

A Model for Urban Commuting in a Multicenter City

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A model for simulating and calculating cost consequences due to urban commuting in a multicenter city is presented in this paper. The model is applied to two alternative city master plans for a Swedish city, Västerås. Illustrative results are reported.

Introduction

Quite extensive research efforts have been devoted to the problems of urban commuting (Quarmby, 1967; De Donnea, 1971 and 1972; Mc Gillivray, 1972, a and b; Domencich-Mc Fadden, 1975; Andersson, 1981; Ben-Akiva-Lerman, 1985). Most models deal with monocentric cities. The objective of this article is to present a model for the more realistic case of a multicenter city. In this context commuting is simulated and evaluated for two alternative city master plans. Thus, the model aims at analysis of cost consequences in the transportation sector due to alternative city master plans.

The model was applied to a real case: the city of Västerås in central Sweden with ten separate work-place centers. Each commuter's choice of work-place center is simulated according to an attraction rule for choosing among possible work-place centers. Commuting in the city is simulated on the assumption that each individual commuter chooses mode and route so as to minimize his commuting costs. This means that each individual will choose the mode and route of commuting that minimizes his individual commuting cost, including commuting time, to a particular work-

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place center. The commuting modes considered are walking, bicycle, car or bus. The degree to which the various modes of commuting are used is to a large extent determined by the private costs incurred. Thus, in the model, parking fees and bus fares influence the distribution of commuting among the modes. Congestion is endogenously determined in the model and influences the commuting pattern.

The model consists of two parts: (a) the assignment of workers to the different work-place centers and (b) the worker's choice of commuting mode and route. These two parts are strongly connected. An interactive procedure is used so that results from one part of the model can become input data in the other part. Interactions come to an end when the results obtained in one part of the model equal the data previously assumed, within some error margin. Once the model has simulated the assignment of the commuters and distribution of commuting among the available modes, total transportation costs are calculated. Both parts of the model are described separately below.

Model for Assigning Residential Nodes to Work-Place Centers

The objective of this model is to assign the workers who live in various residential blocks (= city nodes) to the different work-place centers. All workers belonging to a particular income class at a given residential node are assumed to work at the same work-place center. The working population at a residential node will choose the work-place center for which the quotient between the number of employment opportunities at the work-place center and the community costs is the largest. This rule is an attraction rule for allocating the working population among different work-place centers and is a kind of gravity rule commonly used in the literature. According to this rule, a worker will be more willing to work in the nearest work-place center and in a center with a larger number of employment opportunities. Due to the existence of indivisibilities in the given data about the working population in the residential nodes, another assumption for assignment of workers is introduced. For a given work-place center the node with the largest number of workers among nodes with equal commuting costs will be attracted first, i.e., priority will be given to working there.

The number of employment opportunities at a particular work-place center must be at least as great as the number of workers employed there. These conditions may be expressed mathematically as follows:

The attraction value AV_{np} for people belonging to income class ic and living at residential node n to work at work-place center p is given by the formula:

$$AV_{np} = \frac{\overline{WP}_{ic}^p WP_{nic}}{c_{bn}^p} \quad (1)$$

where WP_{nic} is the working population living at node n and belonging to income class ic , giving priority to nodes with bigger populations,

\overline{WP}_{ic}^p is the number of employment opportunities for people of income class ic at work-place center p ,

c_{bn}^p are the individual costs for commuting from node n to work-place center p (expression to be given later on the text).

Figure 1 illustrates the gravity rule used.

For every work-place center p , the number of work opportunities must be sufficient for the people who choose to work there, i.e.

$$\sum WP_{n \in N_p^{ic}} \leq \overline{WP}_{ic}^p \quad (2)$$

where the sum is extended to the set of nodes N_p whose inhabitants are assigned to work in work-place center p .¹

This work-place assignment is based on the following method: The workers at each node will choose the work-place center with the greatest attraction value, if sufficient work opportunities still exists. If there are not enough jobs, they will choose another work-place center among those with enough employment opportunities that gives the highest value for the attraction ratio. The ranking among nodes in order to give priority to

1. In order to find a solution to this assignment problem, the number of all work opportunities must be smaller than the total working population.

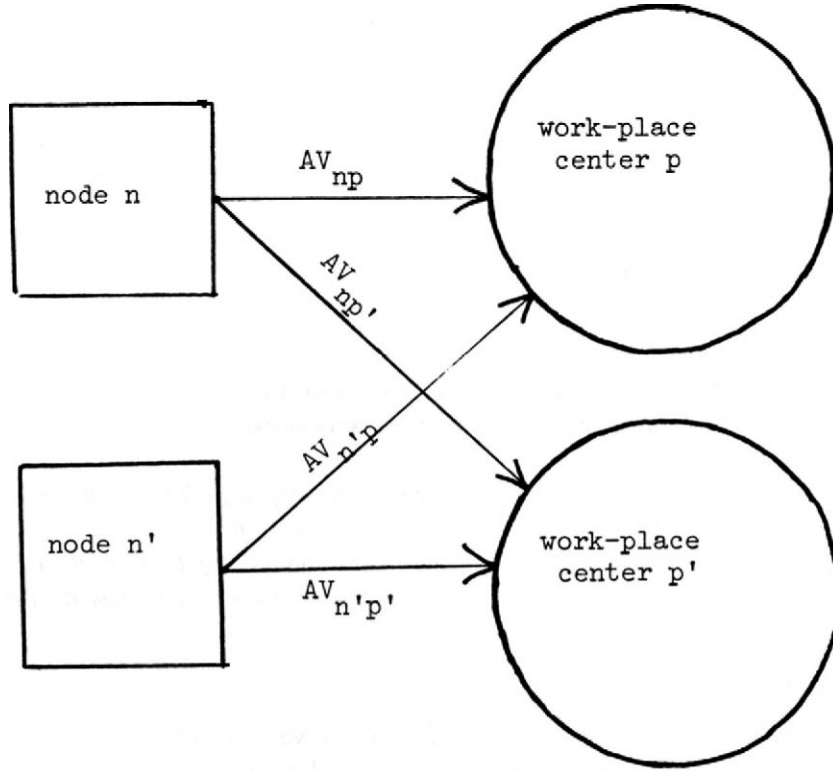


Figure 1. Assignment of residential nodes to work-place centers.

selection of a work-place center will be in decreasing order with respect to their attraction values.

In mathematical terms the above method can be expressed as follows:

- Form the matrix $\{AV_{np}\}$ with dimension $N \times P$, where N is the total number of residential nodes and P the total number of work-place centers.
- A step-by-step algorithm is then applied to this matrix $\{AV_{np}\}$
 - (a) Find the greatest element of the matrix $\{AV_{np}\}$ as defined by row n_k and column p_k .
 - (b) Check whether the number of workers living at node n_k is smaller than the current employment capacity at work-place center p_k , i.e.

$$WP_{nic} \leq \overline{WP}_{ic}^p \quad (3)$$

(c) If equation (3) holds, the following steps are applied:

1. Suppress row n_k of the matrix $\{AV_{np}\}_k$. The resultant matrix will be denoted by $\{AV_{np}\}_{k+1}$
2. Update the value of current employment opportunities at the work-place center p_k , i.e.

$$\text{new } \overline{WP}_{ic}^{pk} = \text{old } \overline{WP}_{ic}^{pk} - WP_{n_k ic}$$

3. Increase k by one unit : new $k = \text{old } k + 1$
4. Go to step (a)

(d) If equation (3) does not hold, then

1. Find the greatest value of $AV_{n_k p}$ in row n_k of matrix $\{AV_{np}\}_k$ subject to the condition $WP_{n_k ic} \leq \overline{WP}_{ic}^p$ i.e., for the workers living at node n_k , choose the most attractive work-place center with sufficient employment opportunities.
2. The above value of p is denoted by p_k .
3. Go to step 1. under Section C above.

This procedure is successively repeated until all the rows have been suppressed, i.e., all the workers at residential nodes have been assigned to the work-place centers or, in other words, when k reaches the value of N .

Model for Commuting

General Characteristics

In order to apply the assignment model described above, values of individual commuting costs from each node $n(c_{bnp})$ to work-place center p must be known. This calls for a model which simulates the commuting pattern in the city. The main assumptions of this model are as follows.

Commuting is assumed to occur only during morning and evening peak periods of given duration. There are four modes of commuting: walking, bicycling, car or bus. Those who commute by walking are assumed to walk the straight-line distance between the residential node and the work-place center. The maximum walking distance is given as data. The commuter who bicycles also uses the minimum distance (straight line) as a commuting route, but at a different speed than the walking commuter. The maximum walking distance gives the boundary between the use of these two commuting modes.

Car commuting is used by an average given "pool" of individuals per car. The commuting route is composed of local roads (leading from the residential node to the main road) and main roads explicitly defined as data. Congestion on the local roads is assumed to be zero and speed constant. Car velocity along the main road varies with actual traffic congestion. The degree of congestion is computed endogenously in the model to include the simultaneous effect of bus and car traffic.

Bus commuting is assumed to be composed of the following stages: walking from a residential node to a bus stop, riding the bus to the bus stop at the work-place center and walking between this stop and the work-place.

Individual Commuting Costs

At each point in time t_a , each worker who lives at residential node n will choose a mode j and commuting route r to some work-place center p . The choice of commuting trip will be made so as to minimize individual commuting costs. Thus, if $c_{bj}(a, n)$ represents the costs related to an individual commuting trip using mode j and route r , then we define $c_{bj}(a, n)$ as the minimum commuting costs for all possible routes using mode j .

$$c_{bj}(a, n) = \min_r c_{bj}^r(a, n) \quad (4)$$

The worker living at node n will then choose the commuting mode that minimizes commuting costs so that,

$$c_b(a, n) = \min_j c_{bj}(a, n) \quad (5)$$

The $c_b(a, n)$ are called the implied individual commuting costs.² Time costs are included and congestion is explicitly taken into account and endogenously determined. The cost items considered in the individual costs for commuting modes are:

Walking: time costs

Bicycling: time costs and costs for the bicycle

By car: time costs, depreciation and maintenance costs for the car, gasoline, oil and tires,

By bus: time costs and bus fare.

2. This individual commuting cost $c_b(a, n)$ was denoted in the preceding paragraph by c_{bn}^p without explicitly considering the current time intersection t_a .

The calculations and most of these cost items are straightforward. However, the procedure for calculating some particular items is shown in Appendix A.

Treatment of Interdependence

It is important to emphasize that these two models are strongly interconnected. Results for some variables (link variables) from one model are used as input in the other. So, in the model 1, i.e., the model for assigning residential nodes to work-place centers, it is necessary to know the commuting cost c_{bn} at each residential node n . This is a result of model 2, i.e. the model for commuting. On the other hand, this model needs the work-place center selected for the workers at each node n as input data. This data is given by the model 1 and influences both commuting congestion and individual commuting costs. An iterative procedure is applied in order to handle this interdependence. Starting with some initial values for c_{bn} , model 1 can be used and its results introduced as data in model 2, which in turn produces new values for c_{bn} to be used again in model 1. The iterations will cease when the results obtained in two consecutive iterations are approximately equal. The values which represent conditions in the initial city as accurately as possible are used as starting values for the iterations, i.e., the model parameters are calibrated. Also, in order to achieve convergence and avoid bang-bang situations, the values of these link variables used as input in one iteration step are not directly the results of the previous step, but the mean value of the two previous steps.

Data

City Master Plans

The models presented above were imbedded in a general model for city master planning for the city of Västerås, Sweden. The city master plans were studied on the basis of this model. So, this is one rationale for the choice of the two particular submodels presented here. Some results related to the cost consequences of urban commuting due to these city master plans are presented in the "Results" Section. City master plan I, called the satellite alternative, corresponds to the construction of residences located

mainly in the outskirts and satellites of Västerås. City master plan II, denoted the urban renewal alternative, is characterized by intensive construction in the built-up area of the inner city. This means that many old residences in the inner city will be demolished and replaced by new ones. Actual commuting in Västerås and the two city master plans are described briefly below.

Data on Work-Place Centers

The city of Västerås has ten separate work-place centers of quite varying sizes. The central business district (CBD) is the dominant work-place center, although the old central industrial area also provides many jobs. Among the new centers, the northern industrial area is very important. Two of the satellites have work-place centers, but they are rather small.

In conjunction with the development of employment opportunities, an increase in the population of Västerås is expected for the first five to ten years of the study, after which a slight decrease in the population level is expected. The relationship between working and total population is assumed to be nearly constant during the time period covered by the study.

The forecasted changes for the work-place centers presented in Table 1 are assumed to be the same for the prescribed city master plans under evaluation.

No changes in the number and locations of work-place centers are expected as compared to the initial situation. The data for employment opportunities forecasted for each work-place center for 1981 to 2000 are the most probable values (see Table 1). The proportion between the different employment opportunities available at each work-place center for the different income classes are the same as in the initial city (see Andersson-Samartin-Martinez 1983).

Data on the Transportation System

As the central area of Västerås is rather small, walking and bicycling may be used extensively for commuting. There is also a well-organized layout for bicycling in Västerås with lanes that are separated from the other traffic by tunnels at intersections (except in the CBD). The present road network is adequate for the most part. Routes for local buses run through almost the entire inner city, and express buses connect the satellites to the CBD.

Table 1. Employment Opportunities at the Work-Place Center's Västerås.

Work-place center	1981	1985	1990	1995	2000
WPC 1. Old Business District	17,996	17,490	16,865	16,240	15,040
WPC 2. Old Industrial Area	7,604	7,660	7,655	7,650	7,675
WPC 3. Old Hospital Area	4,003	4,075	4,248	4,420	4,710
WPC 4. Eastern industrial area	1,548	1,550	1,600	1,650	1,650
WPC 5. Airport area	300	100	150	200	300
WPC 6. Southwestern industrial area	4,786	5,230	5,177	5,125	5,150
WPC 7. Tillberga work-place center	934	950	975	1,000	1,000
WPC 8. New hospital area	498	750	825	900	1,000
WPC 9. Skultuna work-place center	746	750	725	700	700
WPC 10. New northern industrial area	8,524	9,680	9,765	9,850	9,800
Total	46,939	48,235	47,985	47,735	47,025

The existing transportation routes for buses and cars are simulated by means of transportation nodes. A pair of transportation nodes defines a link. Consecutive links are connected to form routes between residential nodes and work-place centers, where the computed congestion along the route (in cars/hour) is indicated. All of these values are known for each city node at the initial point in time.

The same commuting modes are available in both city master plans: walking, bicycling, car and bus commuting. A new set of road links will be created to connect planned new areas to the existing road network. No additional bus lines will be added, although some of the existing lines will be given longer routes to connect these new areas. The cost of new road links and changes in bus routes will vary in the different city master plans. For instance, in "satellite alternative" I, it will be necessary to rebuild the road from a satellite to a transportation node in the road system of the central city. In the "urban renewal policy alternative" I, such costs can be saved because it is assumed that the existing road system can be used. On

Table 2. Observed and Simulated Commuting Patterns in Västerås.

Mode of commuting	Observed commuting pattern for 1975*	Simulated commuting pattern for 1980-81 (Alt. I)
Walking	6079	4521
Bicycling	9583	10716
By car	25055	26147
By bus	11553	11745

* *Source:* Road and Traffic Office, Västerås. The pattern observed 1975 was used as if it was valid for 1980.

the other hand, this case will involve additional congestion costs which will be calculated in the model. The model is calibrated so that the situation at the initial point in time as modelled actually corresponds to the initial situation in the city. This means that the various parameters of the model must be given values such that the output values from the model for the initial year are close to the observed values. As an example, Table 2 shows the observed and simulated values for commuting in Västerås. As can be seen, the correspondence is quite good. The values of the different parameters obtained from the above calibration have been used as input data in the model.

Calculation of Transportation Costs

Transportation costs as consequences of two different city master plans were also calculated. The calculations include the following cost items: roads, sidewalks, bicycle lanes, parking places, bus stops, bicycles, cars, buses, operating costs for cars and buses such as for gasoline and maintenance, commuting time, etc. Transportation costs for the two city master plans are calculated for the time horizon 1980–2000. Those investment costs which fall within the time horizon are discounted to present values and added to the discounted operating costs. Some of these cost calculations are presented in Appendix B.

Results

Some illustrative results concerning the urban commuting model are presented below for the two alternative city master plans: satellite alternative I and urban renewal alternative II.

Table 3. Transportation Costs for Two City Master Plans for Västerås (SEK millions).

	Alt I	Alt II	Alt II-Alt I
Costs for roads	100	54	-46
Costs for commuting	1144	1118	-26
Transportation costs	1244	1172	-72

Costs

Table 3 indicates that urban renewal alternative II is, as expected, cheaper than the satellite alternative as far as transportation costs are concerned. The reasons for this are:

- Fewer new roads are necessary according to urban renewal alternative II; the existing road system in the inner city can be used more intensively.
- The average commuting distance is less in alternative II.

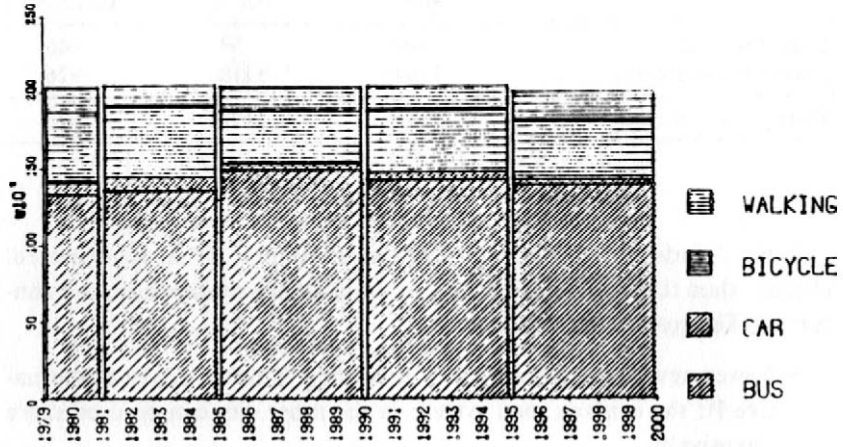
Thus, the larger congestion costs in alternative II do not outweigh the costs of the longer distances in alternative I.

The commuting pattern

The future commuting pattern is also simulated for the two city master plans. Figure 2 shows the commuting pattern in the different work-place centers for alternative I and how it varies during the simulation period. It can be observed, for example, that the dominant mode of commuting to the CBD (WPC 1) will be by bus, followed by bicycling and walking, while car commuting is dominant for WPC 10. Figure 3 shows the distributions among the different work-place centers for alternatives I and II for the final period, 1995–2000. The patterns are quite similar in the two alternatives. The following common characteristics can be observed:

1. Bus commuters are strongly concentrated to the CBD (WPC 1).
2. Most of the car commuters are concentrated to WPC 2, WPC 3, WPC 6 and, in particular, WPC 10.
3. Bicycle commuters are concentrated to the three central work-place centers, WPC 1, WPC 2 and WPC 3, as well as to WPC 10 and satellite WPC 9.
4. Walking commuters are also concentrated to the three central work-place centers.

COMMUTING TO WPC. 1



COMMUTING TO WPC. 10

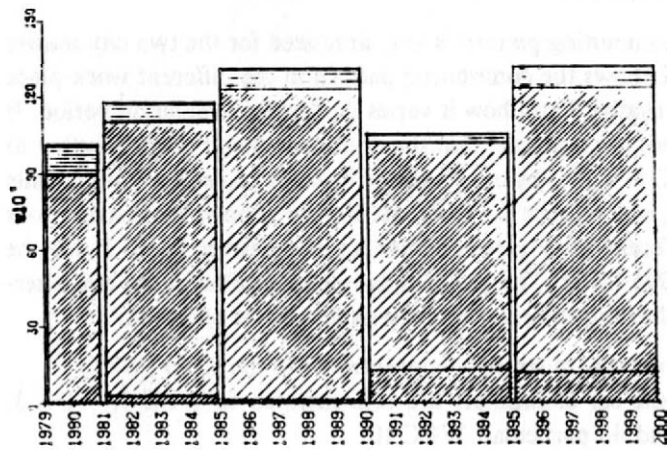


Figure 2. Commuting pattern over time. Alternative I.

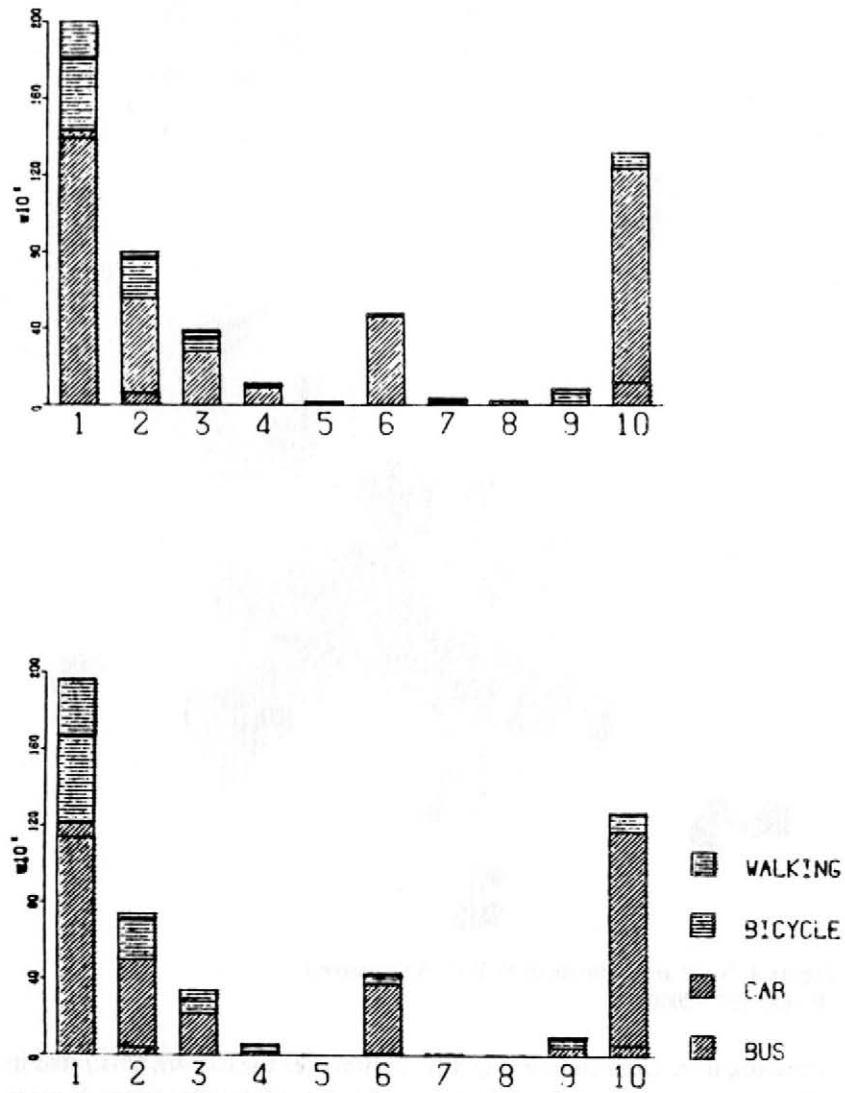


Figure 3. Commuting pattern for different work-place centers. Alternatives I and II. (Period 1995-2000)

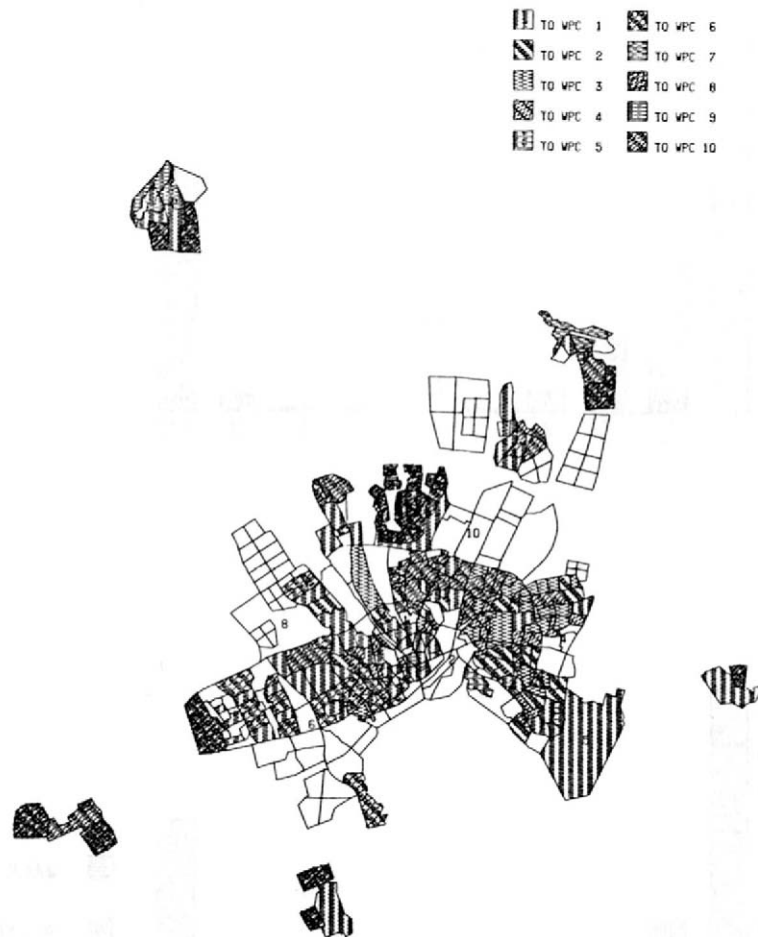


Figure 4. Node assignments to W.P.C. Alternative I.
(Period 1995-2000)

The assignment and commuting distribution over the city are illustrated in Figure 4, which shows the assignment of residential nodes to work-place centers for alternative I during the last period 1995–2000.

Conclusions

In an analysis of urban commuting two interdependent aspects should be considered simultaneously, i.e., where and how each worker in a resi-

dential area commutes. Two different parts of a model are required in order to examine these aspects, one related to the assignment problem and the other to the choice of commuting mode and route.

Simulation of the assignment problem in a multicenter city is a complicated problem. A gravity rule may provide a rough simplification. The choice of commuting mode and route can be simulated on the basis of the concept of minimum commuting costs. These two simulations were integrated into a single commuting model and applied to a real city. Once the commuting pattern has been simulated, total commuting costs for the city can be evaluated in economic terms.

Appendix A. Computation of Individual Commuting Costs

Parking fee

The value of parking fees is given by initial data as starting values and then found by calibration. The units are SEK/vehicle and day.

Congestion costs

The formula used to calculate the car commuter's congestion cost is based on the well-known fact that velocity decreases as flows increase; see Walters (1961) and (1968). For simplicity it will be assumed that there is a linear relationship up to the maximum flow per lane, so that

$$v_3 = \bar{v}_{30} - \bar{v}_{31}f \quad (\text{A.1})$$

where \bar{v}_{30} and \bar{v}_{31} are given coefficients.*

The congestion value, f (the flow of cars per lane per unit of time) is found by assuming that $f = f_0 + f_c$, where f_0 represents the congestion due to traffic other than commuting traffic and f_c represents the congestion due to commuting. A single car driving on a link increases the congestion by

$$\frac{1}{\bar{T}_{rush}} \quad (\text{A.2})$$

where \bar{T}_{rush} is the length of the peak period. The marginal congestion cost x_3^1 will then be (time loss per commuter of an additional car commuter) \times (number of car commuters delayed because of the marginal car commuter) \times (time value of car commuting).

*Here the variables known as data are denoted by a bar.

The first factor is equal to

$$d_3 \frac{1}{\bar{v}_3} + \frac{1}{\bar{v}_3 + \Delta v_3}$$

where

- d_3 is the commuting distance
- \bar{v}_3 is the average actual velocity
- Δv_3 is the decrease in the velocity of a car due to the marginal commuter.

The second factor is obtained from the expression:

$$\frac{NC \cdot \bar{n}_c t_3}{T_{rush}}$$

where

- NC is the number of cars moving along the link
- \bar{n}_c is the average number of commuters per car
- t_3 is the time required to travel the distance d_3 .

The third factor is the given time value of car commuting, i.e., \bar{p}_{t3} . Then the expression for the marginal cost of congestion is

$$x_3^1 = d_3 \left(-\frac{1}{\bar{v}_3} + \frac{1}{\bar{v}_3 + \Delta v_3} \right) \frac{NC \bar{n}_c t_3}{T_{rush}} \bar{p}_{t3} \quad (A.3)$$

Bus time costs and bus fare

The time costs include costs for the time required to walk to the bus stop, wait for the bus, ride the bus to the desired bus stop and walk from there to the workplace center. These costs, except for waiting costs, are found merely by dividing the distance by the velocity and multiplying by the value of time. When the intervals between bus departures are short ($\Delta t_3 < 10$ minutes), commuters can be assumed to arrive uniformly. In this case, the average waiting time is half of the time interval between buses. The bus fare should take into account time losses to the commuters in a bus due to the marginal bus commuter. These time losses are calculated in the following way:

$$x_4 = \frac{\bar{q}_4}{2} (\bar{t}_c + \bar{t}_1) \bar{p}_{t1} + (\bar{t}_c + \bar{t}_1) p_d \quad (A.4)$$

where

- \bar{q}_4 is the capacity of the bus
- \bar{t}_e is the boarding time for one person
- \bar{t}_1 is the deboarding time for one person
- \bar{p}_{t1} is the value of time
- \bar{p}_d is the cost of manning a bus.

The values of \bar{t}_e and \bar{t}_1 were obtained by calibration, under the assumption that x_4 is the actual bus fare for Västerås.

Appendix B. Data on Commuting Costs

The following costs refer to the year 1979.

Walking costs

The only cost item to be considered for walking is time. The time value of walking is SEK 16/hour; see Bruzelius (1979).

Bicycling costs

The cost items for bicycling are time plus the cost of using a bicycle. The time value used for bicycling was SEK 23.84.

Costs for commuting by car

The following cost items are included in the operating costs for commuting by car:

- \bar{p}_{t3} time costs
- \bar{p}^{dm} depreciation and maintenance
- \bar{p}^{got} gasoline, oil and tires

where

$$p_{t3} = \text{SEK } 8/\text{hour; see Bruzelius, (1979).}$$

Car costs are $p_3 = \bar{p}^{dm} + \bar{p}^{got}$, which may be expressed as a function of the flow, f , as follows; see Andersson-Samartin (1979).

$$p_3^1 = 0.1718 \times 10^{-12} v_3^2 - 18774 \times 10^{-9} v_3 + 10.5882 \times 10^{-4}$$

$$\text{and } v_3 = \bar{v}_{30} - \bar{v}_{31} f$$

where p_3 is expressed in SEK/hour, v_3 in m/hour and f in cars/hour.

The coefficients \bar{v}_{30} and \bar{v}_{31} depend on the type (local, secondary and main) of road.

Costs for commuting by bus

The following items are included:

- \bar{p}_{t4} time costs
- \bar{p}^{dm} depreciation and maintenance costs
- \bar{p}^{fo} fuel and oil costs
- \bar{p}_d costs of manning buses.

The following cost data are used for buses with a capacity of 50 seats:

$$\bar{p}_4^{dm} + \bar{p}_4^{fo} = 3.68 + 0.38 = \text{SEK } 4.06/\text{km}$$

and

$$\bar{p}_d = \text{SEK } 60/\text{hour.}$$

The velocity of an express bus, \bar{v}_4 , is assumed to be the same as the velocity of a car on the main roads.

A bus is assumed to correspond to 2.5 cars in traffic, i.e., a conversion factor, of 2.5 is applied in order to obtain a homogeneous flow of vehicles, including buses, through traffic per unit of time.

Express buses run from the satellites to the CBD. Buses on the bus routes in the inner city are local buses. Thus, when calculating the net velocity of these buses, the time required for stops has to be subtracted. The distance between two consecutive bus stops for internal bus lines, d_4 , is assumed to be 300 meters.

References

- Andersson, R., Samartin, A., *Interdependence among Housing, Heating and Transportation in Cities*, Swedish Council for Building Research, D9:1979.
- Andersson, R., Interdependence between Housing and Transportation, *Journal of Advanced Transportation* Vol 15n No 2 1981.
- Andersson, R., Samartin, A., *The Economic Evaluation of Master City Plans: A Pilot Study of Västerås, Sweden*, Volumes 1 and 2, Swedish Council for Building Research, D18 (1983) and D19 (1983).
- Ben-Akiva, M.-Lerman, S.R., *Discrete Choice Analysis: An Application to Predict Travel Demand*. MIT Press, 1985.
- Bruzelius, N., *The Value of Travel Time. Theory and Measurement*. Department of Economics, University of Stockholm, 1979.
- De Donnea, F.X., *The Determinants of Transport Mode Choice in Dutch Cities*, Rotterdam University Press, Rotterdam 1971.
- De Donnea, F.X., Consumer Behavior Transport Mode Choice and Value of Time: Some Micro-Economic Models, *Regional and Urban Economics*, vol. 1, pp. 355-82, 1972.

- Domenicich, T.A., Mc Fadden, D., *Urban Travel Demand: A Behavioral Analysis*, North-Holland, Amsterdam 1975.
- Mc Gillivray, T.A., Mode Split and the Value of Travel Time, *Transportation Research*, vol. 6 pp. 309-16. 1972, a.
- Mc Gillivray, R.G., Binary Choice of Urban Transportation Mode in the San Francisco Region, *Econometrica*, vol. 40, pp. 827-48 1972, b.
- Quarmby, D.A., Choice of Travel Mode for the Journey to Work: Some Findings, *Journal of Transport Economics and Policy*, vol. 1 pp. 1-42, 1967.
- Walters, A.A., "The Theory and Measurement of Private and Social Costs of Highway Congestion", *Econometrica*, 1961. Vol. 29.
- Walters, A.A., *The Economics of Road User Charges*, 1968. The Johns Hopkins Press, Baltimore, Maryland.