An interpolation tool for aeroelastic data transfer problems

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Motivation

Problem: Transfer deformations between meshes

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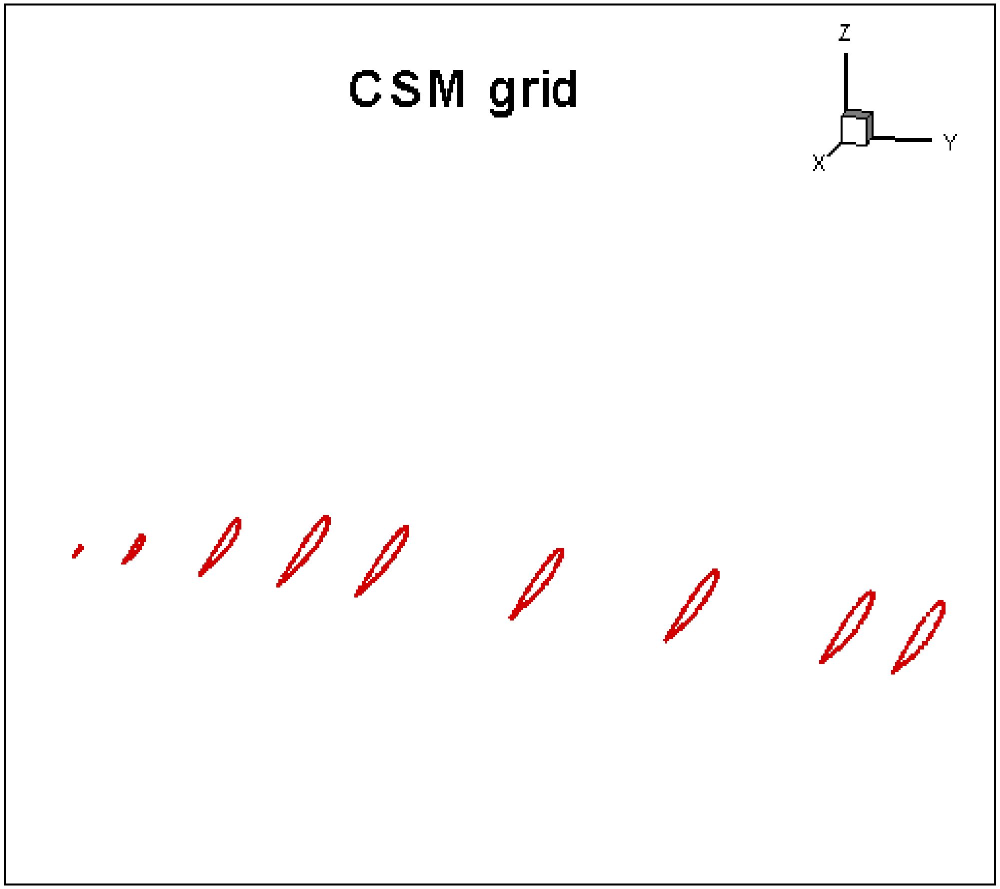
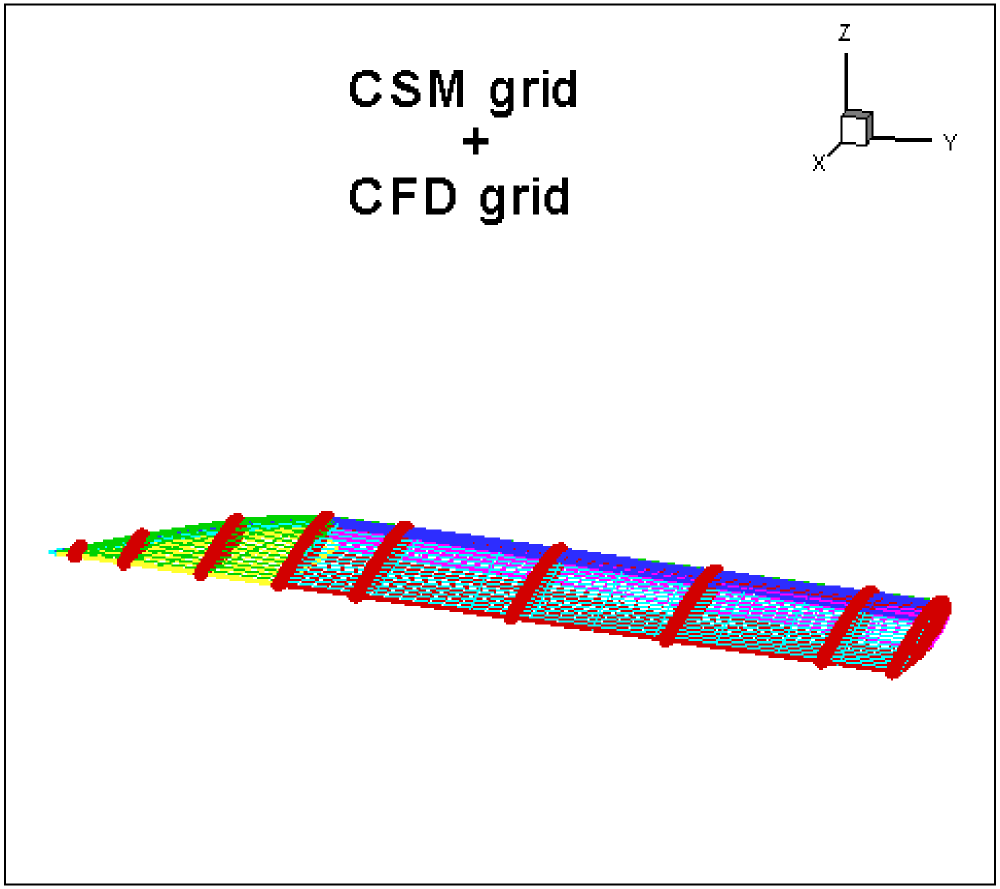
- Transfer deformations from a structural mesh (CSM grid) to an aerodynamic mesh (CFD grid)

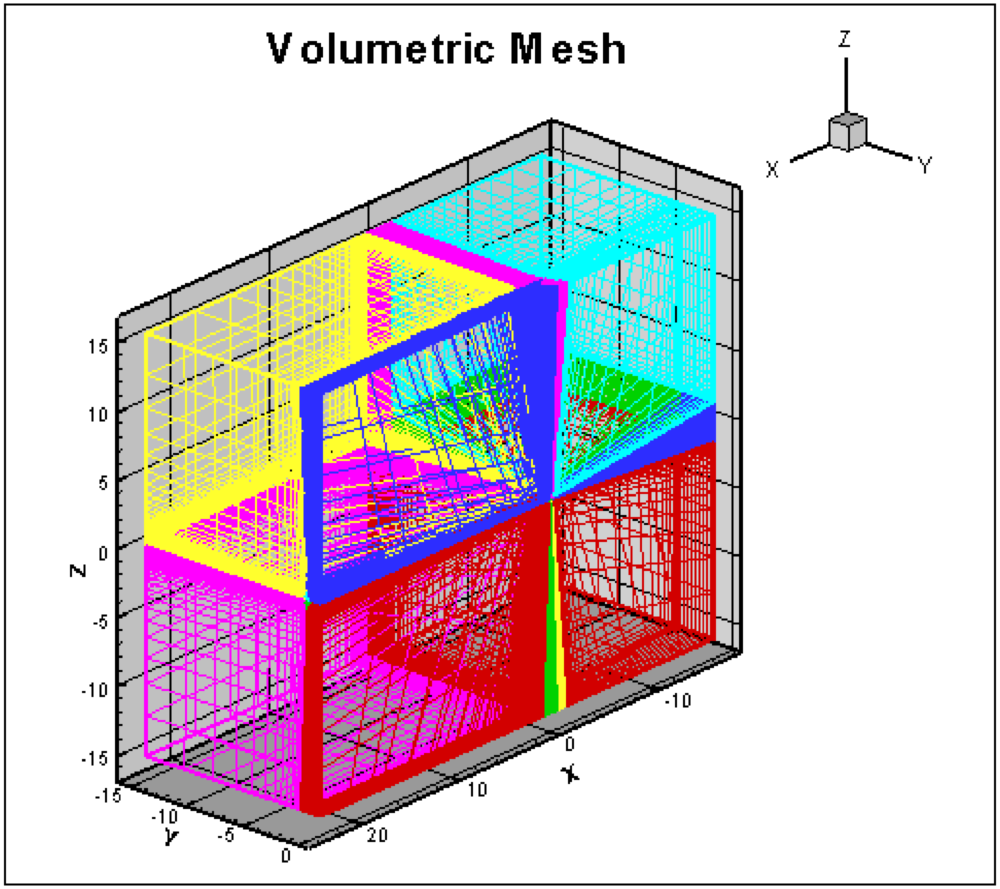


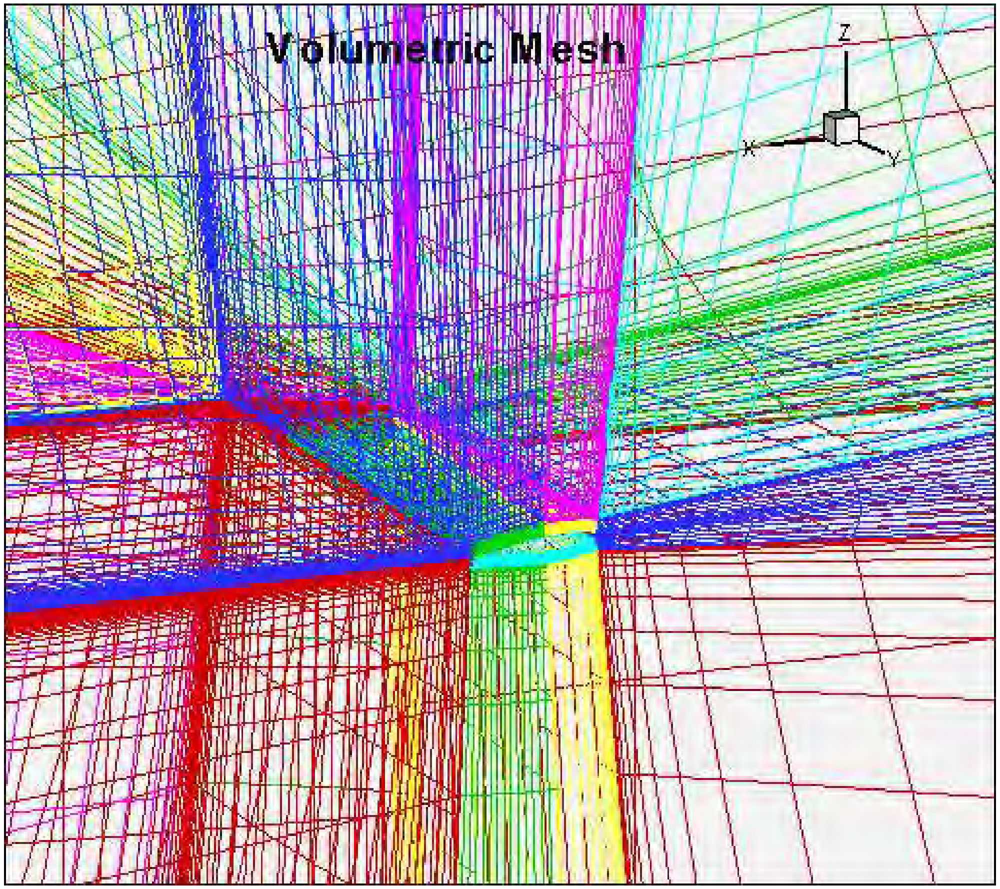


Motivation

Problem: Transfer deformations between meshes

- Transfer deformations from an aerodynamic mesh (CFM grid) to a volumetric mesh (CFD grid)





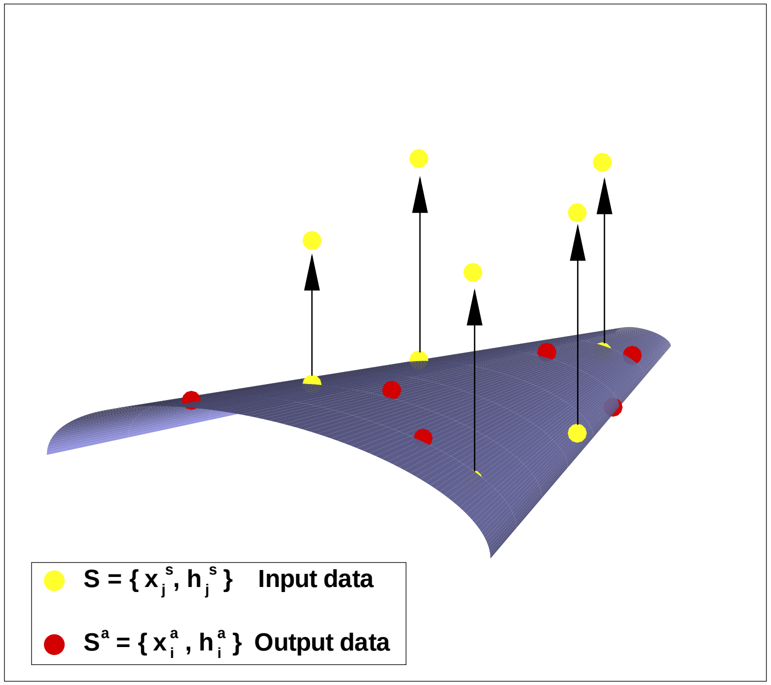
Motivation

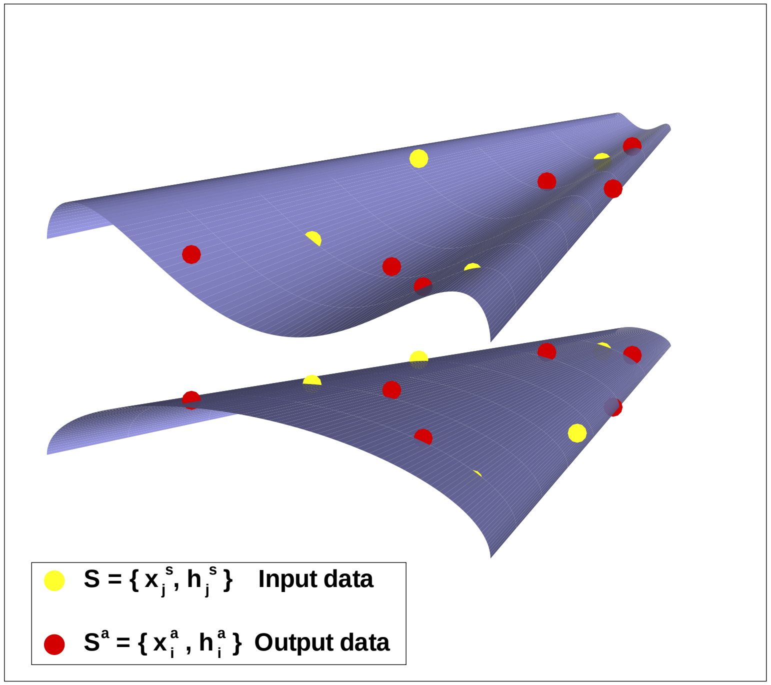
- Transfer (***using an interpolator***) deformations from a structural mesh to an aerodynamic or surface mesh.
  - Low computational cost.
  - Smooth representation.
  - Mesh quality conservation.
  - Applicability to any 3D data set (any kind of 3D meshes: structured, multiblock structured, unstructured and hybrid).

Interpolation

Interpolation

- Given N_s centers $\{x_1^s, \ldots, x_{N_s}^s\}$ and their displacements $\{h_1^s, \ldots, h_{N_s}^s\}$, and N_a evaluation nodes $\{x_1^a, \ldots, x_{N_a}^a\}$
- The problem consists in obtaining the displacements $\{h_1^a, \ldots, h_{N_a}^a\}$ via interpolation methods, in a smooth and regular way.





RBF Interpolation method

Reconstruct a continuous spatial distribution $h(\bar{x})$ using the discrete values $\bar{x}_i^s$

$$h(\bar{x}) = \sum_{i=1}^{N_s} w_i \, \Phi(||\bar{x} - \bar{x}_i^s||) + \Pi(\bar{x})$$

where

- w_i are the coefficients.
- Φ is a basis function which is radial with respect to the Euclidean distance (***Radial Basis Function***)
- Π is a m degree polynomial that depends on the Φ function.

Requirements (Rendall & Allen, 2008)

- ***Interpolation condition*** $h_i^s \equiv h(\bar{x}_i^s)$
- ***Side condition*** $\sum_{i=1}^{N_s} w_i \, q(\bar{x}_i) = 0 \quad \deg(q) \leq \deg(\Pi)$
  - To recover translations and rotations.
  - To conserve forces and moments.
- ***Zero degree polynomial***
  - To avoid transfer of fictitious displacements

$$\Pi(\bar{x}) = \gamma_0 \Longrightarrow \sum_{i=1}^{N_s} w_i = 0$$

RBF Interpolation

- Coefficients computation

$$\left.\begin{array}{l} h_i^s = h(\bar{x}_i^s) \quad i = 1, \dots, N_s \\ \sum w_i = 0 \end{array}\right\} \Longrightarrow \text{System of } N_s + 1 \text{ equations}$$

$$\begin{pmatrix} 0 \\ h_1^s \\ h_2^s \\ \vdots \\ h_{N_s}^s \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 & \dots & 1 \\ 1 & \Phi_{s_1 s_1} & \Phi_{s_1 s_2} & \cdots & \Phi_{s_1 s_{N_s}} \\ 1 & \Phi_{s_2 s_1} & \Phi_{s_2 s_2} & \cdots & \Phi_{s_2 s_{N_s}} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \Phi_{s_{N_s} s_1} & \Phi_{s_{N_s} s_2} & \cdots & \Phi_{s_{N_s} s_{N_s}} \end{pmatrix} \begin{pmatrix} \gamma_0 \\ w_1 \\ w_2 \\ \vdots \\ w_{N_s} \end{pmatrix}$$

$$\boxed{\bar{h}^s = C_{ss}\,\bar{\omega}}$$

RBF Interpolation

- Applying to evaluation nodes

$$h_i^a = h(\bar{x}_i^a) = \sum_{k=1}^{N_s} w_k \, \Phi(||\bar{x}_i^a - \bar{x}_k^s||) + \gamma_0 \quad i = 1, \ldots, N_a$$

$$\begin{pmatrix} h_1^a \\ h_2^a \\ \vdots \\ h_{N_a}^a \end{pmatrix} = \begin{pmatrix} 1 & \Phi_{a_1 s_1} & \Phi_{a_1 s_2} & \cdots & \Phi_{a_1 s_{N_s}} \\ 1 & \Phi_{a_2 s_1} & \Phi_{a_2 s_2} & \cdots & \Phi_{a_2 s_{N_s}} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \Phi_{a_{N_a} s_1} & \Phi_{a_{N_a} s_2} & \cdots & \Phi_{a_{N_a} s_{N_s}} \end{pmatrix} \begin{pmatrix} \gamma_0 \\ w_1 \\ w_2 \\ \vdots \\ w_{N_s} \end{pmatrix}$$

$$\boxed{\bar{h}^a = A_{as} \, \bar{\omega}}$$

Applying the interpolator

- ***Strategy # 1:*** G-matrix calculation

$$\begin{aligned} \bar{h}^s &= C_{ss}\,\bar{\omega} \\ \bar{h}^a &= A_{as}\,\bar{\omega} \end{aligned} \quad \Longrightarrow \quad \bar{h}^a = A_{as}\,C_{ss}^{-1}\,\bar{h}^s = G\,\bar{h}^s$$

Applying the interpolator

- Strategy # 1: G-matrix calculation

$$\left.\begin{array}{l}\bar{h}^s = C_{ss}\,\bar{\omega} \\ \bar{h}^a = A_{as}\,\bar{\omega}\end{array}\right. \Longrightarrow \bar{h}^a = A_{as}\,C_{ss}^{-1}\,\bar{h}^s = G\,\bar{h}^s$$

- ***Strategy # 2:*** Solving linear algebraic system
  - Calculate the $\bar{\omega}$ vector of coefficients
  - Construct matrix A_{as}
  - Calculate the new values $\bar{h}^a = A_{as}\,\bar{\omega}$

Radial Basis Functions

Function	*Definition* $\Phi(\bar{x})$
Volume Spline	$\|\|\bar{x}\|\|$
Wendland $\mathcal{C}^0$	$(1-\|\|\bar{x}\|\|)_+^2$
Wendland $\mathcal{C}^2$	$(1-\|\|\bar{x}\|\|)_+^4\,(4\|\|\bar{x}\|\|+1)$

Test cases

Test cases

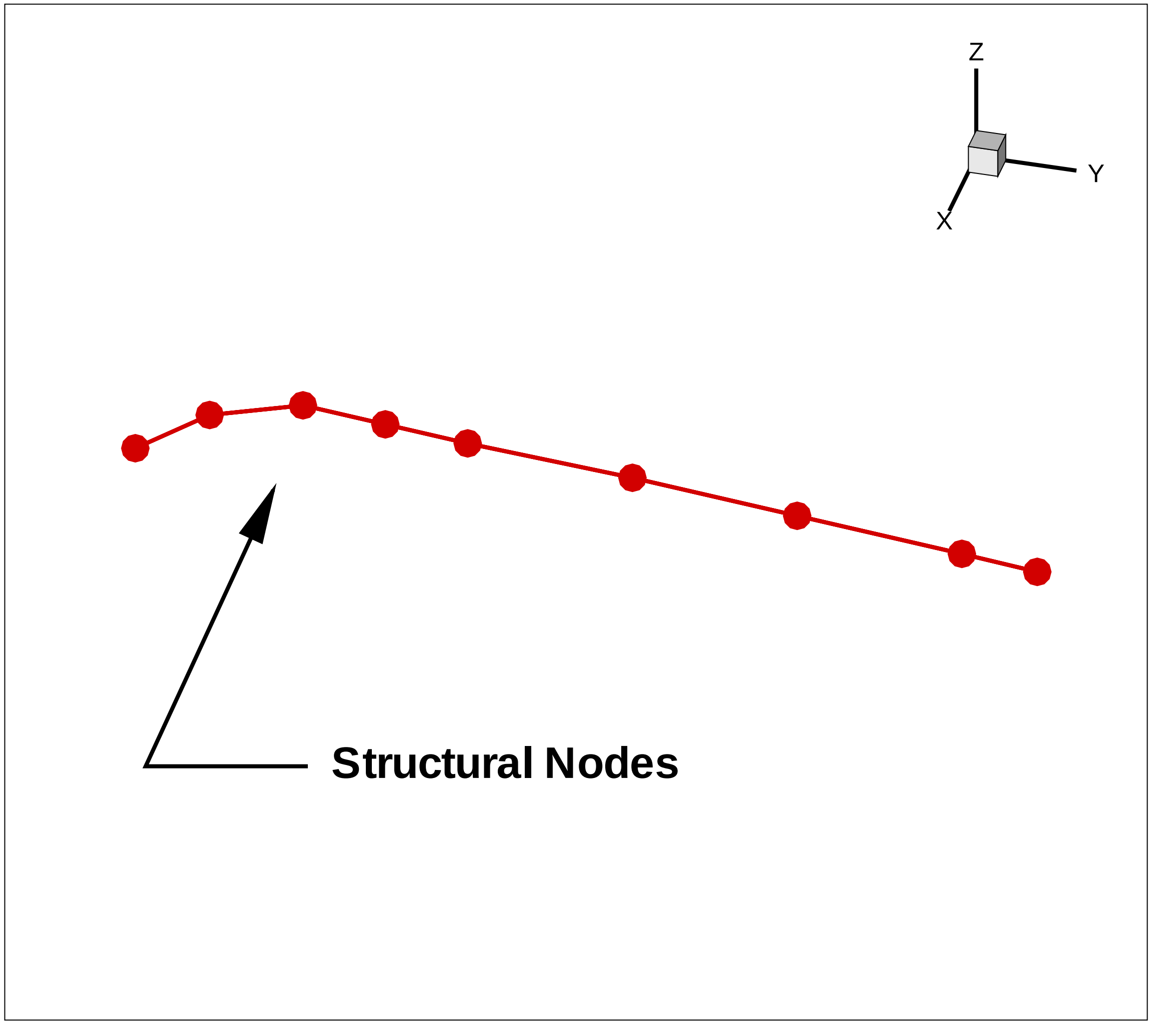
- **FE model**
  - Transfer deformations from the structural mesh to the aerodynamical mesh.

Test cases

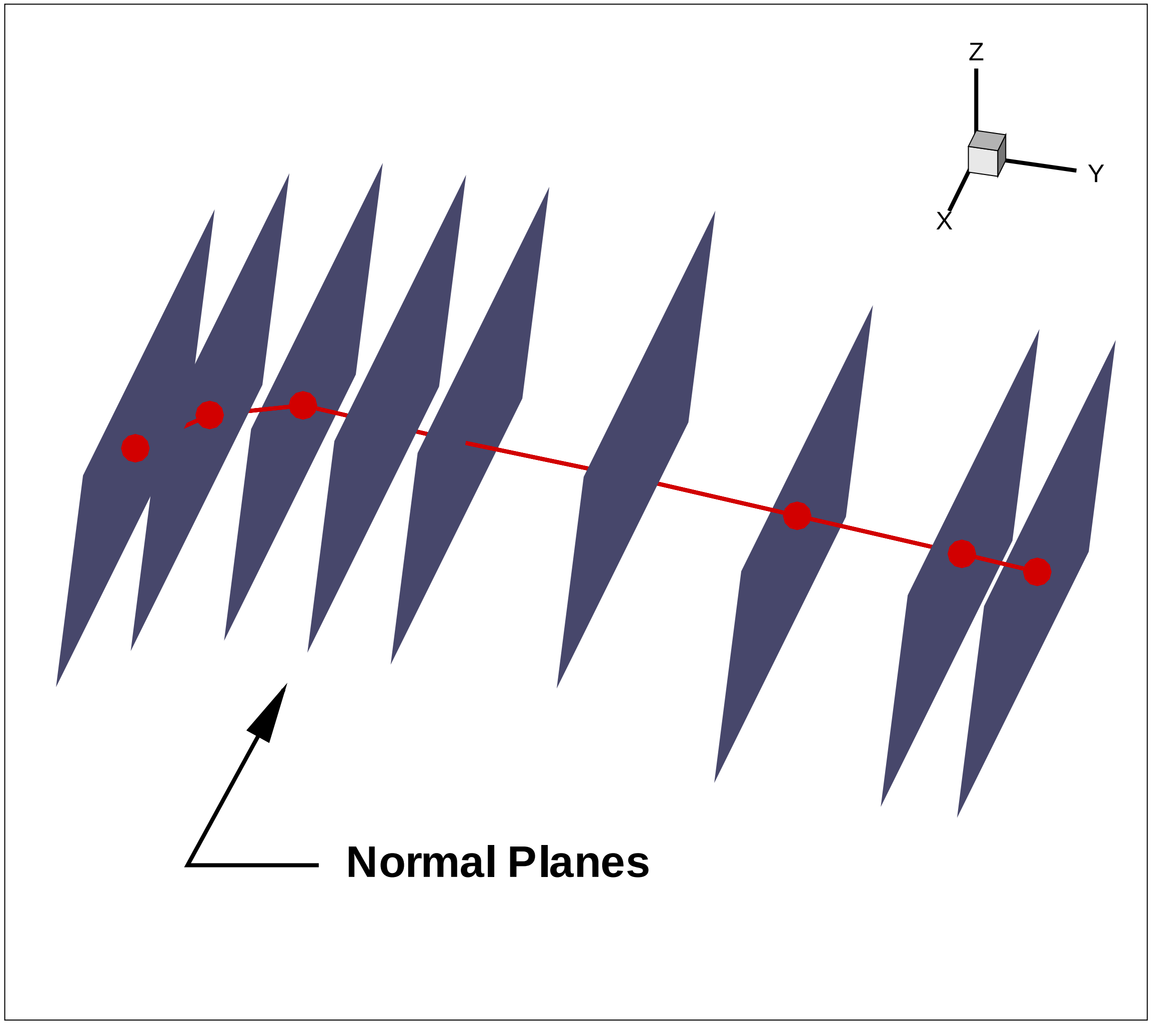
- **FE model**
  - Transfer deformations from the structural mesh to the aerodynamical mesh.
- **Stick model**
  - Generate a virtual FE structural mesh
    - According to stick nodes
    - Near of the aerodynamic surface
  - Transfer deformations from the stick to the virtual structural mesh.
  - Transfer deformations from the structural mesh to the aerodynamical mesh.

Stick model strategy

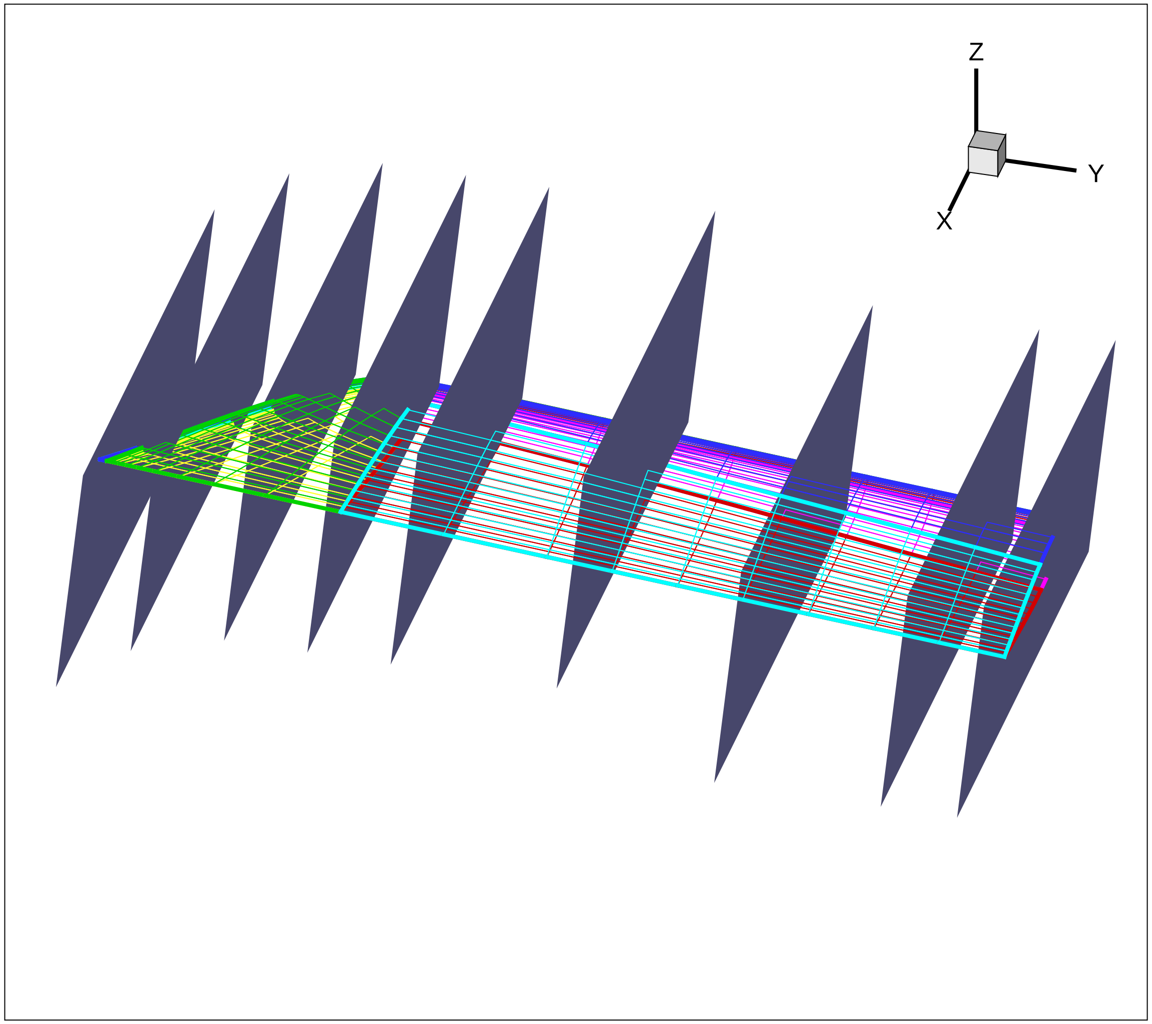
Stick model: Strategy



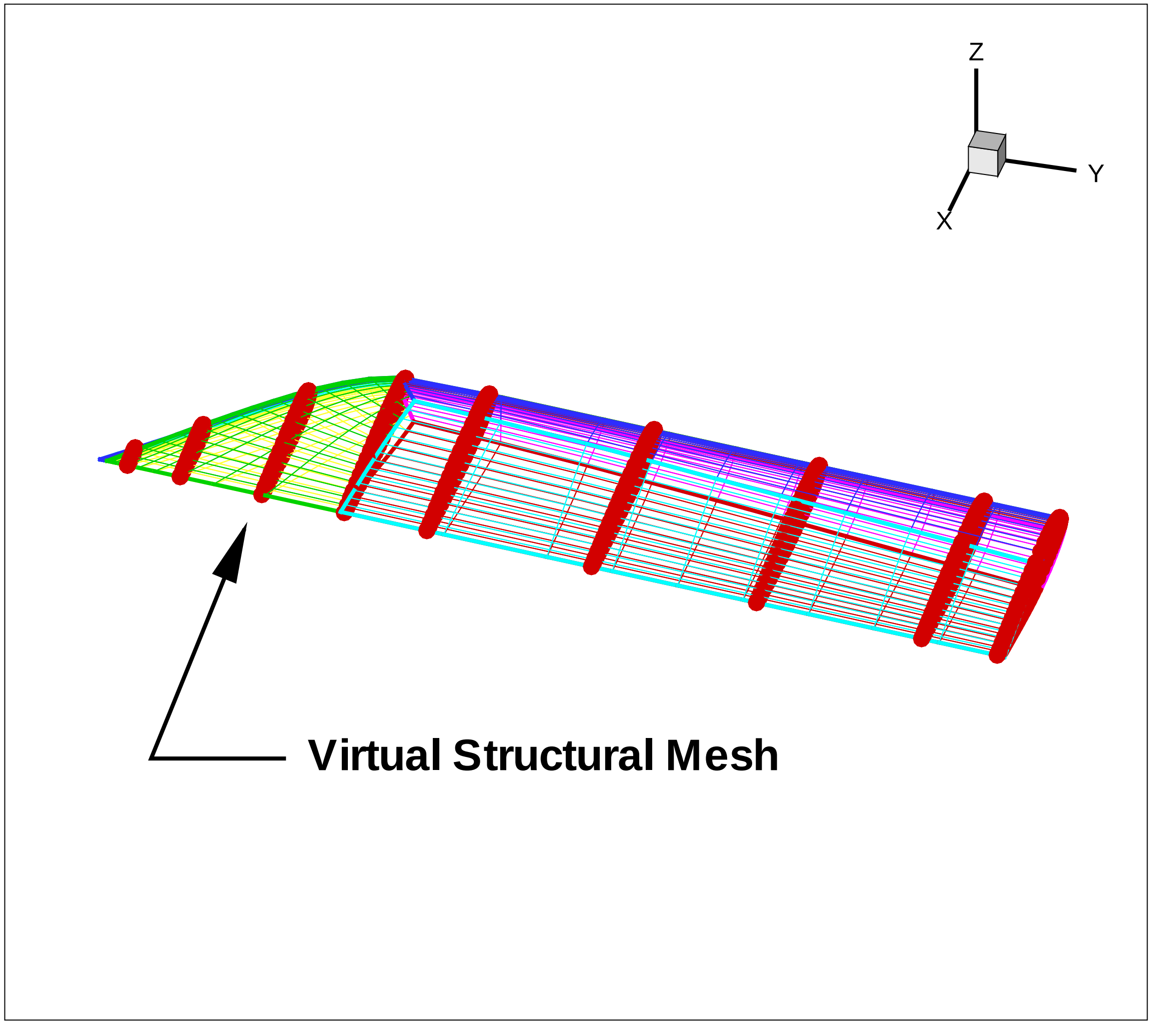
Stick model: Strategy



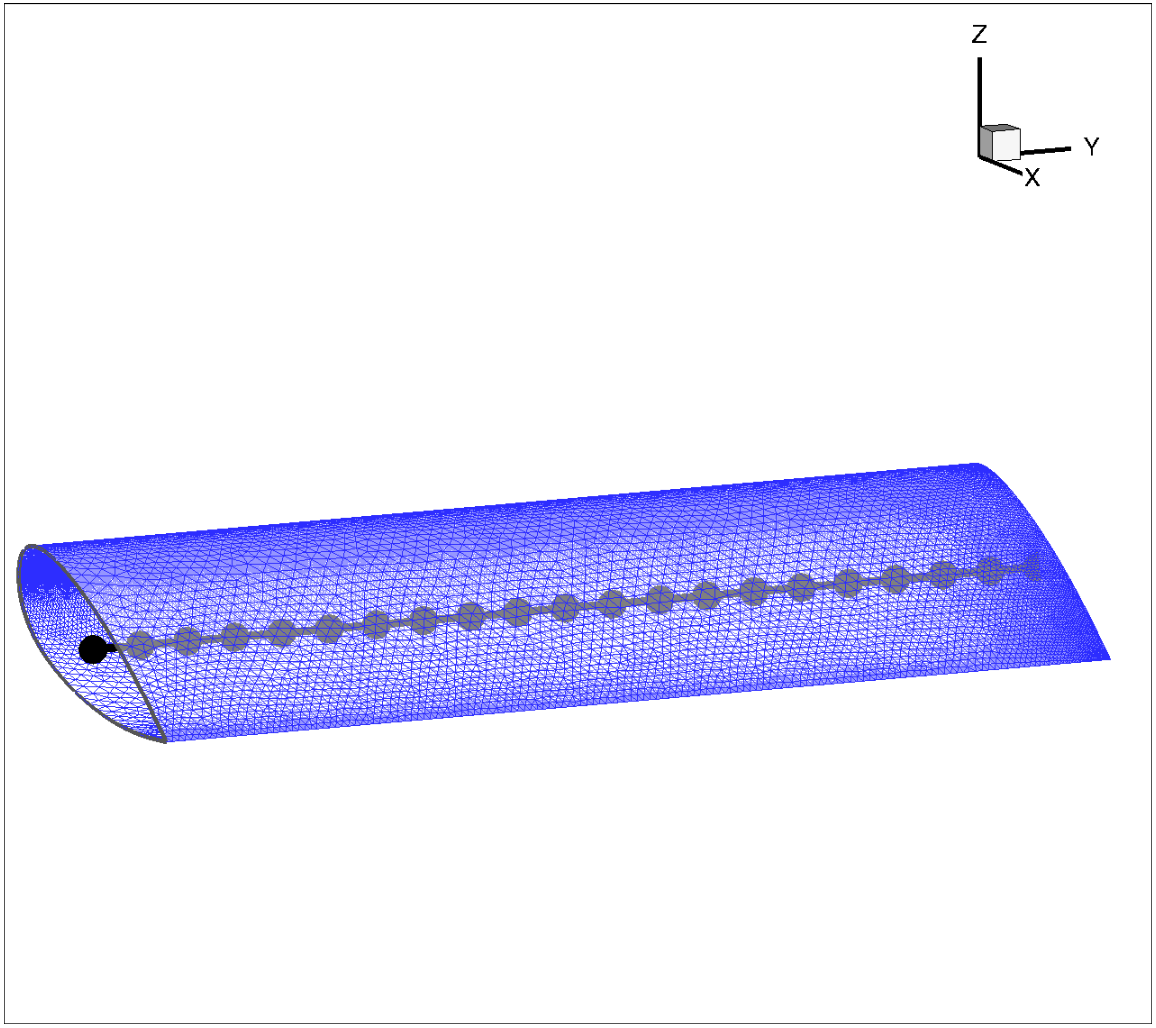
Stick model: Strategy



Stick model: Strategy

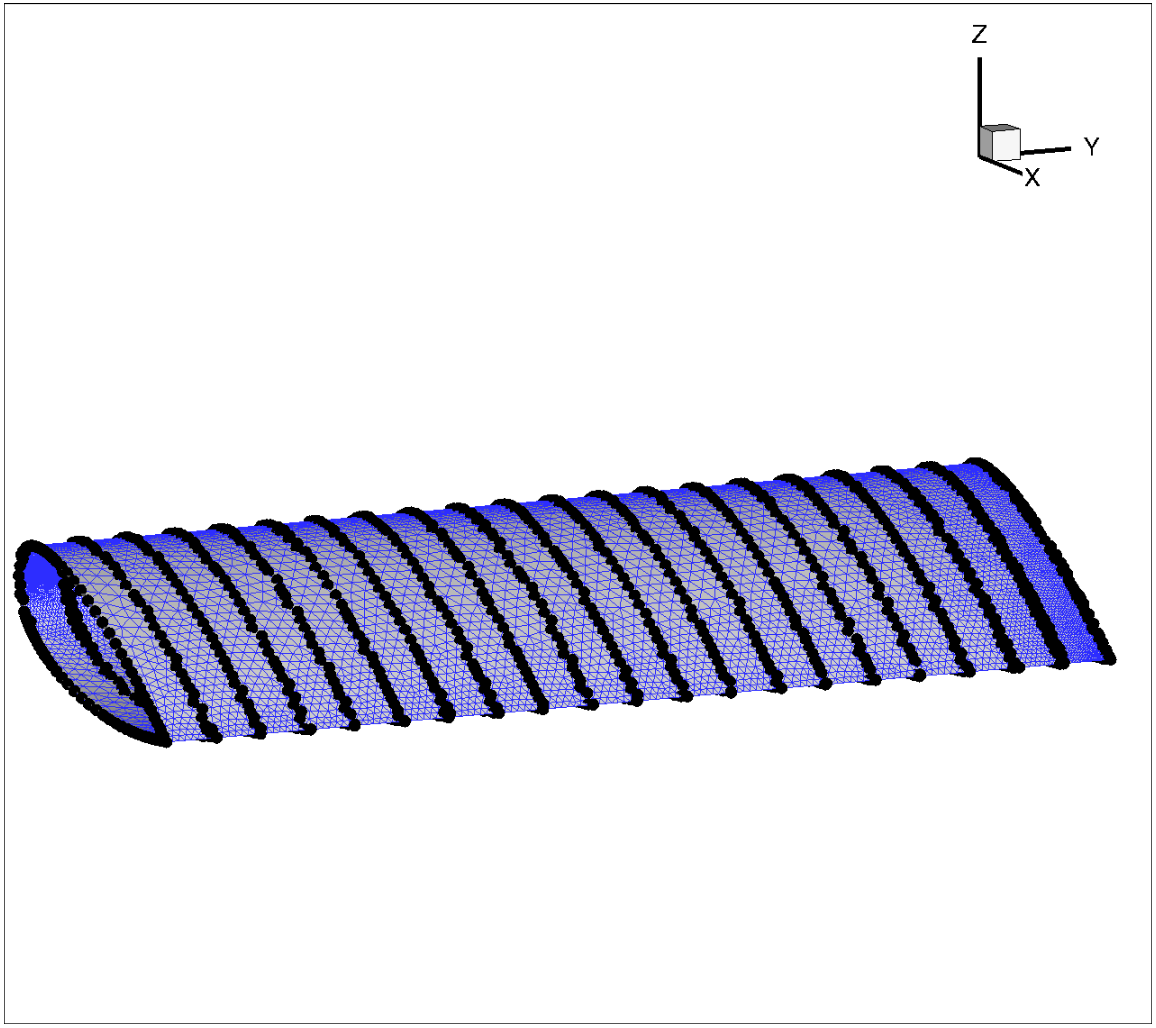


Stick model: NACA0012 wing



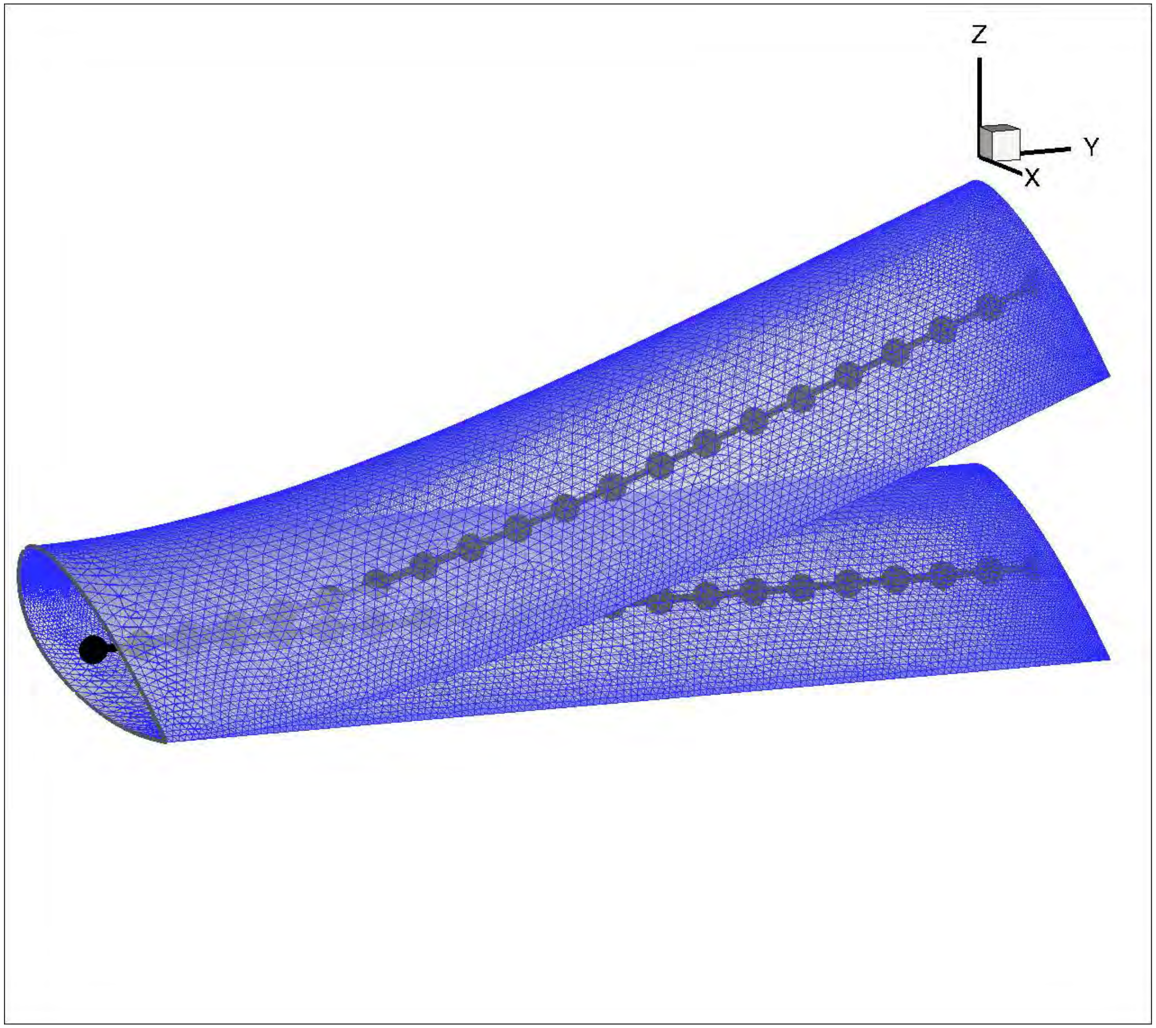
- 21 stick-nodes
- 34007 aerodynamic-nodes and 67918 mesh-elements

Stick model: NACA0012 wing



- Work structure (2666 nodes)

Stick model: NACA0012 wing

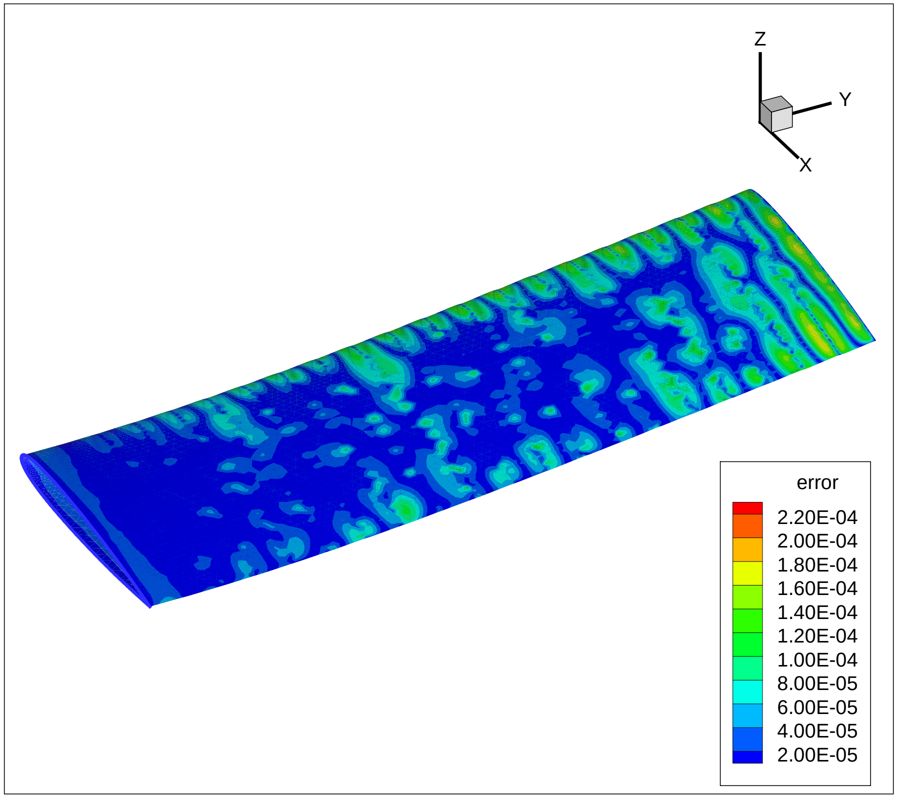


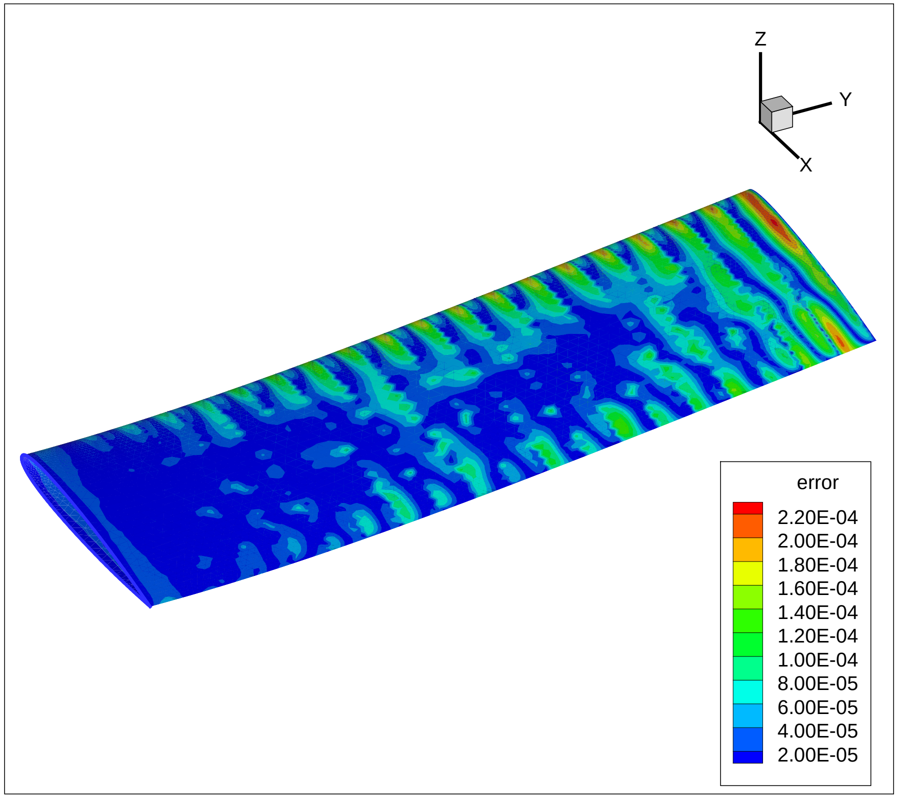
- Deformation

$$\eta(y) = \frac{y^2(6L^2 - 4Ly + y^2)}{3L^4} \qquad \eta_{\max} = 10\,\%L$$

Error contour maps

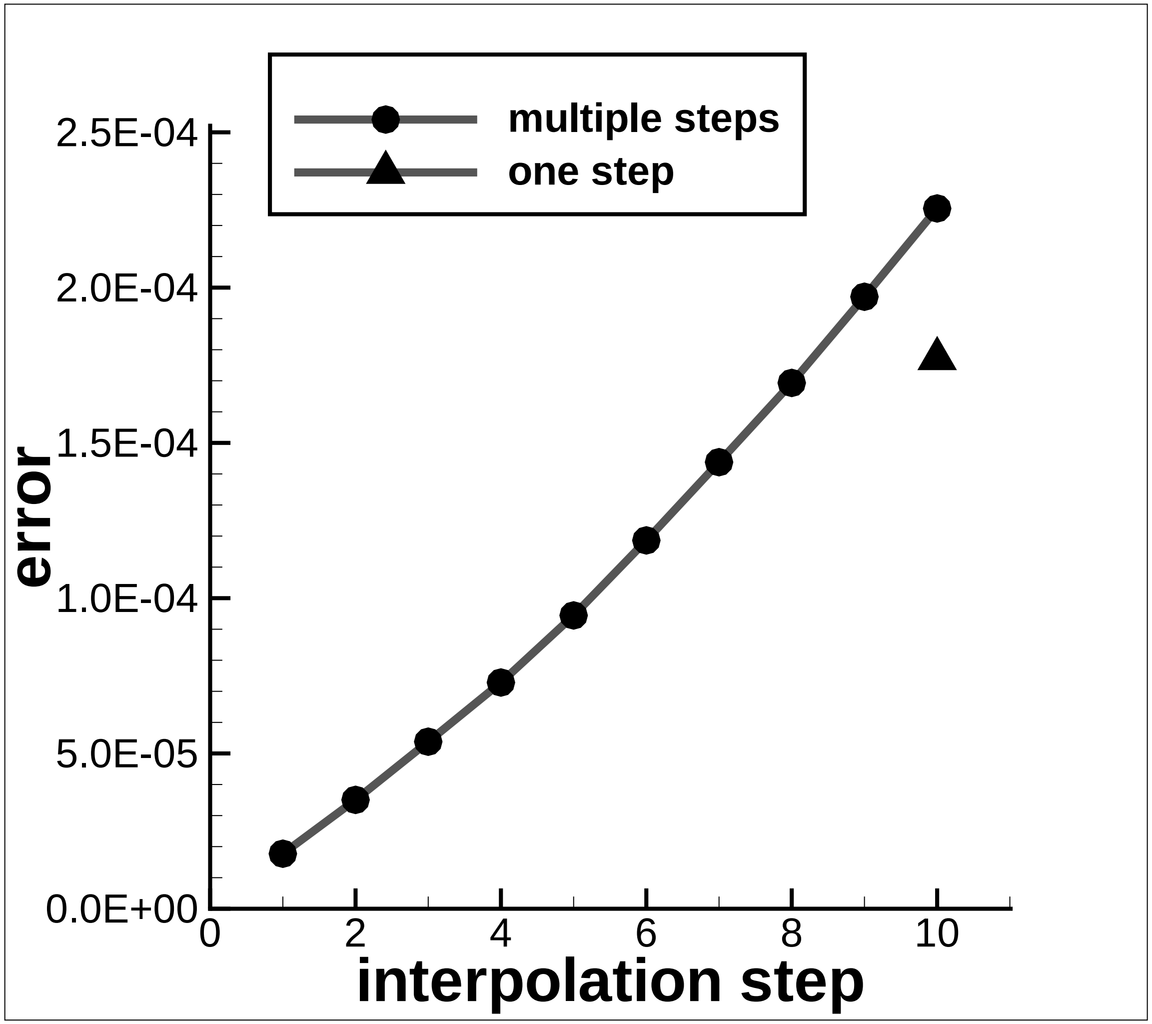
one step computation | multiple steps computation





$$Error_k = ||\boldsymbol{x}_{k,exact} - \boldsymbol{x}_{k,calc}||$$

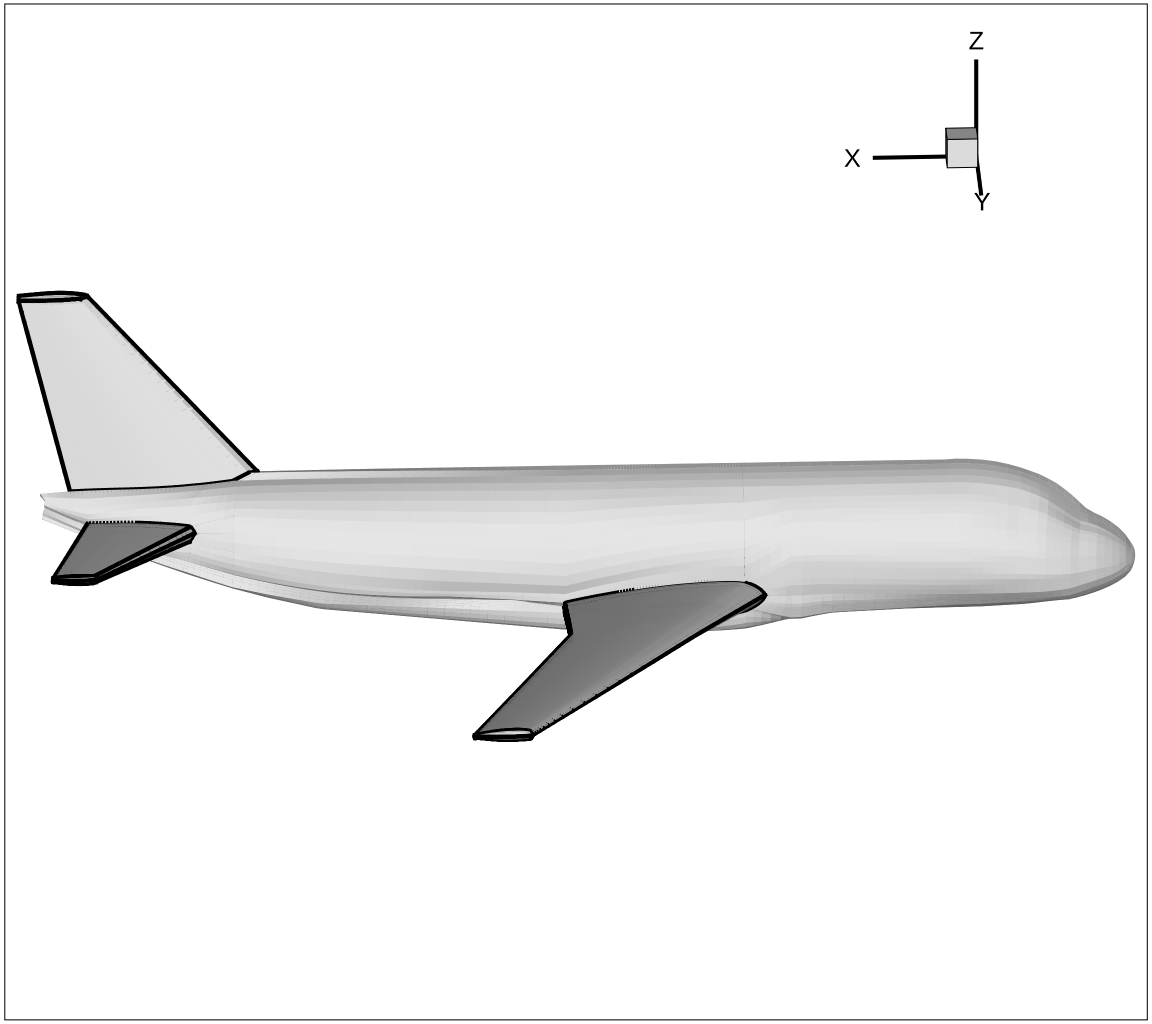
Maximum error estimation



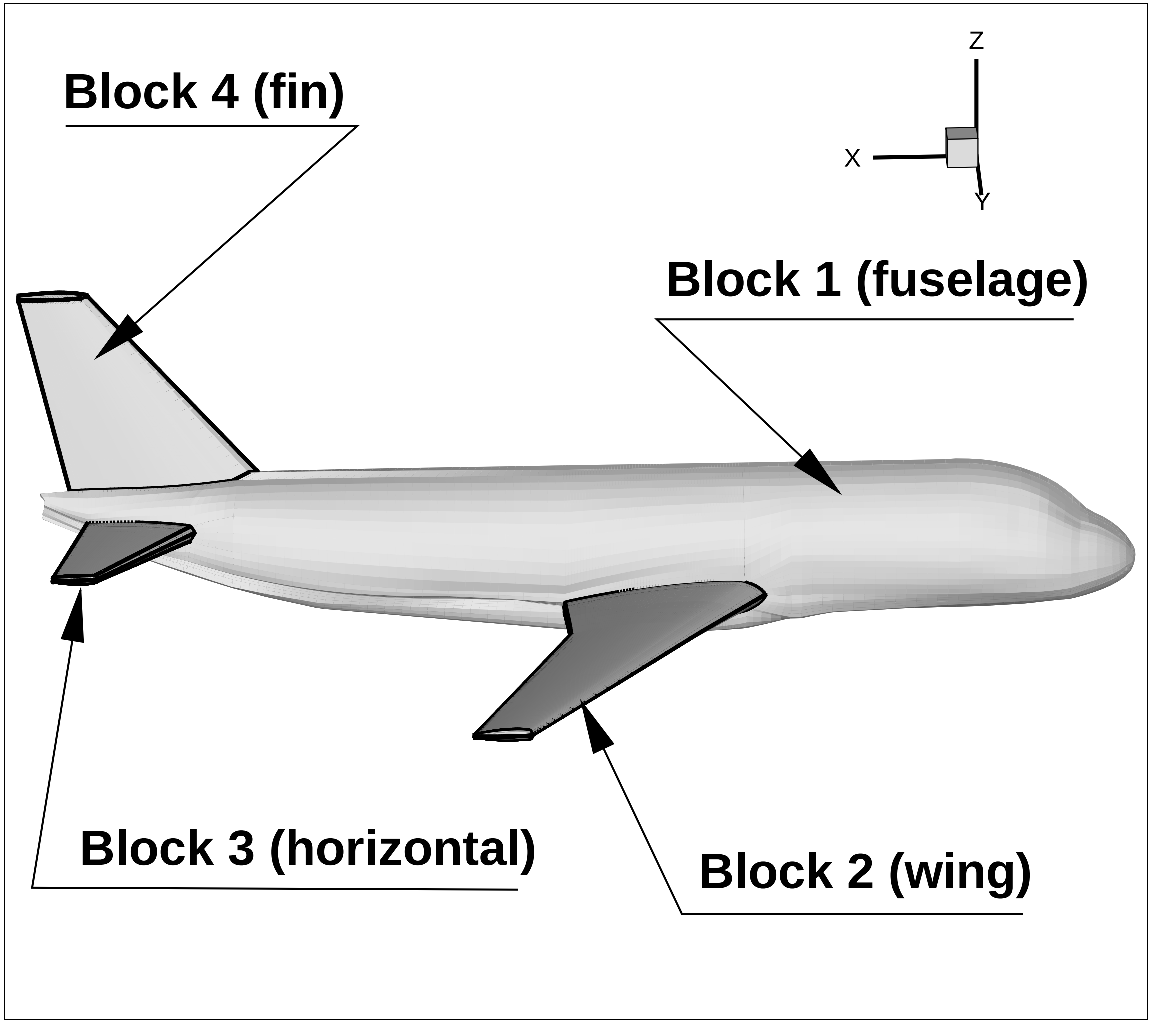
$$Error = \max||\boldsymbol{x}_{k,exact} - \boldsymbol{x}_{k,calc}||$$

Full configuration aircraft

Full configuration aircraft

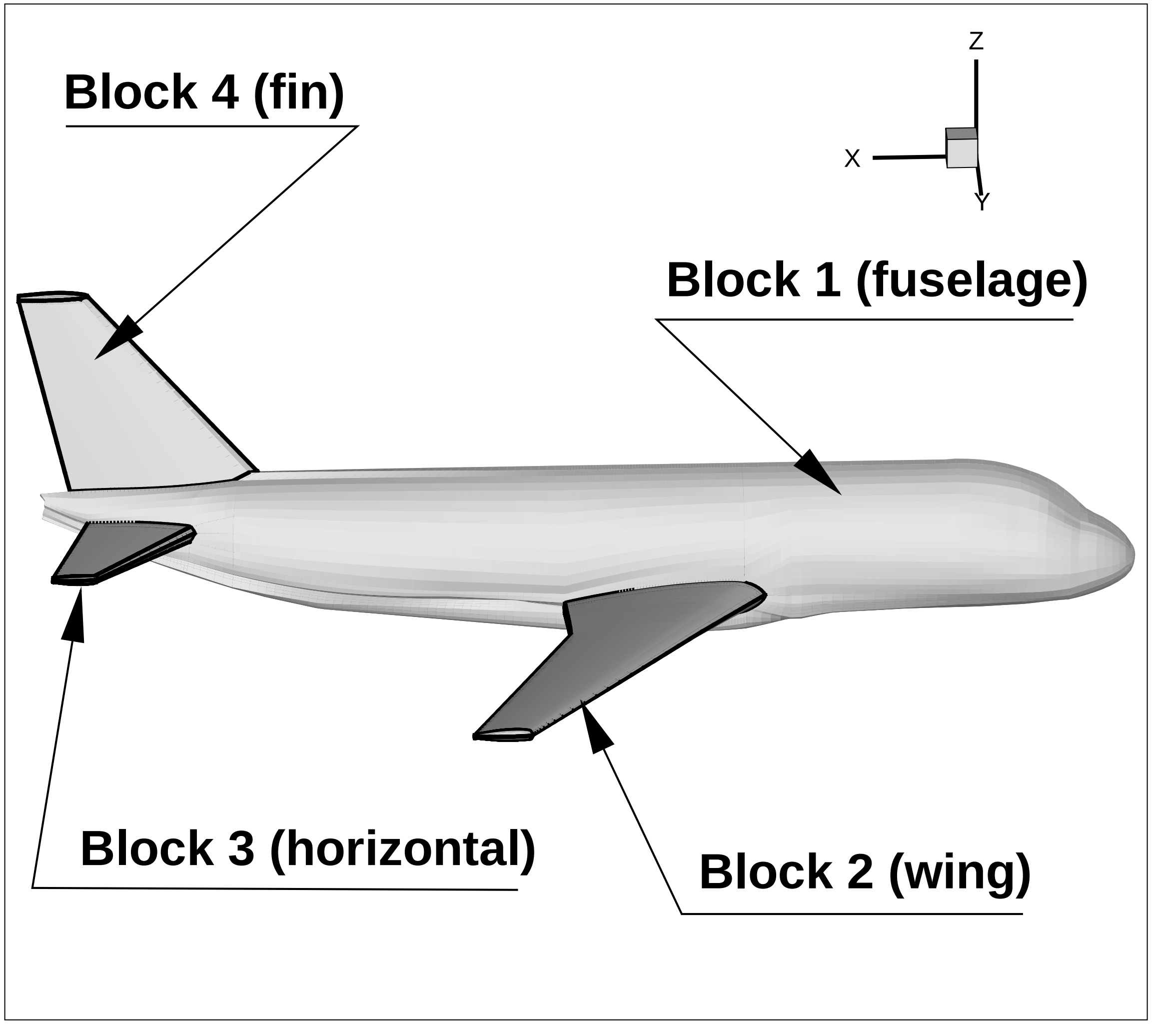


Decomposition in domains or blocks



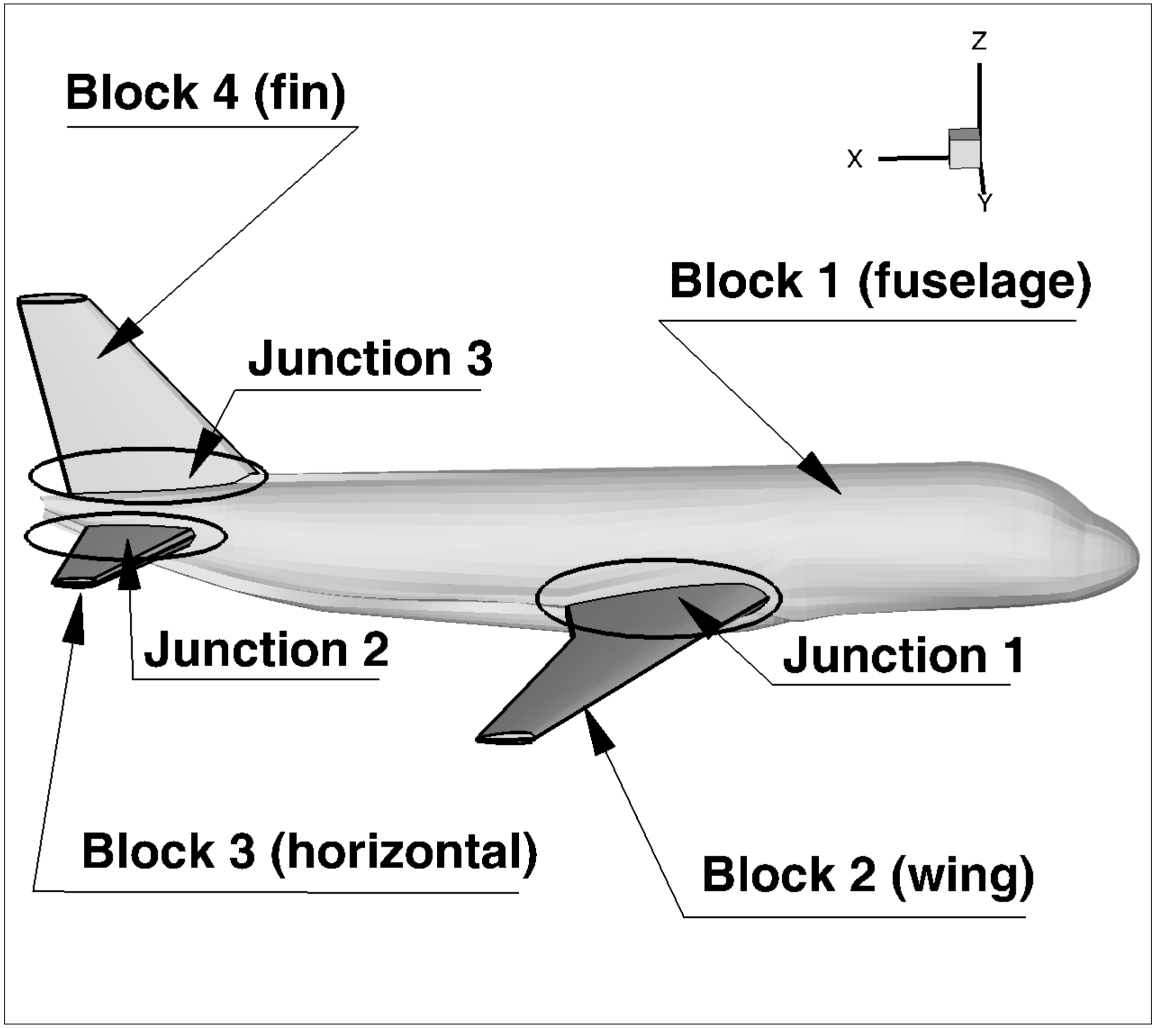
Full configuration aircraft

Deformation by blocks

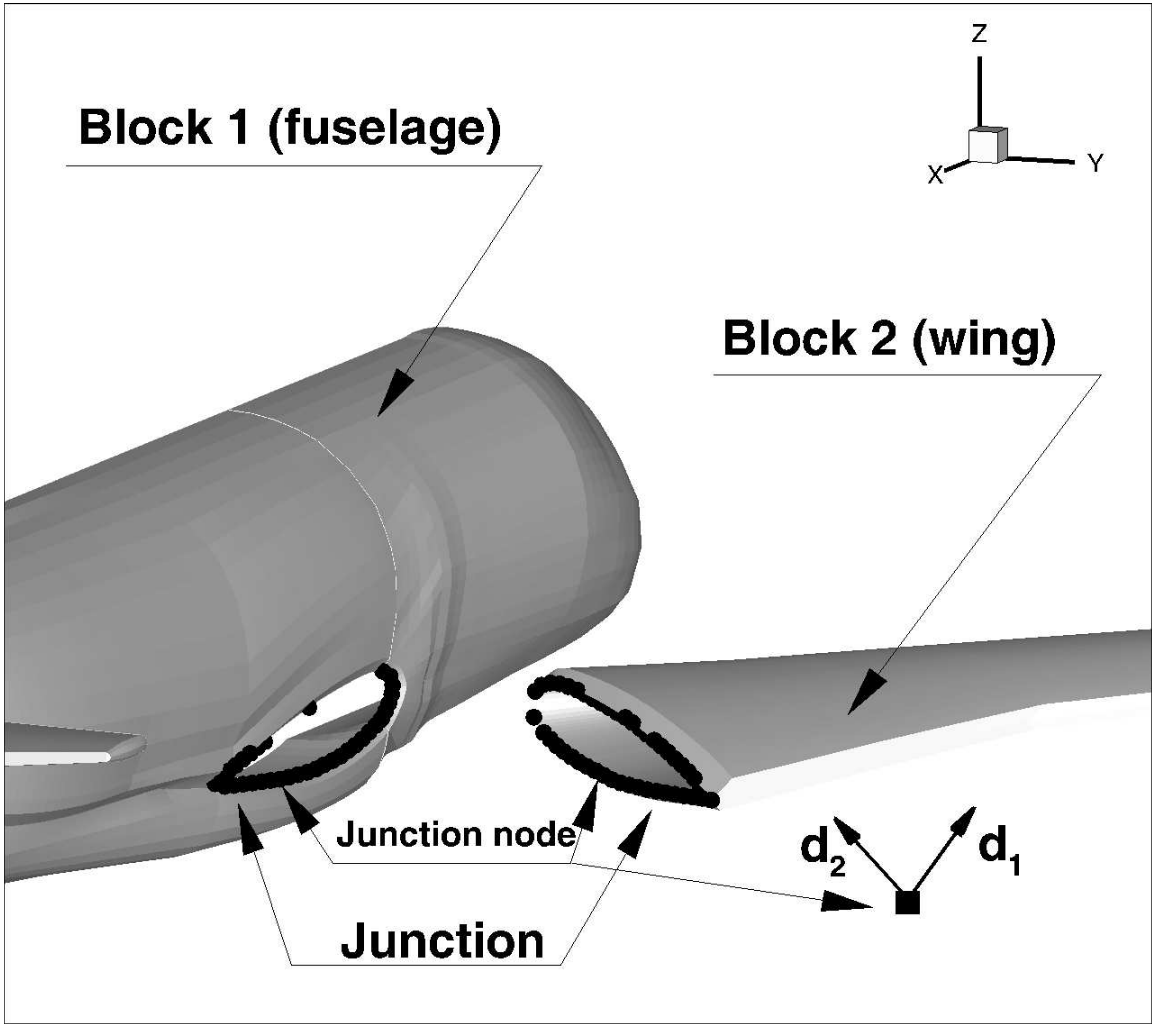


Full configuration aircraft

Correction based on deformed junction nodes

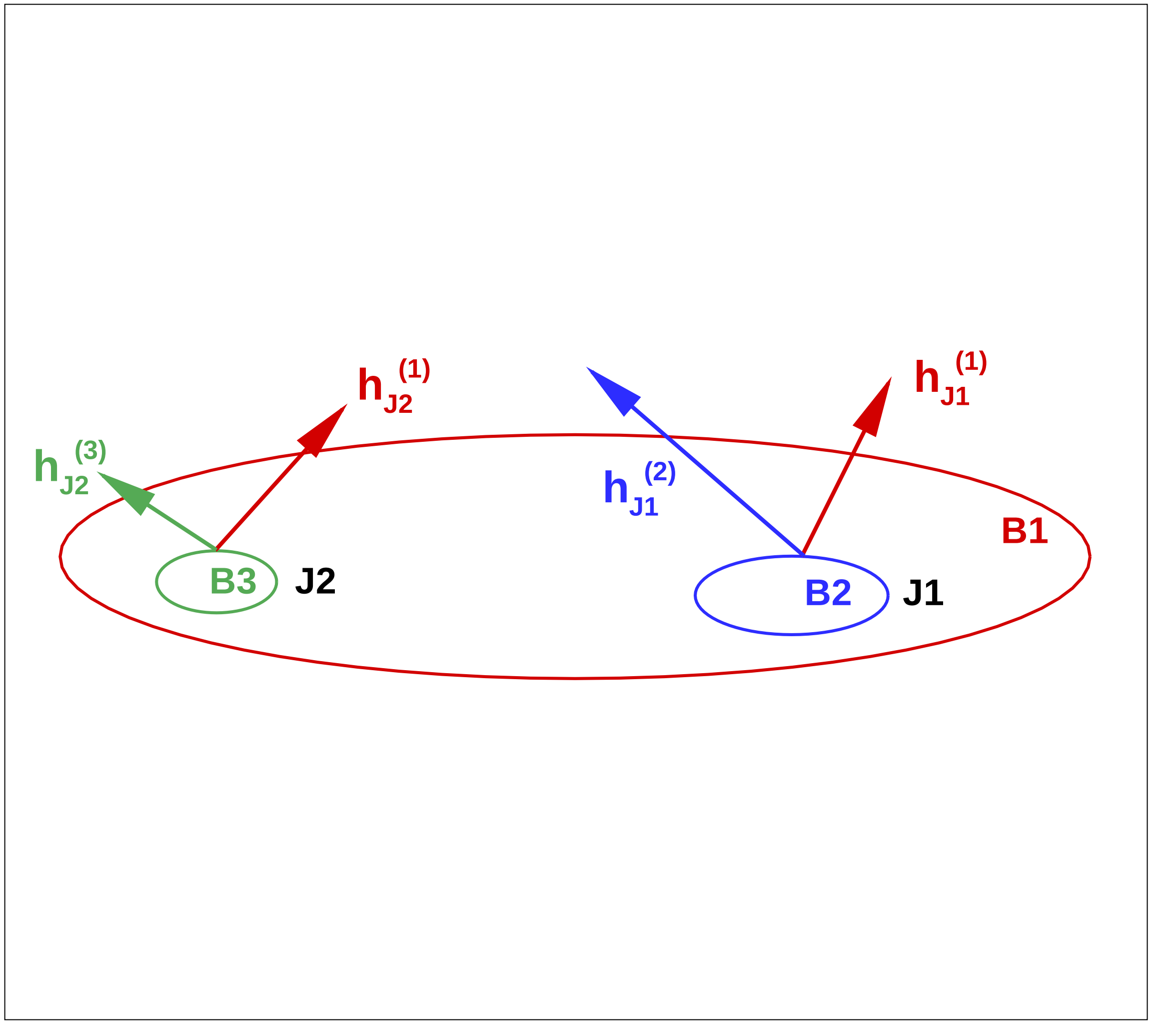


Junctions

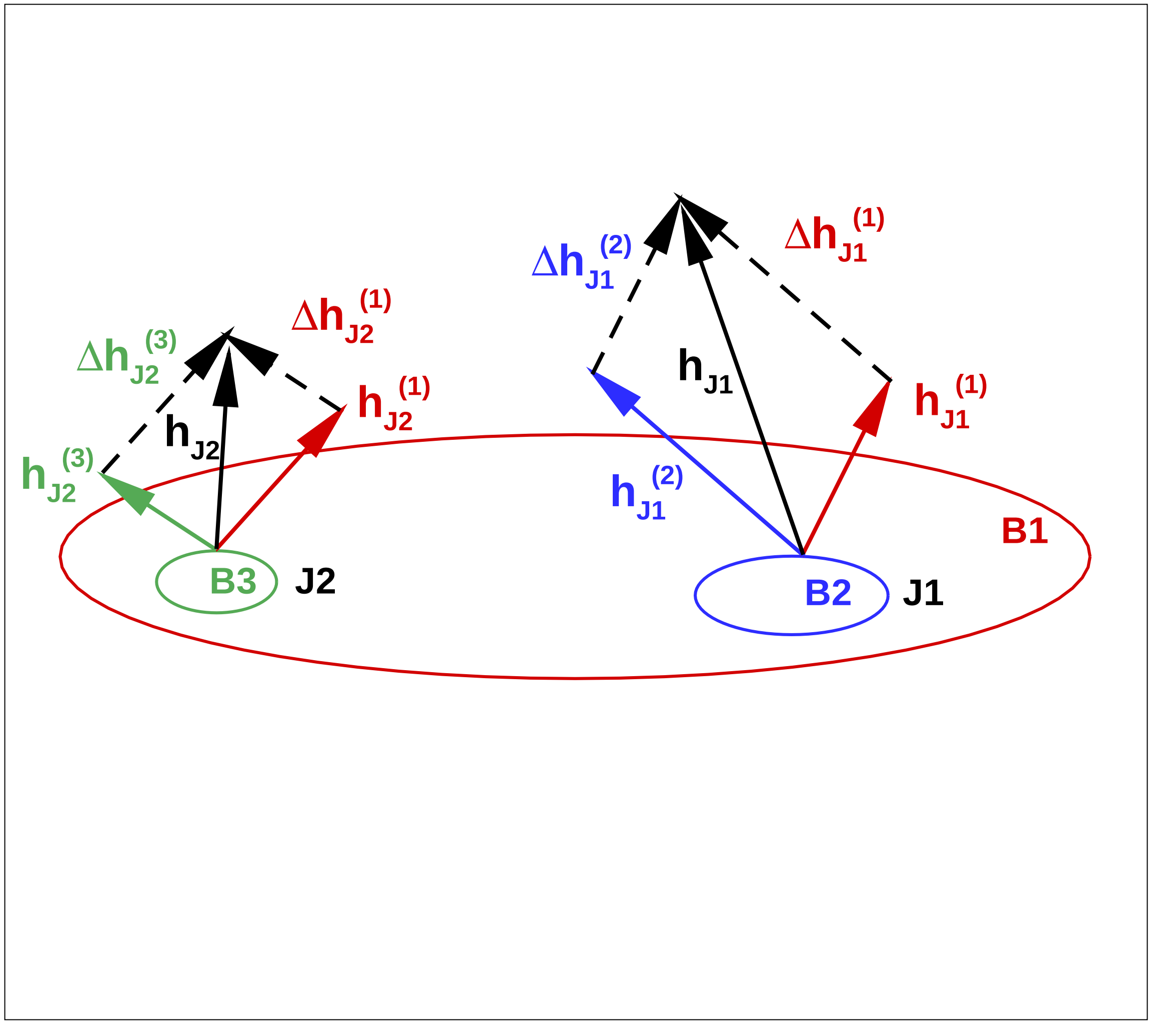


Z
X
Y
Block 1 (fuselage)
Block 2 (wing)
Junction node
d_2
d_1
Junction

Junctions strategy



Junctions strategy



$$h_J = \alpha h_J^{(1)} + (1-\alpha) h_J^{(2)} \qquad 0 \leq \alpha \leq 1$$

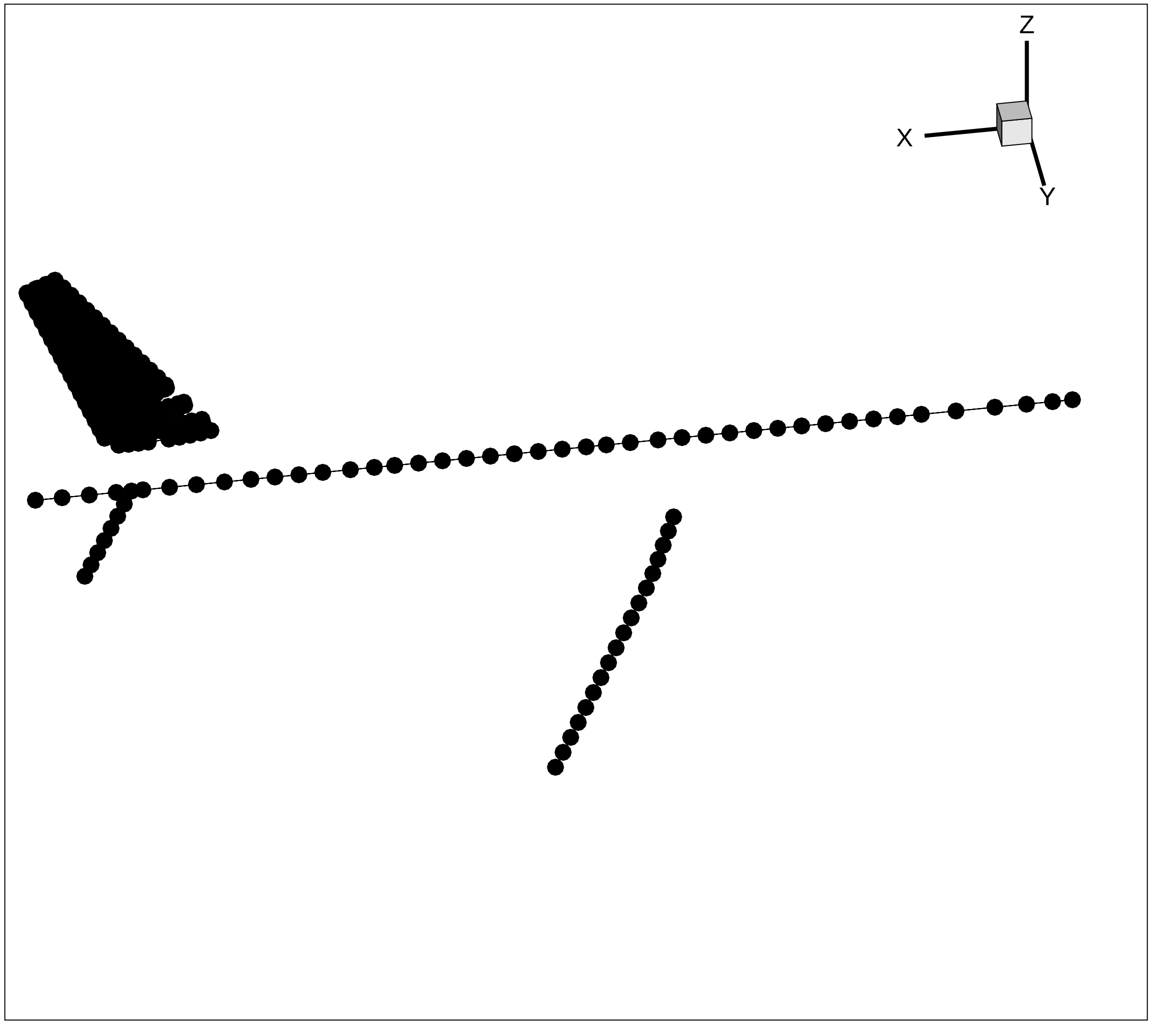
Junctions strategy

$$\left.\begin{array}{ll}\text{Structure:} & \{J1, \Delta h_{J1}^{(1)}\} \cup \{J2, \Delta h_{J2}^{(1)}\} \\ \text{Aerodynamic:} & B1\end{array}\right\} \longrightarrow \Delta h_i^{(1)}$$

$$\left.\begin{array}{ll}\text{Structure:} & \{J1, \Delta h_{J1}^{(2)}\} \\ \text{Aerodynamic:} & B2\end{array}\right\} \longrightarrow \Delta h_i^{(2)}$$

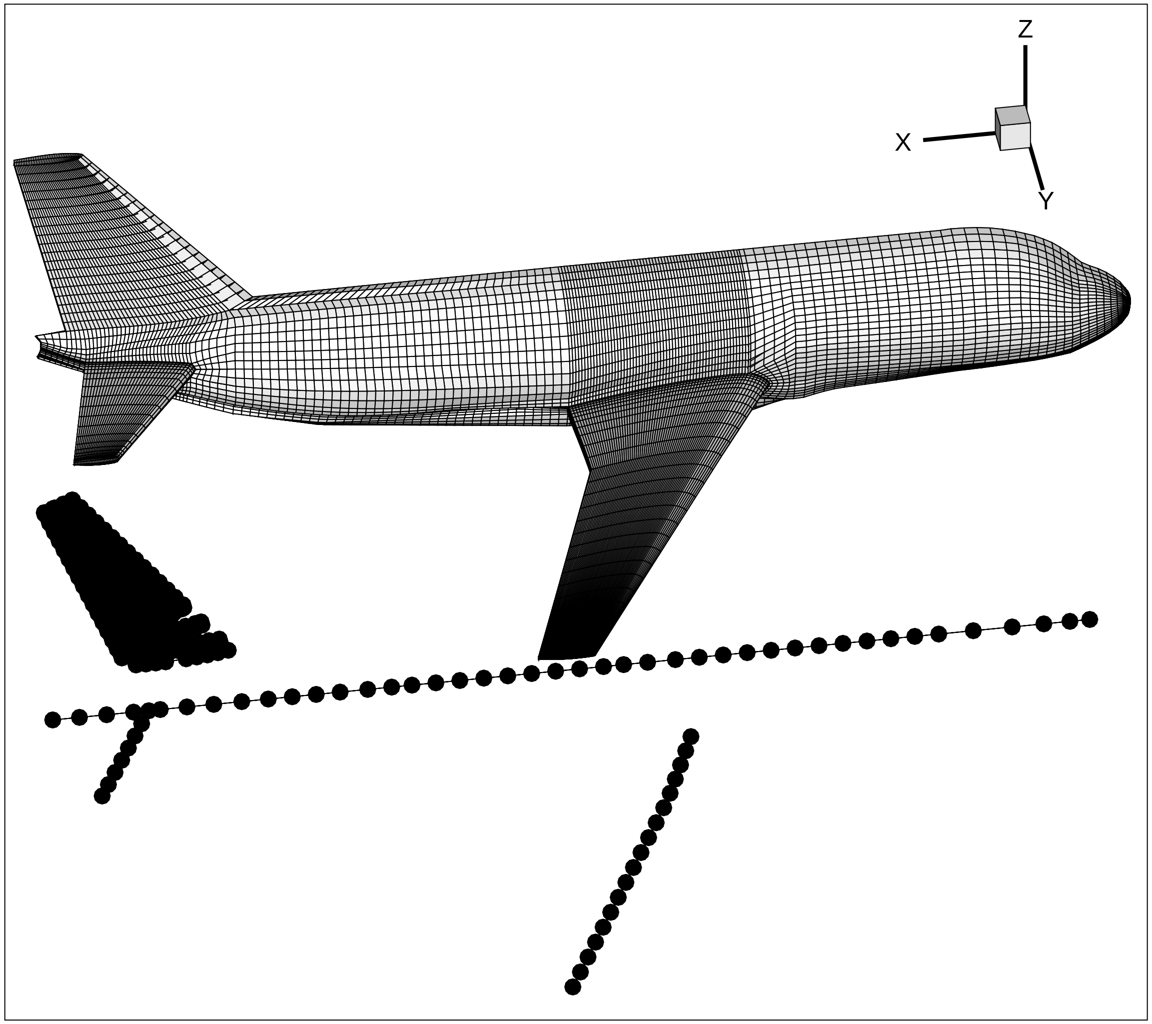
$$\left.\begin{array}{ll}\text{Structure:} & \{J2, \Delta h_{J2}^{(3)}\} \\ \text{Aerodynamic:} & B3\end{array}\right\} \longrightarrow \Delta h_i^{(3)}$$

Stick – FE model: aircraft



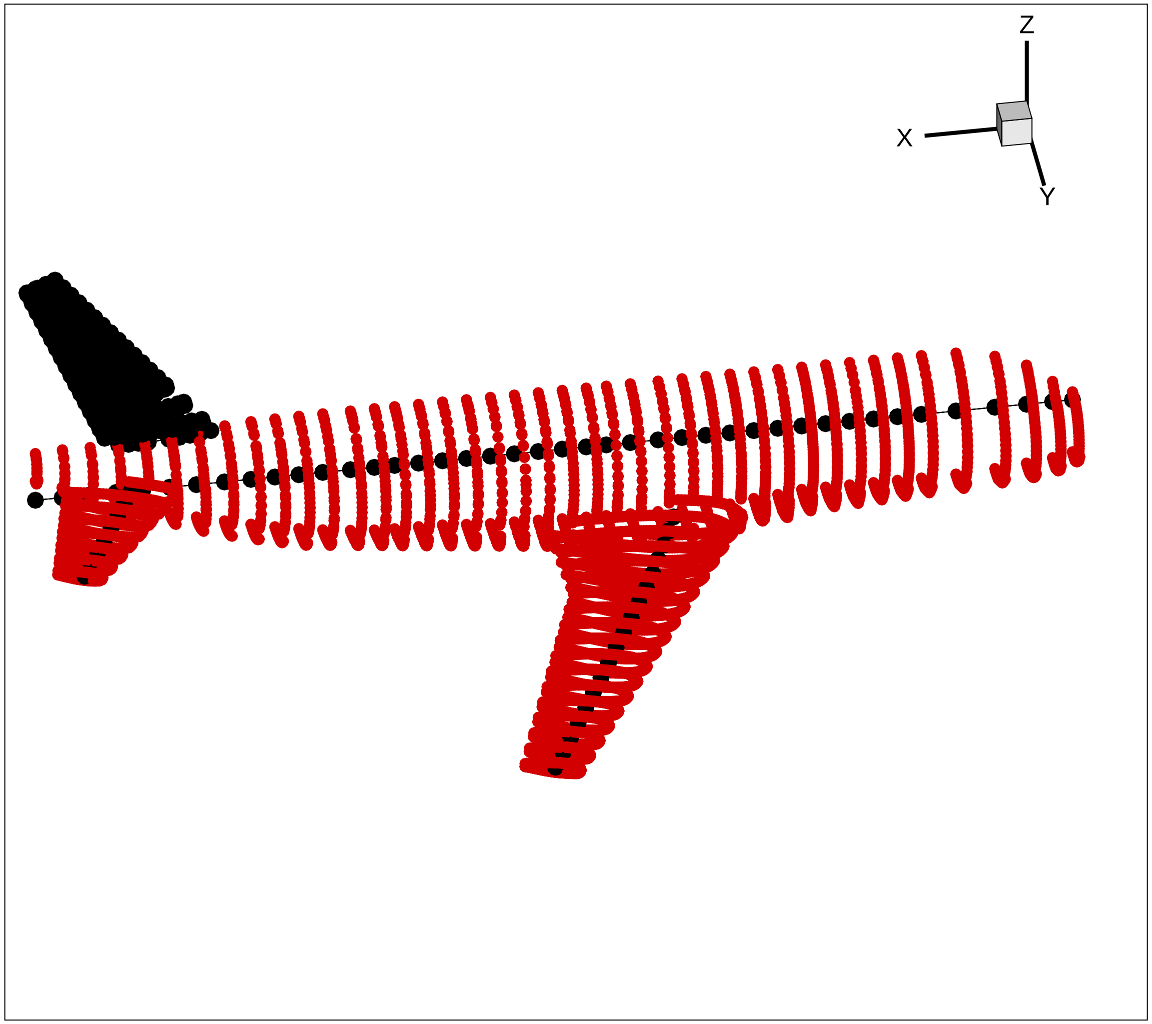
- 68 stick-nodes
- 253 FE-nodes

Stick – FE model: aircraft



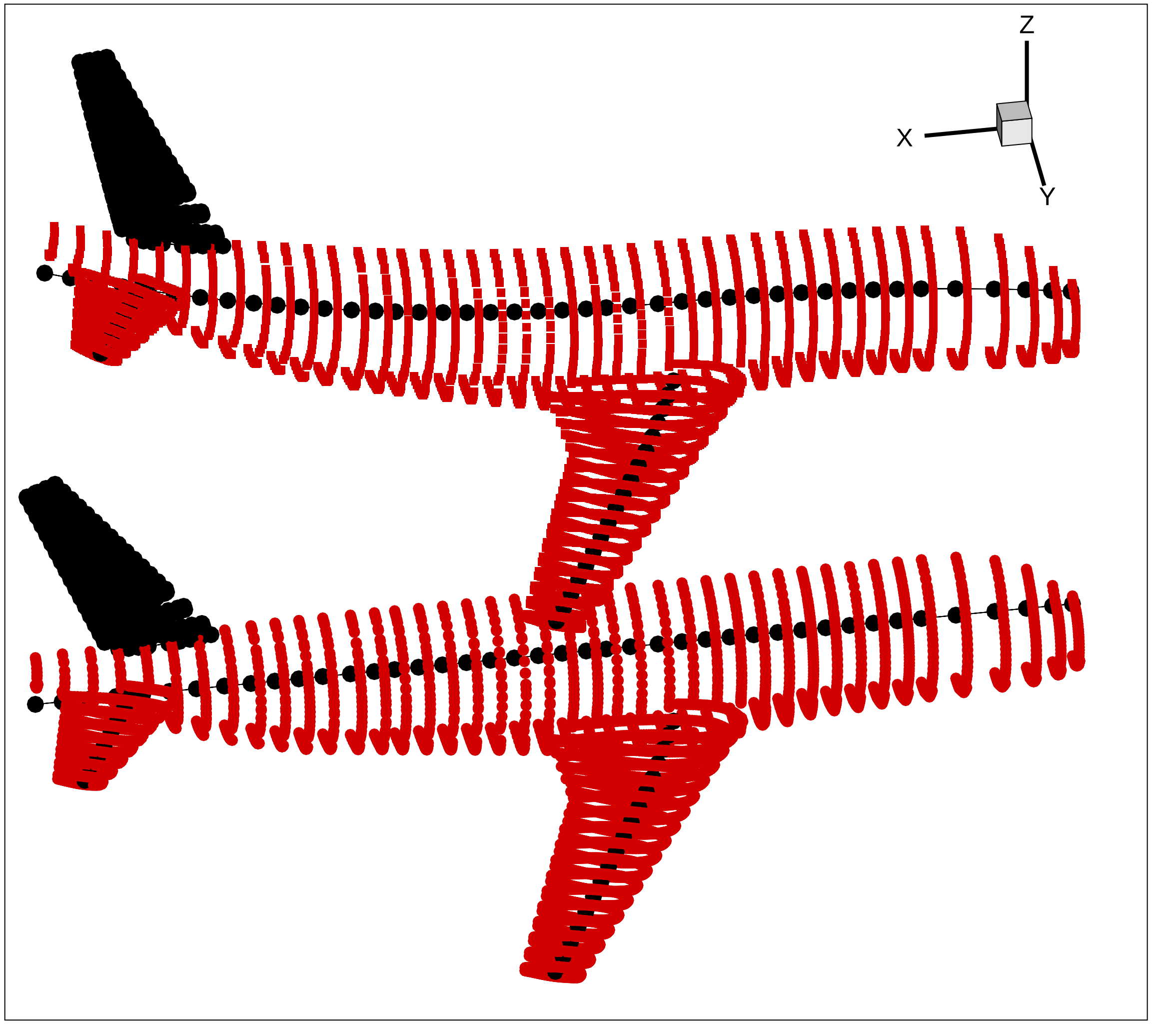
- 67040 aerodynamic-nodes
- Structured mesh

Stick – FE model: aircraft



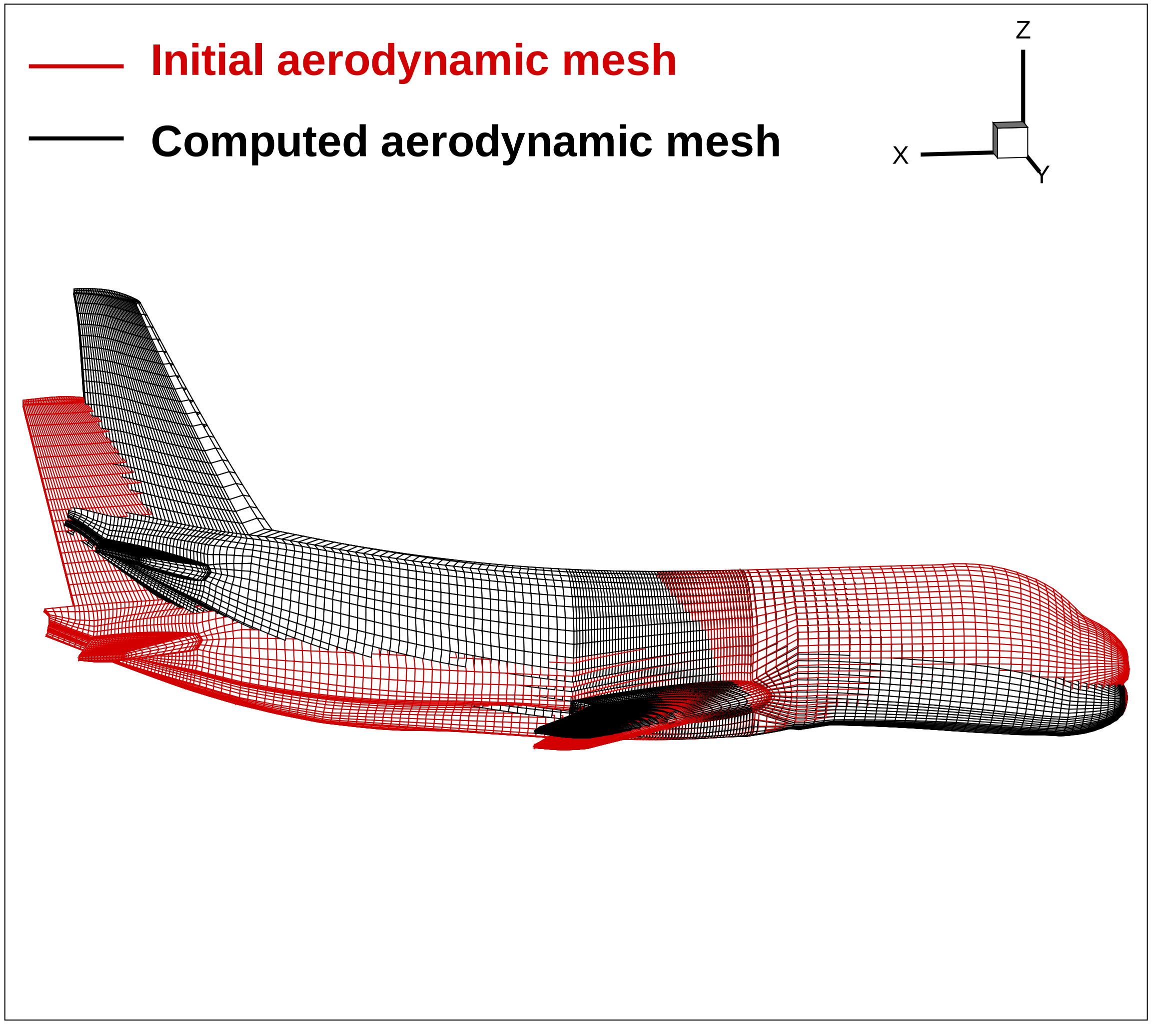
- Work structural mesh (8286 nodes)

Stick – FE model: aircraft



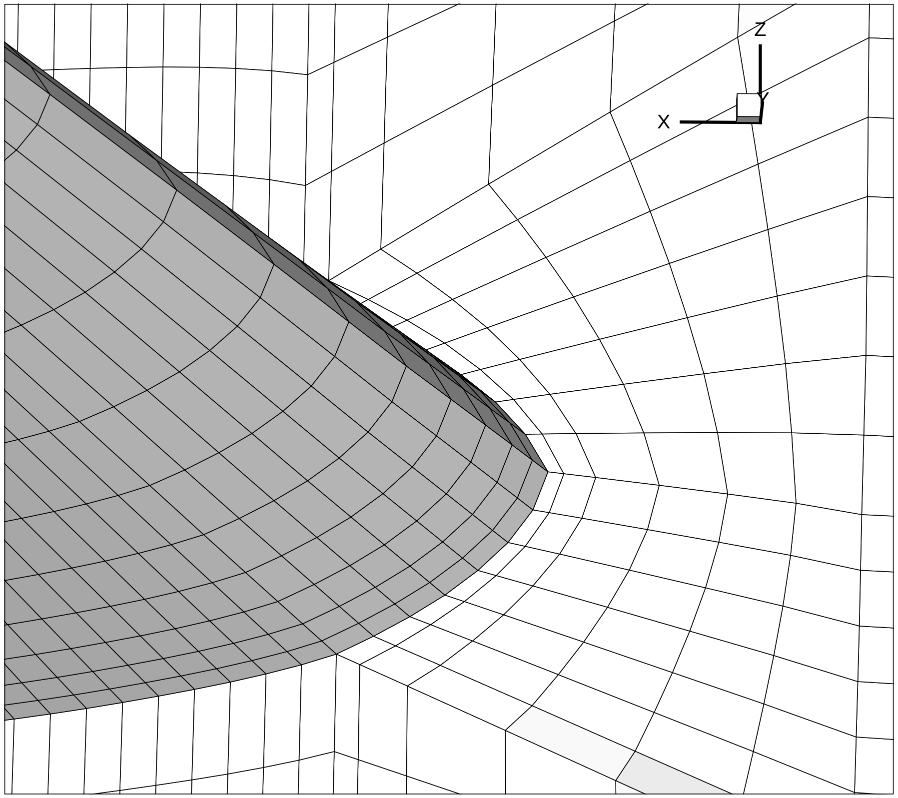
- Deformed work structural mesh

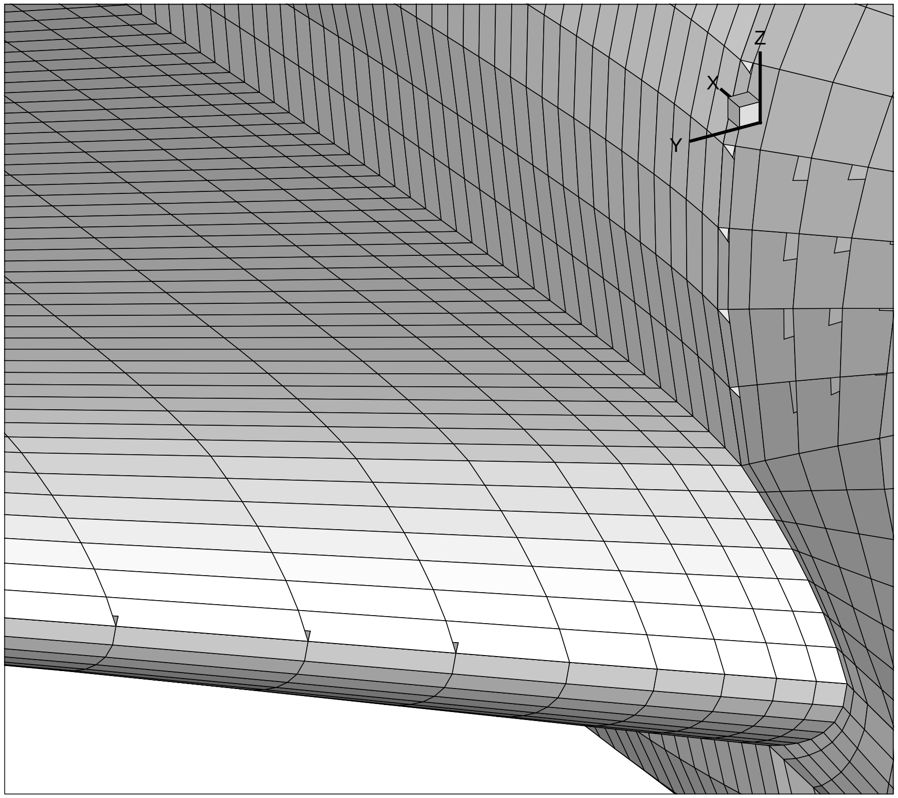
Stick – FE model: aircraft



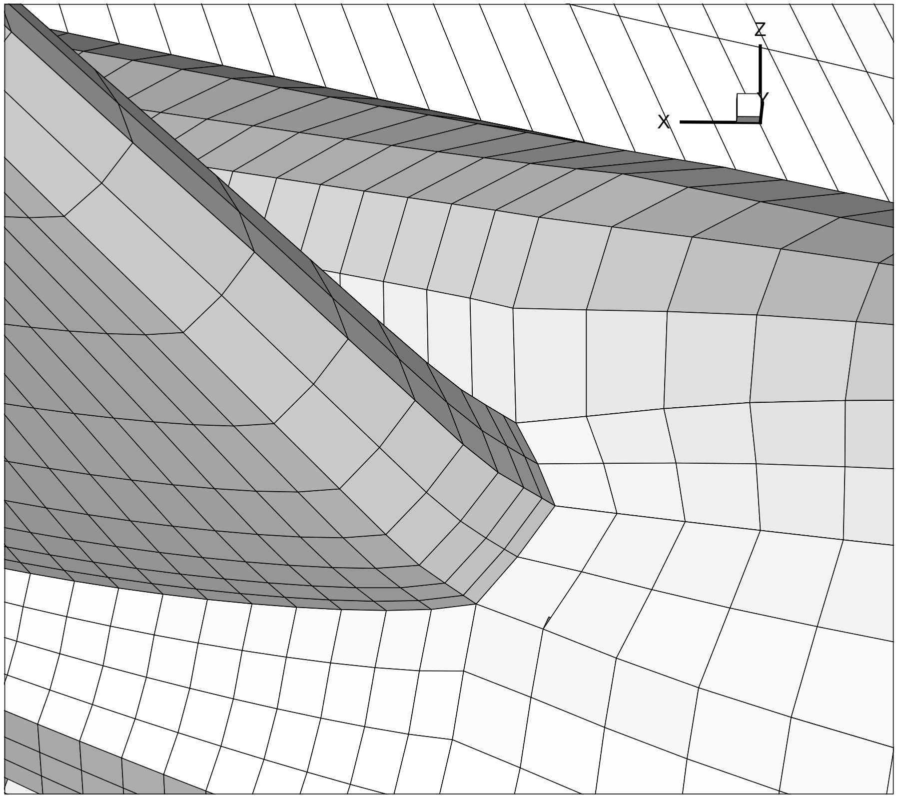
- Interpolated aerodynamic mesh

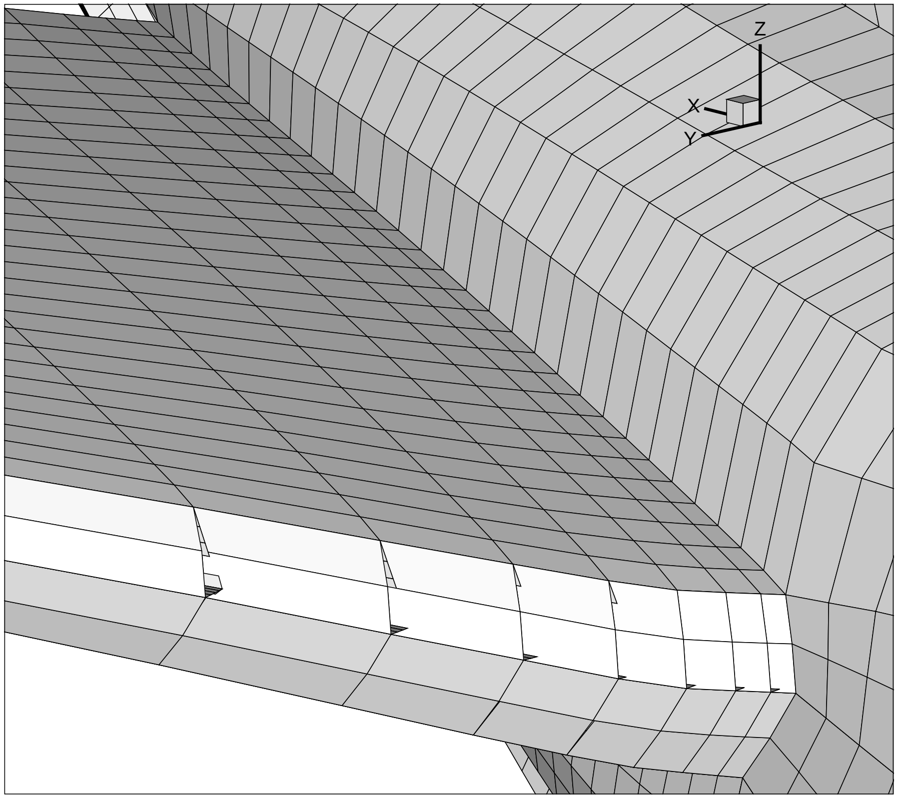
Wing detail



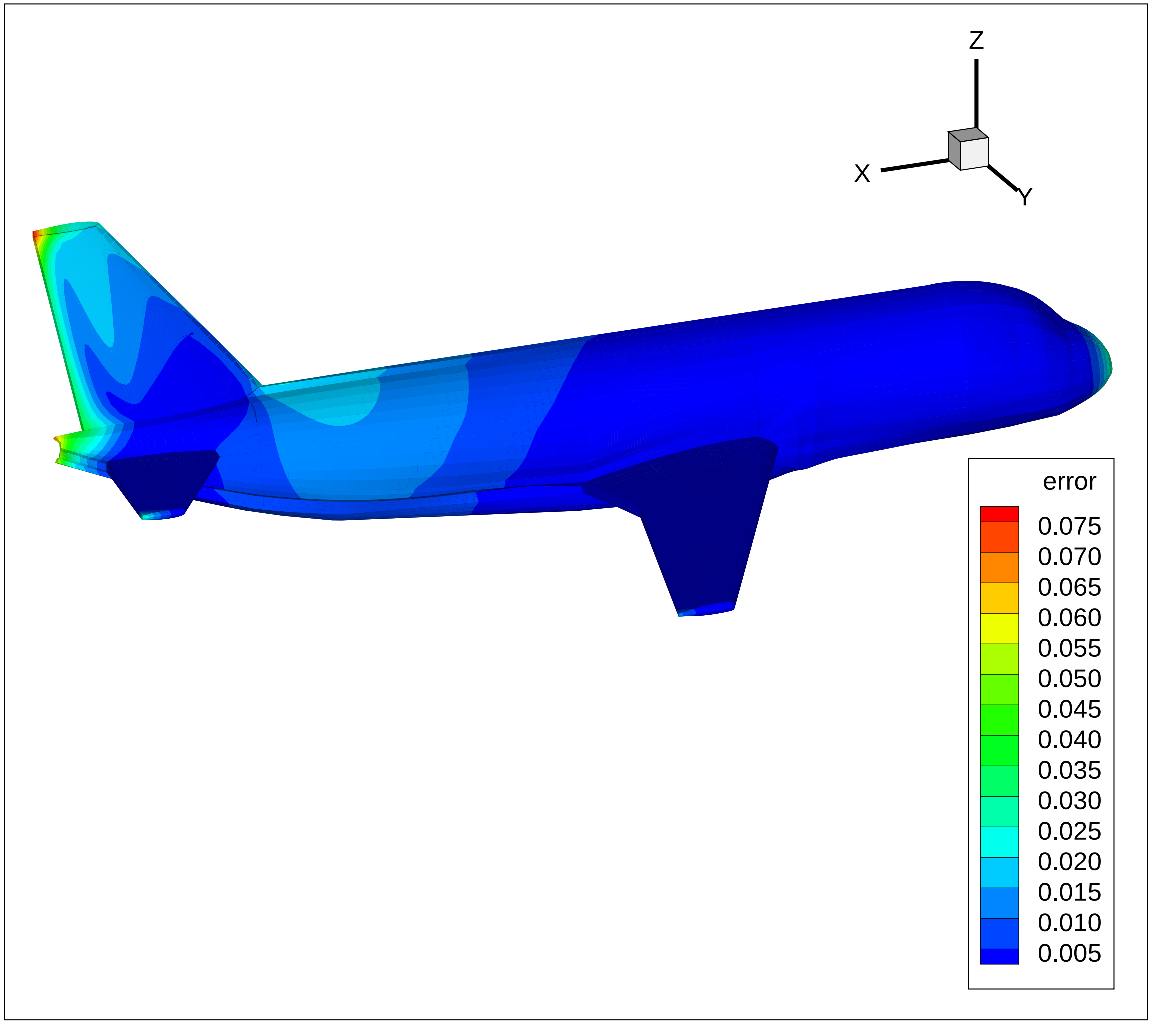


Horizontal stabilizer detail





Error contour map



$$Error_k = ||\boldsymbol{x}_{k,exact} - \boldsymbol{x}_{k,calc}||$$

Conclusions

- An interpolation tool based on radial basis functions has been developed.
- Useful to transfer forces from a structural mesh to an aerodynamic mesh or loads from an aerodynamic mesh to a structural mesh.
- Direct application to any dimension problems, both structured and non structured meshes.

Thank you