

Power Losses Calculations in Windings of Gapped Magnetic Components

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Abstract— A model is proposed for the calculation of the winding losses at the beginning of the design process of high frequency transformers and inductors. Although this kind of losses have been subject of investigation for years, their analytical calculation in gapped components is still limited, and the use of numerical analysis tools, such as finite elements analysis (FEA) tools, are commonly needed for winding characterization. A general 2-D equivalent analytical model for windings losses calculation in gapped magnetic components that shows very good results compared with FEA calculation is presented. The model can be integrated in design and optimization tools in order to evaluate the influence of the gap on the windings at the very early stages of the design process.

Index Terms— Windings losses, windings resistance, magnetic components, air gap.

I. INTRODUCTION

Accurate winding losses estimation in magnetic components used in power electronics applications is a very important task, an imprecise calculation of the equivalent resistances of the windings can lead to under or overestimating the losses in the component, resulting in unexpected temperature rise or a costly component oversize.

Several methods have been developed to accurately predict the losses in windings formed by round conductors [1-6], where the most extended one, often called the Dowell method [1], consists on dividing the windings into portions, considering every portion as an equivalent foil conductor with equal total sectional area and then multiplying the dc resistance of each layer by a corresponding factor to obtain the ac resistance of the winding. This method, however, has considerable errors at high frequencies. To improve it, many authors have developed new models, based on modifications of this method [2-4].

Since Dowell based methods assume an incident one-directional magnetic field over the windings of the component, they do not properly model the winding losses in components in which this assumption cannot be taken, which is the case of components with air gaps. In [5] and [6] exact 2-D methods are used for precisely model winding losses in round conductors but gap effect in winding losses is not taken into account.

In [7], the power loss in a thin conductor layer under a magnetic field transversal to the layer surface is calculated. In

[8], power loss is calculated for round conductors in gapped magnetic components. The solution presented in [7] accounts for planar conductors in gapped magnetic components and provides analytical expressions to determine the magnetic field at a given position from the gap. The mirror-image method described in [9] is used in [8] to calculate the magnetic field in a conductor. Depending on the desired accuracy, a high number of calculations may be required.

Another way to calculate power losses in windings of gapped magnetic components is by means of finite elements analysis (FEA) [10-12]. Although this numerical method is considered very accurate, the cost of commercial tools along with the required time for the component modeling can be an important drawback of this method.

This paper presents an analytical 2-D method for power losses calculation in windings of gapped magnetic components with round wires. FEA simulations are used to verify the accuracy of the proposed model.

II. 2-D ANALYTICAL MODEL

As described in [13], the expression (1) corresponds to the power losses per unit length of a cylindrical conductor under a transverse magnetic field, where ber , bei , ber' and bei' are, respectively, the real and imaginary part of the Bessel functions of first kind and their derivatives, σ and μ the conductivity and permeability of the conductive material, r_0 is the radius of the conductor, H_0 is the magnitude of the transverse field to the conductor and $k = \sqrt{w\sigma\mu}$ where w is the angular frequency.

$$P = \frac{4\pi\mu^2}{\sigma} (kr_0)^2 H_0^2 \frac{ber(kr_0)bei'(kr_0) - ber'(kr_0)bei(kr_0)}{[\mu^2 ber(kr_0)]^2 + [\mu^2 bei(kr_0)]^2} \quad (1)$$

If we consider the fringing field due to an air gap in a magnetic component (see figure 1), H_g , two-directional in the plain rz in cylindrical coordinates, any conductor affected by this fringing field will be exposed to its two components, H_r and H_z , both transverse to the conductor. Then expression (1) results in expressions (2) and (3), and the dissipated power per unit length in the conductor can be expressed as the vectorial sum of (2) and (3).

$$P_r = \frac{4\pi\mu^2}{\sigma} (kr_0)^2 H_r^2 \frac{\text{ber}(kr_0)\text{bei}'(kr_0) - \text{ber}'(kr_0)\text{bei}(kr_0)}{[\mu^2\text{ber}(kr_0)]^2 + [\mu^2\text{bei}(kr_0)]^2} \quad (2)$$

$$P_z = \frac{4\pi\mu^2}{\sigma} (kr_0)^2 H_z^2 \frac{\text{ber}(kr_0)\text{bei}'(kr_0) - \text{ber}'(kr_0)\text{bei}(kr_0)}{[\mu^2\text{ber}(kr_0)]^2 + [\mu^2\text{bei}(kr_0)]^2} \quad (3)$$

$$P = \sqrt{P_r^2 + P_z^2} \quad (4)$$

The field components of H_g can be calculated, according to [7], with (5) and (6) for r and z components respectively, where l_g is half of the air gap length, r and z are the cylindrical coordinates with respect to the origin (see figure 1), m is equal to zero if $r^2 + z^2 > l_g^2$ and 1 if $r^2 + z^2 < l_g^2$ and $H_g \cong 0.9NI/2l_g$ with N and I equal to the number of turns and current through the winding respectively.

$$H_r(r, z) = \frac{H_g}{2\pi} \ln \left[\frac{r^2 + (z - l_g)^2}{r^2 + (z + l_g)^2} \right] \quad (5)$$

$$H_z(r, z) = -\frac{H_g}{\pi} \left[\tan^{-1} \left(\frac{2rl_g}{r^2 + z^2 - l_g^2} \right) + m\pi \right] \quad (6)$$

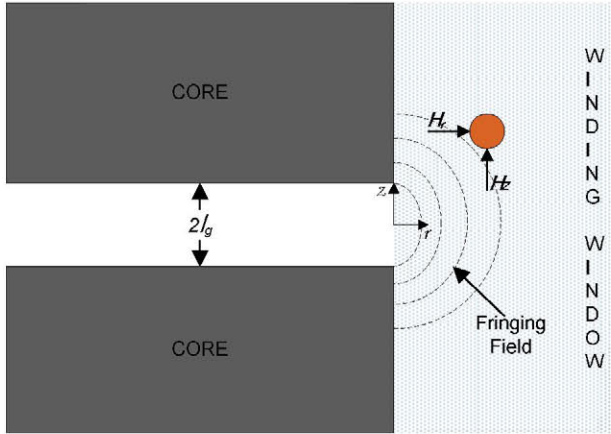


Figure 1. Influence of the fringing field on a conductor in a gapped magnetic component.

However, expression (1) does not account for the effect of near conductors or the effect of the current flowing through the conductor, thus the proximity and skin effects must be added to this expression. This can be done using the simplified expressions for the calculation of the resistance in a cylinder conducting an alternating current and the dissipated power per unit length in a cylinder next to a conductor that carries an alternating current, where in [13] are expressed as

$$R = R_0 \left[1 + \frac{1}{48} \left(\frac{r_0}{\delta} \right)^4 \right]; r_0 < \delta \quad (7)$$

$$R = R_0 \left[\frac{1}{4} + \frac{r_0}{2\delta} + \frac{2}{32} \left(\frac{\delta}{r_0} \right) \right]; r_0 > \delta \quad (8)$$

$$P_{prox}(W/m) = \frac{J_0^2 \pi r^2}{\sigma} \left[1 + \frac{1}{48} \left(\frac{r}{\delta} \right)^4 \left[1 + 12 \left\{ \left(\frac{r}{x} \right)^2 + \frac{1}{6} \sum_{n=2}^{\infty} \frac{1}{(2n-3)} \left(\frac{r}{x} \right)^{2n} \right\} \right] \right] \quad (9)$$

where $R_0 = R_{DC} = l_c / (\sigma A_c)$; l_c and A_c are, respectively, the length and area of the conductor, J_0 is the dc current density and x is the distance between conductors. Then, combining these three expressions together, the total dissipated power per unit length of a cylindrical conductor in a gapped magnetic component can be expressed as the sum of (4) and (9) and its equivalent resistance can be easily obtained just dividing the result by the square of the rms current through the winding.

III. RESULTS

Finite element analysis (FEA) is used to evaluate the accuracy of the proposed model. First, expressions (4) and (9) are evaluated separately and then the complete model is evaluated using different components.

Figure 2 shows an axisymmetric representation of an AWG25 round wire on a gapped RM6 magnetic core. Component 2a is used for the evaluation of equation (4) where the conductor is only affected by the fringing field due to the air gap in the magnetic core. The conductor was placed in different position respect of the air gap (of different lengths) and the dissipated power was calculated according to (4) at different frequencies. Then, the equivalent resistance at the considered frequency is calculated (a current of 1 A was used). A commercial finite element tool (MAXWELL@14 by ANSYS) was used for FEA simulation and the calculated power loss is compared with calculated results in table 1.

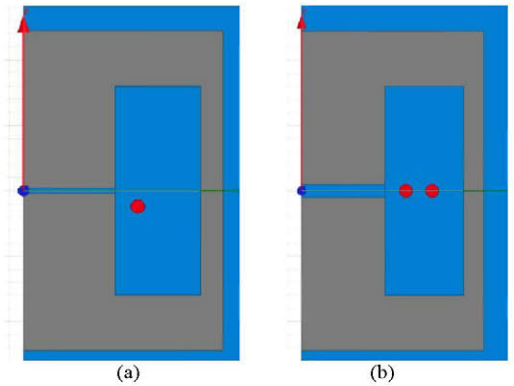


Figure 2. Representation of conductors in a RM6 gapped magnetic component. (a) A single conductor under the influence of the gap and (b) a conductor under the influence of the gap and a near conductor carrying an alternating current.

Table 1. Comparison between analytical and FEA calculated equivalent resistance of a conductor of position (r,z) from the gap.

Conductor position (mm) (r,z)	Frequency (kHz)	Gap length (mm)	Calculated Equivalent Resistance ($m\Omega$)		Relative Error (%)
			Analytical	FEA	
(0.4, 0)	40	0.4	2.2498	2.1014	7.06
(0.8, -0.6)	200.5	0.2	2.7900	2.8751	-2.96
(1.3, 1.5)	100.5	0.15	2.2969	2.4990	-8.09
(0.8, -2.5)	2000	0.7	6.9501	7.8239	-11.17
(2.3, 0.5)	400	0.5	4.5321	4.7936	-5.46

Table 2. Comparison between analytical and FEA calculated equivalent resistance of a conductor in position (r,z) from the gap separated by a distance x from a conductor that carries an AC current of 1A at different frequencies.

Conductor position (mm) (r,z)	Distance between conductors (mm)	Frequency (kHz)	Gap length (mm)	Calculated Equivalent resistance ($m\Omega$)		Relative Error (%)
				Analytical	FEA	
(0.4, 0)	0.6	40	0.4	2.5801	2.6744	-3.53
(0.8, -0.6)	0.7	200.5	0.2	4.9233	5.1620	-4.62
(1.3, 1.5)	0.75	100.5	0.15	3.2902	3.2181	2.24
(0.8, -2.5)	1	400	0.7	4.9995	4.9318	1.37
(1.3, 1.5)	2	100.5	0.5	2.6240	2.6955	-2.65

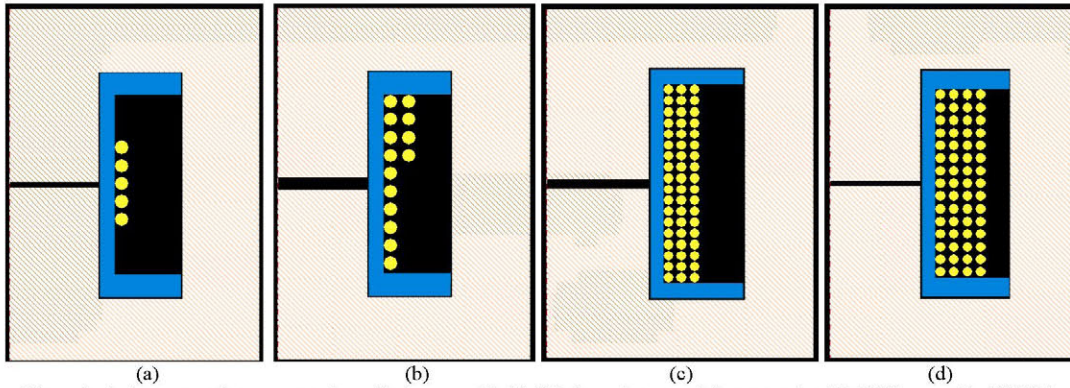


Figure 3. Axis-symmetric representation of inductors with (a) 203.5 μm air gap and four turns in a RM6S/I core with AWG25 wire, (b) 441.97 μm air gap and 14 turns in a RM6S/I core with AWG25 wire, (c) 508.31 μm air gap and 54 turns (18 turns per layer) in a RM10/I core with SWG23 wire and (d) representation of a 60 turns inductor over a RM8/I core (15 turns per layer) with AWG24 wire.

Table 2 shows the comparison between FEA and the described method, in which we calculate the effect of the fringing field plus the proximity effect due to the outer conductor carrying an AC current of 1A in the closest conductor to the air gap in component 2b. As in the previous case, different air gaps, frequencies and positions were used and also different distances between conductors.

As practical cases we use the inductors shown in figure 3 that were modeled in the FEA tool MAXWELL@14. The results obtained from FEA and the analytical method for components 3a, 3b and 3c are compared in table 3. Table 4 shows the results of a family of components derived from the inductor 3d at a frequency of 100 kHz. It can be seen that the proposed 2D analytical model that was used for computing the

equivalent resistance of the windings shows a maximum error of $\pm 20\%$ compared to FEA results.

Table 3. Comparison between analytical and FEA calculated equivalent resistance of inductors 3a, 3b and 3c.

Component	Frequency (kHz)	Equivalent resistance (Ω)		Relative error (%)
		FEA	Analytical	
Inductor 3a	100	0.0229	0.0233	1.72
Inductor 3b	100	0.1351	0.1497	9.75
Inductor 3c	200	9.7000	9.1472	-6.04

Table 4. Comparison between analytical and FEA calculated equivalent resistance of a family of components based on inductor 3d.

Air gap (mm)	Number of layers	Equivalent resistance (Ω)		Relative error (%)
		Analytical	FEA	
0.4	1	0.1373	0.1581	-13.15
	2	0.6425	0.6767	-5.05
	3	1.6839	1.6880	-0.24
	4	3.3456	3.3017	1.33
0.72	1	0.1564	0.1603	-2.43
	2	0.7675	0.6936	10.65
	3	2.0440	1.7400	17.47
	4	4.0924	3.4094	20.03
2.2	1	0.1227	0.1405	-12.67
	2	0.6117	0.5987	2.17
	3	1.6575	1.5182	9.17
	4	3.3745	3.0155	11.90

IV. CONCLUSIONS

An analytical method to calculate winding losses in gapped magnetic components that combines several expressions of the state of the art is proposed. The results of the application of the proposed method in several components are very close of FEA calculations (from which the validity of the model can be derived).

Is important to mention that the power loss must be evaluated in every single turn of the winding, what means that in components with a high number of turns, the required calculation time can be significant, nevertheless, this does not mean that an excessive calculation time is needed in order to apply the method. In fact, in an average computer, for example, the computation time for a 60 turns inductor was less than 20ms, what is much faster than FEA on the same computer (around 7 seconds of computation time). However, and considering that the heaviest part in terms of computation time is the calculation of the proximity effect, a simplification of this model is being studied as future work.

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