Dominance Measuring Methods within MAVT/MAUT with Imprecise Information concerning Decision-Makers' Preferences

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***Abstract*—Dominance measuring methods are an approach for dealing with complex decision-making problems with imprecise information within multi-attribute value/utility theory. These methods are based on the computation of pairwise dominance values and exploit the information in the dominance matrix in different ways to derive measures of dominance intensity and rank the alternatives under consideration. In this paper we review dominance measuring methods proposed in the literature for dealing with imprecise information (intervals, ordinal information or fuzzy numbers) about decision-makers' preferences and their performance in comparison with other existing approaches, like SMAA and SMAA-II or Sarabando and Dias' method.**

I. Introduction

The additive model is widely used within multi-attribute value/utility theory (MAVT/MAUT) to rank alternatives in complex decision-making problems. It is considered a valid approach in many practical situations for the reasons described in [16] and [22].

If we consider a decision-making problem with m alternatives $\{A_1, ..., A_m\}$ and n attributes $\{X_1, ..., X_n\}$, then the functional form of the additive model is

$$v(A_i) = \sum_{j=1}^{n} w_j v_j(x_{ij}),$$

where x_{ij} is the performance over attribute X_j for alternative A_i, and v_j and w_j are the value function and the weight for attribute X_j, respectively. Note that $\sum_{j=1}^{n} w_j = 1$ and $w_j \geq 0$.

However, the information available in most real complex decision-making problems is not precise. Inputs are often described within prescribed bounds or as just satisfying certain relations. Different authors refer to this situation as *decision-making with imprecise information, incomplete information or partial information* ([17], [18]).

Several reasons are given in the literature that justify why a decision-maker (DM) may wish to provide imprecise information ([20], [25]). For example, a DM might prefer not to reveal his/her preferences in public or could feel more comfortable providing a scale to represent attribute importance, or might also have different more or less reliable sources of information.

Many papers on MAVT/MAUT have dealt with imprecise information. Sarabando and Dias [21] provided a brief overview of approaches proposed by different authors within the MAVT/MAUT framework to deal with imprecise information.

One option described in the literature for dealing with imprecision is based on the concepts of *pairwise* and *absolute dominance*.

The use of absolute dominance values is exemplified by the modification of four classical decision rules to encompass an imprecise decision context concerning weights and component values/utilities ([15], [19]): the *maximax* or optimist, the *maximin* or pessimist, the *minimax regret* and the *central value* rules.

A recent approach for dealing with imprecise information is to compute different measures of dominance to derive a ranking of alternatives ([2], [3], [4], [6], [11], [13]). They are known as *dominance measuring methods* (*DMMs*). *DMMs* are based on the computation of a *dominance matrix*, D, including pairwise dominance values, which are exploited in different ways to derive measures of dominance to rank the alternatives under consideration.

Dominance matrix, D, can be defined as follows:

$$D = \begin{pmatrix} - & D_{12} & \cdots & D_{1(m-1)} & D_{1m} \\ D_{21} & - & \cdots & D_{2(m-1)} & D_{2m} \\ D_{31} & D_{32} & \cdots & D_{3(m-1)} & D_{3m} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ D_{m1} & D_{m2} & \cdots & D_{m(m-1)} & - \end{pmatrix},$$

where

$$\begin{array}{ll} D_{kl} = & min\{\mathbf{w}\mathbf{v}_k - \mathbf{w}\mathbf{v}_l\} \\ s.t. & \\ & \mathbf{v}_k = (v_{k1}, \ldots, v_{kn}), \mathbf{v}_l = (v_{l1}, \ldots, v_{ln}) \in V_{kl} \\ & \mathbf{w} = (w_1, ..., w_n) \in W, \end{array} \tag{1}$$

where W and V_{kl} define the feasible region for weights and values associated with the alternatives A_k and A_l over each attribute, respectively. They represent imprecise information.

Note that given two alternatives A_k and A_l, alternative A_k dominates A_l if $D_{kl} \geq 0$, and there exists at least one $\mathbf{w}$, $\mathbf{v}_k$ and $\mathbf{v}_l$ such that the overall value of A_k is strictly greater than that of A_l. This concept of dominance is called *pairwise dominance*.

Imprecision on DM preferences could be represented by *intervals*. Then, the constraint set would be

$$w_j^L \leq w_j \leq w_j^U, j = 1, ..., n,$$
$$v_{kj}^L \leq v_{kj} \leq v_{kj}^U, j = 1, ..., n,$$
$$v_{lj}^L \leq v_{lj} \leq v_{li}^U, j = 1, ..., n.$$

If we consider *ordinal information about weights*, then the DM would provide an attribute importance ranking, arranged in descending order from the most to the least important attribute:

$$\mathbf{w} \in W = \{\mathbf{w} = (w_1, ..., w_n) | w_1 \geq w_2 \geq ... \geq w_n \geq 0\},$$

and $\sum_{j=1}^{n} w_i = 1$.

Ordinal information about the component values/utilities of the alternatives could also be considered ([20], [3]), i.e., the DM provides a ranking of the alternatives in each attribute. Moreover, rankings of the difference between the values of consecutive alternatives could be also taken into account for each attribute. For instance, the DM might consider A_3 to be the best of five alternatives for attribute X_j, followed by A_5, A_4, A_2 and A_1 ($v_j(x_{3j}) \geq v_j(x_{5j}) \geq v_j(x_{4j}) \geq v_j(x_{2j}) \geq v_j(x_{1j})$). Alternatively, the differences between consecutive alternatives might be ranked $\Delta_{j2} \geq \Delta_{j1} \geq \Delta_{j4} \geq \Delta_{j3}$, with $\Delta_{j2} = v_j(x_{5j}) - v_j(x_{4j})$, $\Delta_{j1} = v_j(x_{3j}) - v_j(x_{5j})$, $\Delta_{j4} = v_j(x_{2j}) - v_j(x_{1j})$ and $\Delta_{j3} = v_j(x_{4j}) - v_j(x_{2j})$, as illustrated in Fig. 1.

Trapezoidal or triangular fuzzy numbers could also be used to represent the imprecision or vagueness of DMs' preferences. Then, fuzzy optimization problems would have to be solved to derive the dominance matrix, whose elements would be fuzzy numbers as well.

In Section 2, we give an overview of the *DMMs* proposed in the literature to deal with the three ways to account for imprecision described above. Finally, some conclusions are provided in Section 3.

II. Dominance Measuring Methods: A Review

DMMs are based on the computation of a *dominance matrix*, D, including pairwise dominance values, which are exploited in different ways to derive measures of dominance to rank the alternatives under consideration.

The optimization problems to be solved to derive pairwise dominance values are different depending on how the imprecision concerning DM preferences is represented. Linear programming can be applied for intervals and ordinal information, whereas a fuzzy optimization problem is solved for fuzzy numbers. In this case, the elements in the dominance matrix are fuzzy numbers, too.

A. Intervals or ordinal information

The first *DMM* was proposed by Ahn and Park [4]. It was applicable when the imprecision concerning the DM preferences was represented by both intervals or ordinal information.

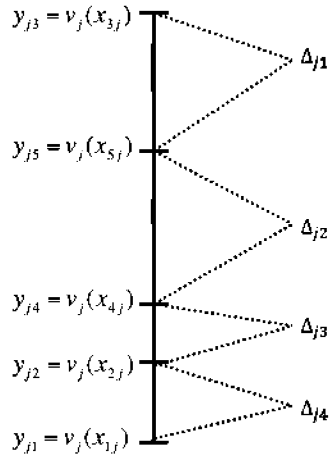

Fig. 1. Ranking of alternatives and differences between consecutive alternatives for the attribute X_j.

Ahn and Park compute a dominating measure $\phi_k^+ = \sum_{j=1, j\neq k}^{m} D_{kj}$ and a dominated measure $\phi_k^- = \sum_{j=1, j\neq k}^{m} D_{jk}$ for each alternative A_k, and then derive a *net dominance* as $\phi_k = \phi_k^+ - \phi_k^-$.

Ahn and Park proposed two ranking methods for these measures: ranking the alternatives according to either ϕ_k^+ or ϕ_k values (denoted as the *AP1* and *AP2* methods, respectively).

Two new *DMMs* were proposed in [9] and [11]. The first one, *DIM1*, was based on the same idea as implemented by Ahn and Park. It also computed dominating and dominated measures but they were combined into a *dominance intensity* rather than a net dominance index, which was used as a measure of the strength of preference.

DIM1 was implemented as follows:

1) Compute dominating indices for each alternative A_k:

$$DI_{k+}^{row} = \sum_{l=1, l\neq k, D_{kl}>0}^{m} D_{kl},$$

$$DI_{k-}^{row} = \sum_{l=1, l\neq k, D_{kl}<0}^{m} D_{kl}.$$

2) Compute the *dominating intensity* DI_k^{row} for each alternative A_k:

$$DI_k^{row} = \frac{DI_{k+}^{row}}{DI_{k+}^{row} - DI_{k-}^{row}}.$$

3) Compute dominated indices for each alternative A_k:

$$DI_{k+}^{col} = \sum_{l=1, l\neq k, D_{lk}>0}^{m} D_{lk},$$

$$DI_{k-}^{col} = \sum_{l=1, l\neq k, D_{lk}<0}^{m} D_{lk}.$$

4) Compute the *dominated intensity* DI_k^{col} for each alternative A_k:

$$DI_k^{col} = \frac{DI_{k+}^{col}}{DI_{k+}^{col} - DI_{k-}^{col}}.$$

5) Calculate a *global dominance intensity* (GDI) for each alternative A_k, i.e. $GDI_k = DI_k^{row} - DI_k^{col}, k = 1, ..., m$, and rank the alternatives according to them.

DIM1 improves *AP2* by reducing the duplicate information involved in the computations.

The second method, *DIM2*, derives a *global dominance intensity index* to rank alternatives on the basis that

$$D_{kl} \leq \mathbf{w}^T (\mathbf{v}_k - \mathbf{v}_l) \leq -D_{lk}, \forall \mathbf{w} \in W, \mathbf{v}_k, \mathbf{v}_l \in V_{kl}.$$

DIM2 was implemented as follows:

1) If $D_{kl} \geq 0$, then alternative A_k dominates A_l, and the dominance intensity of A_k over A_l (DI_{kl}) is 1, i.e., $DI_{kl} = 1$.
   Else ($D_{kl} < 0$):
   - If $D_{lk} \geq 0$, then alternative A_l dominates A_k, and $DI_{kl} = 0$.
   - Else ($D_{lk} < 0$), $DI_{kl} = \frac{-D_{lk}}{-D_{lk} - D_{kl}}$.
2) Calculate a *global dominance intensity* (GDI) for each alternative A_k, i.e. $GDI_k = \sum_{l=1,\ l \neq k}^{m} DI_{kl}$, and rank the alternatives according to them.

Simulation studies were carried out in [11] to compare *DIM1* and *DIM2* methods with other existing approaches (the above modification of four classical decision rules, *SMAA* [7], *SMAA-2* [8] and the *AP1* and *AP2* methods) when the imprecision concerning the inputs represented by value intervals.

We use two measures of efficacy, the hit ratio and the rank-order correlation ([4],[5]). The hit ratio is the proportion of all cases in which the method selects the same best alternative as in the TRUE ranking. Rank-order correlation represents how similar the overall alternative ranking structures are in the TRUE ranking and in the ranking derived from the method. It is calculated using Kendall's τ ([26]):

$$\tau = 1 - \frac{2 \times (\text{number of pairwise preference violations})}{\text{total number of pair preferences}}$$
$$= \frac{S}{m(m-1)/2},$$

where S is the difference between the number of concordant (ordered equally) and discordant (ordered differently) pairs and m is the total number of alternatives.

The results of simulation studies showed that *DIM2* performs better than the *AP1* method and the adaptation of classical decision rules. Although *SMAA-2* slightly outperforms *DIM2*, *DIM2* is applicable when incomplete information about weights is expressed not just as weight intervals but also as weights satisfying linear or non-linear constraints, weights represented by fuzzy numbers or weights fitting normal probability distributions.

The performance of *DIM1* and *DIM2* were also compared in [13] with other existing approaches (surrogate weighting methods, which select a weight vector from a set of admissible weights to represent the set [5], [24]; modified classical decision rules and the *AP1* and *AP2* methods) when ordinal information represents imprecision concerning weights. As regards average hit ratios, *DIM2* and *ROC* outperform the other methods and, according to the *paired-samples t-test*, there is no significant difference between the two. However, *ROC* can be only applied when ordinal relations regarding attribute weights are provided.

Note that *ROC* (*rank-order centroid weights*) is a surrogate weighting method in which $w_j = \frac{1/j}{\sum_{k=1}^{n} 1/k}, j = 1, ..., n$, n being the number of attributes.

A new *DMM*, *DIM3*, was proposed in [10] and [12]. It was based on *DIM2* and, specifically, on the fact that $\mathbf{w}^T (\mathbf{v}_k - \mathbf{v}_l) \in [D_{kl}, -D_{lk}], \forall \mathbf{w} \in W, \mathbf{v}_k, \mathbf{v}_l \in V_{kl}$. *DIM3* incorporates the distance from the interval $[D_{kl}, -D_{lk}]$ to 0 to derive a *dominance intensity measure* to rank the alternatives under consideration:

1) If $D_{kl} \geq 0$, then alternative A_k dominates A_l, and the dominance intensity of A_k over A_l is $DI_{kl} = d([D_{kl}, -D_{lk}], 0)$.
   Else ($D_{kl} < 0$):
   - If $D_{lk} \geq 0$, then alternative A_l dominates A_k, and $DI_{kl} = -d([D_{kl}, -D_{lk}], 0)$.
   - Else ($D_{lk} < 0$),

$$DI_{kl} = [\frac{-D_{lk}}{-D_{lk} - D_{kl}} - \frac{-D_{kl}}{-D_{lk} - D_{kl}}] \times$$
$$\times d([D_{kl}, -D_{lk}], 0).$$

2) Calculate a global dominance intensity (GDI) for each alternative A_k, i.e. $GDI_k = \sum_{l=1,\ l \neq k}^{m} DI_{kl}$, and rank the alternatives according to them.

New extensions of *DIM2* and *DIM3* were proposed in [2]. In the *first extension*, the dominance intensities derived in *DIM2* and *DIM3* are weighted according to the distance between the central weight vector (w_j^c) and the weight vector (w_j^{*lk}) associated with the optimal D_{lk}, when solving the corresponding optimization problem. Note that in the case of interval weights the central weight vector is composed of the midpoints of the different weight intervals.

The aim of these extensions is to attach more importance to weight vectors closer to the central weight vector. To do this, the *DIM2* and *DIM3* methods must be applied, and the derived dominance intensities are then weighted using the following expression:

$$DI_{kl}^* = \frac{DI_{kl}}{d(w_j^c, w_j^{*lk})}.$$

The *second extension* is similar to the above in that weight vectors close to the central weight vector are given more importance. However, the weighting is now applied not to the dominance intensity values but to the pairwise dominance values in D. Then, *DIM2* and *DIM3* are applied as described before.

More recently, the same idea was extended concerning imprecision in the DMs' preferences regarding both weights and component values/utilities. Instead of computing the pairwise dominance values (D_{kl}), the method calculates:

$$v_{kl} = \sum_{j=1}^{n} w_j^c v_{kj}^c - \sum_{j=1}^{n} w_j^c v_{lj}^c,$$

where $(w_1^c, ..., w_n^c)$ is the centroid or center of gravity of the polytope representing the weight space, and $(v_{k1}^c, v_{l1}^c), ..., (v_{kn}^c, v_{ln}^c)$ are the centroids or centers of gravity of the polytopes in the n attributes delimited by the constraints accounting for alternatives A_k and A_l. Note that the centroid is considered as the most representative point that verifies the constraints delimiting the polytope. Moreover, $D_{kl} \leq v_{kl} \leq -D_{lk}$.

The centroid of the polytope associated with constraints on component values in attribute X_j for the alternatives A_k and A_l is

$$\mathbf{v}_j^c = (v_{kj}^c, v_{lj}^c) = \frac{\int_{[0,1]^2} V_j^{kl} dv}{\int_{[0,1]^2} dv},$$

where V_j^{kl} is the set of constraints concerning component values in attribute X_j for alternatives A_k and A_l. Note that $V_j^{kl} \subset V_j$, which includes the constraints concerning component values in attribute X_j for all the alternatives.

As it would be very simplistic to represent a constraint set as just a point, an interval centered on the central value is built as follows:

$$I_{kl} = [I_{kl}^L, I_{kl}^U],$$

where $I_{kl}^L = v_{kl}^c - m_{kl}$ and $I_{kl}^U = v_{kl}^c + m_{kl}$, and

$$m_{kl} = min\{(-D_{lk} - v_{kl}^c), (v_{kl}^c - D_{kl})\}.$$

Then the intervals $I_{kl} = [I_{kl}^L, I_{kl}^U]$ rather than $[D_{kl}, -D_{lk}]$ are used to compute the dominance intensities in *DIM3*.

In [3] the method is applied to deal with ordinal information in both weights and component values. The DM provides an attribute importance ranking. Besides, the method takes into account a ranking of the alternatives in each attribute and also a ranking of the difference of values between consecutive alternatives, as in Fig. 1. Rather than intervals, however, triangular fuzzy numbers are built:

$$\widetilde{I}_{kl} = (I_{kl}^L, v_{kl}^c, I_{kl}^U),$$

with the membership function (see Fig 2.)

$$\mu_{\widetilde{I}_{kl}}(x) = \begin{cases} \frac{x - I_{kl}^L}{v_{kl}^c - I_{kl}^L}, & \text{if } I_{kl}^L \leq x \leq v_{kl}^c \\ 1, & \text{if } x = v_{kl}^c \\ \frac{x - I_{kl}^U}{v_{kl}^c - I_{kl}^U}, & \text{if } v_{kl}^c \leq x \leq I_{kl}^U \\ 0, & \text{otherwise.} \end{cases}$$

Then the *dominance intensities* are computed as follows:

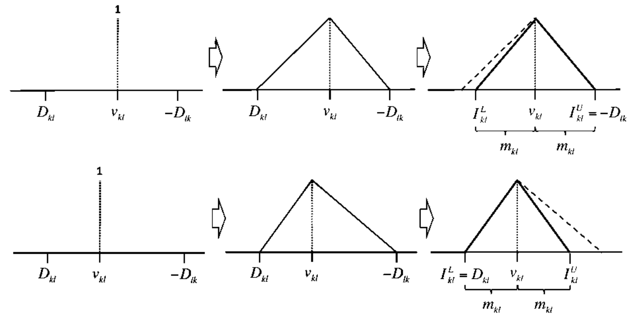

Fig. 2. Building $\widetilde{I}_{kl}$

- If $v_{kl}^c \geq 0$, then $DI_{kl} = d(\widetilde{I}_{kl}, 0, f)$, where d refers to Tran and Duckstein's distance [23], and f is a weight function for differentiating risk-averse, risk-neutral or risk-prone DMs, as explained later.
- Else $(v_{kl}^c < 0)$, $DI_{kl} = -d(\widetilde{I}_{kl}, 0, f)$.

We compute a *dominance intensity measure* for each alternative A_k, $DIM_k = \sum_{l=1, l\neq k}^{m} DI_{kl}$, and rank alternatives according to DIM_k values.

As mentioned above, the method is based on Tran and Duckstein's distance between fuzzy numbers, using the *generalization of the left and right fuzzy numbers* (*GLRFN*), whose membership function is defined as

$$\mu_{\widetilde{a}}(x) = \begin{cases} L\left(\frac{a_2 - x}{a_2 - a_1}\right) & \text{if} \quad a_1 \leq x \leq a_2 \\ 1 & \text{if} \quad a_2 \leq x \leq a_3 \\ R\left(\frac{x - a_3}{a_4 - a_3}\right) & \text{if} \quad a_3 \leq x \leq a_4 \\ 0 & \text{otherwise,} \end{cases}$$

where L and R are strictly decreasing functions defined in $[0, 1]$ and satisfying the conditions:

$$\begin{array}{lll} L(x) = R(x) = 1 & \text{if} & x \leq 0 \\ L(x) = R(x) = 0 & \text{if} & x \geq 0. \end{array}$$

Tran and Duckstein's distance can be expressed for the particular case of the distance from a triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3)$ to a constant (specifically 0) by:

If $f(\alpha) = \alpha$, then

$$\begin{aligned} d^2(\tilde{a}, 0, f) = \quad & a_2^2 + \tfrac{1}{3} a_2(a_3 + a_1) + \\ & \tfrac{1}{18}[(a_3 - a_2)^2 + (a_2 - a_1)^2] - \\ & -\tfrac{1}{18}[(a_2 - a_1)(a_3 - a_2)]. \end{aligned}$$

If $f(\alpha) = 1$, then

$$\begin{aligned} d^2(\tilde{a}, 0, f) = \quad & a_2^2 + \tfrac{1}{2} a_2(a_3 + a_1) + \\ & \tfrac{1}{9}[(a_3 - a_2)^2 + (a_2 - a_1)^2] - \\ & -\tfrac{1}{9}[(a_2 - a_1)(a_3 - a_2)]. \end{aligned}$$

If $f(\alpha) = \alpha^2$, then

$$\begin{aligned} d^2(\tilde{a}, 0, f) = \quad & a_2^2 + \tfrac{1}{4} a_2(a_3 + a_1) + \\ & \tfrac{1}{144}[(a_3 - a_2)^2 + (a_2 - a_1)^2] - \\ & -\tfrac{1}{96}[(a_2 - a_1)(a_3 - a_2)]. \end{aligned}$$

Note that the function $f(\alpha)$, which serves as a weight function, is positive continuous in $[0, 1]$, the distance being computed as a weighted sum of distances between two intervals along all of the α-cuts from 0 to 1.

Thanks to the presence of function f, DM participation is flexible. For example, when the DM is risk-neutral, $f(\alpha) = \alpha$ is a reasonable?assumption. A risk-averse DM might want to attach more weight to information at a higher α level by using other functions, such as $f(\alpha) = \alpha^2$ or a higher power of α. A constant ($f(\alpha) = 1$), or even a decreasing f function, can be utilized for a risk-prone DM. Tran and Duckstein's distance for the risk-prone and risk-averse DM can be found in [6].

The results of Monte Carlo simulation techniques demonstrate that the proposed method is clearly better than the dominance measuring methods described above. Its performance is very similar to the method proposed by Sarabando and Dias [21], which was likewise developed to deal with ordinal information about DMs' preferences. The *paired-samples t*-test showed that there was no significant difference between the two for a neutral, risk-prone and risk-averse DM. Sarabando and Dias' method is less computationally demanding, but is only applicable in the discussed imprecise decision-making situation.

B. Fuzzy numbers

In [6] a *DMM* was proposed accounting for weights described by means of trapezoidal fuzzy numbers. The method was based on fuzzy pairwise dominances and on Tran and Duckstein's distance between fuzzy numbers, using the *GLRFN*. Trapezoidal fuzzy numbers are a special case of *GLRFN* with $L(x) = R(x) = 1 - x$.

Note that for the particular case of the distance of a trapezoidal fuzzy number $\tilde{a} = (a_1, a_2, a_3, a_4)$ to a constant (specifically 0), Tran and Duckstein's distance is:

$$\begin{aligned} d^2(\widetilde{a}, 0, f) = \; & \left(\tfrac{a_2+a_3}{2}\right)^2 + \tfrac{1}{3}\left(\tfrac{a_2+a_3}{2}\right)\left[(a_4 - a_3) - (a_2 - a_1)\right] \\ & + \tfrac{2}{3}\left(\tfrac{a_3-a_2}{2}\right)^2 \\ & + \tfrac{1}{9}\left(\tfrac{a_3-a_2}{2}\right)\left[(a_4 - a_3) + (a_2 - a_1)\right] \\ & + \tfrac{1}{18}\left[(a_4 - a_3)^2 + (a_2 - a_1)^2\right] \\ & - \tfrac{1}{18}\left[(a_2 - a_1)(a_4 - a_3)\right], \end{aligned}$$

when $f(\alpha) = \alpha$, i.e. for a risk-neutral DM.

First, fuzzy optimization problems have to be solved to derive the dominance matrix, D, whose elements are trapezoidal fuzzy numbers. The objective function in (1) can now be represented by

$$\begin{aligned} \widetilde{D}_{kl} &= \sum_{j=1}^{n} \widetilde{w}_j z^*_{klj} = \sum_{j=1}^{n} (w_{j1}, w_{j2}, w_{j3}, w_{j4}) z^*_{klj} \\ &= (D_{kl1}, D_{kl2}, D_{kl3}, D_{kl4}), \end{aligned}$$

where z^*_{klj} is the optimum value of the following optimization problem:

$$\begin{aligned} \min z_{klj} \quad & = u_j(x_{kj}) - u_j(x_{lj}) \\ s.t. \quad & \\ & x^L_{kj} \leq x_{kj} \leq x^U_{kj},\ j = 1, \ldots, n \\ & x^L_{lj} \leq x_{lj} \leq x^U_{lj},\ j = 1, \ldots, n \\ & u^L_j(x_{kj}) \leq u_j(x_{kj}) \leq u^U_j(x_{kj}), j = 1, \ldots, n \\ & u^L_j(x_{lj}) \leq u_j(x_{lj}) \leq u^U_j(x_{lj}), j = 1, \ldots, n. \end{aligned} \tag{2}$$

The optimal solution of problem (2) can be determined very simply for certain types of utility functions [14]. Specifically, if the utility function is monotonically increasing or decreasing, then $z^*_{klj} = u^L_j(x^L_{kj}) - u^U_j(x^U_{lj})$ or $z^*_{klj} = u^U_j(x^U_{kj}) - u^L_j(x^L_{lj})$, respectively.

Next, the strength of dominance of alternative A_k is computed by adding the trapezoidal fuzzy numbers in the kth row of D,

$$\begin{aligned} \widetilde{d}_k = (d_{k1}, d_{k2}, d_{k3}, d_{k4}) = \sum_{l=1, l\neq k}^{m} \widetilde{D}_{kl} = \\ = \left(\sum_{l=1, l\neq k}^{m} D_{kl1}, \sum_{l=1, l\neq k}^{m} D_{kl2}, \sum_{l=1, l\neq k}^{m} D_{kl3}, \sum_{l=1, l\neq k}^{m} D_{kl4} \right). \end{aligned}$$

Finally, a *dominance intensity*, DI_k, for each alternative A_k is computed as the proportion of the positive part of the fuzzy number $\widetilde{d}_k$ by the distance of the fuzzy number to zero. Specifically, the dominance intensity for alternative A_k is computed according to the location of $\widetilde{d}_k$ as follows:

1) If all of $\widetilde{d}_k$ is located to the left of zero, then DI_k is minus the distance of $\widetilde{d}_k$ to zero, because there is no positive part in $\widetilde{d}_k$.
2) If all of $\widetilde{d}_k$ is located to the right of zero, then DI_k is the distance of $\widetilde{d}_k$ to zero, because there is no negative part in $\widetilde{d}_k$.
3) If $\widetilde{d}_k$ includes the zero in its base, then the fuzzy number will have a part on the right of zero ($\widetilde{d}^R_k$) and another part on the left of zero ($\widetilde{d}^L_k$), see e.g. Fig 3.
   DI_k is the proportion that represents $\widetilde{d}^R_k$ with respect to $\widetilde{d}_k$ by the distance of $\widetilde{d}_k$ to zero less the proportion that represents $\widetilde{d}^R_k$ with respect to $\widetilde{d}_k$ by the distance of $\widetilde{d}_k$ to zero.

Monte Carlo simulation techniques proved that the method performs well for different imprecision levels, the hit ratio and Kendall's correlation values being higher than 84% and 80%, respectively, in the worst case (20% imprecision). As expected, the results were better than for similar studies accounting for weight intervals, but not as good as when triangular rather than trapezoidal fuzzy weights are used, since triangular fuzzy weights provide more meaningful information about the weights.

III. Conclusion

Dominance measuring methods (*DMMs*) have been shown to be a valid approach for dealing with complex decision-making problems with imprecise information within multi-attribute value/utility theory. *DMMs* are based on the computation of pairwise dominance values and exploit the information

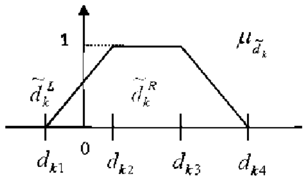

Fig. 3. Example of location of $\widetilde{d}_k$

in the dominance matrix in different ways to derive measures of dominance intensity and rank the alternatives under consideration.

Different *DMMs* for dealing with imprecise information about decision-maker preferences represented in different ways, like intervals, ordinal information, normal distributions or fuzzy numbers can be found in the literature.

The performance of these methods has been compared with other existing approaches, like surrogate weighting methods, adapted classical decision rules, *SMAA* and *SMAA-2* methods or Sarabando and Dias' method. To do this, two efficiency measures, the hit ratio and the rank-order correlation, have been taken into account.

The results of the applied Monte Carlo simulation techniques demonstrate that *DMM* and *ROC* outperform the other methods when ordinal information represents imprecision concerning weights and that there is no significant difference between the two. However, *ROC* can be only applied when ordinal relations regarding attribute weights are provided.

DIM2 performs better than the *AP1* method and the adaptation of classical decision rules when weight intervals are considered. Although *SMAA-2* slightly outperforms *DIM2*, *DIM2* is applicable when incomplete information about weights is expressed not just as weight intervals but also as weights satisfying linear or non-linear constraints, weights represented by fuzzy numbers or weights fitting normal probability distributions.

Besides, there is no significant difference between the corresponding *DMM* and the method proposed by Sarabando and Dias for a neutral, risk-prone and risk-averse DM when dealing with ordinal information in both weights and component values.

Finally, Monte Carlo simulation techniques also proved that the *DMM* performs well for different imprecision levels when weights are described by means of trapezoidal fuzzy numbers, again for a neutral, risk-prone and risk-averse DM.

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