Mesh movement strategy based on octree decomposition

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Motivation

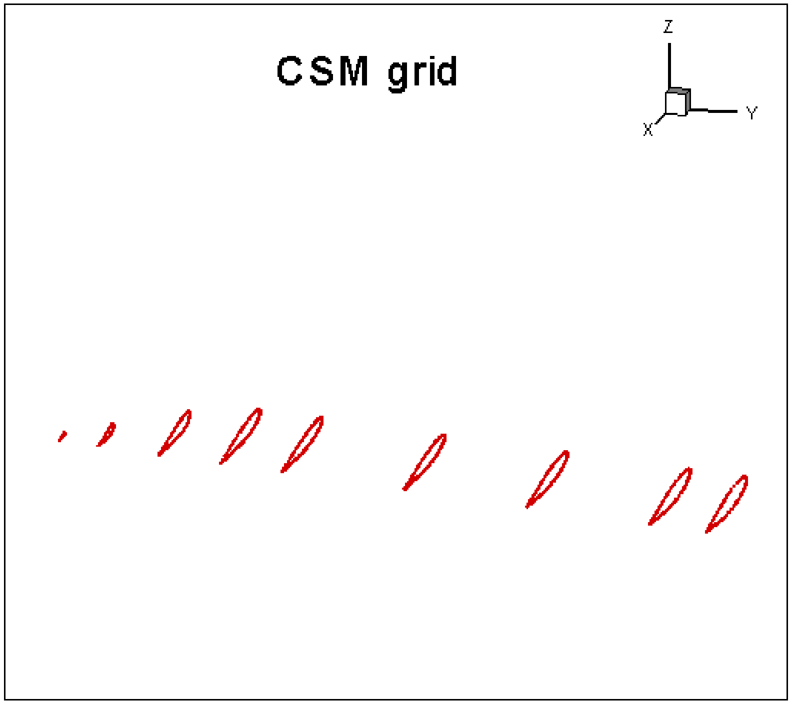
Problem: Transfer deformations between meshes

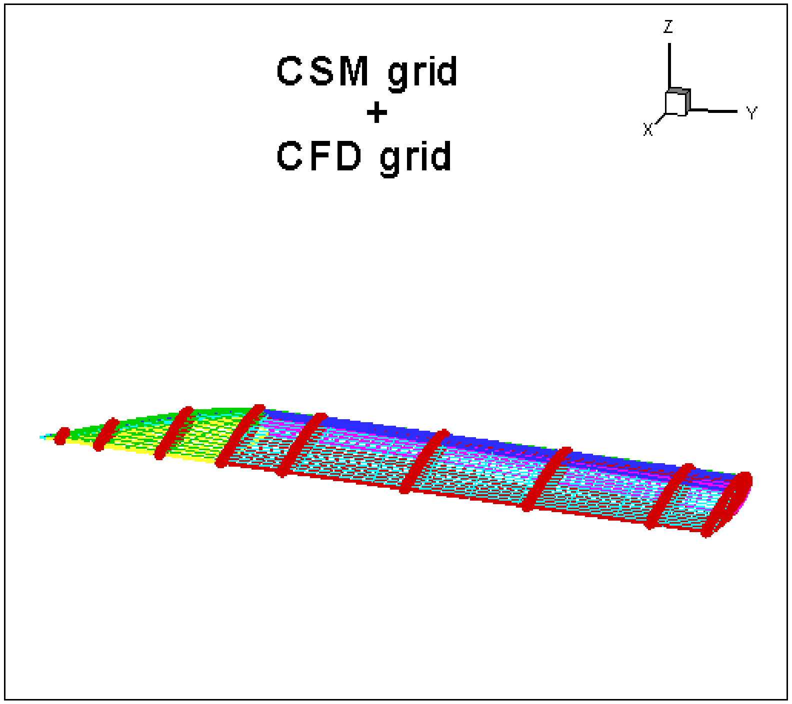
- ***BodyTransfer***
  - Transfer deformations from a structural mesh to an aerodynamic or surface mesh.

- ***MeshMove***
  - Transfer deformations from an aerodynamic mesh to a volumetric mesh.

Objetives

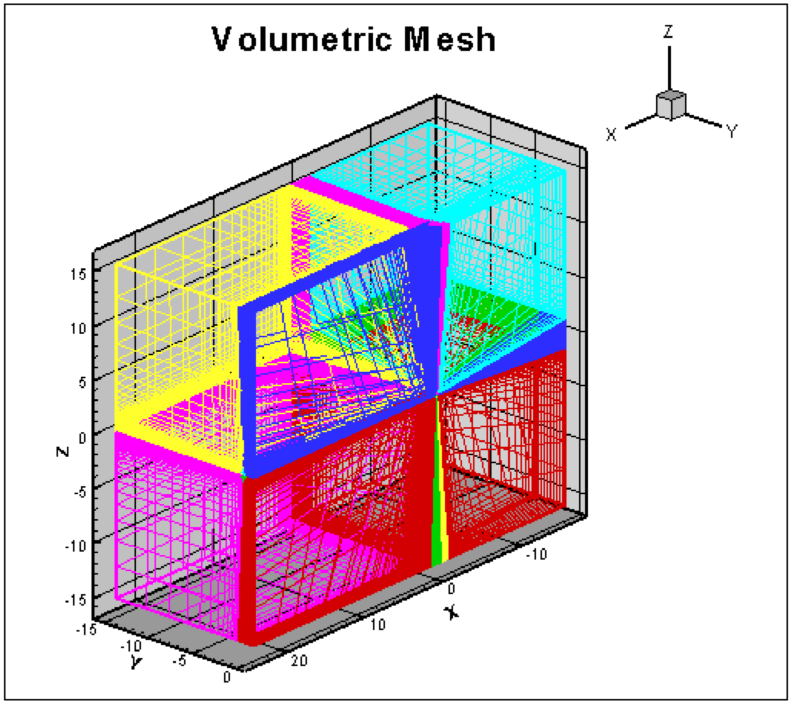
- Transfer deformations from a structural mesh (CSM grid) to an aerodynamic mesh (CFD grid)

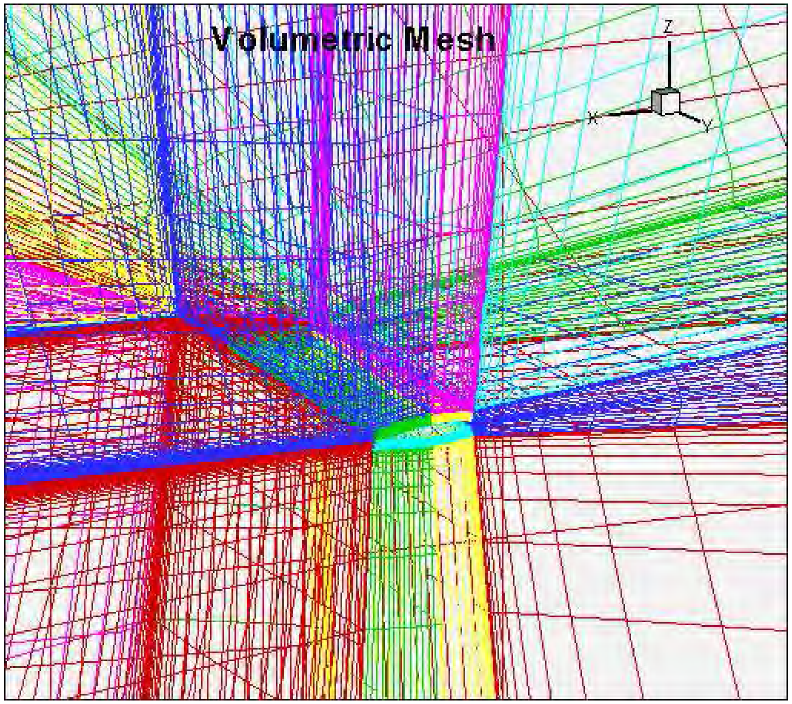




Objetives

- Transfer deformations from an aerodynamic mesh (CFM grid) to a volumetric mesh (CFD grid)





Objetives

- Transfer deformations from the boundary mesh to the volumetric mesh for a wide range of perturbations.
- Efficiency in computational resources (time and memory).
- Mesh quality preservation.
- Applicability to any 3D data set (any kind of 3D meshes: structured, multiblock structured, unstructured and hybrid).

MeshMove tool

- ***Methodology:***
  - Interpolation of deformations with ***Radial Basis Functions***.
- ***Strategy:***
  - Definition of interpolation domains that cover the whole mesh.

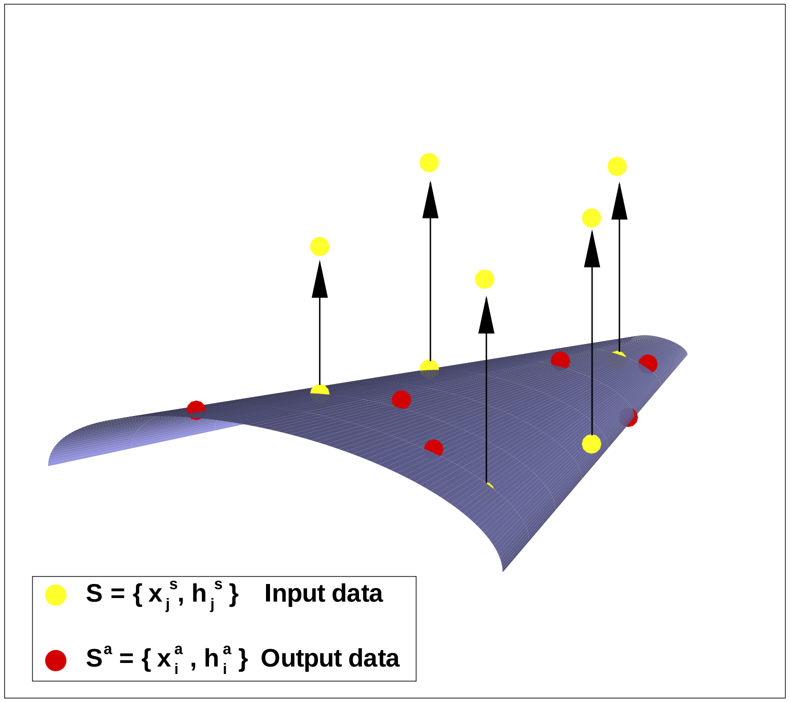
    Generation
    Management
    } of domains with ***Octree data structure***

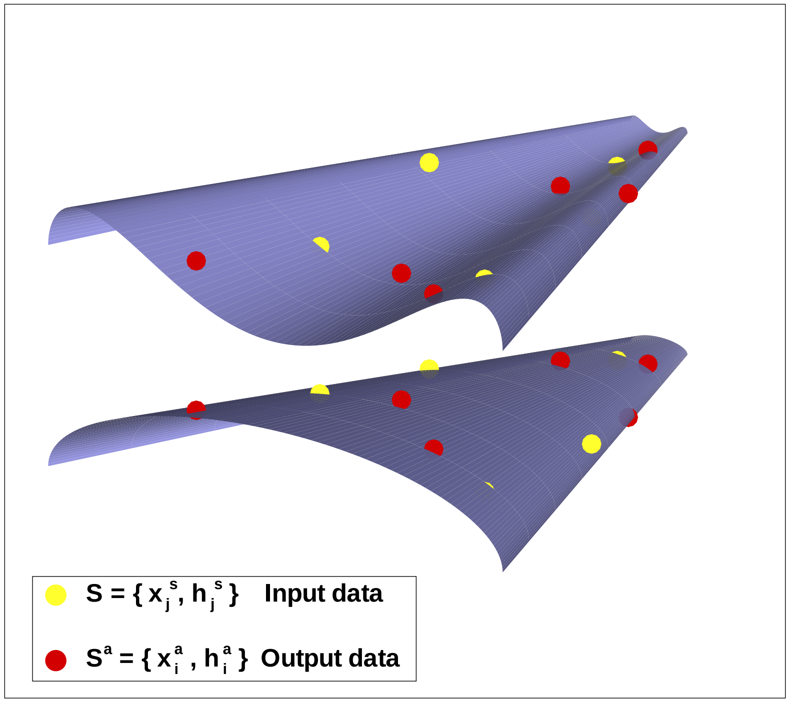
  - Definition of an optimal ordering to the domain deformation sequence.

Interpolation

Interpolation

- Given N_s centers $\{x_1^s, \ldots, x_{N_s}^s\}$ and their displacements $\{h_1^s, \ldots, h_{N_s}^s\}$, and N_a evaluation nodes $\{x_1^a, \ldots, x_{N_a}^a\}$
- The problem consists on obtaining the displacements $\{h_1^a, \ldots, h_{N_a}^a\}$ via interpolation methods, in a smooth an regular way





RBF Interpolation method

Reconstruct a continuous spatial distribution $h(\bar{x})$ using the discrete values $\bar{x}_i^s$

$$h(\bar{x}) = \sum_{i=1}^{N_s} w_i \, \Phi(||\bar{x} - \bar{x}_i^s||) + \Pi(\bar{x})$$

where

- w_i are the coefficients.
- Φ is a fixed basis function which is radial with respect to the Euclidean distance (***Radial Basis Function***)
- Π is a m degree polynomial that depends on the Φ function.

Requirements

- ***Interpolation condition***

$$h_i^s \equiv h(\bar{x}_i^s)$$

- ***Zero condition***

$$\sum_{i=1}^{N_s} w_i \, q(\bar{x}_i) = 0$$

for all polynomials q with a degree $\deg(q) \leq \deg(\Pi)$

- To avoid transfer of fictitious displacements, zero degree polynomials are required

$$\Pi(\bar{x}) = \gamma_0 \Longrightarrow \sum_{i=1}^{N_s} w_i = 0$$

RBF Interpolation

- Coefficients computation

$$\left.\begin{array}{l} h_i^s = h(\bar{x}_i^s) \quad i = 1,\ldots,N_s \\ \sum w_i = 0 \end{array}\right\} \Longrightarrow$$ System of $N_s + 1$ equations

$$\begin{pmatrix} 0 \\ h_1^s \\ h_2^s \\ \vdots \\ h_{N_s}^s \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 & \ldots & 1 \\ 1 & \Phi_{s_1 s_1} & \Phi_{s_1 s_2} & \cdots & \Phi_{s_1 s_{N_s}} \\ 1 & \Phi_{s_2 s_1} & \Phi_{s_2 s_2} & \cdots & \Phi_{s_2 s_{N_s}} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \Phi_{s_{N_s} s_1} & \Phi_{s_{N_s} s_2} & \cdots & \Phi_{s_{N_s} s_{N_s}} \end{pmatrix} \begin{pmatrix} \gamma_0 \\ w_1 \\ w_2 \\ \vdots \\ w_{N_s} \end{pmatrix}$$

$$\boxed{\bar{h}^s = C_{ss}\,\bar{\omega}}$$

RBF Interpolation

- Applying to evaluation nodes

$$h_i^a = h(\bar{x}_i^a) = \sum_{k=1}^{N_s} w_k \, \Phi(||\bar{x}_i^a - \bar{x}_k^s||) + \gamma_0 \quad i = 1, \ldots, N_a$$

$$\begin{pmatrix} h_1^a \\ h_2^a \\ \vdots \\ h_{N_a}^a \end{pmatrix} = \begin{pmatrix} 1 & \Phi_{a_1 s_1} & \Phi_{a_1 s_2} & \cdots & \Phi_{a_1 s_{N_s}} \\ 1 & \Phi_{a_2 s_1} & \Phi_{a_2 s_2} & \cdots & \Phi_{a_2 s_{N_s}} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \Phi_{a_{N_a} s_1} & \Phi_{a_{N_a} s_2} & \cdots & \Phi_{a_{N_a} s_{N_s}} \end{pmatrix} \begin{pmatrix} \gamma_0 \\ w_1 \\ w_2 \\ \vdots \\ w_{N_s} \end{pmatrix}$$

$$\boxed{\bar{h}^a = A_{as} \, \bar{\omega} = A_{as} \, C_{ss}^{-1} \, \bar{h}^s}$$

Radial Basis Functions

Function	***Definition*** $\Phi(\bar{x})$
Volume Spline	$\|\|\bar{x}\|\|$
Wendland $\mathcal{C}^0$	$(1-\|\|\bar{x}\|\|)^2_+$
Wendland $\mathcal{C}^2$	$(1-\|\|\bar{x}\|\|)^4_+\,(4\|\|\bar{x}\|\|+1)$
Wendland $\mathcal{C}^4$	$(1-\|\|\bar{x}\|\|)^6_+\,(35\|\|\bar{x}\|\|^2+18\|\|\bar{x}\|\|+3)$

RBF interpolation method

- ***Advantages***
  - Valid for any dimensional problem.
  - Translation and rotation invariant.
  - Smooth representation.
  - Possibility to manage any kind of 3D data (only depend on the distance between nodes).
- ***Disadvantages***
  - Time and memory consumption.

Mesh movement methodology

MeshMove tool

- ***Problems***
  - ***Numerical viability***
    - Mesh size
  - ***Accuracy***
    - Distance between centers and evaluation nodes
- ***Strategy***
  - ***Generate local interpolation domains***
    - Enough centers to guarantee a reliable interpolation,
    - Not too many to ensure a manageable C_{ss} matrix size.

Interpolation domains

- Non-disjoint sets of centers and evaluation nodes, lin-ked by their proximity.

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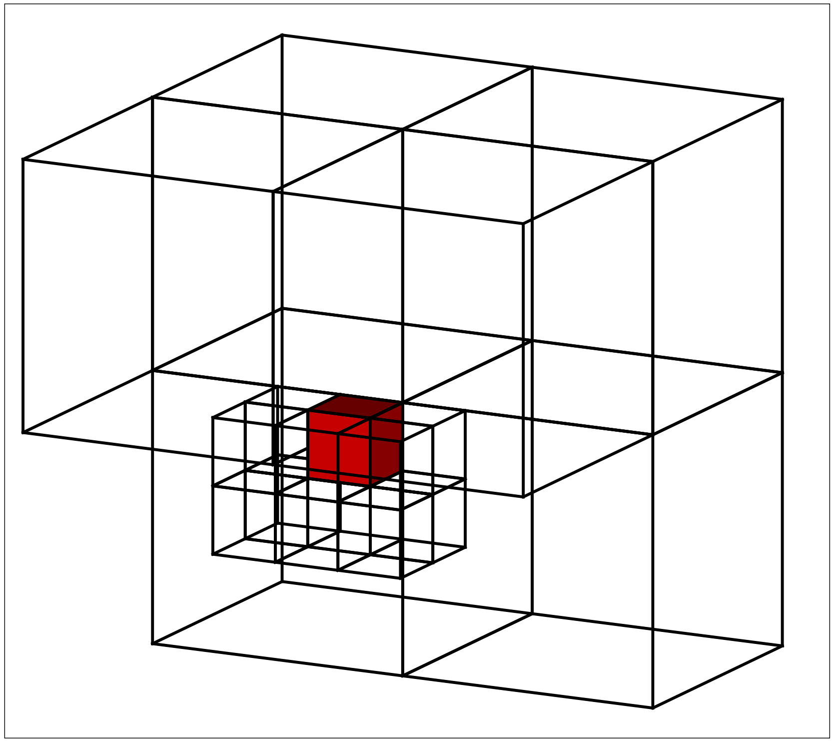
Octree data structure



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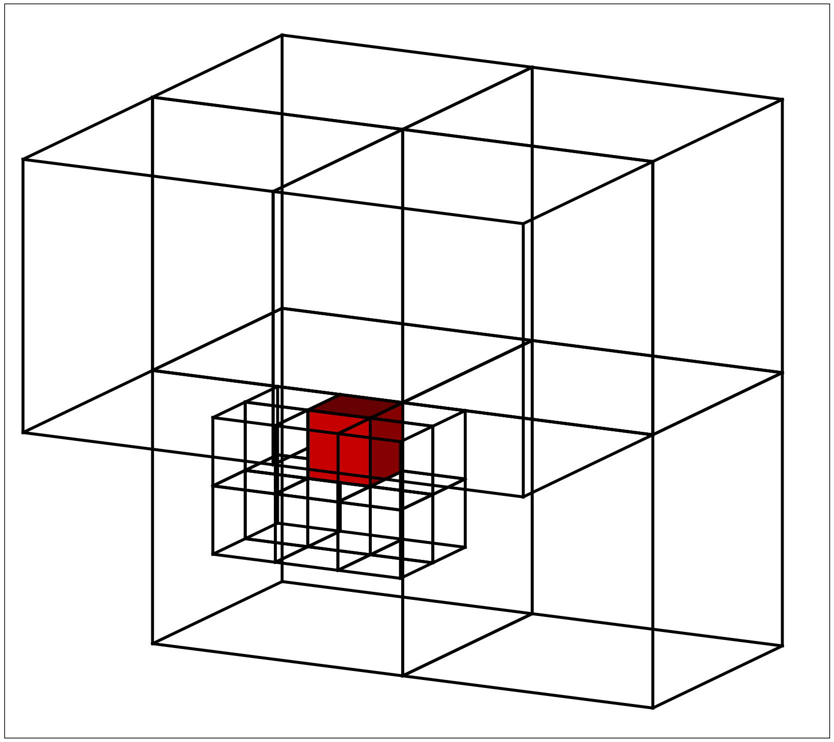


- The whole mesh is recovered by the union of all domains.

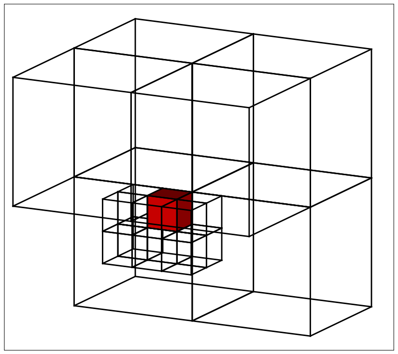
Interpolation domains

- Each domain represents a full interpolation problem.

- A domain is characterized by
  - The centers and evaluation nodes that form its own C_{ss} and A_{as} matrices.
  - Its position index in the ordered sequence of domains.

Centers and evaluation nodes

- An interpolation domain is made up of:
  - one kernel cube containing the evaluation nodes
  - and all neighbour cubes



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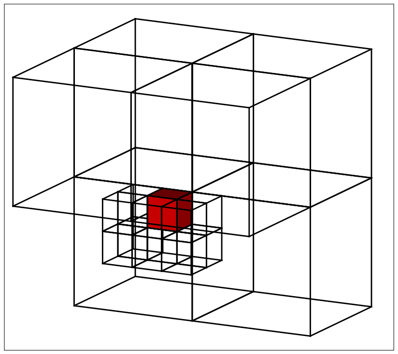
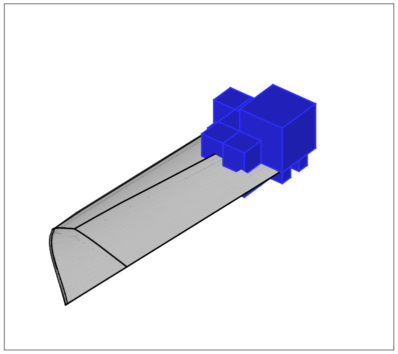


- As the interpolation is going on the domain's sequence
  - the evaluation nodes of the neighbour cubes already deformed, act as centers.

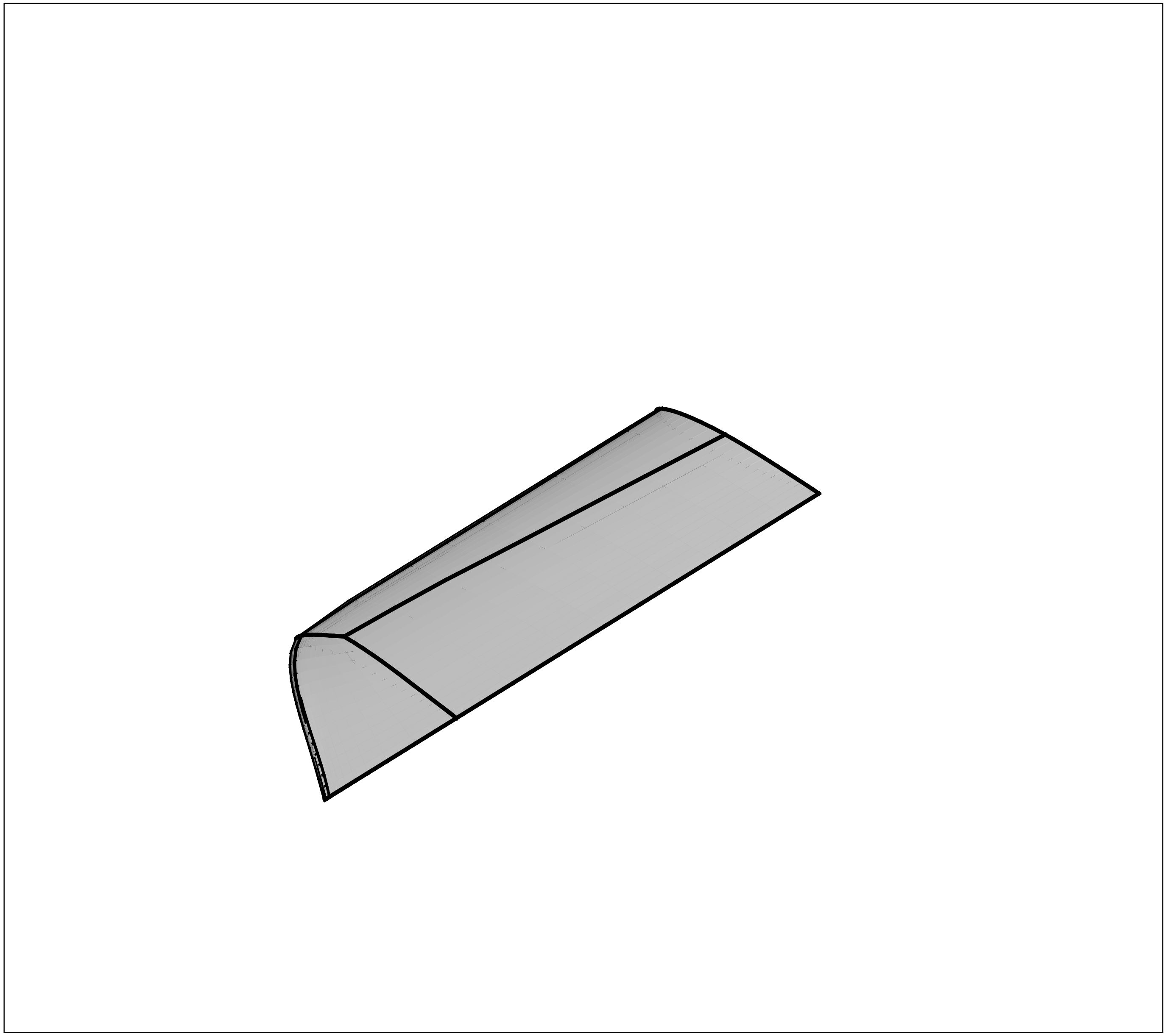


Interpolation sequence

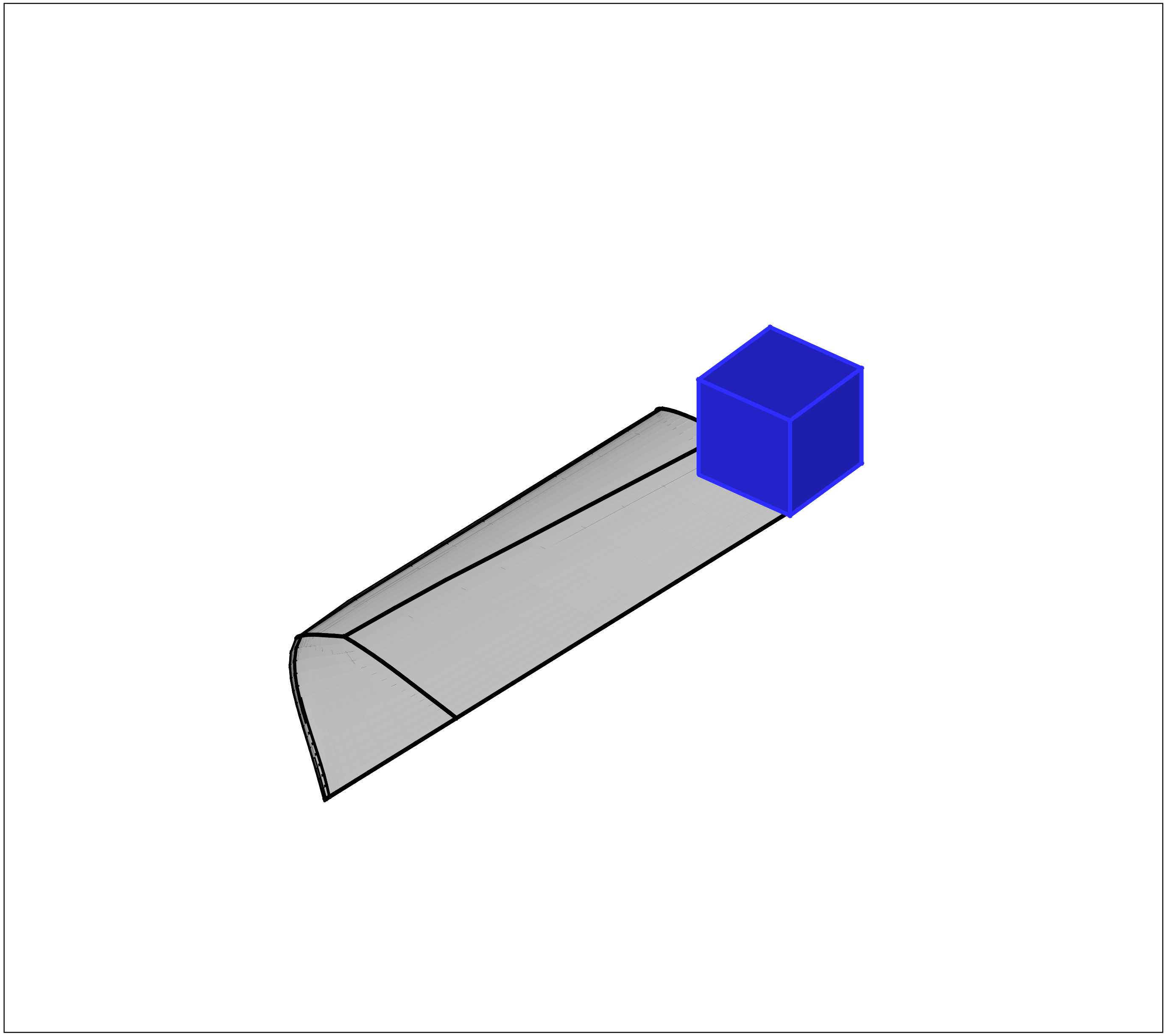
- ***Advancing front strategy***
  - Information travels through concentric layers surrounding the boundary.

- ***From the surface to the farfield***
  - Domains whose kernel contains surface nodes make up the first layer.
  - Once the first domain has been chosen, it is possible to cover the whole boundary travelling across neighbouring kernels.

Interpolation sequence



Interpolation sequence



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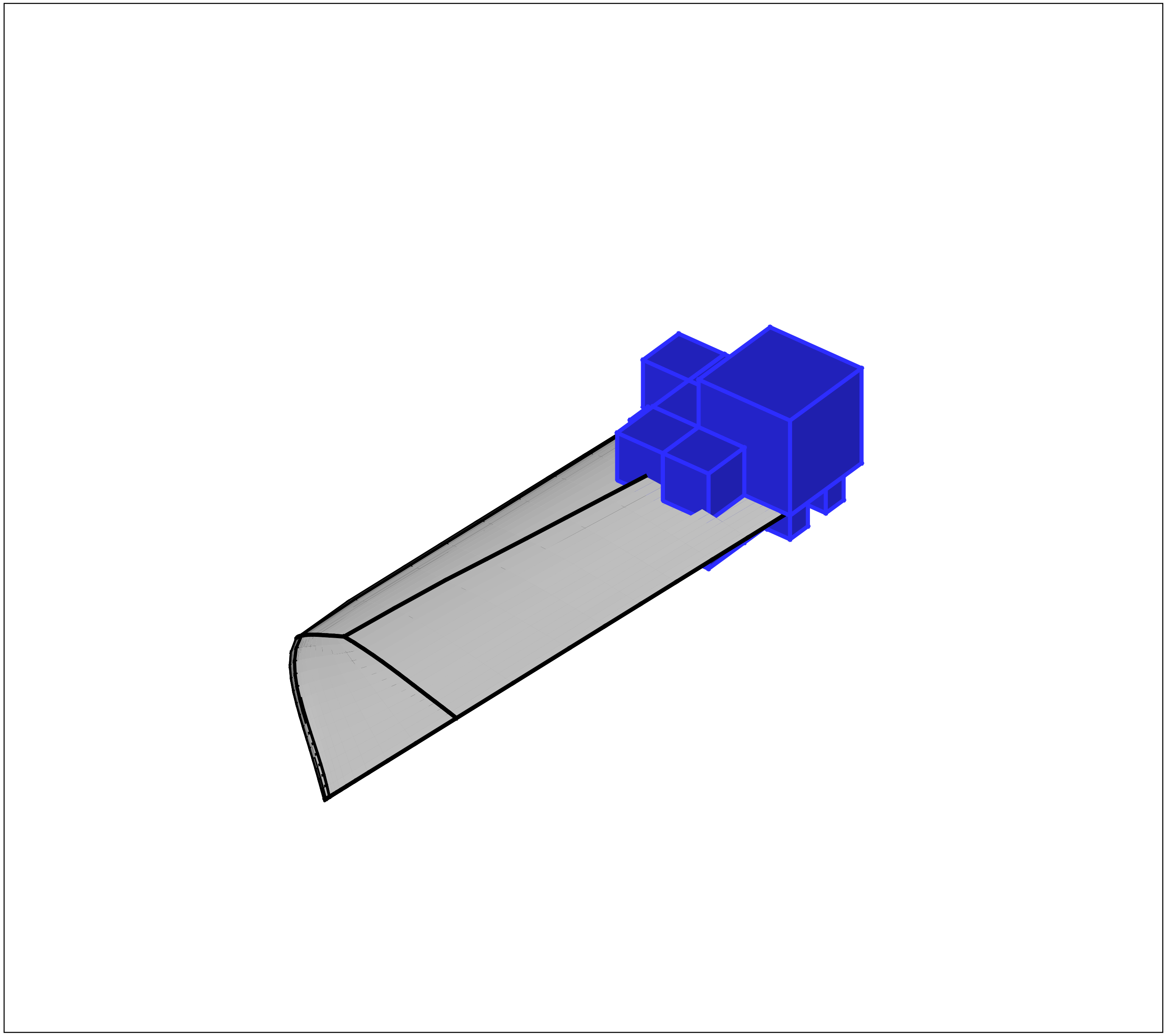
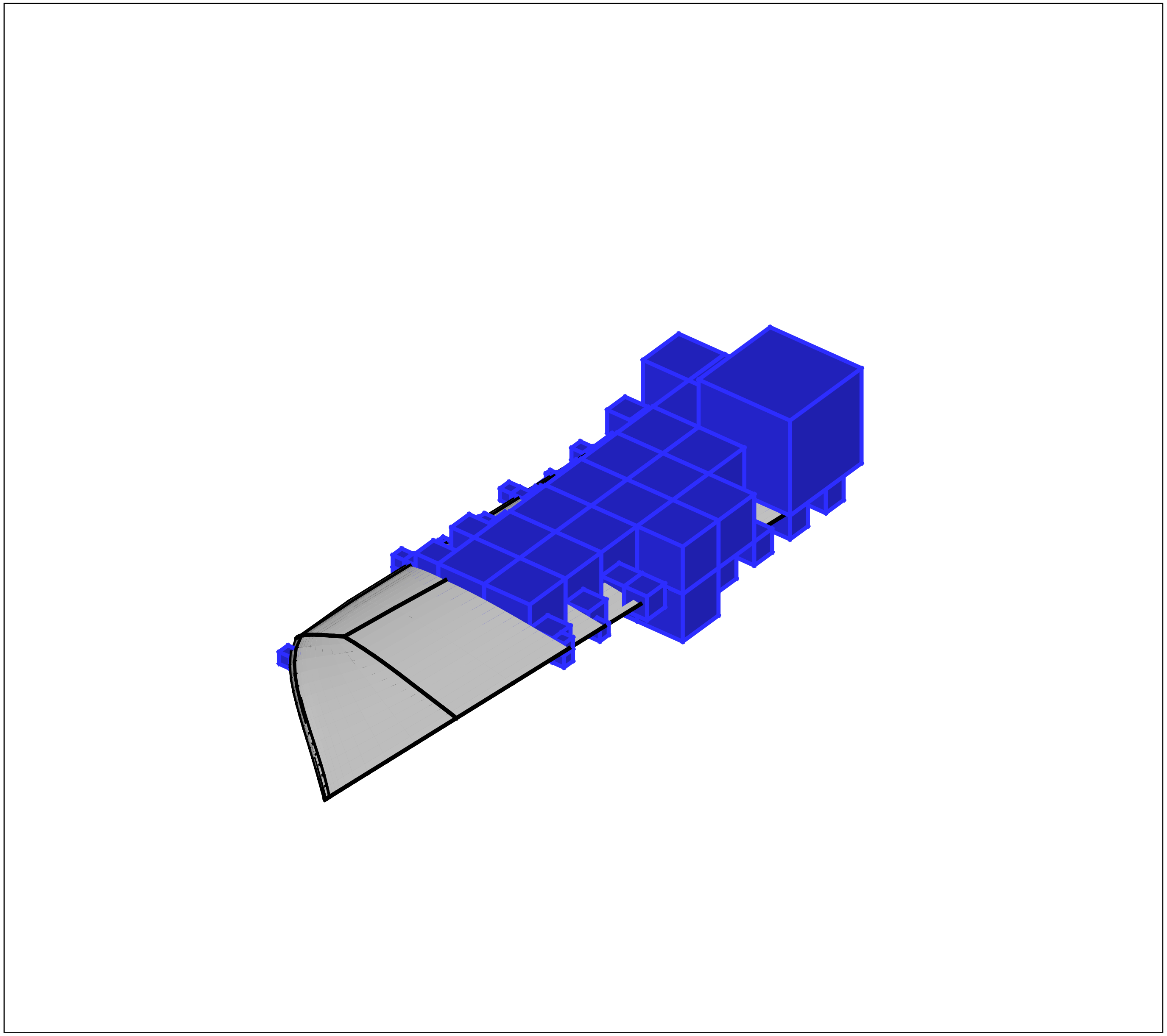
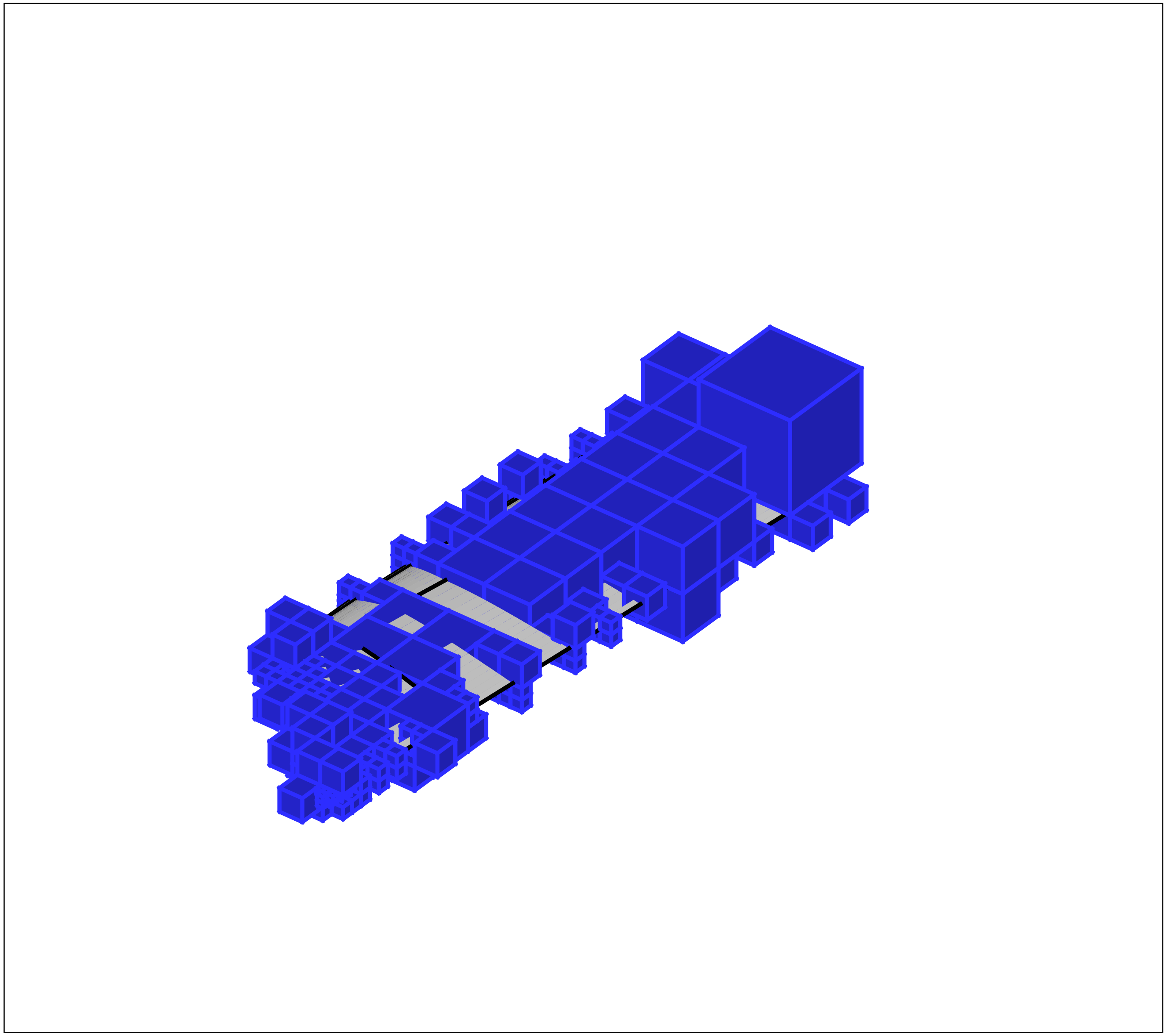


Interpolation sequence



Interpolation sequence



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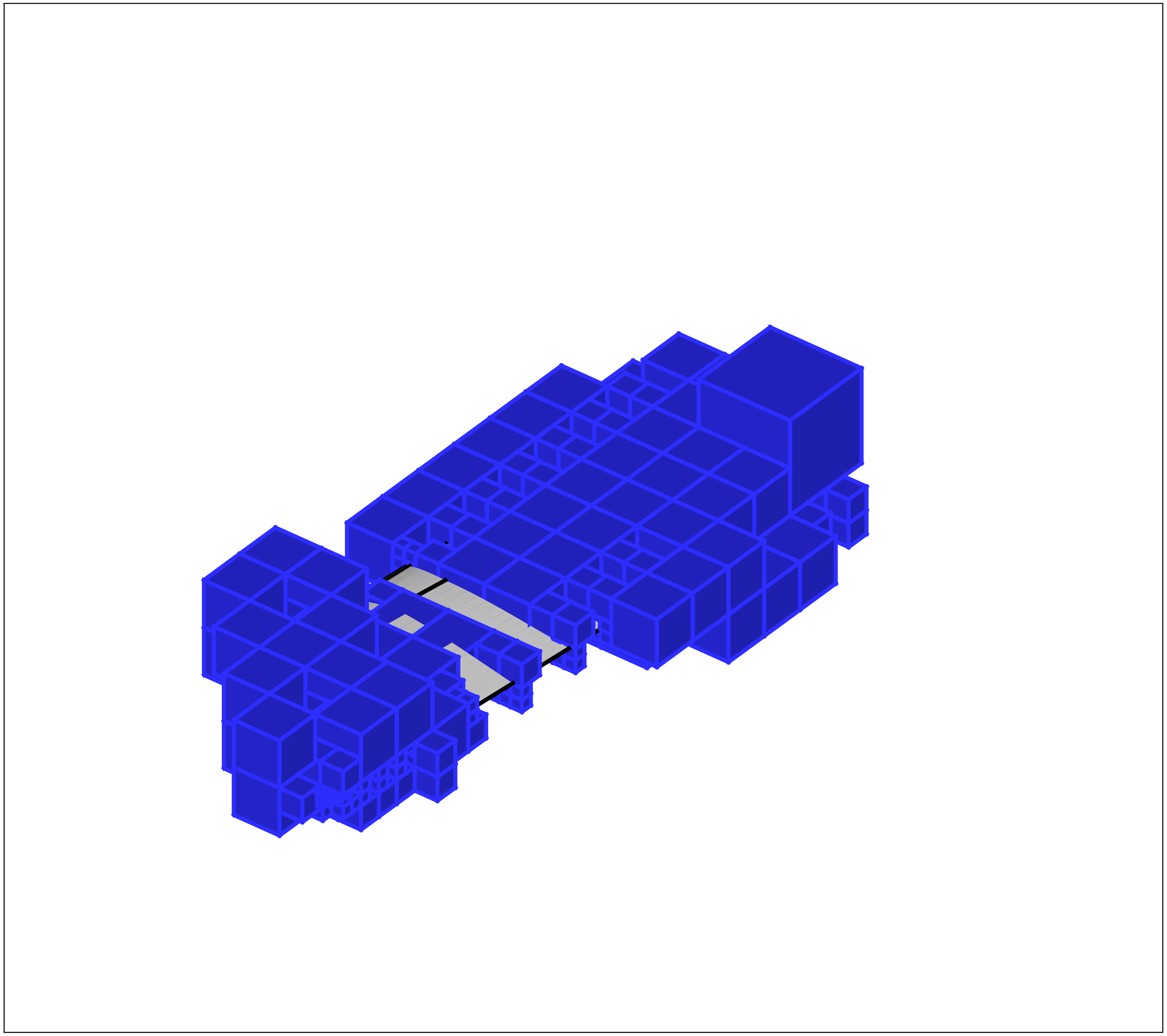

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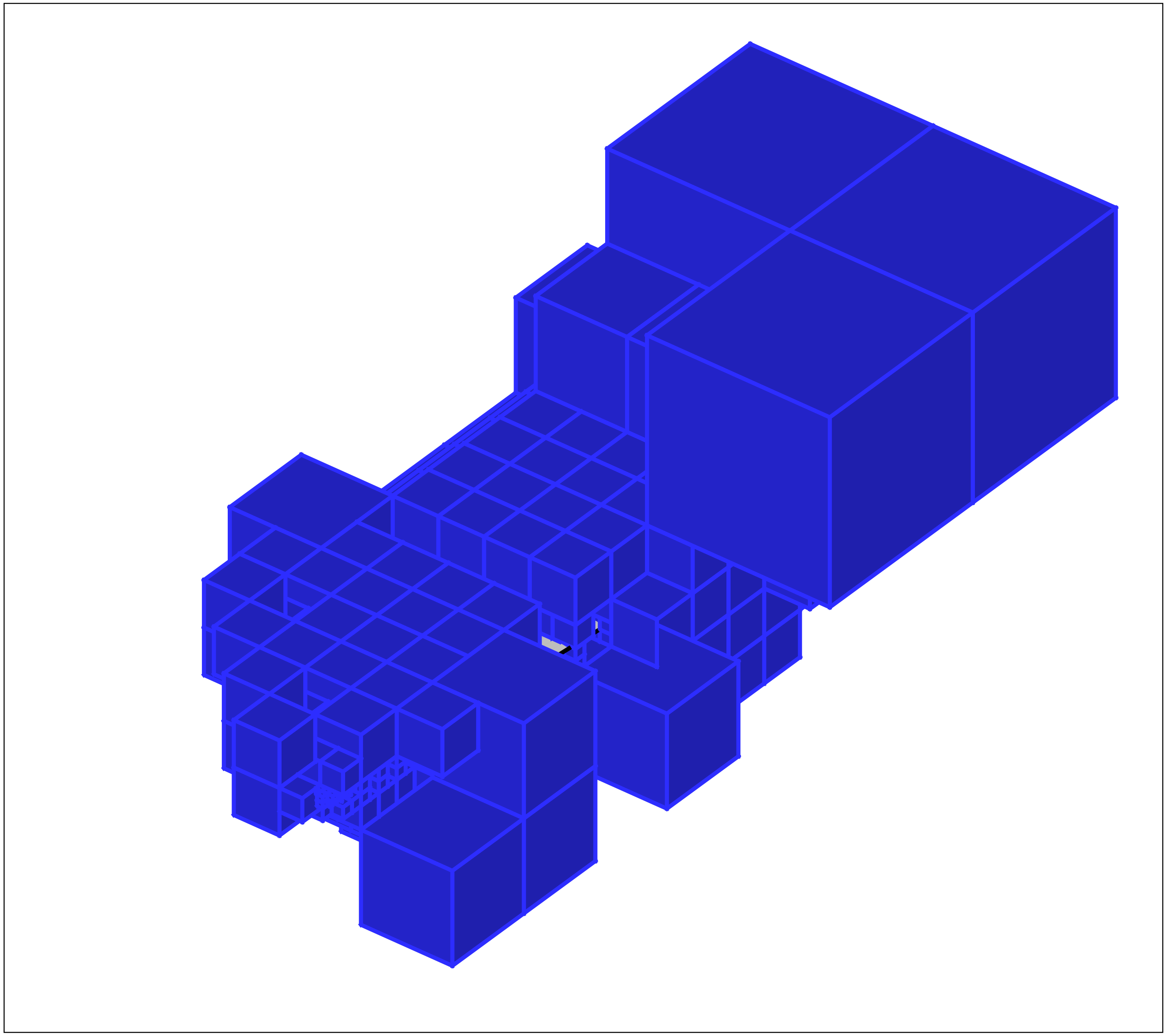

Computational strategy

- ***Computational bottlenecks***
  - Number of interpolation domains.
  - Maximum size of the interpolation matrix in each domain.

- ***Solution***
  - Two user-provided parameters.

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- *Max. number of nodes within an octree cube.*
  - It controls the number of interpolation domains
  - It is related with the preproccessing time

Computational strategy

- ***Computational bottlenecks***
  - Number of interpolation domains.
  - Maximum size of the interpolation matrix in each domain.
- ***Solution***
  - Two user-provided parameters.
- *Max. number of centers within an interpolation domain.*
  - It controls the size of the interpolation matrix C_{ss}
  - It is related with the evaluation time

Mesh quality metrics

Mesh quality metrics

Two quality algebraic metrics have been incorporated to measure the quality of the deformed mesh in order to:

- Prove the validity of the methodology.
- Stop the computation cycle: Maximum allowed deformation
  - when any of the quality parameters go below a prescribed threshold (degenerated mesh).

Mesh quality metrics

- ***Relative size metric***

$$f_{size} = \min\{\tau, \frac{1}{\tau}\}$$

Degenerated deformed element $\Longrightarrow f_{size} < 0$

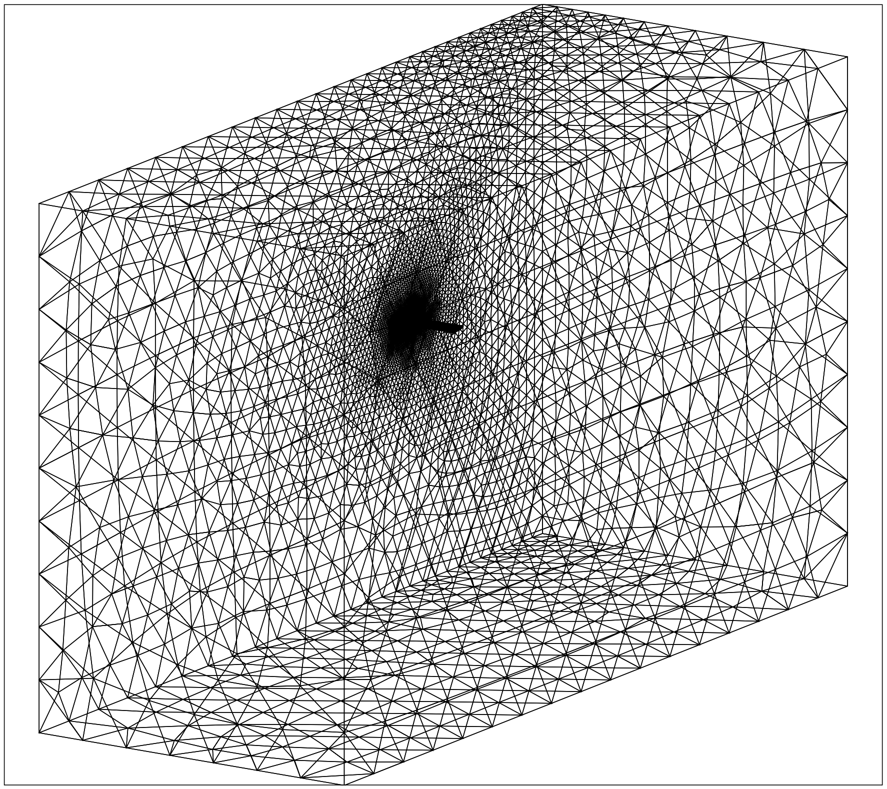
- ***Shape metric***

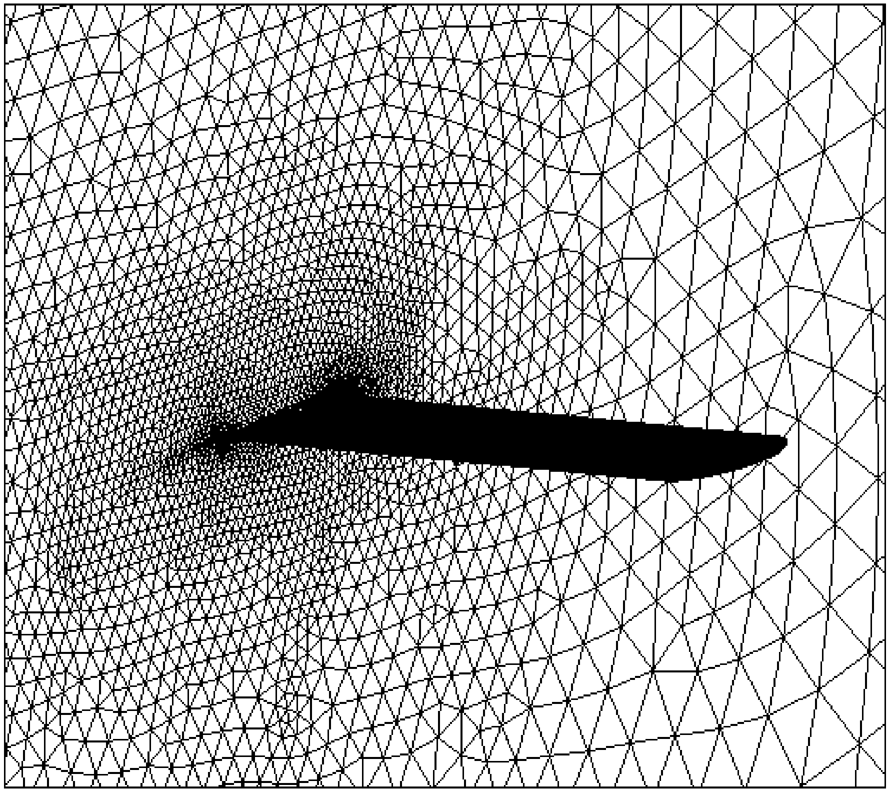
f_{shape} combination of skew metric and element edge-length ratios

Three edges at one vertex coplanar $\Longleftrightarrow f_{shape} = 0$

Numerical results

Inviscid mesh. NACA0012





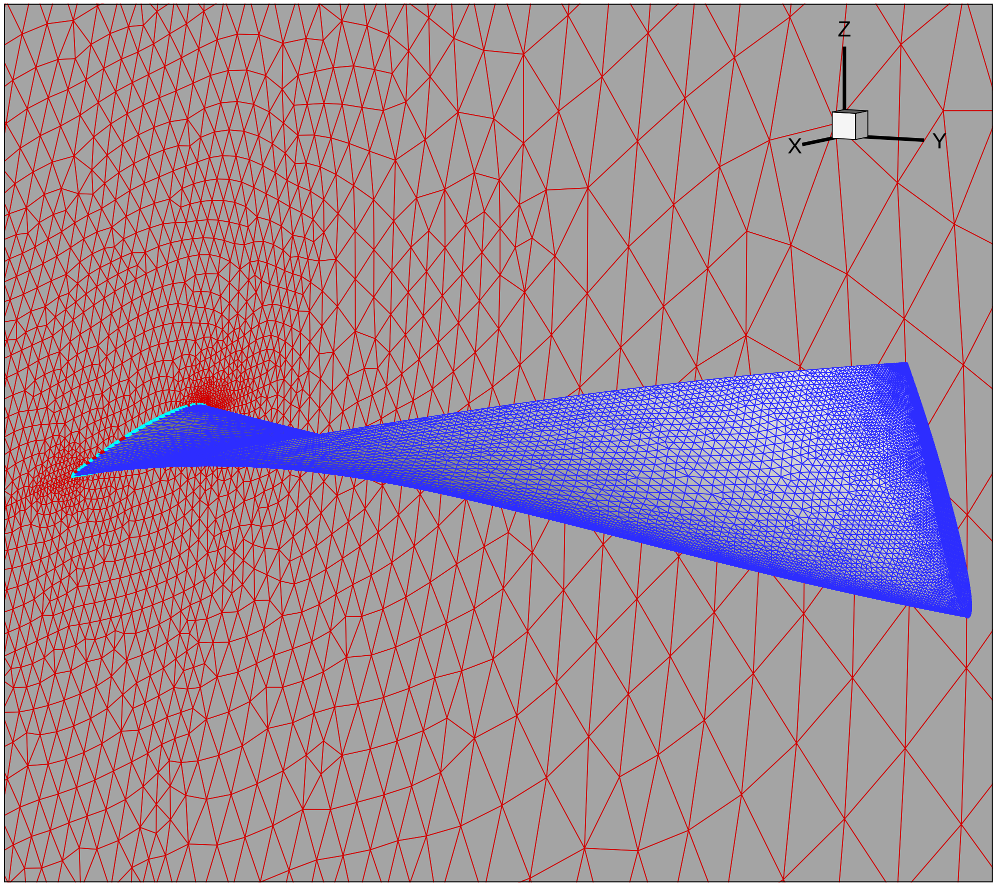
- 34.000 surface mesh nodes
- 180.000 volumetric mesh nodes

Types of deformations

Torsion

$$\varphi(y) = \frac{y}{L}\,\varphi_L$$

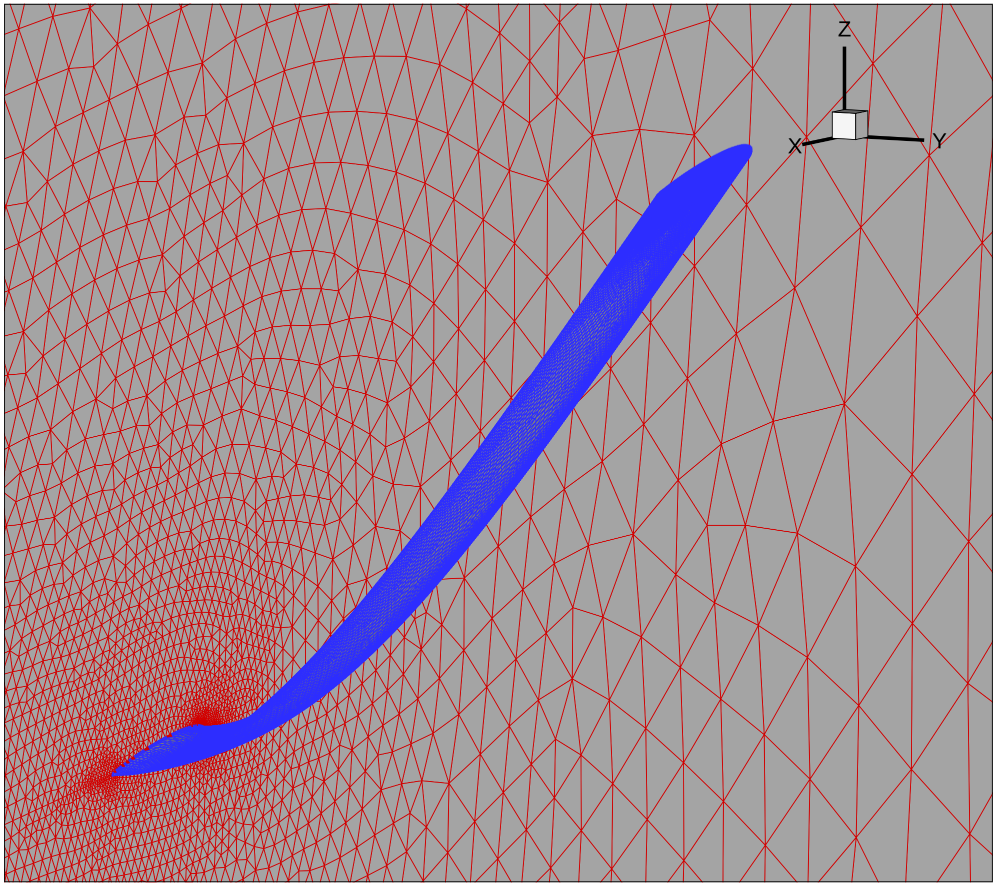
$\varphi_L \equiv$ twist at wing tip



Bending

$$\eta(y) = \frac{y^2(6L^2 - 4Ly + y^2)}{3L^4}$$

$\eta \equiv$ vertical displacement



Robustness tests

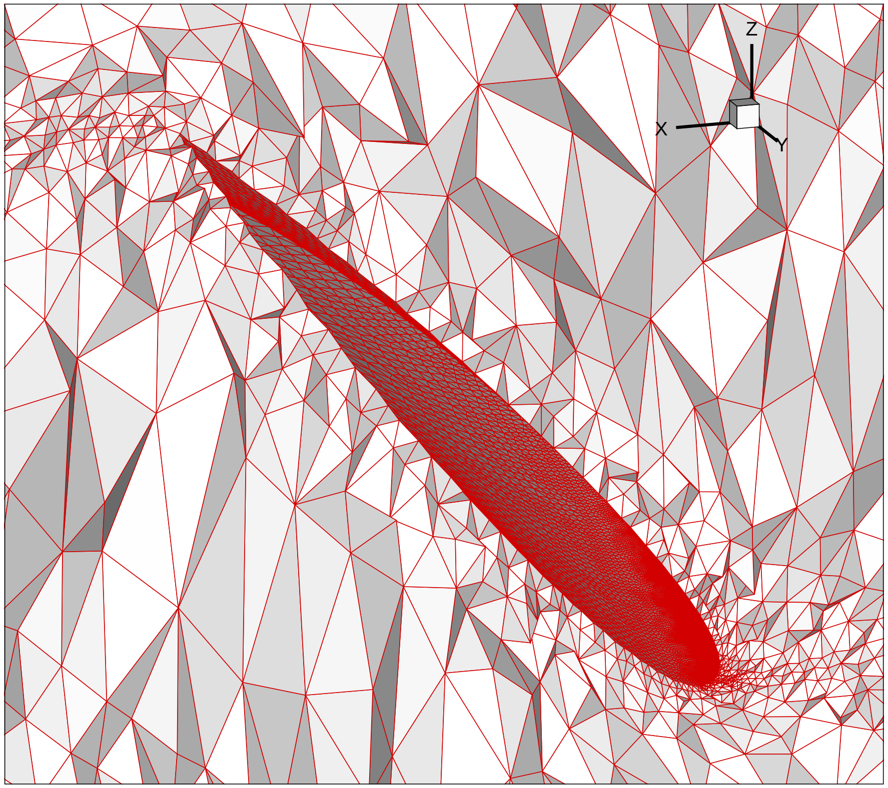
- Maximum deformation running the algorithm once.
- Maximum deformation running it iteratively.

Test	spline	Wend. $\mathcal{C}^0$	Wend. $\mathcal{C}^2$
$\varphi_{L_{max}}$ one step	51^{o}	37^{o}	34^{o}
$\varphi_{L_{max}}$ multiple steps	100^{o}	65^{o}	50^{o}
η_{max} one step	$54\,\%L$	$38\,\%L$	$34\,\%L$
η_{max} multiple steps	$100\,\%L$	$100\,\%L$	$100\,\%L$

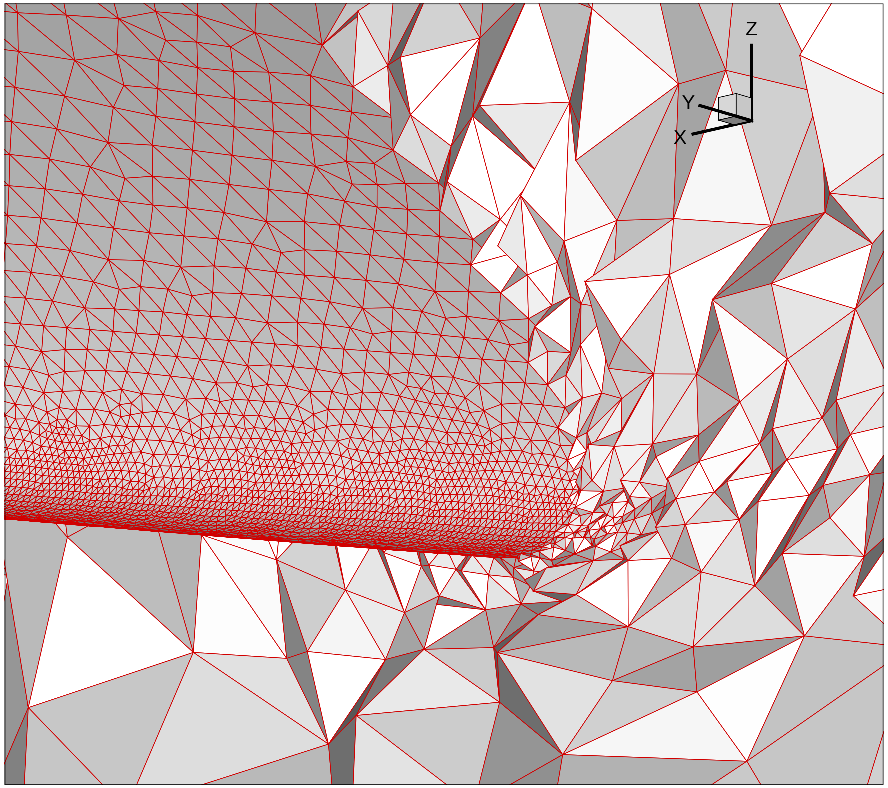
- ***Mesh quality condition***: $\left.\begin{array}{l} f_{size} > 0 \\ f_{shape} > 0 \end{array}\right\}$ Both close to 1

$70\,\%$ twist at wing tip

Close to wing tip ($80\,\%$ span)



Leading edge



Mean values of quality parameters:

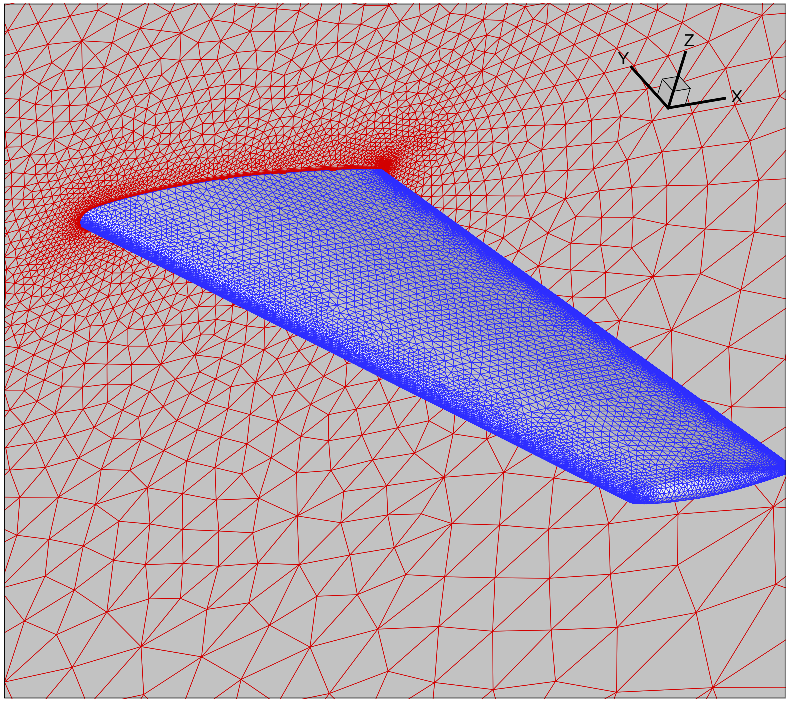
$$f_{size} = 0.996 \qquad f_{shape} = 0.9996$$

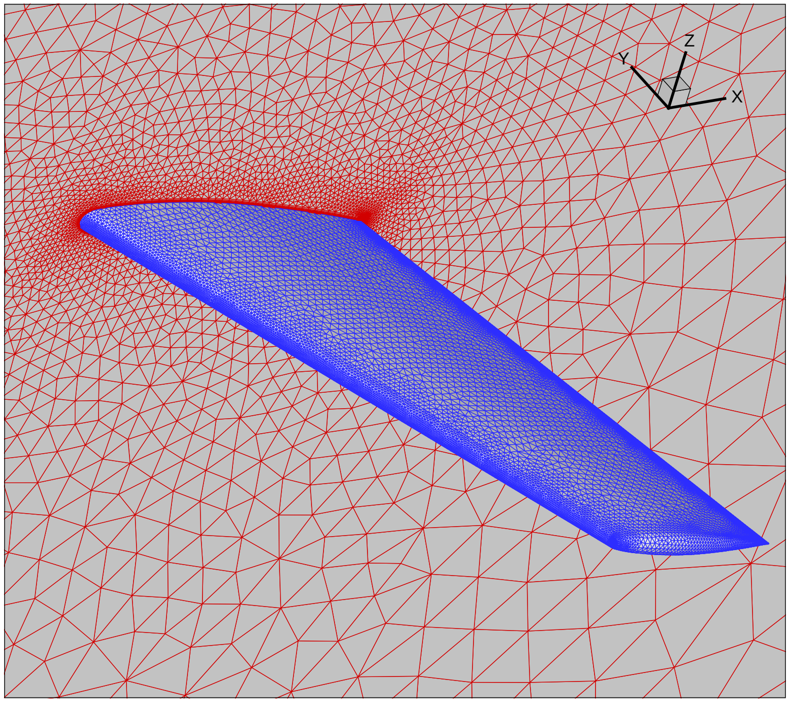
Viscous mesh. ONERA M6

Original mesh

Deformed mesh

(Wing rotation of 10º)





- 43.200 surface mesh nodes
- 1.500.000 volumetric mesh nodes

Quality mesh test

Two flow computations have been carried on using DLR_TAU code (Mach=0.2, Re = 11.2×10^6)

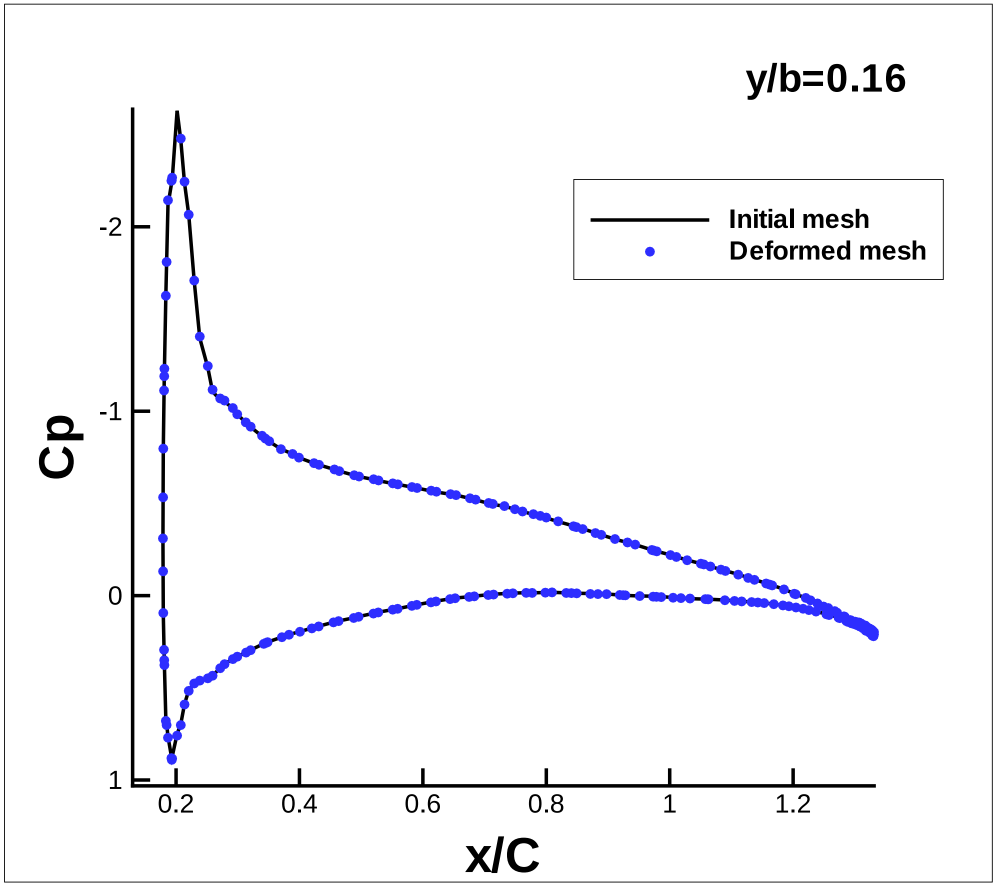
1. Over the original mesh with $\alpha = 10$º
2. Over the deformed mesh with $\alpha = 0$º

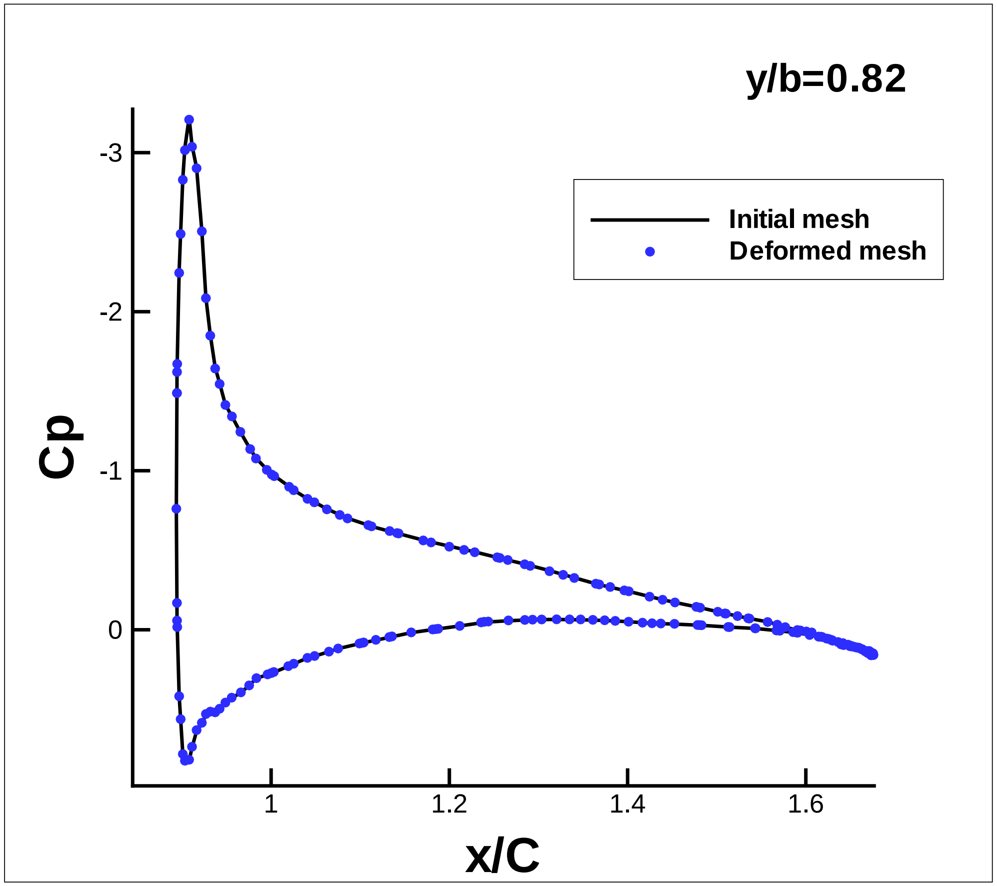
- Mean values of quality parameters:

$$f_{size} = 0.984 \qquad f_{shape} = 0.995$$

Quality mesh test

Pressure coefficient distributions at sections of wing





Conclusions

- An general interpolation tool based on RBFs together with an advancing front strategy for moving 3D meshes has been developed.
- It can be applied to any kind of meshes (structured or unstructured).
- It is robust, efficient and preserves the quality of the original mesh for very large deformation.
- It can be parallelized because of the inherent domain decomposition strategy.

Thank you