A GRADIENT-ENHANCED CONTINUUM DAMAGE MODEL WITH APPLICATION TO FIBRE-REINFORCED TISSUES AT FINITE STRAINS

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A non-local gradient-based damage formulation within a geometrically non-linear setting is presented. The hyperelastic constitutive response at local material point level is governed by a strain energy function which is additively composed by an isotropic neo-Hookean matrix and by an anisotropic fibre-reinforced material based on the model proposed by [3]. The neo-Hookean matrix is assumed to behave elastically while only the anisotropic contribution is affected by the damage, which is governed by a scalar $[1-d]$-type damage formulation. Following the concepts in [2] and [1], the local free energy function is enhanced by a gradient-term of the non-local damage variable which, itself, is introduced as an additional independent variable. In order to link the local and non-local damage field variables, κ and ϕ respectively, a penalisation term is also incorporated within the free energy function, which reads

$$\Psi = \Psi^{\mathrm{iso}} + f_{\mathrm{d}}(\kappa)\,\Psi^{\mathrm{ani}} + \frac{c_{\mathrm{d}}}{2}\,\nabla_{\boldsymbol{X}}\phi \cdot \boldsymbol{C}^{-1} \cdot \nabla_{\boldsymbol{X}}\phi + \frac{\beta_{\mathrm{d}}}{2}\,[\phi - \kappa]^2 \tag{1}$$

Based on the principle of minimum total potential energy, a coupled system of Euler-Lagrange equations, i.e. the balance of linear momentum and the balance of the non-local damage field, is obtained and solved in weak form. Two different three-dimensional finite element discretisations are implemented, i.e. Q1Q1 and Q2Q1 hexahedral elements. Following this framework a finite element simulation of a two-layered fibre-reinforced artery-like tube subjected to internal pressure is performed. This idealised setting is commonly used to assess basic capabilities of particular constitutive models for arteries,

see e.g. [4] and references cited therein. The results show that the proposed gradient-enhanced damage model is able to reproduce the loss of stiffness due to fibre-degradation in an artery when the blood pressure reaches values far beyond the physiological range. These results are of great importance within biomechanical applications provided they can serve as a basis for planning of medical interventions such as angioplasty.

Point	u_r [mm]	p [kPa]
①	1.17	30.39
②	1.26	48.38
③	1.40	8.58

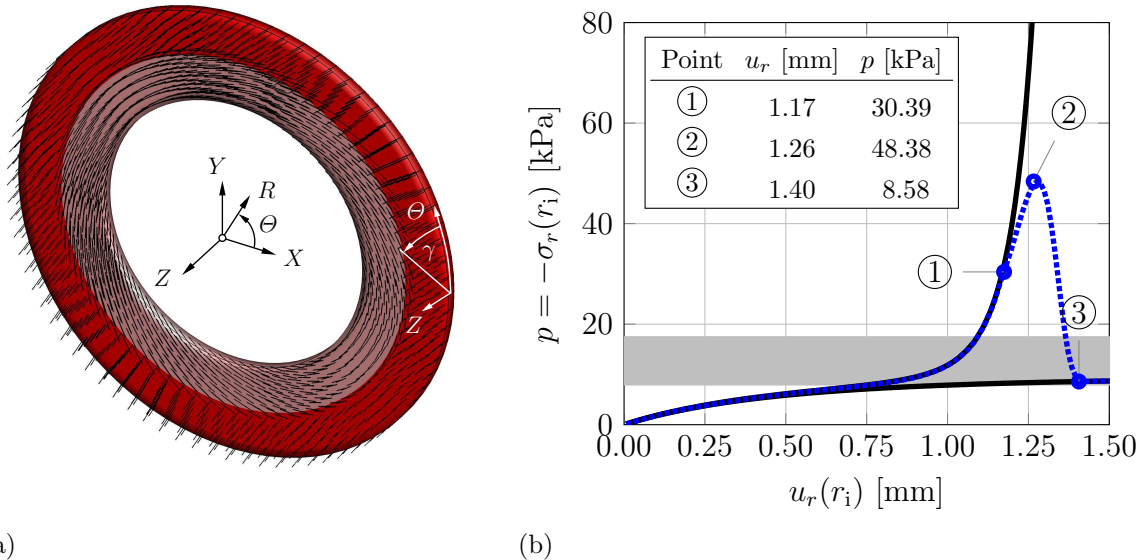(a)   (b)

Figure 1: Two-layered fibre-reinforced artery-like tube subjected to internal pressure. (a) Finite element mesh with fiber orientations. Media (inner layer) and adventitia (outer layer) are highlighted in different colors. (b) Load-displacement response for the inflated tube: pressure p versus radial displacement $u_r(r_i)$ at the inner radius of the tube. The lower and upper solid black curves represent the purely elastic response of the neo-Hookean ground substance and the fibre-reinforced material, respectively. The blue curve shows the mechanical response captured by the gradient-enhanced damage model. The gray-shaded region represents the physiological range for blood pressure.

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