Nonlinear effects in optical fibers - v1

Miguel A. Muriel

UPM-ETSIT-MUIT-CFOP

Nonlinear effects in optical fibers

1) Introduction
2) Causes
3) Parameters
4) Fundamental processes
5) Types
6) Envelope nonlinear equation

1-Introduction

-The propagation properties depend on the optical signal, traveling along the optical fiber

-The presence of an optical field can modify optical properties of the medium

$$I\left[\frac{W}{m^2}\right]=\frac{P[W]}{A[m^2]}$$

$$I \propto |E|^2$$

-High optical intensity → Nonlinear regime→ Interaction between light and a nonlinear medium.

2-Causes

- Increase of capacity through WDM technologies → many channels in fiber

- Optical transmitters are more powerful

- Optical amplifiers

- $B\uparrow \rightarrow T_B\downarrow$ → high peak power

- Low temporal dispersion → pulses are widened very little → they keep their peak power along transmission

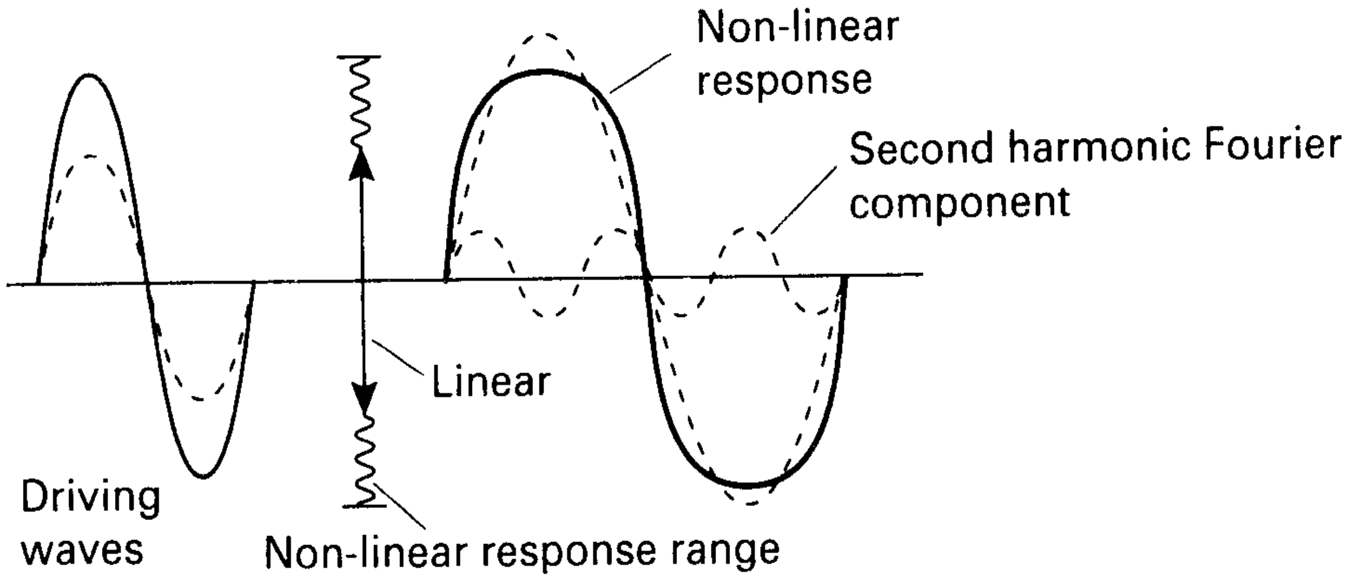
- Reduced dimension of core fiber→ high power density → 1 mW in SMF → 12 MW/m^2.

Linear Media

-Linear displacement of electrons with the incident field
-The reemitted wave has the same frequency as the incident

Nonlinear Media

-Nonlinear displacement of electrons with the incident field
-The reemitted wave has the same frequency as the incident, together with harmonics.



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Linear regime

- Output power proportional to the input power

- Phase change proportional to the effective refractive index

- New wavelengths aren´t generated

Nonlinear regime

- Output power NOT proportional to input power → extra attenuation

- Phase change NOT proportional to effective refraction index → generation of chirp pulse

- New wavelengths are generated → different carriers interacting between themselves → crosstalk and distortion

3-Parameters

1- Effective core area

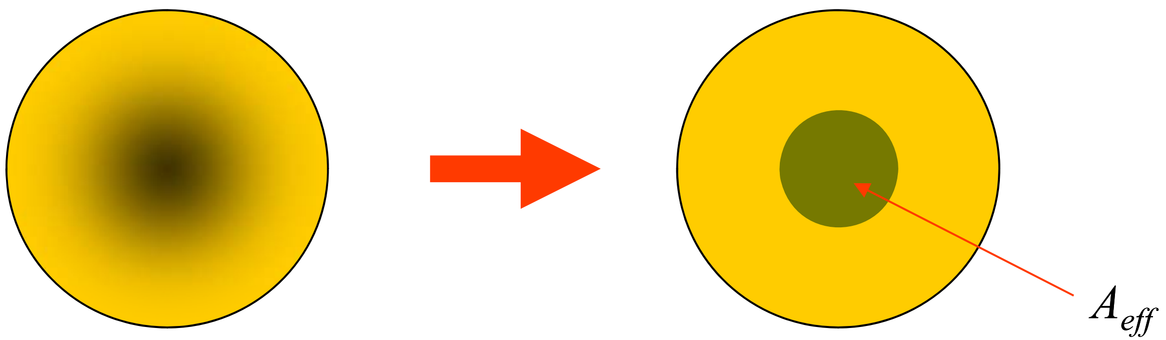
2- Effective length

3-1- Effective core area (A_{eff})

- Non uniform density of field and power in the transversal section of the fiber

- Effective area → Equivalent area with uniform density

$$A_{eff} = \frac{\left(\int I dA\right)^2}{\int I^2 dA} \qquad \begin{cases} \text{Cartesian} \rightarrow dA = dxdy \\ \text{Cylindrical} \rightarrow dA = 2\pi r dr \end{cases}$$



- SMF, fundamental mode LP_{01}

$$E(r) = e^{-\left(\frac{r}{w}\right)^2}$$

$$I(r) \propto \left|E(r)\right|^2 = e^{-2\left(\frac{r}{w}\right)^2}$$

$w \rightarrow$ Mode field radius

$MFD = 2w \rightarrow$ Mode field diameter

$$\boxed{A_{eff} = \pi w^2 = \frac{\pi \cdot MFD^2}{4}}$$

Effective core area of various fiber types (typical values)

ITU-T fibre type	A_{eff} @ 1550 nm (μm²)
G.652 (SMF)	80
G.653 (DSF)	50
G.654 (CSF)	90
G.655 (NZDSF)	55 (D>0), 60 (D<0)
(DCF)	20

SMF: Single Mode Fiber

DSF: Dispersion Shifted Fiber

CSF: Cutoff Shifted Fiber

NZDSF: Non-Zero DSF

DCF: Dispersion Compensating Fiber

- $A_{eff}\uparrow \rightarrow P_{th}\uparrow$

- The DSF fiber presents the smallest effective area among the line fibers.

- Among special fibers, the effective area is particularly small in DCF →Caution when fixing the DCM input power levels in dispersion compensated links.

- In MM fibers A_{eff} is estimated as core section.

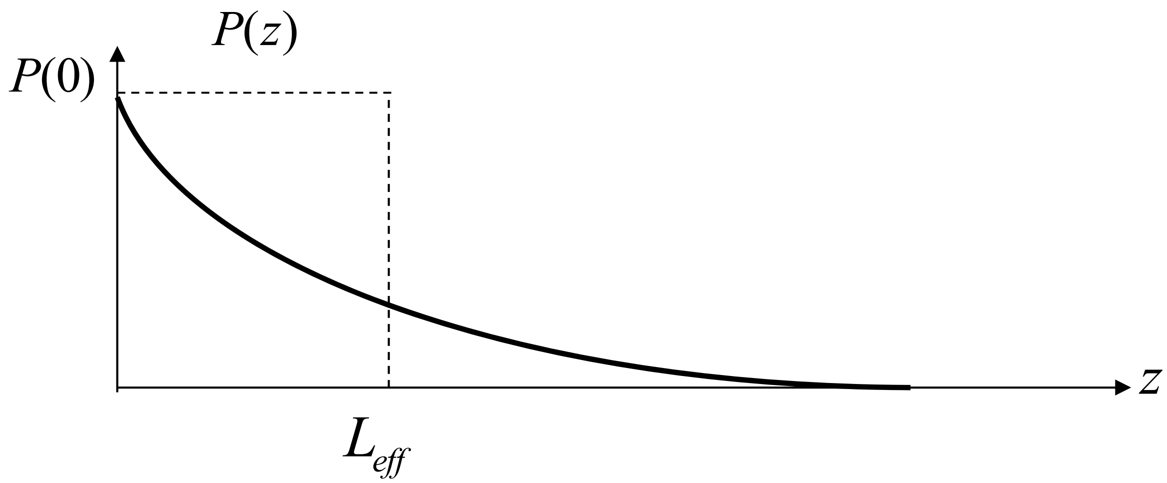
3-2- Effective length (L_{eff})

- Signal power decreases exponentially with distance z

$$P(z) = P(0)e^{-\alpha_f z}$$

$$\alpha[dB/km)] = 4,343\,\alpha_f[1/km] \rightarrow \alpha_f \approx 0,23\,\alpha$$

- Effective interaction length L_{eff} is the length of a fiber with zero attenuation, which has the same nonlinear impact as a fiber with attenuation α



$$P(0)L_{eff} = \int_{z=0}^{L} P(z)dz \quad \rightarrow \quad \boxed{L_{eff} = \frac{1-e^{-\alpha_f L}}{\alpha_f}}$$

$$\alpha_f L \ll 1 \rightarrow L_{eff} \approx L$$

$$\alpha_f L \gg 1 \rightarrow L_{eff} \approx \frac{1}{\alpha_f}$$

$$\lim_{L\to\infty} L_{eff} = \frac{1}{\alpha_f}$$

$$\text{Example} \rightarrow L = 200 Km,\ \alpha = 0,22\,[dB/km)]$$

$$\rightarrow \alpha_f \approx 0,05\,[1/km]$$

$$\rightarrow \alpha_f L = (0,23)(0,22)(200) = 10,12$$

$$\rightarrow L_{eff} \approx \frac{1}{\alpha_f} = 20\ Km$$

4) Fundamental processes

In silica fibers there are two fundamental processes:

1) Stimulated Scattering

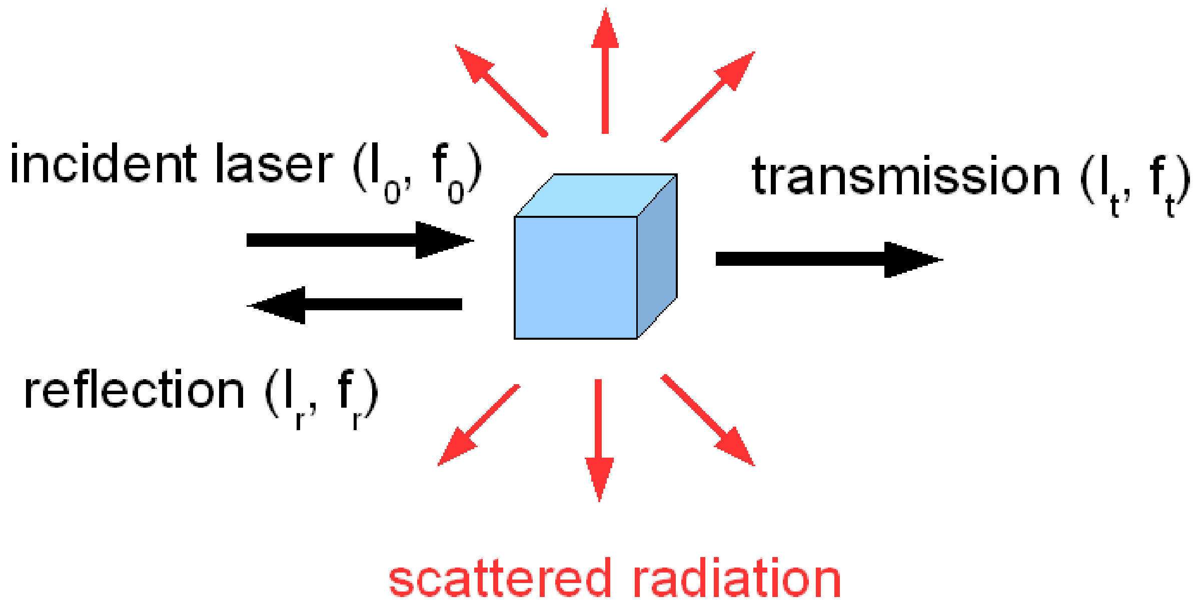
Molecular vibration modes of SiO_2

$$\nu_{\text{photon(OUT)}} = \nu_{\text{photon(IN)}} \pm \nu_{\text{phonon}}$$

2) Optical Kerr effect

Variation of refraction index with optical intensity

4-1- Stimulated Scattering



Rayleigh (ν_0)

- Particles
- Local refractive index variations

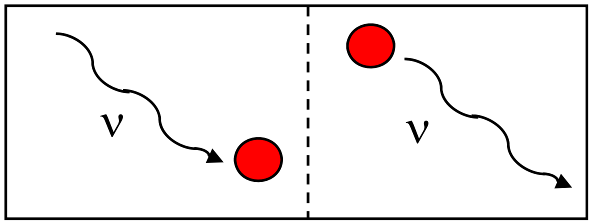
Brillouin ($\nu_0 \pm \nu_B$)

- Density fluctuations
- Acoustic waves
- Acoustic phonons
- $\nu_B \sim$ GHz

Raman ($\nu_0 \pm \nu_R$)

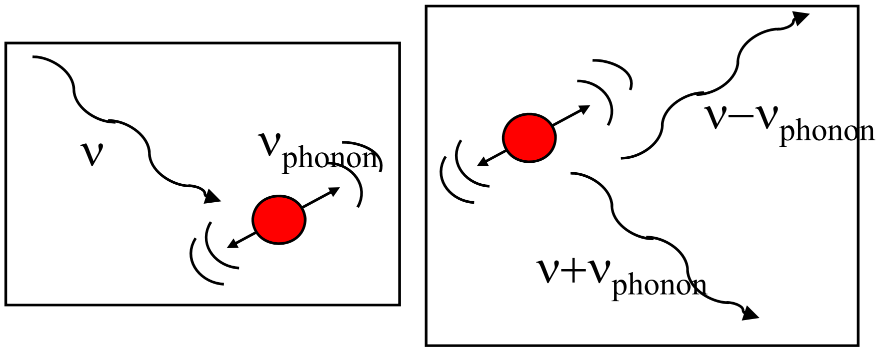
- Molecular vibrations
- Molecular rotations
- Electronic transitions
- Optical phonons
- $\nu_R \sim$ THz

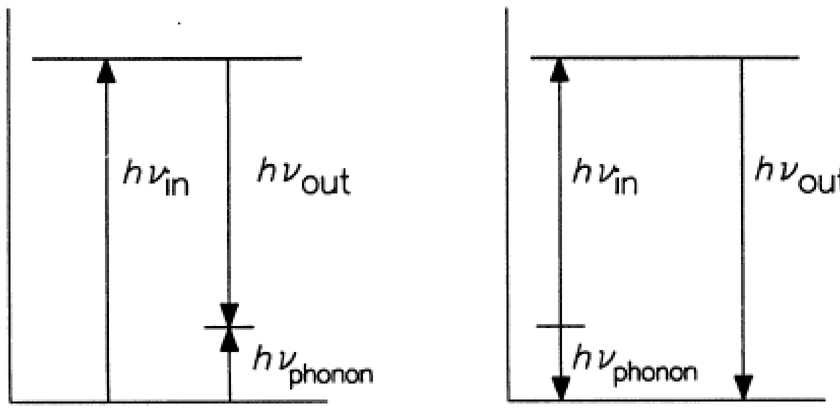
Rayleigh



Brillouin

Raman





(a) Stokes (b) Anti–Stokes

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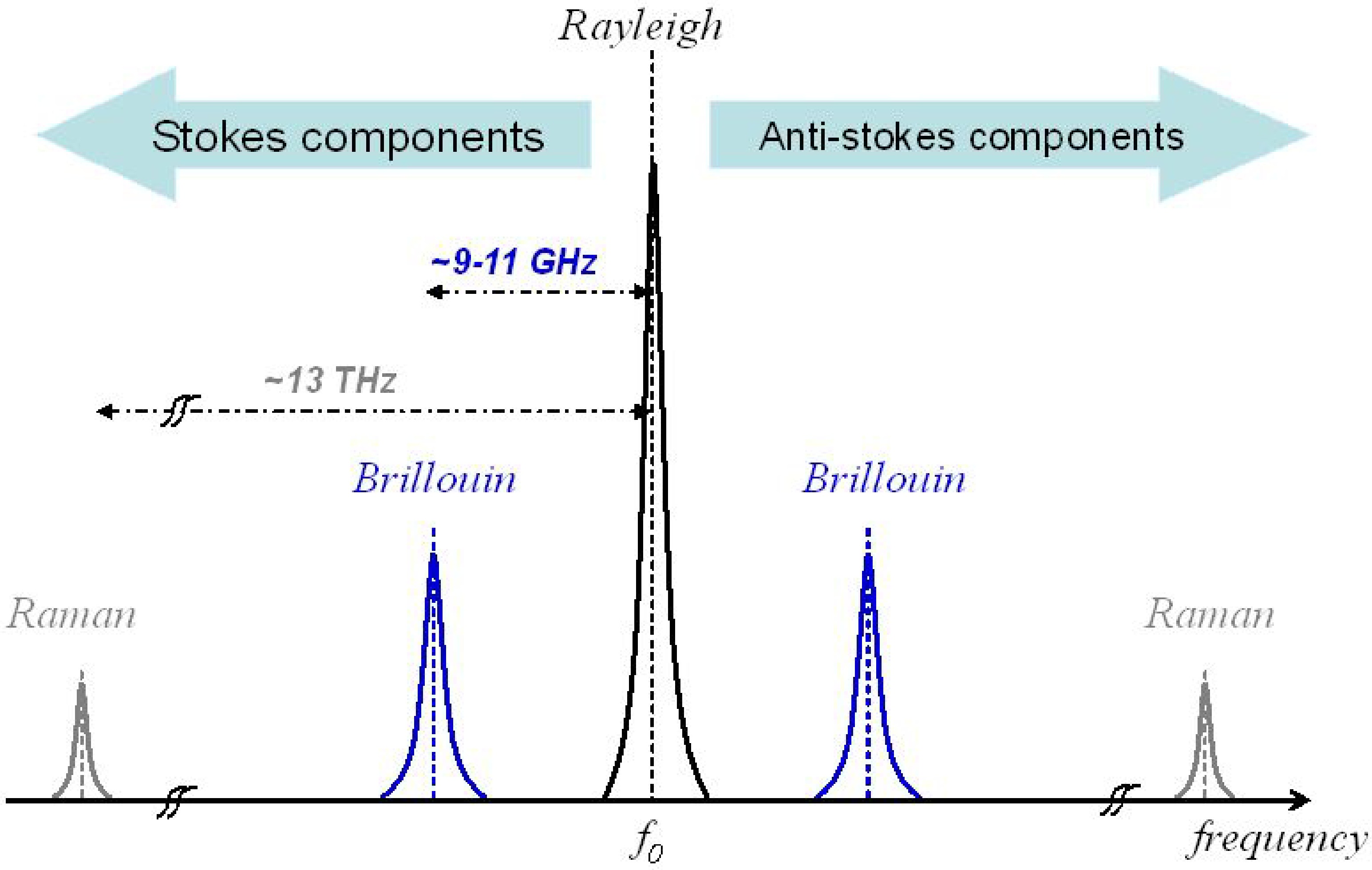
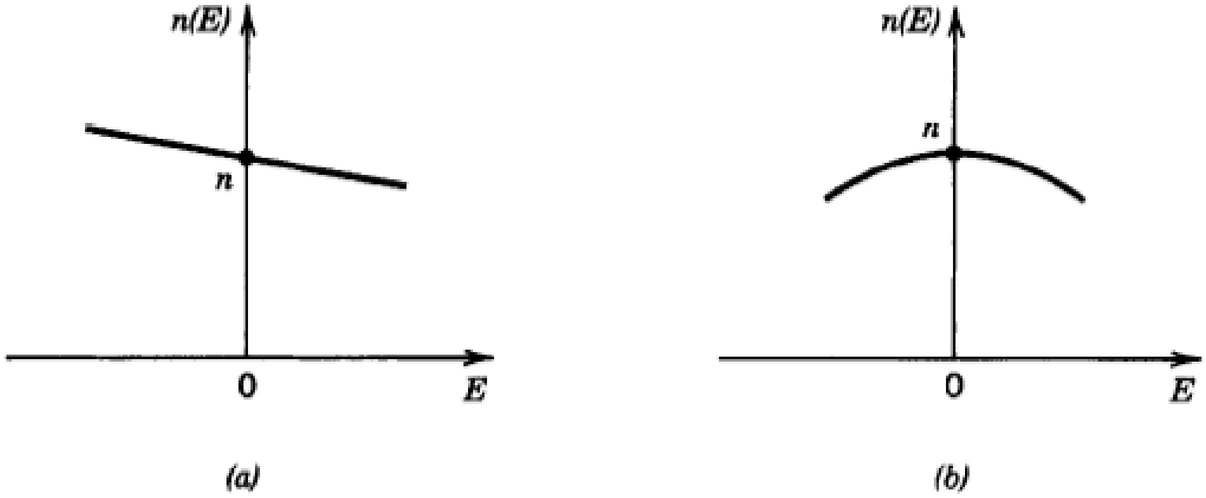


http://www.intechopen.com/

4-2- Optical Kerr effect



Dependence of the refractive index on the electric field: (*a*) Pockels medium; (*b*) Kerr medium.

$$n(E) = \underbrace{n(0)}_{n_0} + \underbrace{\left.\frac{dn(E)}{dE}\right|_{(E=0)}}_{-n_1} E + \frac{1}{2}\underbrace{\left.\frac{d^2n(E)}{dE^2}\right|_{(E=0)}}_{2n_2} E^2$$

$$n(E) = n_0 - n_1E + n_2E^2 \begin{cases} n(E) = n_0 - n_1E & \leftarrow \text{ Pockels Effect} \\ n(E) = n_0 + n_2E^2 & \leftarrow \text{Kerr Effect} \end{cases}$$

(*Fundamentals of Photonics, 2nd Ed.,* Saleh and Teich, 2007)

The refractive index depends on the optical field power.

$$n(E) = n_0 + n_2 E^2$$

$$n_2 = (2,2 \cdots 3,4) \cdot 10^{-20} \frac{(m)^2}{(W)}$$

$$E^2 \propto I = \frac{P}{A_{eff}}$$

$$n = n_0 + n_2 I = n_0 + n_2 \frac{P}{A_{eff}}$$

Nonlinear coefficient

$$\gamma = \frac{k_0 n_2}{A_{eff}}$$

In standard single mode fiber with:

$$A_{eff} = 80\left(\mu m\right)^2$$

$$n_2 = 2{,}35 \cdot 10^{-20} \frac{(m)^2}{(W)}$$

$$\lambda = 1{,}55\left(\mu m\right)$$

$$\gamma = 1{,}76 \frac{1}{(W \cdot km)}$$

Parameter/Fiber Type	SSMF	DSF	DCF
γ / W^{-1} km^{-1}	1,2	1,76	8,3

5) Types

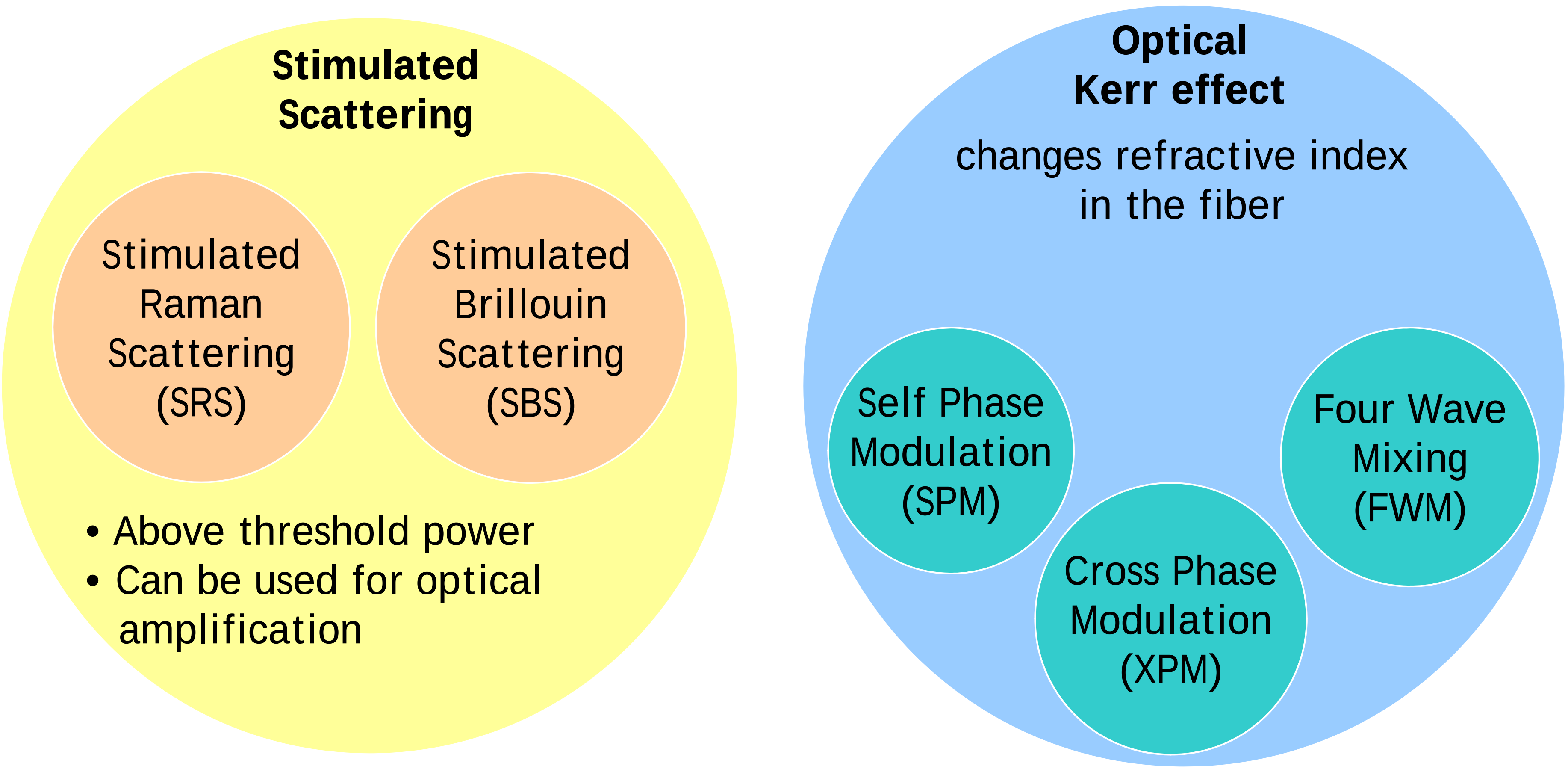
1- SBS Stimulated Brillouin Scattering

2- SRS Stimulated Raman Scattering

3- SPM Self-Phase Modulation

4- XPM Cross-Phase Modulation

5- FWM Four-Wave Mixing


Stimulated
Scattering
Stimulated
Raman
Scattering
(SRS)
Stimulated
Brillouin
Scattering
(SBS)
• Above threshold power
• Can be used for optical
amplification
Optical
Kerr effect
changes refractive index
in the fiber
Self Phase
Modulation
(SPM)
Cross Phase
Modulation
(XPM)
Four Wave
Mixing
(FWM)

5-1-SBS (Stimulated Brillouin Scattering)

Brillouin scattering is a result of the interaction between the optical wave and the propagating density fluctuations in the fiber, due to thermo-elastic motions of the molecules, that can be regarded as acoustic waves, traveling through the fiber at the speed of sound.

A strong pump wave generates a refractive index modulation (Bragg grating) in the material via the effect of electrostriction.

A part of the pump wave is backscattered at the refractive index modulation (Bragg grating), this is the Stokes wave.

$$\underbrace{\nu_{\mathrm{P}}}_{\text{Pump}} = \underbrace{\nu_{\mathrm{S}}}_{\substack{\text{Doppler shifted}\\ \text{Scattered}}} + \underbrace{\nu_{\mathrm{B}}}_{\substack{\text{Density (Acoustic)}\\ \text{Brillouin}}}$$

$$\nu_{\mathrm{P}} = \frac{c}{\lambda_{\mathrm{P}}}$$

$$\nu_{\mathrm{B}}$$

$$\nu_{\mathrm{S}} = \nu_{\mathrm{P}} - \nu_{\mathrm{B}}$$

Frequency shift of the back scattered wave for:

Pump wavelength, $\lambda_P = 1550$ nm

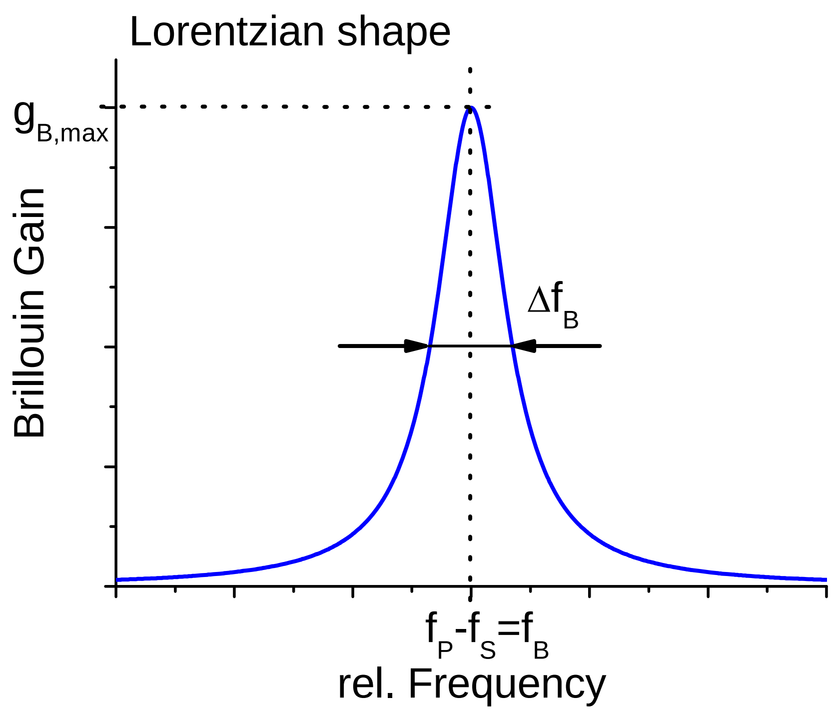
Refractive index, $n = 1.45$

Sound velocity, $\upsilon_a = 5.96$ km/s

$$\nu_{\mathrm{B}} = \frac{2\upsilon_a n}{\lambda_{\mathrm{P}}} = 11.15\ \text{GHz}$$

- $\Delta\nu_B$ → Full width half maximum (FWHM) of Brillouin Gain depends on the pump power

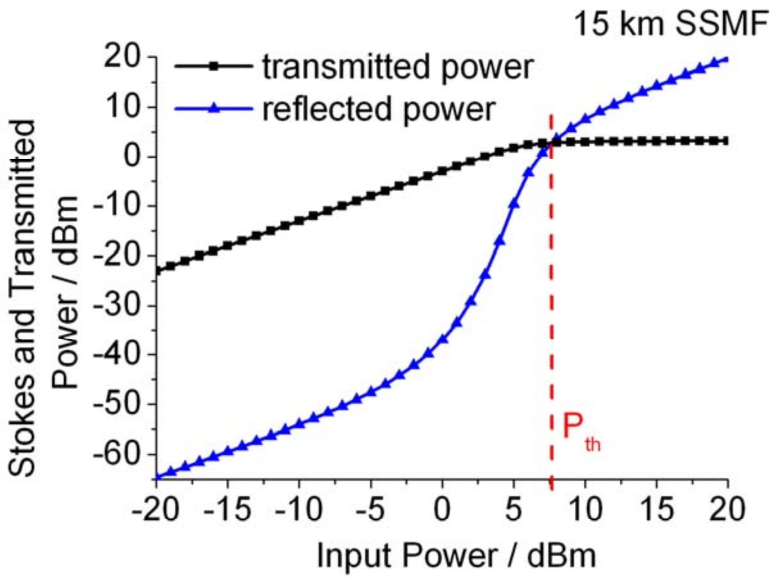
- $\Delta\nu_B$ (Typical) ≈ 50 MHz @ 1550 nm



Threshold Power

$$P_{th} = 21\frac{KA_{eff}}{g_B L_{eff}}\left[1+\frac{\Delta\nu_P}{\Delta\nu_B}\right]$$

- g_B → Brillouin-gain coefficient ($\sim 5\cdot 10^{-11}$ m/W)
- K → Constant determined by the polarization ($1 \le K \le 2$)
- $\Delta\nu_B$, $\Delta\nu_P$ → Brillouin and pump spectral bandwidth



5-2- SRS Stimulated Raman Scattering

- Raman scattering is a result of the interaction between the optical wave and the molecules of the material. Therefore, no phase matching condition is needed

- The scattered wave could be shifted down (Stokes) or up (anti-Stokes) in frequency. In optical fibers, the intensity of the downshifted wave is much higher

- Scattering mainly in the propagation direction

- The channels with a higher carrier frequency deliver a part of their power to the channels with a lower carrier frequency

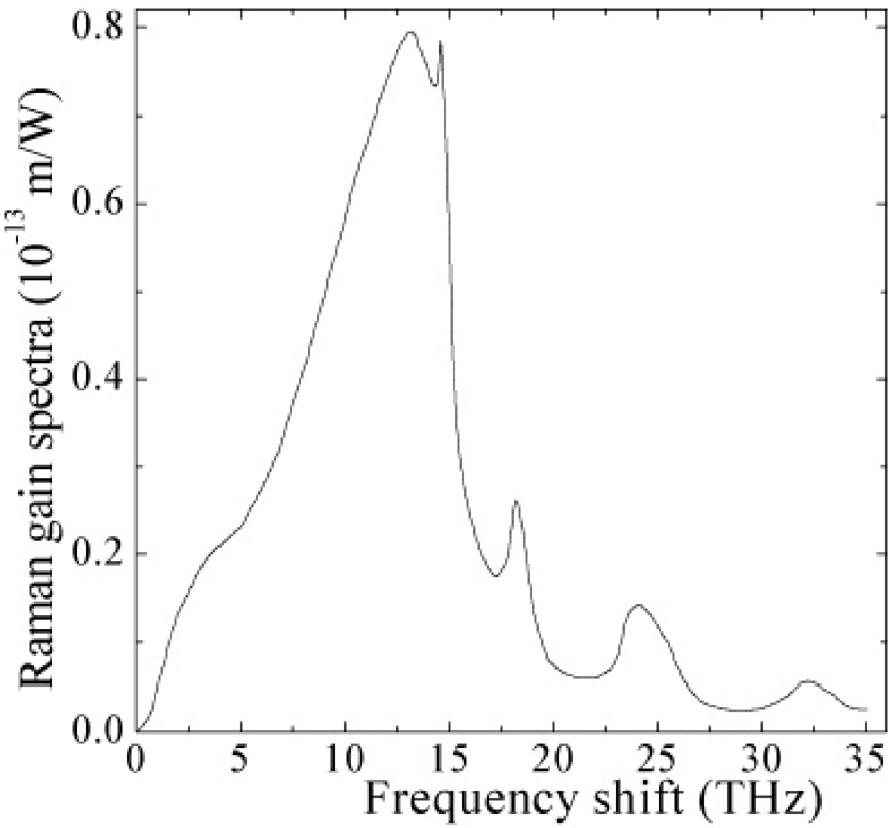
- In terms of wavelength, the channel with higher wavelength is amplified at the expense of the channel with the lower carrier wavelength

- The Raman gain in optical fibers is very broad (~40 THz)

- The maximum of the frequency shift is at ~ 13 THz (105 nm)

$\nu_{\text{optical phonon}}(\nu_R) \approx 13.2 \text{ THz}$

$\Delta\nu_R \approx 6 \text{ Thz}$



(*Nonlinear Fiber Optics, 5th Ed.,* Agrawal, 2010)

Threshold Power

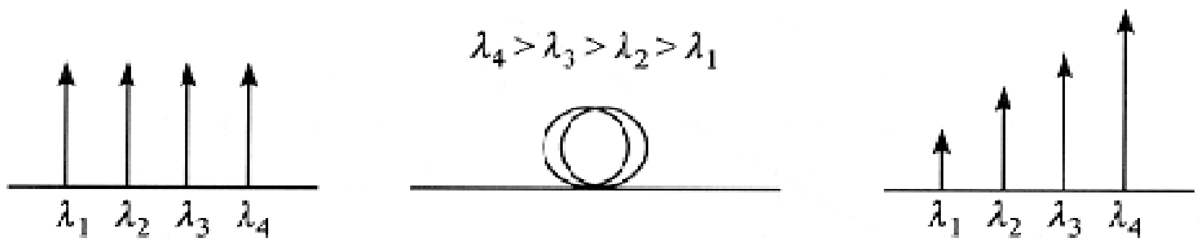
$$P_{th} \cong \frac{16}{g_R L_{eff}} A_{eff}$$

- g_R → Peak Raman Gain (~ $8 \cdot 10^{-14}$ m/W)

P_{th} ~ 570 mW (@ 1.55 μm) con $\Delta\nu_R \approx 16$ Thz

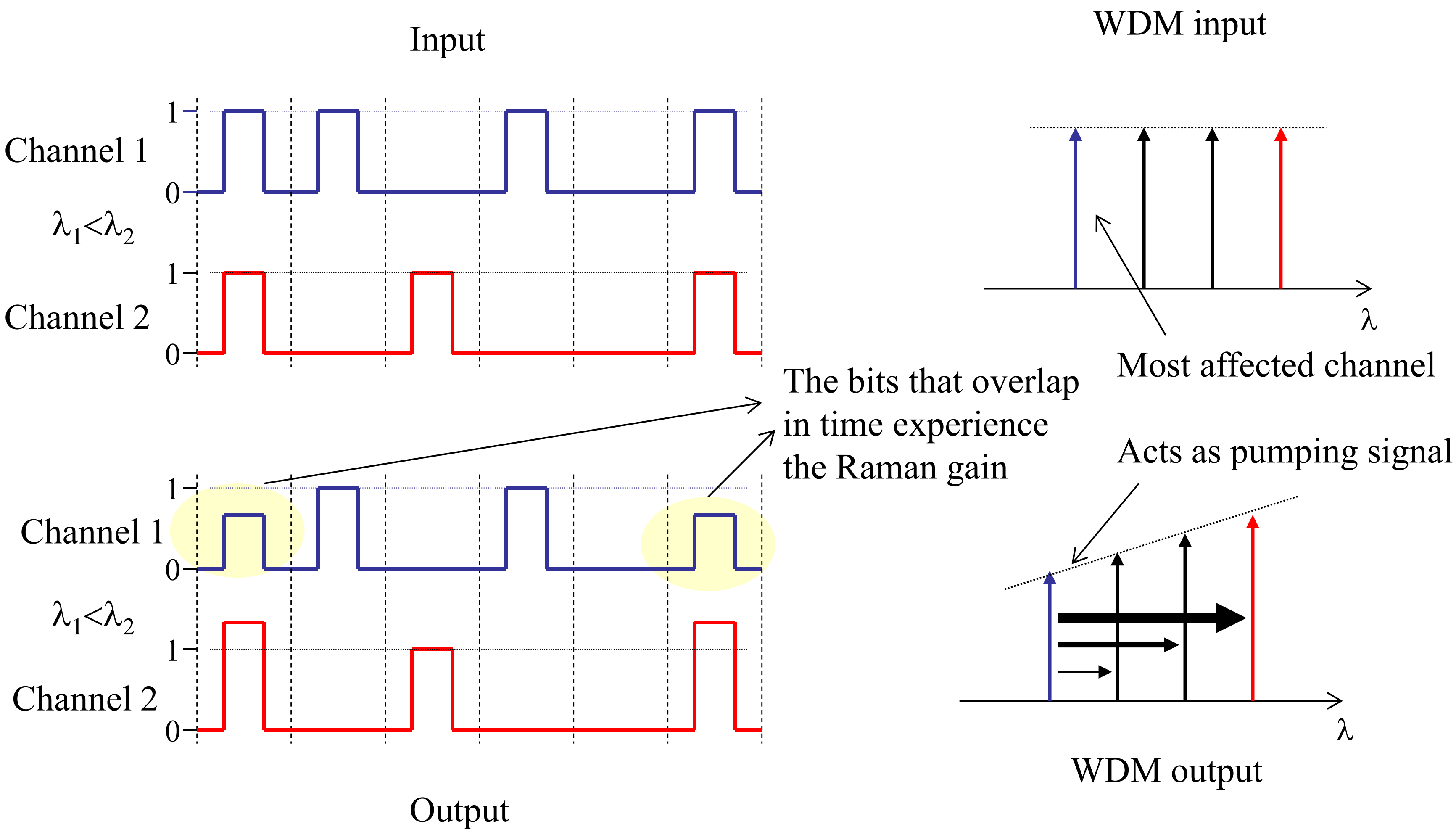
Input signals of equal power

Relative output signals



SRS Effect

SRS effect in WDM



5-3- SPM   Self-Phase Modulation

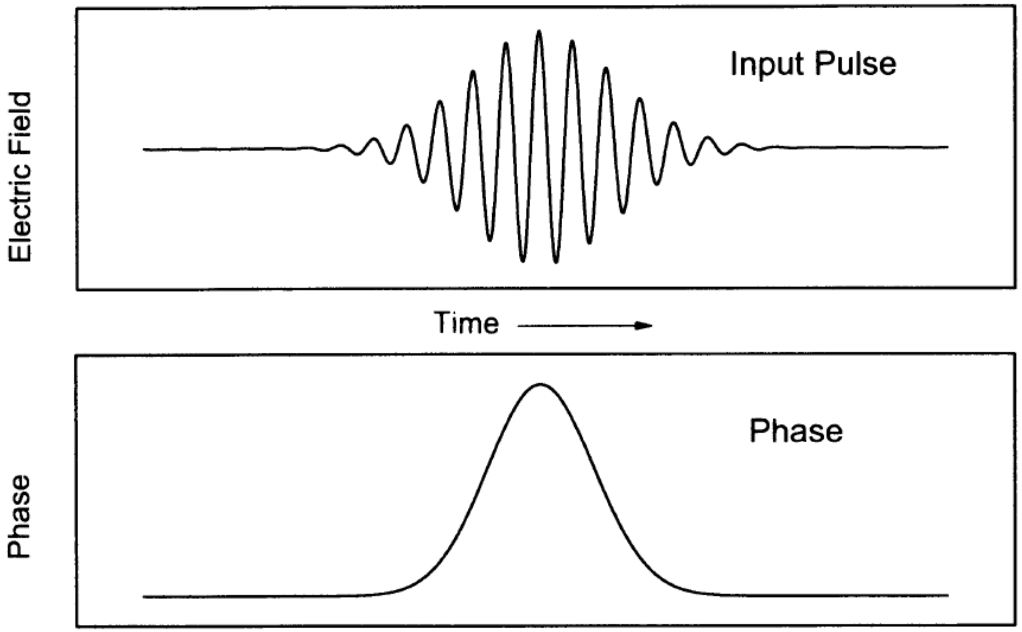
- SPM leads to a phase alteration of the wave due to its own intensity.

- Phase alteration causes a spectral broadening of the pulses

- Together with fiber dispersion, spectral broadening can be translated into an alteration of the temporal width of the pulse

-In the case of normal dispersion ($D < 0$) the pulse is additionally stretched, whereas it will be compressed for an anomalous dispersion ($D > 0$)

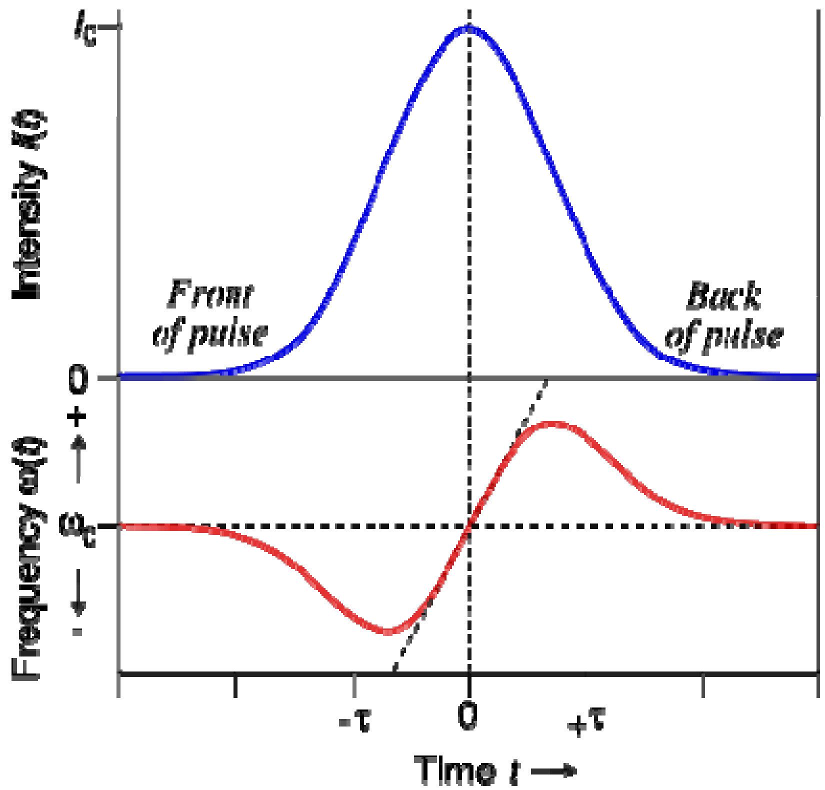
- Important in single-channel systems (Intra-SPM)



$$n(t) = n_0 + n_2 I(t) = n_0 + n_2 \frac{P(t)}{A_{eff}}$$

$$\phi(t) = \omega_0 t - k_0 n(t) L = \omega_0 t - k_0 n_0 L - \underbrace{\gamma P(t) L}_{\text{Nonlinear}}$$

www.wikipedia.org



$$\omega(t) = \frac{d\phi(t)}{dt} = \omega_0 - \underbrace{\gamma L \frac{dP(t)}{dt}}_{\Delta\omega_{NL}(t)}$$

www.wikipedia.org

5-4- XPM, CPM   Cross-Phase Modulation

- Cross-phase modulation (XPM, or sometimes, CPM) is similar to SPM, but the origin of the spectral broadening of the pulses are the other pulses propagating at the same time in the waveguide; they will mutually influence each other, via the alteration of the intensity-dependent refractive index.

- The refractive index that a wave experiences can be altered by the intensities of all other waves propagating in the fiber.

- A pulse at one wavelength has an influence on a pulse at another wavelength. This effect can be exploited for optical switching and signal processing, but lead to a degradation of a communication system performance.

- This is especially important for WDM systems where a huge number of pulses with different carrier wavelengths will be transmitted in one fiber (Inter-SPM)

- The XPM is the fundamental effect that determines the capacity of optical transmission systems.

$$\phi_{NL}(t) = k_0 n_2 (I_a(t) + c_{ab} I_b(t))L$$

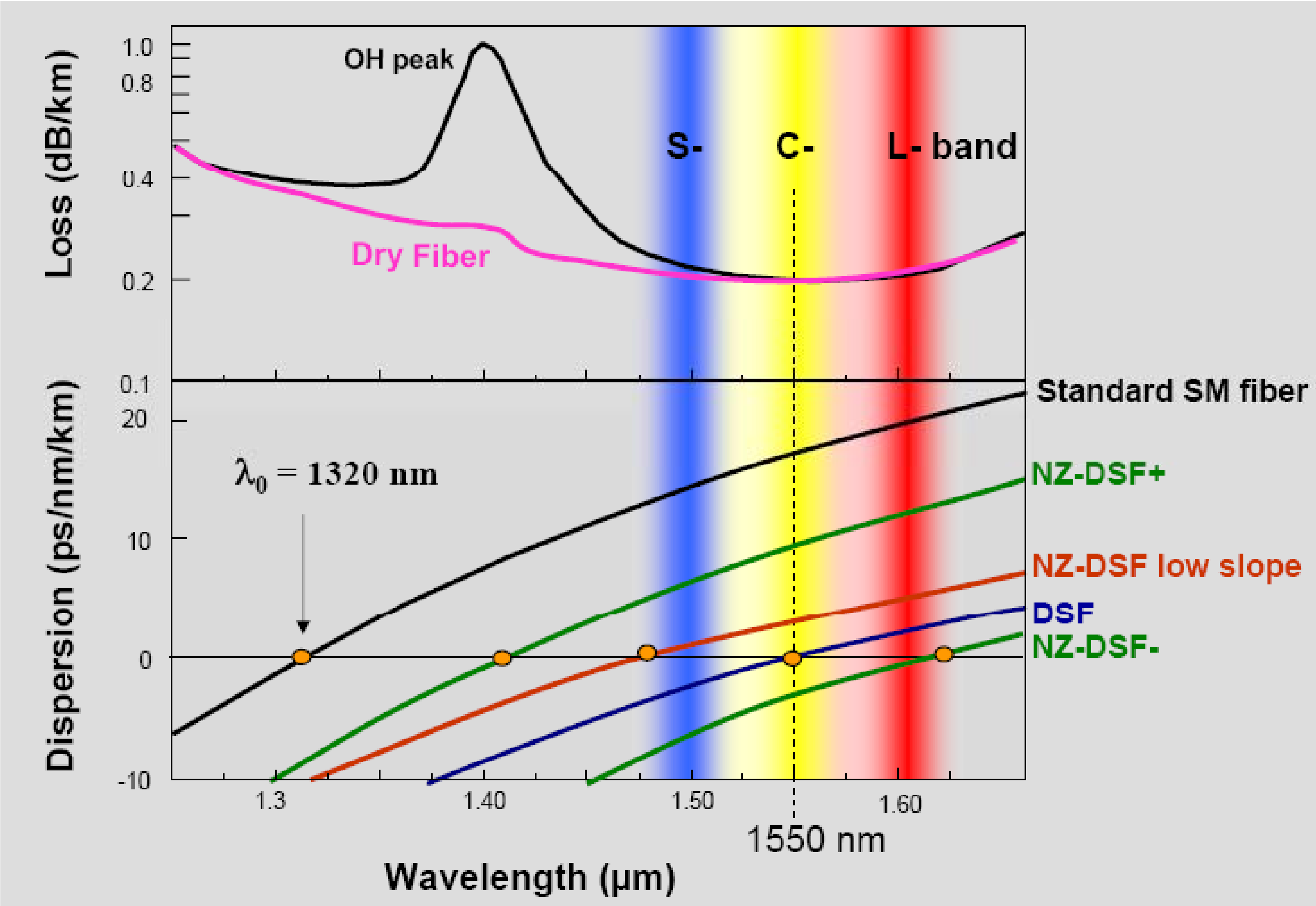
$$\omega_{NL}(t) = -\gamma L\left(\frac{dP_a(t)}{dt} + c_{ab}\frac{dP_b(t)}{dt}\right)$$

- It is important that there in no coincidence of pulses $\rightarrow$

$\rightarrow$ Different velocities $v_g \rightarrow$ Different β_1

$$\rightarrow \frac{d\beta_1}{d\omega} = \beta_2 \neq 0 \rightarrow D \neq 0$$

$\rightarrow D\uparrow\rightarrow$ XPM $\downarrow\rightarrow$ NZ-DSF fiber



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5-5- FWM Four-Wave Mixing

- Four-wave-mixing (FWM), or sometimes four-photon-mixing (FPM), describes a nonlinear optical effect at which four waves or photons interact with each other due to the third order nonlinearity of the material

-As a result, new waves with sum and difference frequencies are generated during the propagation in the waveguide

- It is comparable to intermodulation in electrical communication systems

- For WDM systems in dispersion-shifted fibers, FWM is the most important nonlinear effect

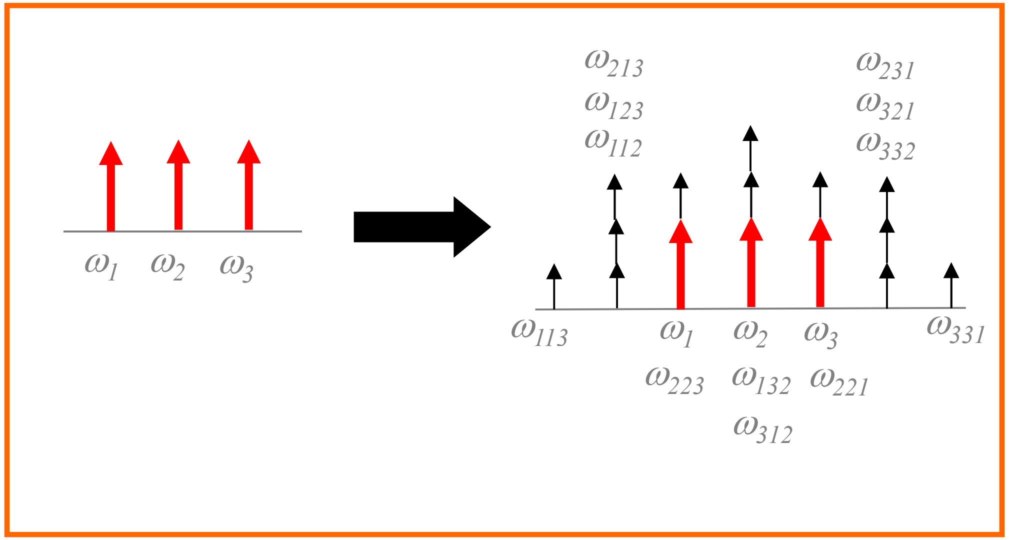
- FWM leads to a reduction of the SNR and, hence, an increase of the BER

-Inter-SPM → Important in multichannel systems (N ≥ 3)

-With phase matching → 3 tones interaction → 4th tone

$$\omega_4 = \omega_1 + \omega_2 - \omega_3$$

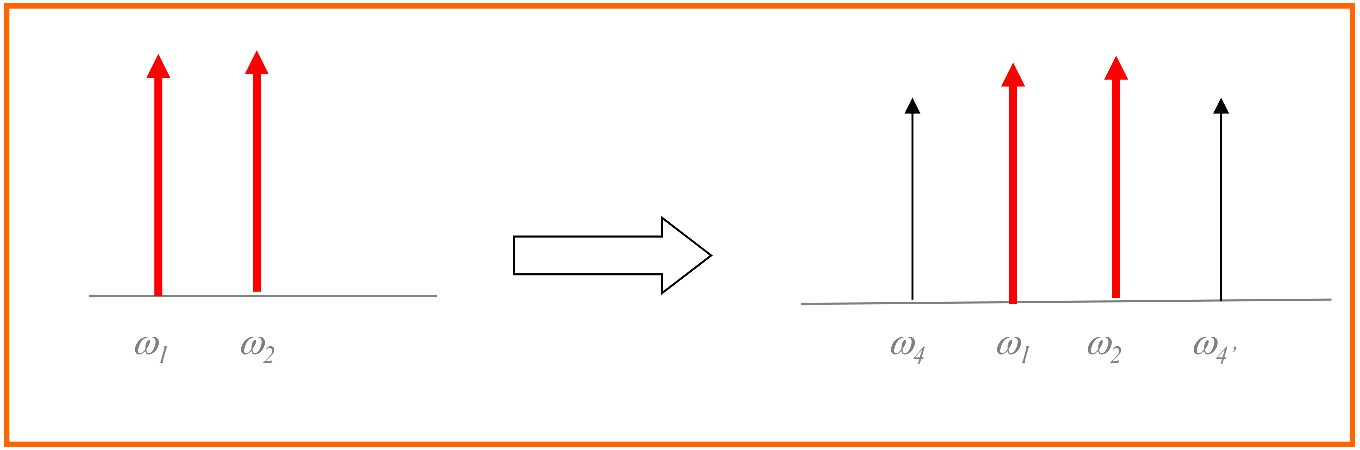
-They propagate in the same direction

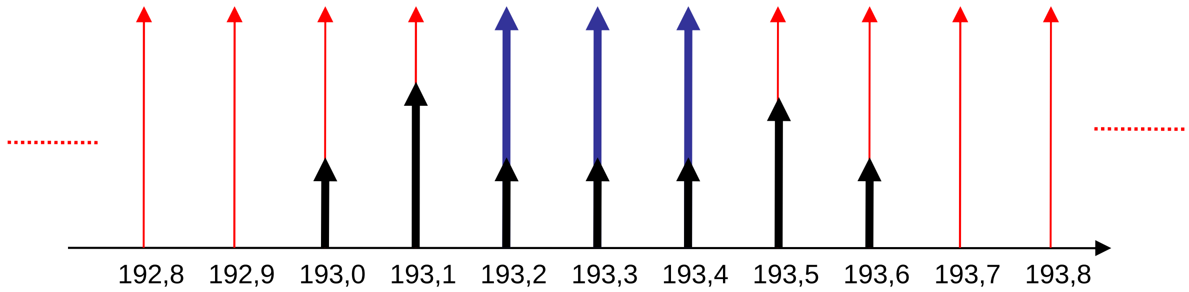


With two frequencies ω_1 y ω_2 $\quad (\Delta\omega = \omega_2 - \omega_1)$

$$\omega_4 = 2\omega_1 - \omega_2 = \omega_1 - \Delta\omega$$

$$\omega_4' = 2\omega_2 - \omega_1 = \omega_2 + \Delta\omega$$





Optical frequencies (THz)

$M = N^2 (N-1)/2$

N = nº of channels

M = nº of new frequencies

FWM Efficiency

$$\eta \propto \left[\frac{n_2 P}{A_{eff} D (\Delta\lambda)^2} \right]^2$$

5-6- Envelope nonlinear equation

Nonlinear propagation constant $\beta(\omega)$

$$\beta(\omega) = k_0 \left(\underbrace{n_0}_{Linear} + \underbrace{n_2 \frac{P(t)}{A_{eff}}}_{Nonlinear} \right) = k_0 n_0 + \underbrace{\frac{k_0 n_2}{A_{eff}}}_{\gamma} \left| A \right|^2$$

γ →Nonlinear coefficient (Kerr effect)

$$\boxed{\frac{\partial A}{\partial z} + j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} = \underbrace{j\gamma\left|A\right|^2 A}_{\text{SPM}}}$$

-This equation is referred as the Nonlinear Schrödinger (NLS) equation because it resembles the Schrödinger equation with a nonlinear potential term

-The NLS equation is a fundamental equation of the nonlinear science

- In the general case of $\beta_3 \neq 0$ and $\alpha_f \neq 0$, this equation is called the generalized (or extended) NLS equation

$$\frac{\partial A}{\partial z} + j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} - \frac{\beta_3}{6}\frac{\partial^3 A}{\partial t^3} + \frac{\alpha_f}{2}A = \underbrace{j\gamma\left|A\right|^2 A}_{\text{SPM}}$$

In the more general case of three simultaneous signals, as in WDM:

$$\frac{\partial A_1}{\partial z} + \underbrace{j\frac{\beta_2}{2}\frac{\partial^2 A_1}{\partial t^2}}_{\substack{1^{st}\ \text{order}\\ \text{GVD}}} - \underbrace{\frac{\beta_3}{6}\frac{\partial^3 A_1}{\partial t^3}}_{\substack{2^{nd}\ \text{order}\\ \text{GVD}}} + \underbrace{\frac{\alpha_f}{2}A_1}_{\text{Attenuation}} = \underbrace{j\gamma\left|A_1\right|^2 A_1}_{\text{SPM}} + \underbrace{j2\gamma\left(\left|A_2\right|^2 + \left|A_3\right|^2\right)A_1}_{\text{XPM}} + \underbrace{j\gamma\left|A_2\right|^2 A_3^*}_{\text{FWM}}$$