset of L services will be only realized. Note that equations (17)-(18) limit the services' departures and the planning period $[0, \tau]$. If the planning period $[0, \tau]$ should be restricted, α coefficients should be selected considering that $t_0^\omega + \sum_{i=1}^n \alpha_i^\omega = \tau$. This constraint yields a timetable in which no train will leave its station outside the planning period. Also constraint (15) must be linearised for time period $D_q^l - \tau_\omega$. These constraints are not shown due to space limitations.

Rolling Stock and fleet capacity. These constraints impose that two services realised by the same train cannot be realised during the same time period. The first service must end and the time needed for preparing the train must be spent before realizing the next service. The notation for these constraints is as follows. A train realizes a set of time ordered services. Given a service l denote l^+ as the next service realized by the same train. Denote L_F as the last services realized by all the trains.

$$D_{p_{l^+}^0}^{l^+} - A_{p_l^F}^l \geq \rho_{ll^+}; \quad \forall l \in L - L_F \tag{22}$$

where $p_{l^+}^0$ is the first track traversed by service l^+ , which is the successor of l , p_l^F is the last track of service l and ρ_{ll^+} is the minimum necessary time for a train, which realized service l , to be prepared for service l^+ . Rolling stock decisions determine the capacity of services and the possibility of serving passengers. This relationship is modelled as follows:

$$\sum_{(q,r) \in W_l} F_{(q,r)}^l + \sum_{\omega \in W_{lq}^+} F_\omega^l \leq \sum_{c \in C} \kappa_c^l K_c^l; \quad \forall l, \forall q \in Q_l \tag{23}$$

where W_{lq}^+ are all the origin-destination pairs served by line l with their origin before station q and their destination after station q and W_l is the set of origin-destination pairs served by line l . Finally, each service is operated only by a unique composition:

$$\sum_{c \in C} K_c^l = 1; \quad \forall l \in L \tag{24}$$

2.2.6. Objective function

This model considers the maximization of the profit, that is, income of ticket sales minus operating costs.

$$\text{maximize: } Z = \sum_{l \in L} \sum_{\omega \in W_l} \psi_\omega^l F_\omega^l - \sum_{l \in L} \sum_{c \in C} \pi_c^l K_c^l$$

$$\text{subject to: (1)-(24)} \tag{P}$$

2.3. Meaning of variables F_ω^l

In the model formulation F_ω^l is interpreted as the number of users of origin-destination pair ω who use service l , but this is not completely correct. Model [P] represents the operator's point of view, not user's. In this framework,

these variables represent the *seat capacity allocation* to make the railway company obtain the maximum profit. To illustrate this fact the example shown in Figure 1 is considered. Given a train of line 1 which realizes service $r \rightarrow q \rightarrow s$. Assume an intense demand in origin-destination pair $r \rightarrow q$. Therefore, the train could be completely filled at station r with passengers travelling to q , not leaving any ticket to other passengers. This train would leave all the passengers in station q and would realize its travel with empty seats, even though there are users demanding these seats in the origin station. To avoid this situation the model assigns a number of seats per each demand and service.

3. Computational experiments

In this section, we study the Spanish Madrid-Seville corridor using the proposed model. The complete description of the problem has been uploaded to <http://bit.ly/1eTmnPK> due to space limitations. The model has been solved using CPLEX 12.5.1.0 using a computer with the following specifications: 2 x AMD Opteron 4226 6 cores 2, 7GHz and 12 GB RAM 1600 MHZ. The experiment has been carried out during 24 h 12 min 32 sec, obtaining an objective function value of 209362.7 euros. The relative optimality gap of this solution is 1.9, indicating that a best solution of 604886.3 euros could exist. This result shows that the current solution could be meaningfully improved. However, the computational cost is considerable. The problem is composed of 42 potential services. The solution found schedules 20 services in simple composition and 3 in double composition; 19 trains services are not scheduled. The time-station diagram is shown in Figure 4. The thickest line represents services assigned with double composition and the rest the simple composition ones. Note that some services are practically simultaneous. This fact indicates the possibility of the deletion of some services in the best solution.

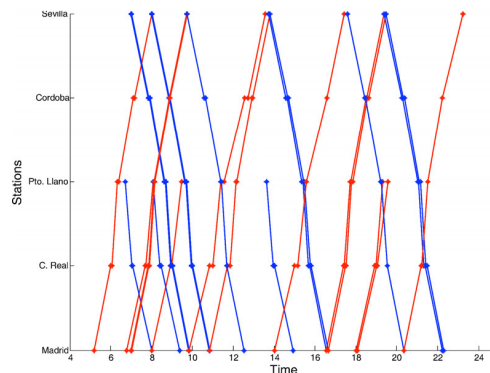


Figure 4: Time-station diagram of the solution

The potential and attracted demand is represented with respect to time to illustrate the novel characteristics of the model. Figure 5 shows two of the 20 origin-destination pairs of the example and Figure 6 represents the same idea but at each station (Puertollano station has been omitted because of the similarity with Ciudad Real station). It is remarkable that there is a part of the potential demand which: 1) is not attracted by the planned schedule, 2) or that it is attended with a delay of one hour ($\tau_\omega = 1$ has been considered in the experiment for each pair ω), 3) or, on the other hand, the train which could attend the demand is full.

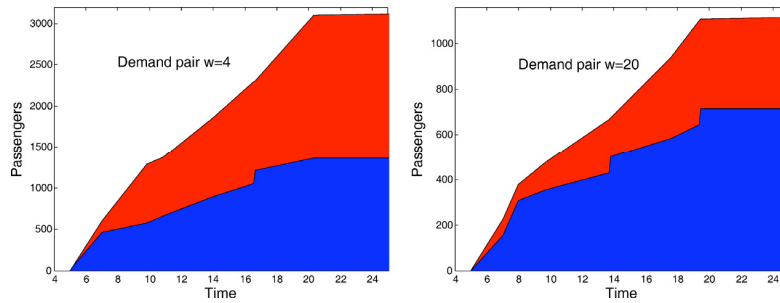


Figure 5: Demand attracted example in pairs $\omega = 4$ and $\omega = 20$

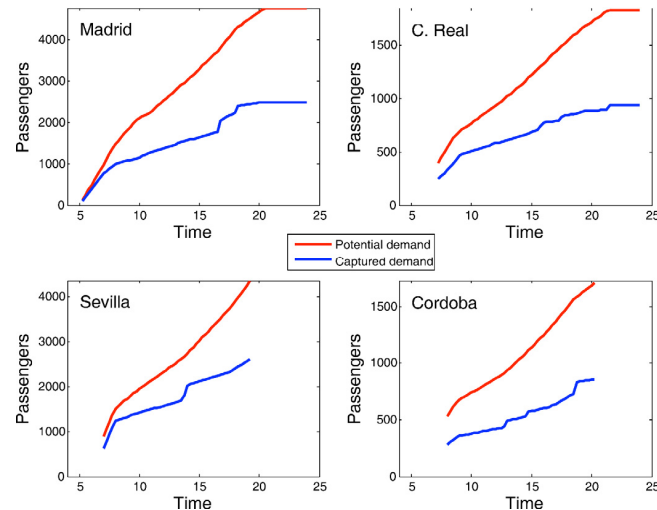


Figure 6: Potential and attracted demand in stations with respect to time

4. Conclusions

In this paper, we present an integrated scheduling and rolling stock model. We formulate the model as a mixed-integer linear programming model. The novel aspect of this formulation is the modelling of the time instant in which an user wants to travel.

The real case of Madrid-Seville corridor with 42 services has been solved. The computational cost of the example has been 24 hours obtaining a relative gap of 1.9. These results evidence that it is necessary to develop decomposition algorithms for this problem, which will be the next task to be tackled.

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References

- Cadarso, L. & Marín, A., (2012). Integration of timetable planning and rolling stock in rapid transit networks. *Annals of Operations Research* 199 (1), 113–135.
- Espinosa-Aranda, J.L. & García-Ródenas, R.,(2013). A demand-based weighted train delay approach for rescheduling railway networks in real time. *Journal of Rail Transport Planning and Management* 3 (1-2), 1–13.