To map a given airfoil into an approximate circle ϵ can be computed by relation (8) truncated to ϵ^2:

$$\epsilon = 1 - \sqrt{1 - 2t/c} \tag{9}$$

The coordinates of the airfoil are stretched such that the trailing edge is located at [2.,0.] and the leading edge at [(−2. −4ϵ^2),0.]. The transformed coordinates are then calculated by means of the inverse relation (5):

$$z = \zeta \pm \sqrt{\xi^2 - 4}/2 \tag{10}$$

The sign of the square root depends on the quadrant.

The transformed airfoil has the shape shown on Fig. 3 for a NACA 65012_1 wing section. The structure of the matrix for the linear system is now quite satisfactory, with a strong main diagonal and no large off-diagonal terms.

Once the solution has been computed in the z plane, the physical solution can be easily obtained by transforming the coordinates and vectors according to relation (5). The local pressure coefficients computed by the present method are presented in Fig. 4. The pressure loop at the trailing edge has completely disappeared, and a large increase in the load coefficient is observed. Typical results for 10-deg incidence are listed in Table 1.[5]

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Cancellation Zone in Supersonic Lifting Wing Theory

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Introduction

BASING their work on a linear theory, Evvard[1] and Krasilshchikova[2,3] independently developed an expression that yields the perturbation generated by a thin lifting wing of arbitrary planform flying at supersonic speed on a point placed on the wing plane inside its planform,[1] or both on and above the wing plane.[2] This point must be influenced by two leading edges, one supersonic and the other partially subsonic. Although these authors followed different approaches, their methods concur in showing the existence of a perfectly defined cancellation zone.

In this Note, the Evvard approach is generalized to the case solved by Krasilshchikova. Circumventing the latter's lengthy and somewhat complex approach, Evvard's simple method seems to be useful at least for educational purposes.

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The Proof

Consider the configuration shown in Fig. 1. In a lifting problem, the perturbation velocity potential at point N, placed on the wing plane but outside its planform and wake, must vanish; that is,

$$\psi(x_N, y_N, 0) = -\frac{1}{\pi} \iint_{S_1+S_2} \frac{w(x_0,y_0)\mathrm{d}x_0\mathrm{d}y_0}{\sqrt{(x_N-x_0)^2 - \beta^2(y_N-y_0)^2}} = 0 \tag{1}$$

where x_0, y_0 represent the source-point coordinates, $\beta = (M_\infty^2 - 1)^{1/2}$, and M_∞ is the freestream Mach number. w is the vertical velocity, which will be denoted w_i or w_o for source points placed inside or outside the planform, respectively. Transformation to the characteristic coordinate system r,s defined by

$$r = x - \beta y, \quad s = x + \beta y, \quad \tilde{z} = \beta z, \quad \frac{\partial(x_0,y_0)}{\partial(r_0,s_0)} = \frac{1}{2\beta}$$

$$r_0 = x_0 - \beta y_0, \quad s_0 = x_0 + \beta y_0 \tag{2}$$

leads to

$$\iint_{S_1+S_2} \frac{w(r_0,s_0)\mathrm{d}r_0\mathrm{d}s_0}{\sqrt{r_N - r_0}\sqrt{s_N - s_0}} = \int_0^{r_N} \frac{\mathrm{d}r_0}{\sqrt{r_N - r_0}}$$

$$\left[\int_{s_0 = B'O(r_0)}^{s_0 = OB(r_0)} \frac{w_i(r_0,s_0)\mathrm{d}s_0}{\sqrt{s_N - s_0}} + \int_{s_0 = OB(r_0)}^{s_0 = s_N} \frac{w_o(r_0,s_0)\mathrm{d}s_0}{\sqrt{s_N - s_0}} \right] = 0$$

(3)

The right-hand side of Eq. (3) is an Abel equation equated to zero, so that the terms in brackets should vanish for $r_0 = r_N$; i.e.,

$$\int_{s_0 = OB(r_N)}^{s_0 = s_N} \frac{w_o(r_0,s_0)\mathrm{d}s_0}{\sqrt{s_N - s_0}} = -\int_{s_0 = B'O(r_N)}^{s_0 = OB(r_N)} \frac{w_i(r_0,s_0)\mathrm{d}s_0}{\sqrt{s_N - s_0}} \tag{4}$$

as the integrand is known to have no singularities in the region of integration. This equation is the basis of the determination of the cancellation zone. The direct Krasilshchikova approach requires the inversion of the integral equation (4), followed by a double integration, as in Eq. (1). These steps, however, are not essential for the demonstration and, as shown in Ref. 1 for the case of a point on the wing planform, Eq. (4) can be used to determine the cancellation zone for a point $P(x,y,z)$ placed outside the wing, which is the purpose of this Note.

Let us calculate the perturbation potential at P. The region of integration is divided into three parts, S_0, S_1, S_2, as shown in Fig. 2. $P'(x,y,0)$ is the projection of P over the wing plane, and the curve S is the intersection of the fore cone of P with the wing plane. In characteristic coordinates, the perturbation potential reads

$$\psi(r,s,\tilde{z}) = -\frac{1}{2\pi\beta} \iint_{S_0+S_1+S_2} \frac{w(r_0,s_0)\mathrm{d}r_0\mathrm{d}s_0}{\sqrt{(r-r_0)(s-s_0) - \tilde{z}^2}} \tag{5}$$

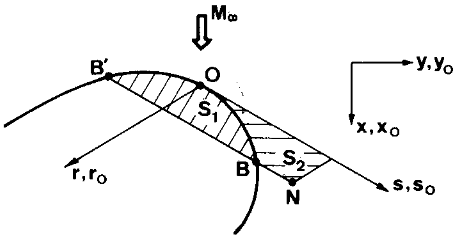

Fig. 1 Regions of integration inside (S_1) and outside (S_2) the wing planform employed in the calculation of the perturbation potential at point N, placed in the wing plane.

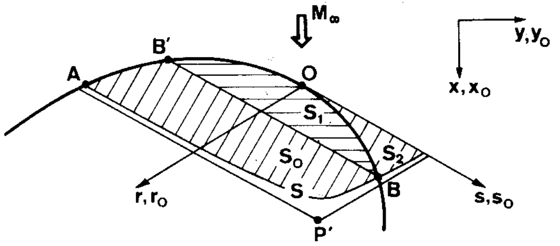

Fig. 2 Regions of integration inside (S_0 and S_1) and outside (S_2) the wing planform employed in the calculation of the perturbation potential at point P, whose projection over the wing plane is P'.

We will show that contributions from S_1 and S_2 cancel each other. To this aim, these contributions can be written as

$$I = -\frac{1}{2\pi\beta}\int_0^{r_B}\frac{\mathrm{d}r_0}{\sqrt{r-r_0}}\left[\int_{s_0=B'O(r_0)}^{s_0=OB(r_0)}\frac{w_i(r_0,s_0)\mathrm{d}s_0}{R} + \int_{s_0=OB(r_0)}^{s_0=S(r_0)}\frac{w_o(r_0,s_0)\mathrm{d}s_0}{R}\right] \tag{6}$$

where $R^2 = s - s_0 - \bar{z}^2/(r-r_0)$ and $S(r_0) = s - \bar{z}^2/(r-r_0)$, as $S(r_0)$ is the value of s_0 for which R vanishes. Equation (6) reduces to

$$I = -\frac{1}{2\pi\beta}\int_0^{r_B}\frac{\mathrm{d}r_0}{\sqrt{r-r_0}}\left[\int_{s_0=B'O(r_0)}^{s_0=OB(r_0)}\frac{w_i(r_0,s_0)\mathrm{d}s_0}{\sqrt{S(r_0)-s_0}} + \int_{s_0=OB(r_0)}^{s_0=S(r_0)}\frac{w_o(r_0,s_0)\mathrm{d}s_0}{\sqrt{S(r_0)-s_0}}\right] \tag{7}$$

The integral I is zero because the terms within the brackets coincide with Eq. (4). This term states that ψ is zero at the point $[r_0,S(r_0)]$, which is placed over the part of the hyperbola S lying outside the planform. Therefore, S_0 is the only region contributing to the integral in Eq. (5). The effect of the source points placed in the regions S_1 and S_2 is to cancel each other. This occurs because along each line $r_0 =$ constant of the region of integration, the relationship (4) guarantees the compensation of effects produced on P by both segments of this line lying in S_1 and S_2, respectively.

Once the cancellation zone is determined, the velocity perturbation component u can be calculated in the usual way[1-4] and the extension to the calculation of u at a point influenced by the wing wake produced at a subsonic trailing edge is obvious.

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Transonic Airfoil Calculations Including Wind Tunnel Wall-Interference Effects

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Introduction

MEANINGFUL comparisons of experiment and Navier-Stokes calculations for airfoils at transonic speeds require a proper account of wind tunnel wall interference effects. Levy[1] demonstrated this by using a tangency condition at the walls of a solid-wall tunnel. King and Johnson[2] employed a pressure boundary condition utilizing measured pressures in the flowfield as boundary data. In both cases, inclusion of wall-interference effects resulted in shock positions close to that observed experimentally. Free-air results characteristically placed the shock too far downstream.

While qualitatively correct, the results of Ref. 2 were unsatisfactory because of the finite-difference meshes employed. In particular, surface pressures exhibited significant irregularities that were mesh dependent and which limited interpretation of the results. The purpose of this Note is to present additional calculations performed on finer, better designed meshes, so that a truer representation of wall effects may be demonstrated.

Numerical Procedure

The basic technique employed is the numerical method developed by Steger[3] for the Reynolds-averaged time-dependent compressible Navier-Stokes equations. The method comprises the following elements: 1) transformation of the governing equations to a generalized body-fitted coordinate system; 2) the thin-layer approximation for the viscous terms; 3) the algebraic turbulence model of Baldwin and Lomax[4]; and 4) use of the second-order-accurate factored-implicit algorithm of Beam and Warming.[5,6]

To account for wind tunnel wall interference, the Steger code was modified by incorporating a pressure boundary condition (PBC) along the upper and lower computational boundaries. The pressures imposed are those measured experimentally at locations one chord above and below the airfoil. Because the flows for the conditions tested were entirely subsonic at these locations, inviscid homentropic flow is a reasonable assumption. Using the notation of Steger,[3] the equations along the outer boundary are

$$p/p_\infty = (p/p_\infty)_{\mathrm{meas}} \tag{1}$$

$$\rho = (p/p_\infty)^{1/\gamma} \tag{2}$$

$$\tfrac{1}{2}(u^2+v^2) = (1-a^2)/(\gamma-1) + \tfrac{1}{2}u_\infty^2 \tag{3}$$