AN ANALYSIS OF INSTALLATION OF SUBMARINE PIPELINES

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1. INTRODUCTION

There are different methods of construction of outfall pipelines, all of them have to solve the problem of placing a tube over a known location in sea bed. This process has sometimes to be done in difficult conditions as waves, current or depths greater than 30 metres, where a diver can not go safely beyond. Also the placement of the pipeline must be carried out without any damage to the tube, therefore a close control of the deflections and stresses in the structure must be performed. The importance of this control should be not diminished because a damage during the construction would imply a very difficult and expensive repair, that should be avoided with a proper design of the construction process.

This paper is focused in the analysis of the tube during its placement according to a very well known construction method consisting in placing the tube from a boat, where all the connections between consecutive tube segments are performed, and also the whole process is controlled. This method is used for outfall as well as offshore pipelines, and it will be described in Section 2

2. CONSTRUCTION METHOD

The tube is supposed to be of polyethylene or its derivates and it will be placed along the sea fond according to the procedure known as progressive inundation, i.e. a controlled inundation of the tube. Several variants of this procedure exist but here only the one using an auxiliary ship as a device to sweep away the tube from the shore, where the tube is filling in a controlled way by water, will be described. At this floating device the successive tube segments are joined and the last one will be pulled down to the water. Therefore several phases can be distinguished in this procedure (Figure 1):

Phase 1. Put in place the first tube segment. The empty tube is sweep away by the floating platform, i.e. during this phase the tube is supported at the shore and at the floating platform.

Phase 2. In each of these successive and identical phases the tube is inundated an incremental length and also if it is necessary the distance of the floating platform to the shore is increased,

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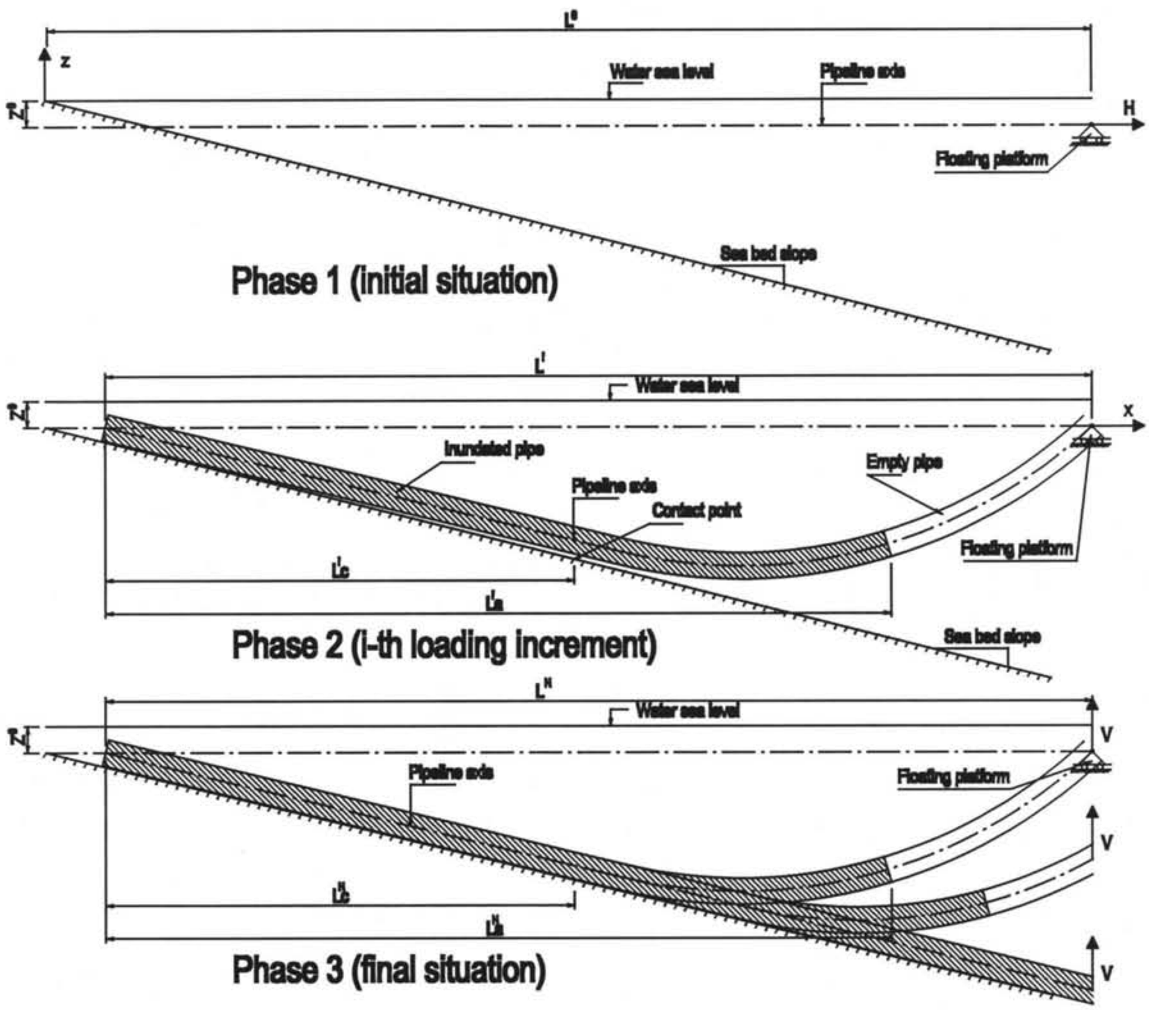

Figure 1.- Construction phases

in such a way a part of the tube is resting along the bottom sea without any stresses and deformations and the other part of the tube can also be considered to be subdivided into two parts: one completely filled by water and the another, the closer one to the ship, empty. Additional tubes segments are joined at the platform device according it is needed.

Phase 3. This last phase consists in a vertical transport of the tube offshore end from the surface sea level until the bottom sea. This operation is carried out by a simultaneous progressive water filling of the tube and a drop of its offshore end.

3. ANALYSIS. HYPOTHESIS

The main objective of the analysis to be presented here is to find the stresses, forces and displacements produced along the tube during its placement on the sea bed according to the just described procedure. Due to the complexity of this structural problem several simplifying hypothesis are usually introduced. They are summarized as follows:

First the material shows a linear and elastic behaviour. This hypothesis assumed moderated stresses are produced during the construction.

- Dynamic effects due to the sea waves and other variable actions are not considered.
- The tube is modelled as an 1-D beam in a vertical plane. This simplification is reasonable due to the small transversal dimensions of the tube in comparison to its length.
- Beam-column effects are negligible, i.e. the stresses produced by the axial forces are small in comparison to the ones due to the bending moments.

Despite of these simplifying hypothesis the structural analysis to be carried is highly non linear, because during the placement of the tube large displacements are produced and also the presence of up lift forces on the tube due to the hydrostatical pressures, that are dependent on the tube geometry respect to the sea level is another source of non linearity. In addition, the existence of the sea bed introduces another linearity, known as an unilateral contact problem.

As a consequence, the analysis of the tube during its construction should be carried out in a successive way following the order of the construction phases until the whole tube length is placed along the sea bed. Therefore it is not possible to study a particular construction phase independently on the previous ones, because the geometry and stress distribution in the tube at initial time of a current construction phase are function of the structural response to the loads of the previous phases.

From the above considerations it can be concluded that the analysis should be an incremental one, i.e. the loading should be introduced in the analysis as a set of small incremental values which sum is the total loading. In each load increment the geometry and the stresses of the tube are the results of the previous load increment. The analysis in the load increment under study can be considered linear and then to be carried out using the tangent stiffness matrix. Although this matrix is dependent on the geometry of the beam, if the load increment is small it can be supposed to be constant trough out the whole load increment. Once the structural analysis in the load increment is finished the geometry and the stresses of the tube are updated by adding the results obtained in the current increment load to the previous values. Finally the updated values are used as starting ones for the next load increment analysis.

The presence of the sea bed introduces a structural contact problem and it can be treated by checking in each incremental analysis the occurrence of the contact between the tube and the sea bed. It can be simulated if contact occurs by connecting to the tube unilateral springs with large spring constants. These springs can only react in case of compression, then if a separation between the tube and the marine soil is produced then no reaction exists.

4. ANALYSIS. METHODOLOGY

The different phases of the tube construction can be modelled by means a sequential analysis procedure, i.e. starting from an given initial tube state, defined by its position and stresses distribution, the analysis proceeds along successive states, simulating the different construction phases, until the final state is reached, i.e. until the whole tube length is placed along the sea bed. Each construction phase is defined by a loading change, produced by an additional inundated length of the tube. In each analysis step the initial geometry used is the final one obtained in the previous analysis step. In the current analysis step the additional loading is introduced by a set of small incremental values. For these incremental loading values a linear analysis using the consistent stiffness matrix is carried out. This matrix is dependent on the geometry at the end of the incremental load and therefore it is unknown. For that reason the procedure used is an iterative one, i.e. a successive set of linear analysis is carried out until the assumed geometry at the beginning of the analysis coincides with the final one. An approximate expression of the consistent stiffness matrix is presented in appendix. By application of the just summarized methodology to each increment of loading the final results of the current step analysis are obtained.

In the above described incremental-iterative analysis procedure the following notation for the loading step i is used in order to define the initial tube geometry (Figure 1):

L_c^i: Horizontal length of contact between the tube and the sea bed.

L_c^i: Horizontal length of inundated tube.

L^i: Horizontal length between the two extremes of tube, i.e. between the sections tube at shore and at floating platform.

A result of the analysis in an incremental step, in addition to the displacements, stresses and forces produced along the built tube, is the geometry at the end of the step analysis i. These final values (or initial values of the next incremental step $i+1$) are denoted by L_c^{i+1}, L_a^{i+1} and L^{i+1}. Then, for an arbitrary increment of the inundated tube length ΔL_a the parameters L_c^{i+1}, $L_a^{i+1}=L_a+\Delta L_a^{i+1}$ and L^{i+1} have to be obtained, in such a way that at this final position no separation between the tube and the sea bed appears.

In order to reach some insight about the tube structural behaviour during its construction the sea bed will be simplified as a plane of constant slope m and in this way the study of the construction phases 1 and 2 can be reduced only to the general one shown in Figure 2. In this figure coordinate axis and notation are introduced.

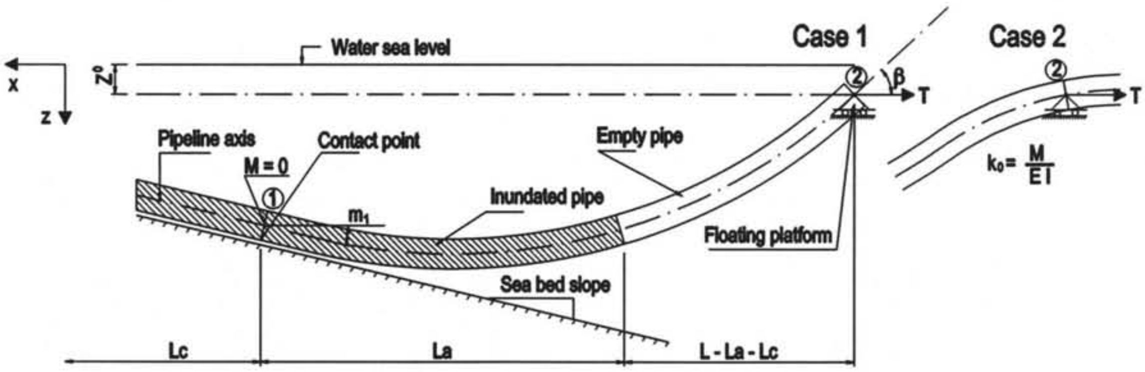

Figure 2.- Pipeline geometry of a typical case

Then the following conditions hold:

$$H = mL_c - R, \quad L_c \geq \frac{1}{m}(R + z_0) \tag{1}$$

where z_0 is the floating depth of the emptied tube.

The problem to be solved consists in finding the lengths $L_a = L_a(H)$ and $L = L(H)$ such that the tube will be lied on the sea bed with null bending moment M. In the another tube end the actual boundary conditions have to be introduced. Two typical cases are considered for these boundary conditions:

(a) Case 1.

The formulation of the problem of finding the stresses and displacements of the elastic beam in the framework of a large displacement analysis can be simplified as follows:

The pipeline is simply supported in the platform with a tension T, and a known slope at this end is produced by the floating platform ramp (Figure 2).

First, the beam is at the initial position $z=z_0$. That is, the beam is horizontally floating without any inundated portion. Then the value z_0 is defined by $p_e(z)=p_0$, i.e. as the equilibrium position of the tube subjected to the uplift load $p_e(z)$, function of the depth, and to the tube weight p_0, sum of the empty tube plus an additional dead load. (see appendix)

Finally, the position of the beam has to fulfil the following conditions:

At end 1 $(x=L_c)$:

$$u(x) = 0 \qquad w(x) = H - z_0$$
$$tg[\alpha(x)] = -m \qquad M(x) = 0 \tag{2}$$

with $u(x)$, $w(x)$ and $\alpha(x)$ the total displacements and rotation of section x and $M(x)$ the bending moment.

At end 2 $(x=L)$:

$$w(x) = 0 \qquad tg[\alpha(x)] = tg(\beta) \qquad T(x) = T \tag{3}$$

with $T(x)$ the axial force at section x.

(b) Case 2.

In this case the pipeline has at its end 2 a specified curvature k_0 produced by the floating platform ramp and also there the tension T is given. The formulation of this case follows similar line as in case 1. The initial position of the beam is supposed as before to be horizontal and the boundary conditions at the end 1 are identical as the previous case. At the offshore end the conditions are now the following ones (Figure 2):

At end 2 $(x=L)$:

$$w(x) = 0 \qquad \frac{M(x)}{EI} = k_0 \qquad T(x) = T \tag{4}$$

i.e. the bending moment is now the datum at this end instead of the slope.

The change from the initial to the final position of the pipeline is produced by filling with water a length L_a of the tube. This length has to be determined by the already stated boundary conditions. To this aim an iterative procedure is applied. That is, assuming an initial value of L_a a non linear analysis is carried out and the fulfilment of the boundary conditions is checked. If they are not coincident with the previous ones, the L_a value is modified until some final values are found such that the boundary conditions at both ends of the beam, are satisfied within a maximum allowable error.

The analysis for each value of the submerged length L_a is performed by using an incremental iterative process, that is, the water loading $p_a - p_0$, the depth H, and the slope at both ends (case 1) or the slope at end 1 and the moment at end 2 (case 2), are divided in N steps. In every loading step the following actions are applied: force distribution given by $\frac{p_a - p_0}{N}$, an imposed vertical displacement of $\frac{H}{N}$ and in the case 1, rotations $\frac{tg^{-1}(m_j)}{N}$, with $j=1$, 2 at both ends of the beam or in case 2 a rotation $\frac{tg^{-1}(m_j)}{N}$ at end $j=1$ and a moment M_j over N at end $j=2$. As it has been pointed out in each load step a linear analysis is performed using a tangential stiffness matrix that depends on the geometry and it is continually redefined according to the deflected pipeline of the previous load step.

In summary, in the iterative incremental analysis just described the tube is assumed to be full encastred at end 1 (displacements and rotations imposed) and a vertical roller at end 2 subjected to an horizontal force and a imposed rotation (case 1) or to an imposed bending moment (case 2).

The number of loading steps N to be used in the analysis should be large enough in order to assure a linear solution at each step and a limited amount of accumulative error in the final solution.

It should observed that the described analysis does not simulate the construction procedure, because its objective is to find the geometry and the stress distribution of the pipeline in the construction state defined by the values $L_c^i = \frac{1}{m}(H+R)$, $L_a^i = L_a(H)$ and $L^i = L(H)$. In this way the feasibility of the pipeline placement procedure can be checked. In fact, this method of analysis allow us to study the stresses occurring in the tube when the part of the pipeline considered in the analysis is joined to the segment of the pipeline already submerged and rested along the sea bed at a section with zero bending moment. Moreover from the knowledge of the functions $L_a(H)$ and $L(H)$ is possible to check the maximum and minimum stresses appearing during the whole process of construction.

Then, only for the case $H=0$ the analysis just described represents the actual construction of the phase 1 but in other cases is not so, but the computed stresses and geometry are the same as the ones produced in the actual pipeline construction.

In a similar way can be treated the analysis of the phase 3. In this case the following incremental iterative procedure is used:

At initial position of this phase the stresses distribution and the geometry of the pipeline are known, i.e., the values of the parameters $L_a(H)$ and $L(H)$, with H the depth at section $x=L_c$ or section of contact between the bed sea and the tube. The final position of this phase corresponds to the pipeline standing on the sea bed. This is achieved by applying to the pipeline initial position a set of N incremental actions and in each of them it is supposed zero bending moment at section $x=L_c$. The actions considered in this phase are the following ones: First the pulling force T is removed by introduction N incremental steps of a force with value $-\frac{T}{N}$. Second, the roller is simultaneously eliminated by similar technique, i.e. by N load steps of vertical forces of value $-\frac{V}{N}$, where V is the vertical reaction at initial position of the phase. During the application of these loads the length of inundated pipeline varies in such way that the bending moment at section x=L_c be zero. Finally, the angle (case 1) or (the bending moment (case 2) at pipeline end x=L become zero by introducing N increments of angle (case 1) or bending moments (case 2) and the length of the pipeline to be inundated is reached also by N increments of water loads distribution of length $\frac{L-L_a-L_c}{N}$ each. In this way the resulting stresses appearing during the final construction phase are found.

4. NUMERICAL APPLICATION

Following the theory described in the previous section a computer program has been written based in the ANSYS code, that allows us to analyze the stresses occurring in the installation phases of a proposed offshore pipeline to the city of Santander (Spain). The input data considered in the analysis are given in next page. In order to compare the sensitivity of various parameters in the design several thrust forces T has been envisaged, namely T=0, 50, 100 and 150 tons, and a set of different sea depths has been studied H=5, 10, 20 and 40 m. Also two slopes at the offshore end (case 1) has been also considered. For each of these cases the minimum pipeline length has been found by assuming that this minimum occurs when the inundated pipeline length coincides with the total tube length, i.e. when $L-L_a=L-L_c$. That means for longer pipeline lengths and under the same sea depth the inundated lengths are smaller than the total pipeline length. Some results are shown in Figure 3 for a given depth of 40 m, and the variation of the inundated tube length with the sea depth is given by Figure 4

Data

R_0	Inner pipeline radius	0.738	m	L	Pipeline length	500	m
R_1	Outer pipeline radius	0.800	m	p_1	Concentrated dead load	4.158	t
p_0	Empty pipeline weight	0.281	t/m	t	Application length of the dead load	0.900	m
p_a	Water full pipeline weight	1.759	t/m	d	Distance between consecutive axes	3.000	m
p_e	Hydrostatic pressure on pipeline	2.063	t/m	p_{e_1}	Hydrostatic pressure	1.778	t/m²
E	Young's Modulus for tube material	9000	t/m²	m	Mean sea bed slope	0.200	-
σ_a	Design stress	500	t/m²	H	Max. pipeline depth in section 1	38.00	m
σ_e	Elastic stress limit	2500	t/m²	γ_w	Sea water density	1.026	t/m³

5. FINAL REMARKS

Using one simplified structural scheme all but the final one installation phases of an offshore pipeline can be simulated. In this way is possible to study the sensitivity of the different variables in the design. Moreover by using dimensional analysis the above sensitivity study could easily be extended to more general situations pipeline installations i.e. including different material and geometrical properties of the pipeline, sea bed slopes and loads. A useful result of this analysis can be the construction of simple aid design charts.

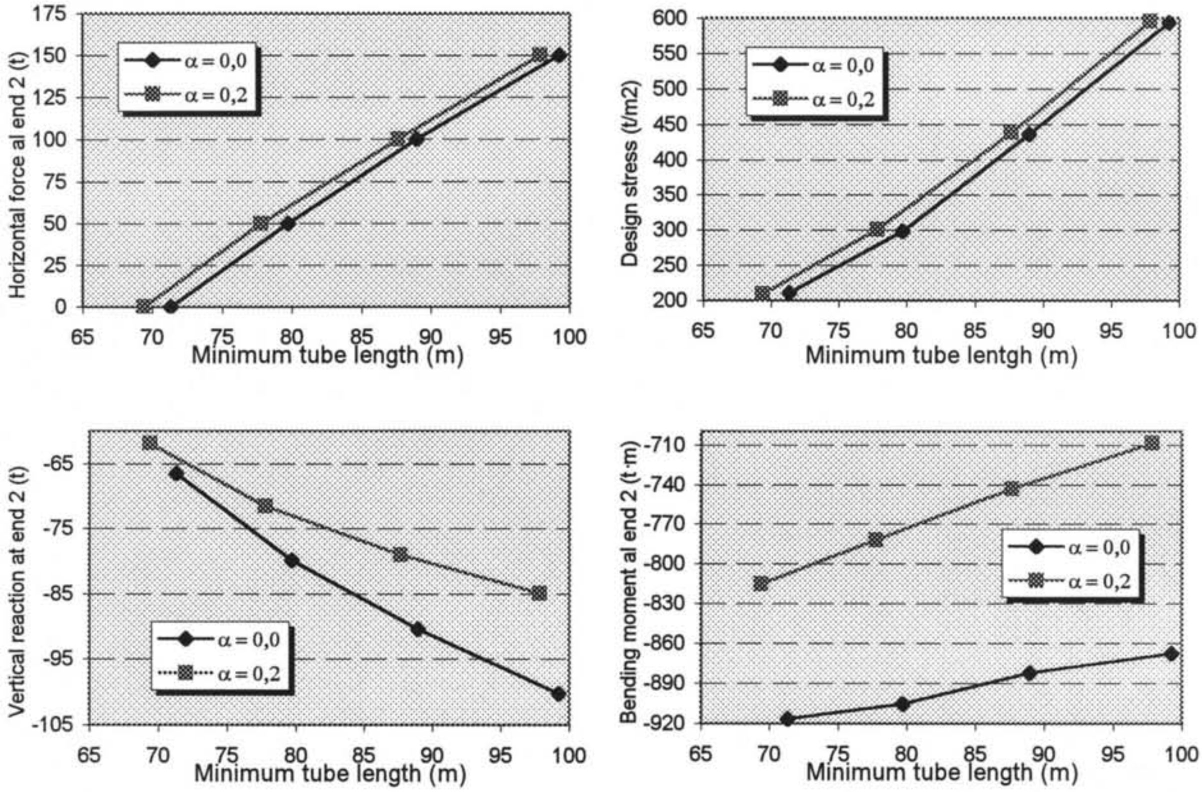

Figure 3.- Some results for a sea depth of 40 m (Maximum length $L_a = L - L_c$)

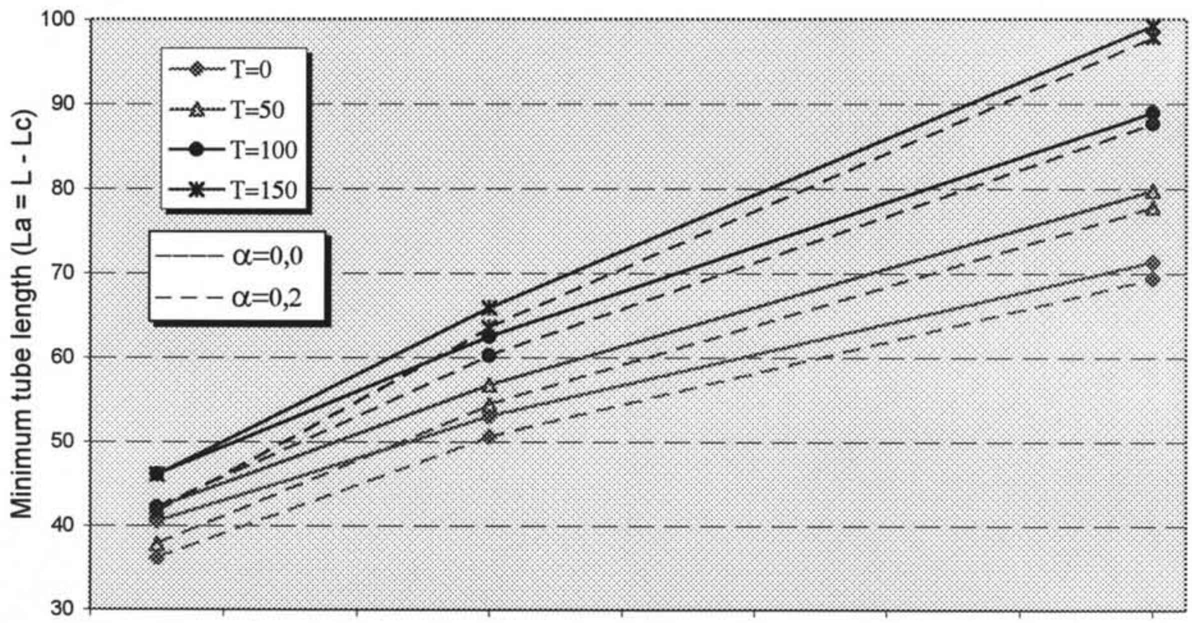

Figure 4.- The inundated tube length as function of the sea depth

6. REFERENCES

1 T.J. Datta and A.K. Basu "Stress analysis of submarine pipelines". Proc. Instn. Civ. Engr, Part 2, 63:833-841, December 1977.

2 J. T. Powers and L.D. Finn "Stress analysis of offshore pipelines during installation" Proc. 1st Conf Offshore Technol. Conf. Paper 1071. Houston 1969.

3 J. Oliver y E. Oñate. "Una formulación para el estudio, por el método de los elementos finitos, de la deformación de tuberias marinas durante su instalación". Revista de Obras Públicas. Oct 1983.

4 B. Ovunc and H. Mallareddy "Stress analysis of offshore pipelines" Proc. 2nd Conf Offshore Technol. Conf. Paper 1222. Houston 1970.

APPENDIX A

A.1. Normal force function $p_e(z)$

The function that describes the force against the submerged depth of the tube is given by:

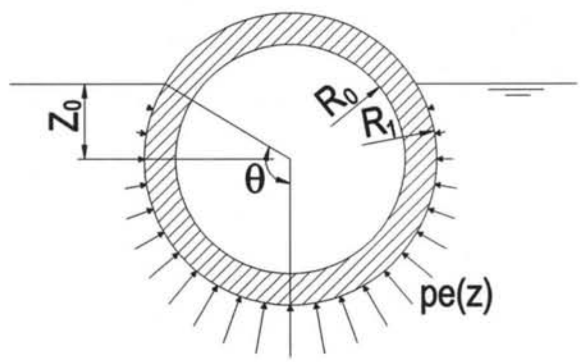

Figure 5.- Normal force function

$$\text{if} \quad -R_1 \le z \le R_1 \qquad p_e(z) = -\gamma R_1^2 [\theta - \sin(\theta)\cos(\theta)] = $$

$$= -\gamma R_1^2 \left[\arccos\left(\frac{z}{R_1}\right) + \pi + \frac{z}{R_1}\sqrt{1 - \frac{z^2}{R_1^2}} \right] \tag{5}$$

$$\text{if} \quad z \le -R_1 \qquad p_e(z) = 0$$

$$\text{if} \quad z \ge R_1 \qquad p_e(z) = -\gamma \pi R_1^2$$

A.2. Tangential force function $k_e(z)$

The function that describes the force against the submerged depth of the tube is given by:

$$\text{if} \quad -R_1 \le z \le R_1 \quad k_e(z) = 2\gamma R_1 \sqrt{1 - \frac{z^2}{R_1^2}}\, dz \tag{6}$$

$$\text{if} \quad z \le -R_1 \quad k_e(z) = 0 \ ; \quad \text{if} \quad z \ge R_1 \quad k_e(z) = 0$$

with $dz = R_1 \sin(\theta)\, d\theta$

A.3. Tangent stiffness matrix of a submerged element beam

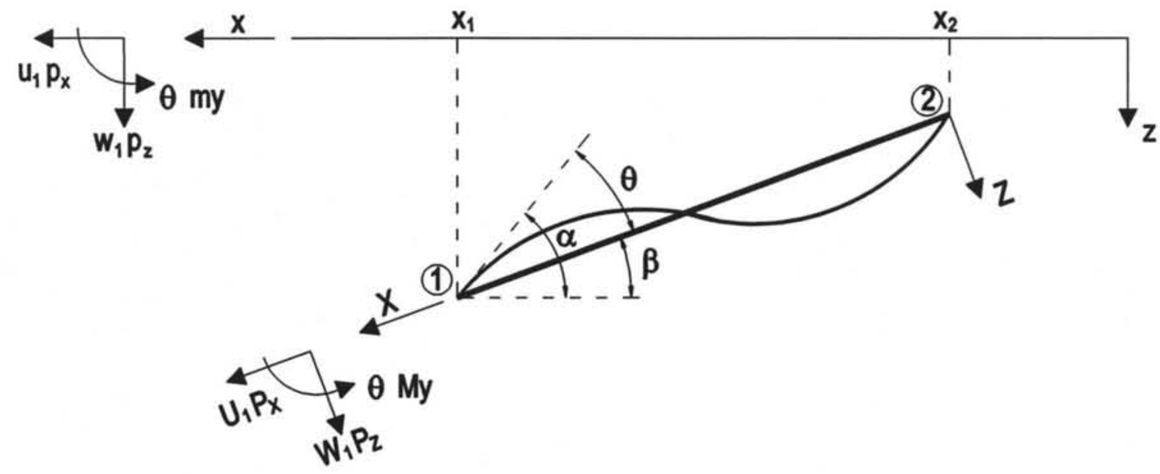

Figure 6.- Current position of pipeline beam element

Given the 1-2 element of Figure 6 with local axis $(X,\ Z)$, and the general axis $(x,\ z)$, the geometrical input data of the current substep $(x_1,\ x_2,\ z_1,\ z_2,\ \alpha_1,\ \alpha_2)$, are the output of the previous one. These data correspond to the coordinates and to the tangents of the rotations of one beam element in which the pipeline has been divided. Both systems of coordinates are related by the equations:

$$X=(x-x_1)c+(z-z_1)s\ ,\quad Z=-(x-x_1)s+(z-z_1)c$$
$$x=x_1+Xc-Zs\ ,\qquad z=z_1+Xs-Zc \tag{7}$$

where $c=\cos\beta$, $s=\sin\beta$ and β is the angle of rotation from local axis to the global axis. The length of the element is $L=\sqrt{(x_2-x_1)^2+(z_2-z_1)^2}$.

The geometry of the pipeline axis at initio of the current substep is:

$$z=z(x)=N_1 z_1+N_2 z_2+N_3\alpha_3+N_4\alpha_4$$
$$\alpha=\alpha(x)=\frac{\partial z(x)}{\partial x}=\frac{1}{l}(N'_1 z_1+N'_2 z_2+N'_3\alpha_3+N'_4\alpha_4) \tag{8}$$

where $\xi=\dfrac{x-x_1}{x_2-x_1}$, $l=(x_2-x_1)$ and $(\,')=\dfrac{\partial}{\partial\xi}$. The functions $N_i(\xi)$ are the hermitian interpolation polynomials.

The tangent stiffness matrix referred to the global axis is defined as follows:

$$\boldsymbol{dp}=\boldsymbol{k_t du} \tag{9}$$

where ***du*** and ***dp*** are the incremental displacement and forces vectors given by the expressions:

$$\boldsymbol{du}=[du_1,du_2,dw_1,dw_2,d\alpha_1,d\alpha_2]^T$$
$$\boldsymbol{dp}=[dp_{x1},dp_{x2},dp_{z1},dpz_2,dm_{y1},dm_{y2}]^T \tag{10}$$

Similar expressions can be written in capital letters for the tangent stiffness matrix referred to the local axis.

The incremental uplift forces distribution produced by a small change of the depth of the pipeline axis are given by equation:

$$dp_e(z) = -k_e(z)\,dz = -k_e(z)\,\boldsymbol{N_z}\,\boldsymbol{du} \tag{11}$$

where:

$$\boldsymbol{N_z} = \left[\frac{\alpha}{l}(x-x_2), -\frac{\alpha}{l}(x-x_1), N_1, N_2, N_3, N_4\right] \tag{12}$$

The equivalent nodal forces at the beam element ends are in local axis:

$$\boldsymbol{P_0} = -\left[L\int_0^L k_e(z)\boldsymbol{N_u^T}\boldsymbol{N_z}\boldsymbol{T}d\eta\right]\boldsymbol{dU} \tag{13}$$

where $\boldsymbol{N_u} = [\overline{N_1}(\eta), \overline{N_2}(\eta), N_i(\eta)]$ $(i=1, 4)$ is the vector containing the linear interpolation functions $\overline{N_1} = 1-\eta$, $\overline{N_2} = \eta$ and the interpolation Hermite functions expressed in the variable $\eta = \frac{X}{L}$. The matrix $\boldsymbol{T}$ is the transformation matrix between the vectors $\boldsymbol{du}$ and $\boldsymbol{dU}$, i.e.

$$\begin{bmatrix} c & 0 & -s & 0 & 0 & 0 \\ 0 & c & 0 & -s & 0 & 0 \\ s & 0 & c & 0 & 0 & 0 \\ 0 & s & 0 & c & 0 & 0 \\ \frac{c^2}{L}\alpha_1 & -\frac{c^2}{L}\alpha_2 & -\frac{1}{L}(1+cs\,\alpha_1) & \frac{1}{L}(1+cs\,\alpha_2) & 1 & 0 \\ \frac{c^2}{L}\alpha_1 & -\frac{c^2}{L}\alpha_2 & -\frac{1}{L}(1+cs\,\alpha_1) & \frac{1}{L}(1+cs\,\alpha_2) & 0 & 1 \end{bmatrix} \tag{14}$$

From the above considerations the following expression of the tangent stiffness matrix in local axis can be obtained:

$$\boldsymbol{K_t} = \boldsymbol{P_0} + \boldsymbol{K_L} \tag{15}$$

where the standard linear stiffness beam matrix has been denoted by $\boldsymbol{K_L}$.

Then the tangent stiffness in global axis is

$$\boldsymbol{k_t} = (\boldsymbol{T^{-1}})^{\boldsymbol{T}}\boldsymbol{K_t}\boldsymbol{T^{-1}} \tag{16}$$