DISTRIBUTED COGNITIVE RADIO SYSTEMS WITH TEMPERATURE-INTERFERENCE CONSTRAINTS AND OVERLAY SCHEME

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ABSTRACT

Cognitive radio represents a promising paradigm to further increase transmission rates in wireless networks, as well as to facilitate the deployment of self-organized networks such as femtocells. Within this framework, secondary users (SU) may exploit the channel under the premise to maintain the quality of service (QoS) on primary users (PU) above a certain level. To achieve this goal, we present a noncooperative game where SU maximize their transmission rates, and may act as well as relays of the PU in order to hold their perceived QoS above the given threshold. In the paper, we analyze the properties of the game within the theory of variational inequalities, and provide an algorithm that converges to one Nash Equilibrium of the game. Finally, we present some simulations and compare the algorithm with another method that does not consider SU acting as relays.

Index Terms— Cognitive radio, variational inequalities, game theory, self-organized networks, small cells

1. INTRODUCTION

Encouraged by an increasing demand, wireless data services have experienced a tremendous growth in the past years, and more is expected to come due to a greater proliferation of user terminals, transmission requirements, and ubiquitous services. For these reasons, self-organized networks that adapt and configure themselves in a changing environment constitute a desirable deployment strategy for operators to increase transmission rates and coverage, while reducing their installation and maintenance costs.

The cognitive radio (CR) paradigm intends to combine the present deployed networks with the future self-organized networks, by establishing a hierarchy on the data services offered to licensed and unlicensed users. While the macrocell networks are set to provide sufficient quality of service (QoS) to the licensed or primary users (PU), the unlicensed or secondary users (SU) may exploit the underused channels without disturbing the PU transmissions. This requires some knowledge of the transmitting channels, specially some measure or estimation of the interference caused to the PU (or QoS perceived), in order to maintain the service unobtrusively. Several techniques for the SU have been proposed to accomplish this, namely interweaving, underlay and overlay transmissions [1, 2, 3], which we introduce next.

The interweaving technique consists on the SU sensing the spectrum holes in the different subchannels, and avoid causing any interference to the PU. The main difficulty of this scheme resides in sensing these gaps accurately, and in predicting the next PU transmission slots. Therefore, it becomes difficult to implement in highly dynamic environments, where the SU communications pair would require to be very precise and agile in switching channels.

On the other hand, in the underlay scheme the SU transmit in the same bands as the PU while satisfying some QoS constraints. This scheme is also transparent to the PU, which just regards the interference as additive noise without affecting communication. However, it has the difficulty for SU to work in low SNR regimes and very short range communications, in order to keep the interference temperature on PU under given thresholds.

Finally, the overlay scheme presents itself as a generalization of the underlay scheme, where SU act as relays for the PU communication (increasing their SINR), and are therefore allowed to further augment the interference level caused to these PU. The technique comes however at the cost of greater integration among SU and PU, at least in order for the PU to decode the relayed message. See [1] for techniques to accomplish this. Similar approaches were also proposed in [4, 5], where the PU leased their own spectrum in exchange for the helping relays.

Within the overlay paradigm, we present a monotone game played among SU that maximize the achievable information rate, while satisfying some QoS constraint on the PU, and where the SU forward information dynamically acting as relays. We study the existence of Nash Equilibrium (NE) solutions, and provide an algorithm that converges to one of these equilibriums. Our contributions on this paper are mainly the generalization of previously studied underlay techniques to the overlay scheme, and showing the capacity gains when comparing both techniques.

Regarding previous work, we build upon the ideas introduced in [2] which proposed a potential game among SU for both underlay and overlay schemes. In their approach, authors introduce a performance function for each player that minimizes interference, but without explicitly regarding the maximization of a capacity formula. Additionally, the interference levels reached at the PU are not constrained below a given level, but rather have to be found through simulation after specifying some weighting parameters. To address this matter, in our approach we explicitly impose feasibility constraints, and maximize capacity on the available subcarriers.

We also base our work in the framework presented in [6] and previously developed through different publications applied to CR scenarios [7, 8, 9], which expand the analysis tools to study monotone games through variational inequalities (VI). In our presentation, we adapt our problem formulation to this theory, and analyze its convergence. Furthermore, we compare our simulation results with the algorithm presented in [6] as the SISO case (underlay paradigm), showing the achievable performance gain in high-interference scenarios, when adopting an overlay transmission scheme.

Finally, other recent related work is the one proposed in [10] where authors present a reinforcement learning algorithm to solve an altruistic game (team game) and a competitive game. Here, the learning process can rely on a formulation with full channel knowledge (all players know all strategies and channel state), or with limited knowledge (players only have access to their utility function). In our algorithm (as well as in [6, 7, 8]), only local information is required, as well as some collaboration from PU indicating how saturated the interference constraints are.

In Section 2 we introduce the system model, the game formulation for SU, and formulate the equivalence to VI. Then, within the VI theory we analyze the existence of solutions and properties. In Section 3 we reformulate the objective functions, and describe an algorithm to find a solution of the game. Finally, on Sections 4 and 5 we present some simulations and the conclusions. During the exposition of the problem we will assume some knowledge of VI and game theory. As definitions of NE, VI solutions, monotonicity of VI and games, P-matrix properties, and uniformly P-functions, we have used the ones presented in [6].

2. SYSTEM MODEL AND PROBLEM FORMULATION

We present here a CR system with P primary users and Q secondary users (players), who transmit in N-parallel Gaussian interference channels. Each SU represents a transmitting pair who tries to maximize their achievable data rate, while PU have assigned transmission channels and QoS requirements. We denote here the channel (cross)transfer coefficients for SU as $H_{ij}^{SS}(k)$ indicating that this is the (squared) channel gain from the j SU transmitter to the i SU receiver on subcarrier k, and we indicate $H_{pj}^{PS}(k)$ for the transfer function from the j SU transmitter to the p PU receiver on subcarrier k. The SU transfer coefficient is therefore referred to as $H_{ii}^{SS}(k)$. The channel is AWGN and we express the noise variances with $\sigma_i^2(k)$. Note that each noise term may include any undecodable signal of PU on SU, and this term will not vary during the resolution of the game since PU do not participate in the power allocation algorithm.

The objective of the game is to find the corresponding power allocation scheme for each player, which engulfs both the power dedicated to the individual data, as well as the relayed transmission. We will refer to the former for user i as $\{p_i(k)\}_{k=1}^N$, and we will indicate the later as $\{p_i^p(k)\}_{k=1,p=1}^{N,P}$, where every SU may relay the data of one, or several PU. Additionally, we will use the vector notation of the previous magnitudes as $\mathbf{p}_i = (p_i(k), (p_i^p(k))_{p=1}^P)_{k=1}^N$, we will refer to the strategies from other users with $\mathbf{p}_{-i} = (\mathbf{p}_j)_{j=1, j\neq i}^Q$, and to the strategies of all users with $\mathbf{p} = (\mathbf{p}_i)_{i=1}^Q$. The achievable transmission rate is then given by the capacity formula,

$$r_i(\mathbf{p}_i, \mathbf{p}_{-i}) = \sum_{k=1}^{N} \log\left(1 + \frac{H_{ii}^{SS}(k)p_i(k)}{\sigma_i^2(k) + \sum_{j\neq i} H_{ij}^{SS}(k)p_j(k)}\right) \tag{1}$$

with total power constraint $\sum_k (p_i(k) + \sum_p p_i^p(k)) \leq P_i^{\max}$, and limiting values $0 \leq p_i(k), p_i^p(k) \leq p_i^{\max}(k)$ for all i and p. In equivalent form the strategy set is

$$\mathcal{P}_i \triangleq \left\{\mathbf{p}_i \in \mathbb{R}_+^N | \mathbf{1}^T\mathbf{p}_i \leq P_i^{\max},\ \mathbf{p}_i \leq \mathbf{p}_i^{\max}\right\}. \tag{2}$$

where the SU transmission and relayed powers are implicitly included in $\mathbf{p}_i$.

In order to satisfy the QoS on every PU, we define a minimum capacity threshold $b_p(k)$ on every subcarrier where a transmission takes place, given by

$$\log\left(1 + \frac{G_p(k) + \sum_{j=1}^Q H_{pj}^{PS}(k)p_j^p(k)}{\sigma_p^2(k) + \sum_{j=1}^Q H_{pj}^{PS}(k)p_j(k)}\right) \geq b_p(k) \tag{3}$$

where $G_p(k)$ represents the PU joint channel gain and power. We can find an equivalent expression by manipulating its terms, getting

$$g_{pk}(\mathbf{p}) = \sum_{j=1}^{Q} H_{pj}^{PS}\left(a_p(k)p_j(k) - p_j^p(k)\right) - I_p(k) \leq 0 \tag{4}$$

where $a_p(k) = e^{b_p(k)} - 1$, and $I_p(k) = G_p(k) - a_p(k)\sigma_p^2$. Equation (4) is linear, limits the total interference with a maximum temperature value $I_p(k)$, and is coupled among all users. Adding these constraints to the feasible region yields:

$$\hat{\mathcal{P}} \triangleq \{\mathcal{P}_i\}_{i=1}^Q \cap \{\mathbf{p} \mid \mathbf{g}_p(\mathbf{p}) \leq 0, \quad \forall p = 1, \ldots, P\} \tag{5}$$

with $\mathbf{g}_p(\mathbf{p}) = (g_{pk}(\mathbf{p}))_{k=1}^N$.

Now we can present the Generalized Nash Equilibrium Problem (GNEP) for all SU as $\mathcal{G}_{\text{gnep}} = \left\langle \hat{\mathcal{P}}, (r_i)_{i=1}^Q \right\rangle$,

$$\begin{array}{ll} \max\limits_{\mathbf{p}_i} & r_i(\mathbf{p}_i, \mathbf{p}_{-i}) \\ \text{s.t.} & \mathbf{p}_i \in \hat{\mathcal{P}}(\mathbf{p}_{-i}) \end{array} \qquad \forall i = 1, \ldots, Q. \tag{6}$$

The variational inequality (VI) associated to $\mathcal{G}_{gnep}$ will allow us to analyze the properties of the game. Defining $\mathbf{F}_i = \nabla_{\mathbf{p}_i} r_i(\mathbf{p}_i, \mathbf{p}_{-i})$, and $\mathbf{F} = (\mathbf{F}_i)_{i=1}^Q$ we can state the following,

Lemma 1. *(from [7]) Let the game $\mathcal{G}_{gnep} = \left\langle \hat{\mathcal{P}}, (r_i)_{i=1}^Q \right\rangle$ and let the variational inequality be defined as VI($\hat{\mathcal{P}}, \mathbf{F}$). Then, if $(\mathbf{p}^*, \lambda^*)$ is a solution of the VI, so that*

$$\begin{array}{l} \mathbf{0} \in \mathbf{F}_i(\mathbf{p}^*) + \sum_{p=1}^P \lambda_p^{*T} \nabla_{\mathbf{p}_i} \mathbf{g}_p(\mathbf{p}^*), \quad \mathbf{p}_i^* \in \mathcal{P}_i,\ \forall i \\ \mathbf{0} \leq \lambda_p^* \perp \mathbf{g}_p(\mathbf{p}^*) \leq \mathbf{0}, \quad \forall p \end{array} \tag{7}$$

it is also a solution of $\mathcal{G}_{gnep}$. We will refer to these points as variational solutions of the game.

In order to establish the existence of solutions for the VI, and consequently to the $\mathcal{G}_{\text{gnep}}$, we need to proof monotonicity of the mapping $\mathbf{F}$ on $\hat{\mathcal{P}}$ from the defined VI. We denote the Jacobian matrix as $\mathbf{JF} = (J_{\mathbf{p}_j}\mathbf{F}_i(\mathbf{p}))_{i,j=1}^{Q}$, with $J_{\mathbf{p}_j}\mathbf{F}_i(\mathbf{p})$ indicating the partial Jacobian matrix with respect to the SU's vector $\mathbf{p}_j$. Then, using the developed framework to analyze the structure of VIs from [7], we introduce the following matrix

$$[\mathbf{JF}_{\text{low}}]_{ij} \triangleq \begin{cases} \inf_{\mathbf{p}\in\hat{\mathcal{P}}}[\mathbf{JF}]_{ii} & \text{if } i=j \\ -\sup_{\mathbf{p}\in\hat{\mathcal{P}}} |[\mathbf{JF}]_{ij}| & \text{otherwise} \end{cases} \quad (8)$$

which applied to our problem has the following structure,

$$[\mathbf{JF}_{\text{low}}(k)]_{ij} = \begin{cases} 1 & \text{if } i=j \text{ and } \frac{\partial \mathbf{F}_i}{\partial p_i(k)} \\ -\frac{|H_{ij}^{SS}(k)|^2}{|H_{jj}^{SS}(k)|^2}\cdot \text{innr}_{ij}(k) & \text{if } i\neq j \text{ and } \frac{\partial \mathbf{F}_i}{\partial p_j(k)} \\ 0 & \text{if } \frac{\partial \mathbf{F}_i}{\partial p_i^p(k)} \text{ or } \frac{\partial \mathbf{F}_i}{\partial p_j^p(k)} \end{cases} \quad (9)$$

with $\text{innr}_{ij}(k) \triangleq \frac{\sigma_j^2(k)+\sum_r H_{jr}^{SS} p_r^{\max}(k)}{\sigma_i^2(k)}$. We can now enunciate the following properties:

Proposition 1. *Given the game $\mathcal{G}_{gnep}$ and its associated VI($\hat{\mathcal{P}},\mathbf{F}$) the following holds*

1. *Suppose matrix $\mathbf{JF}_{low}(k)$ as given in (9) is positive semidefinite, then the VI is monotone, and $\mathcal{G}_{gnep}$ is a monotone GNEP.*
2. *Because $\hat{\mathcal{P}}$ is closed and compact, the VI and $\mathcal{G}_{gnep}$ have a nonempty and compact solution set (also convex if condition 1 is satisfied).*

Proof. We can apply the theory of NEPs to our GNEP as stated in Lemmas 4.2, 4.3 in [7] because Assumption 4.3 [7] is satisfied. Indeed, we have a nonempty, closed and convex subset $\hat{\mathbf{P}}_i$ for every i, the objective functions are twice continuously differentiable and concave, and $\mathbf{g}_p(\mathbf{p})$ is jointly convex and continuously differentiable. Now, we can affirm condition 1 is guaranteed by Proposition 5 (a) in [6], and condition 2 by Theorem 8 (a,b) [6]. □

For the uniqueness of solution of the VI, matrix $\mathbf{JF}$ would be required to be a P-matrix. However, because the Jacobian is zero on variables $p_i^p(k)$, $\mathbf{JF}$ cannot satisfy the conditions, and the VI may have infinite NE. Nonetheless, we add a proximal regularizer to analyze the P-properties of the mapping $\mathbf{F}$, and this will also be useful to guarantee convergence of Algorithm 1. We consider then the modified VI($\hat{\mathcal{P}},\mathbf{F}+\tau(\mathbf{I}-\mathbf{y})$), where $\mathbf{I}$ is the indicator function, $\mathbf{y} = (\mathbf{y}_i)_{i=1}^{Q}$, and $\mathbf{y}_i = (y_i(k),(y_i^p(k))_{p=1}^{P})_{k=1}^{N}$ is a fixed point so that $\mathbf{y}\in\hat{\mathcal{P}}$. Point $\mathbf{y}$ limits the VI to a unique solution, and by updating its value to a closer NE of the original game as in Algorithm 1, it will guarantee reaching such a solution. Note that $\mathbf{y}$ is instrumental into attaining a solution, but it doesn't affect the existence of multiple NE on the original game (6).

Now, based on [7] we define matrices, $\mathbf{\Upsilon}_{\mathbf{F}}^{\text{S}}$ and $\mathbf{\Upsilon}_{\mathbf{F}}^{\text{P}}$

$$[\mathbf{\Upsilon}_{\mathbf{F}}^{\text{S}}]_{ij} \triangleq \begin{cases} \inf_{\mathbf{p}\in\hat{\mathcal{P}}} \lambda_{\text{least}}(J_{p_i(k)}\mathbf{F}_i(\mathbf{p})), & \text{if } i=j \\ -\sup_{\mathbf{p}\in\hat{\mathcal{P}}} \|(J_{p_i(k)}\mathbf{F}_j(\mathbf{p}))\|, & \text{if } i\neq j \end{cases} \quad (10)$$

$$[\mathbf{\Upsilon}_{\mathbf{F}}^{\text{P}}]_{ij} \triangleq \begin{cases} \inf_{\mathbf{p}\in\hat{\mathcal{P}}} \lambda_{\text{least}}(J_{p_i^p(k)}\mathbf{F}_i(\mathbf{p})), & \text{if } i=j \\ -\sup_{\mathbf{p}\in\hat{\mathcal{P}}} \|(J_{p_i^p(k)}\mathbf{F}_j(\mathbf{p}))\|, & \text{if } i\neq j \end{cases} \quad (11)$$

where their definition differs in that the partial derivatives of the Jacobians are relative only to $p_i(k)$, or to $p_i^p(k)$, respectively. Note that the notation λ_{least} means smallest eigenvalue. The purpose behind these definitions is to determine sufficient conditions that would guarantee that the VI($\hat{\mathcal{P}},\mathbf{F}+\tau(\mathbf{I}-\mathbf{y})$) is a P-function. Due to the Cartesian structure of the Jacobian matrix $\mathbf{JF}$, we can affirm that if both matrices $\mathbf{\Upsilon}_{\mathbf{F}}^{\text{S}}$ and $\mathbf{\Upsilon}_{\mathbf{F}}^{\text{P}}$ are P-matrices, then so is the VI.

As analyzed in [6], matrix $\mathbf{\Upsilon}_{\mathbf{F}}^{\text{S}}$ has the form

$$[\mathbf{\Upsilon}_{\mathbf{F}}^{\text{S}}]_{ij} = \begin{cases} 1 & \text{if } i=j \\ -\max_k \left\{ -\frac{H_{ij}^{SS}(k)}{H_{jj}^{SS}(k)}\cdot \text{innr}_{ij}(k) \right\} & \text{if } i\neq j \end{cases} \quad (12)$$

and matrix $\mathbf{\Upsilon}_{\mathbf{F}}^{\text{P}} = 0$ since the Jacobian is zero on those terms. Adding a regularizer as intended, $\mathbf{\Upsilon}_{\mathbf{F},\tau_1}^{\text{S}} = \mathbf{\Upsilon}_{\mathbf{F}}^{\text{S}} + \tau_1\mathbf{I}$ is a P-matrix for

$$\tau_1 > \left[\max_i \left\{ \sum_{j\neq i} \max_k \left\{ \frac{H_{ij}^{SS}(k)}{H_{jj}^{SS}(k)}\cdot \text{innr}_{ij}(k) \right\} \right\} - 1 \right]^+ \quad (13)$$

and matrix $\mathbf{\Upsilon}_{\mathbf{F},\tau_2}^{\text{P}} = \mathbf{\Upsilon}_{\mathbf{F}}^{\text{P}} + \tau_2\mathbf{I}$ is a P-matrix for $\tau_2 > 0$, where we have split $\tau = (\tau_i)_{i=1}^{2}$ to analyze both matrices. We have used the operator $[z]^+ = \max\{z,0\}$.

Summing up, τ_1 as specified on equation (13) and $\tau_2 > 0$ give sufficient conditions for VI($\hat{\mathcal{P}},\mathbf{F}+\tau(\mathbf{I}-\mathbf{y})$) to be a uniformly P-function, so that a unique solution exists on the modified VI for given $\mathbf{y}$.

3. DISTRIBUTED ALGORITHM

We now reformulate the game $\mathcal{G}_{gnep}$ in a more convenient form with the feasible region having a Cartesian structure, so that we can use the decomposition algorithms for Nash Equilibrium Problems (NEPs) as in [6, 7]. We introduce the game $\mathcal{G}_{\lambda}(\mathbf{y})$

$$\begin{array}{ll} \max\limits_{\mathbf{p}_i} & r_i(\mathbf{p}_i,\mathbf{p}_{-i}) - \sum_{p=1}^{P} \lambda_p^T \mathbf{g}_p(\mathbf{p}_i,\mathbf{p}_{-i}) \\ & -\frac{\tau}{2} \parallel \mathbf{p}_i - \mathbf{y}_i \parallel^2 \\ \text{s.t.} & \mathbf{p}_i \in \mathcal{P}_i \end{array} \quad \forall i \quad (14)$$

and furthermore require

$$0 \leq \lambda_p \perp \mathbf{g}_p(\mathbf{p}) \leq \mathbf{0}, \qquad \forall p = 1,\ldots,N \quad (15)$$

where the term $\frac{\tau}{2} \parallel \mathbf{p}_i - \mathbf{y}_i \parallel^2$ has been added to obtain a strongly convex optimization problem, where $\mathbf{y}_i$ is another point in the feasibility region, and τ has to be big enough to guarantee P-uniformity, as given in (13).

This formulation allows for a distributed computation for every player for known coefficients λ_p, since all variables and feasible sets are local. The parameters λ_p can be interpreted as the price paid by the players for using the maximum allowed interference, which due to equation (15) is nonzero only when the resources become scarce.

To solve problem (14), the individual KKT conditions have to be satisfied, therefore

$$\frac{H_{ii}^{SS}(k)}{\sigma_i^2(k)+\sum_{j=1}^{Q} H_{ij}^{SS}(k)p_j(k)} - \tau_1 \left(p_i(k) - y_i(k)\right) - \sum_{p=1}^{P} \lambda_p(k) H_{pi}^{PS}(k) - \mu_i = 0 \tag{16}$$

$$-\tau_2 \left(p_i^p(k) - y_i^p\right) - \sum_p \lambda_p(k) H_{pi}^{PS}(k) - \mu_i = 0 \tag{17}$$

$$0 \leq \mu_i \perp \sum_k (p_i(k) + \sum_p p_i^p(k)) - P_i^{\max} \leq 0 \tag{18}$$

for every user $i = 1, \ldots, Q$. The parameter μ_i is some water-level that has to be determined to satisfy condition (18).

The real positive root from (16) is given by

$$p_i(k) = \frac{1}{2}\left(y_i(k) - \frac{1}{H_k}\right) - \frac{1}{2\tau}\left[\tilde{\mu}_i(k) - \sqrt{\left[\tilde{\mu}_i(k) - \tau_1\left(y_i(k) + \frac{1}{H_k}\right)\right]^2}\right]_0^{p_k^{\max}} \tag{19}$$

which is the same solution as equation (71) in the underlay game from [6], and where we have simplified the notation to $H_k = \frac{H_{ii}^{SS}(k)}{\sigma_i^2(k)+\sum_{j=1}^{Q} H_{ij}^{SS}(k)p_j(k)}$ and $\tilde{\mu}_i(k) = \mu_i + \sum_{p=1}^{P} \lambda_p(k) H_{pi}^{PS}(k)$ for our problem. The used operator is defined as $[z]_a^b = \min\{\max\{z, a\}, b\}$.

Likewise, relayed powers from (17) are given by

$$p_i^p(k) = \left[y_i^p(k) + \frac{1}{\tau_2}\left(\sum_{p=1}^{P} \lambda_p(k) H_{pi}^{PS}(k) - \mu_i\right)\right]_0^{p_k^{\max}} \tag{20}$$

Based on the Projection Algorithm with variable steps from [7], we can formulate Algorithm 1 that determines the best response in variables $\{p_i(k), \{p_i^p(k)\}_p\}_k$ for all SU. Note that on step 2 values $\mathbf{y}^{(n)}$ and $\lambda_p^{(n)}$ are held fixed for the resolution of inner game $\mathcal{G}_{\lambda^{(n)}}(\mathbf{y}^{(n)})$ (as given in (14)), and are later updated for subsequent iterations. Algorithm 2 is an inner loop of the main algorithm that solves $\mathcal{G}_{\lambda^{(n)}}(\mathbf{y}^{(n)})$, whose structure is based on a bisection algorithm presented in [6], and considers the relayed powers towards PU. This algorithm has the novelty of detecting saturated PU constraints, and balancing the available powers both on maximizing user capacity, and helping to lower the cost of interfering PU.

Algorithm 1 Projection Algorithm

1. Set $n \leftarrow 0$
Initialize variables $y_i^{(n)}(k)$, $y_i^{p,(n)}(k)$, $\lambda_p^{(n)}(k)$, $\forall i, p, k$.
2. If stopping criteria is satisfied, stop
3. Determine best response of variables $p_i(k), p_i^p(k)$ $\forall k, i$ using Alg. 2, with fixed values $\lambda_p^{(n)}$, $y_i^{(n)}(k)$, $y_i^{p,(n)}(k)$.
4. Update interference prices

$$\lambda_p^{(n+1)} = \left[\lambda_p^{(n)} + \alpha_n \mathbf{g}_p(\mathbf{p})\right]^+, \qquad \forall p = 1, \ldots, P$$

5. Set $n \leftarrow n + 1$;
Update $y_i^{(n)}(k) \leftarrow p_i(k)$, $y_i^{p,(n)}(k) \leftarrow p_i^p(k)$
Go to step 2.

Algorithm 2 Power allocation for Overlay Wireless Network

1. Choose accuracy level ϵ.
2. Set $\underline{\mu}_i = 0$, and

$$\overline{\mu}_i = \left[\max_k \left\{(H_k + \tau_1 y_i(k) - \sum_p \lambda_p(k) H_{pi}^{PS}(k))\right\}\right]^+$$

3. Set $\mu_i = (\underline{\mu}_i + \overline{\mu}_i)/2$
4. Using (19) and (20) determine $p_i(k)$ and $p_i^p(k)$ $\forall k, p$
5. If $\sum_k (p_i(k) + \sum_p p_i^p(k)) \leq P_i^{\max}$, then $\overline{\mu}_i = \mu_i$ otherwise, $\underline{\mu}_i = \mu_i$.
6. If $\overline{\mu}_i - \underline{\mu}_i > \epsilon$, go to step 3, otherwise stop.

4. SIMULATIONS

For the simulations we have created a scenario of $Q = 10$ SU, $P = 2$ PU and $N = 64$ subcarriers. The transfer function for an OFDM channel is determined by a FIR filter of length L and exponential power profile, with randomly chosen complex Gaussian coefficients which are then normalized, and where the subcarrier coefficients are determined by the squared samples of the FFT of order N. As simulation parameters, we have chosen a level of total power transmission of 10 mW, interference peak level $I_p(k) = 0.8$ mW, and the joint channel and power gain of PU to $G_p(k) = 2$ mW. All channel noises have been set to 1 mW. For the SU to SU channel gain the SNR $= \mathrm{E}\left\{P_i^{\max} H_{ii}^{SS}(k)/\sigma_i^2(k)\right\} = 5$ dB, and for cross-channel gains among different SU INR $= \mathrm{E}\left\{P_i^{\max} H_{ji}^{SS}(k)/\sigma_j^2(k)\right\} = 0$ dB. Finally step size $\alpha_n = 0.1$, τ_1 has been chosen to satisfy equation (13), and $\tau_2 = 1$. In the following figures we have simulated our algorithm which we refer to as “overlay”, and the SISO algorithm from [6], which we refer to as “underlay”.

In Figure 1 we have represented the convergence speed given the previously mentioned simulation parameters, and additionally considering a channel gain from SU to PU of 0 dB for all the cases. We observe that the total sum rate oscillates until it stabilizes, due to temporal violation of the given constraints on the interference caused to PU. Note, that both schemes show this behavior, and have similar convergence speed for the simulated channels.

In Figure 2 we represent the total sum-rate of SU as a function of the increasing channel gain caused from SU to PU. We observe that on lower levels of interference the overlay and underlay algorithms converge to the same sum-rate, but as the interference channel gain increases, the underlay algorithm saturates on a lower sum-rate level so that it does not violate the given constraints. On the other hand, with the overlay scheme, the unused power can be employed on relaying the PU information, and can therefore increase the transmitted information rate.

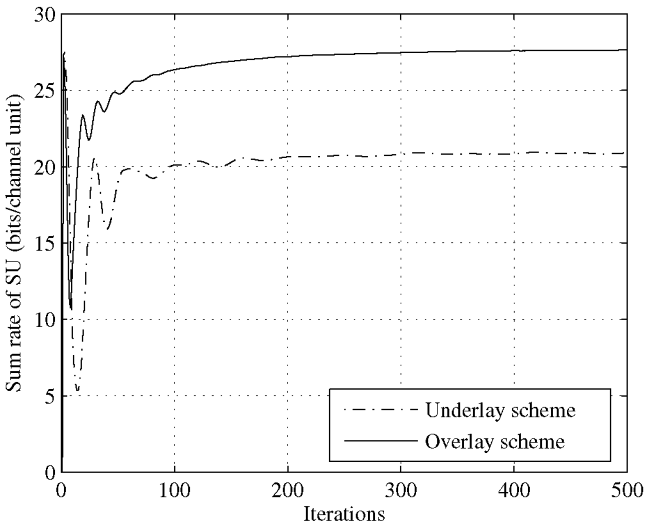

Fig. 1. Example of sum-rate Vs. number of iterations.

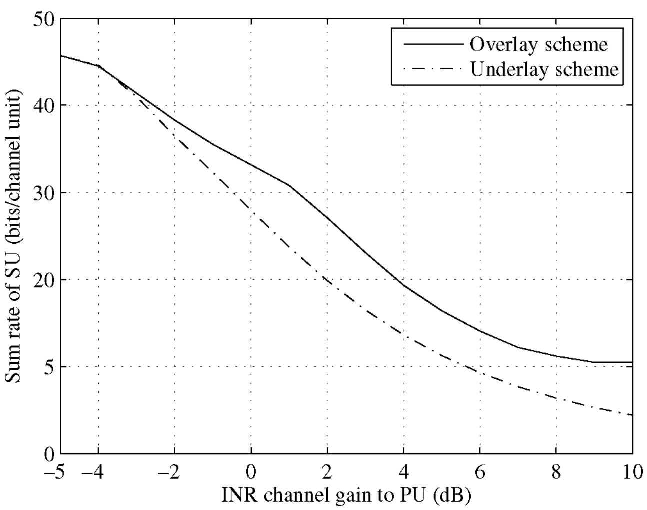

Fig. 2. Channel gain from SU to PU (in dB) Vs. total sum-rate

5. CONCLUSIONS

In this paper we have presented a monotone GNEP which maximizes the SU transmission rates, while holding the capacity level of PU above a given value. This was accomplished using an overlay scheme, where SU may retransmit the PU information as a trade-off to increase the interference level caused. We have analyzed the game through the theory of VI, and provided a convergent algorithm that allocates the transmission powers of SU data, as well as the power used to relay PU information. With our algorithm, we can analyze the achievable gain for SU when cooperating with PU at the expense of more integration.

As future work, NE selection could be implemented and analyzed in the algorithm. Additionally, more realistic information could be incorporated to the model, such as SINR requirements on SU to be able to decode and retransmit the PU data, as well as a more advanced structure of relaying network among SU.

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