Economic Evaluation of School Plans in a Master City Planning

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Abstract

A model is presented for simulation and economic evaluation of school
plans within the framework of master city planning. The model has
been applied to the plans for a Swedish city, Västerås, and some
illustrative results are reported.

1. Introduction

The objective of this paper is to present a method for simulation
and economic evaluation of school plans within the framework of master
city planning. Such planning is required in order to adapt the city
structure to exogenous changes in an efficient way, while taking into
account the strong interdependence among housing, heating, transporta-
tion and other city facilities such as schools. The problems which have
to be considered in master city planning include the choice of locali-
zation for new buildings, types of housing, modes of heating (e.g.,
district heating versus electric heating, etc.), modes of transporta-
tion, the use of existing schools versus building new schools, etc.
For example, when expansion of a city is envisaged, two corner solu-
tions appear, i.e., to build (and perhaps also demolish old buildings)
in the inner city or to construct buildings on the outskirts of the
city. For each of these two extreme cases of city plans, complementary
decisions concerning different city facilities should be considered.
Building on the outskirts usually implies single-family houses, car
commuting and new schools. An urban renewal alternative, on the other
hand, includes multifamily houses, more public transport and use of
already existing schools; cf. Andersson, R. & Samartin, A. (1979).

In addition to the interdependence among complementary city facilities,
uncertainty about the future, the durability and irreversibility of
city structures complicate economic evaluation of the consequences
of building plans; cf. Walters (1968), von Rabenau (1973), Anas (1976)
and Wheaton (1982). These characteristics should be considered explicitly
in a cost-benefit analysis of alternative master city plans.

An important issue in master city planning in many Swedish cities today
involves not only the use of existing schools in built-up areas, but
also where, when and in what size to locate new schools so as to avoid
excessive busing. This problem is accentuated over time due to the
fact that the ageing population lives in the inner city, where existing

schools are located, while families of fertile age usually move to newly developed city areas.

An economic evaluation of different school plans is presented in this paper, with an emphasis on the cost aspects. Due to interdependence problems noted above, such an evaluation should be carried out within a general model for master city planning. A model developed by Andersson-Samartin-Martinez (1983) is summarized in Section 2. The school model is then presented in more detail in Section 3. Some illustrative results obtained from an application to Västerås, a city in central Sweden, are given in Section 4 and some conclusions are drawn in Section 5.

3. Model for master city planning

2.1 General features

City activities as a response to a master city plan under study are simulated to enable evaluation of the consequences in economic terms. For instance, in order to calculate the total costs for commuting, the mix chosen from the available modes of commuting have to be determined first. This implies that the model used must simulate the demand of the individuals who commute. The degree to which the various modes of commuting are used is determined to a large extent by the private costs incurred (parking fees, bus fare, time costs, etc). Therefore the model for master city planning performs two essential tasks: simulation and evaluation. The first task is accomplished by the following set of interlocking models:

- model for the working population
- model for allocation of the inhabitants
- model for housing
- model for transportation
- model for assignment of working place centers
- model for heating
- model for schools
- model for determination of land rent distribution.

These models are all strongly interdependent in the sense that the results from one model may be used as inputs for another. Due to the large number of mathematical operations involved, an iterative computer

program is required.

Some of the data used in one model consists of the results obtained in another model in a previous iteration. The iterations will cease when some kind of equilibrium conditions are reached for all models. In broad terms, iterations are halted when the results obtained in two consecutive iterations are approximately equal.

Once simulation of the city conditions has been accomplished, the economic evaluation can be performed. The economic evaluation is carried out in a single model where all the different cost items are calculated and inserted into an objective function.

The different models are described briefly in the following subsections.

2.2 Model for the working population

In this model it is assumed that among uncertain changes in exogenous variables, those in working opportunities for a city are regarded as the most important changes. The three main steps in this model are:

1. Simulation of the path of working populations' opportunities, η, and the probability of its occurrence, $\pi(\eta)$, for the time span of the study.

    The path, η, and its associated probability, $\pi(\eta)$, are obtained numerically for each intersection point in time, t_a, by assuming a given probability distribution between the upper and lower bounds of the working possibilities. The path, η, is represented by the following vector:

    (2.1) $\eta \equiv WP(t_1), WP(t_2),\ldots, WP(t_a),\ldots, WP(t_{NT})$.

2. Distribution of the working population.s opportunities $(WP(t_a)$ among the different working place centers, using an exogenously given rule.

3. The ratio between the working population and the total population here called s, is given for each city node and intersection time.

2.3 Model for allocation of inhabitants

In general the population of one node is interdependent of the population of the remaining nodes. The population for the various nodes are usually obtained by a minimization procedure. The main purpose of this model is to simulate a reasonable allocation of the population over the city area. Therefore, a strong interdependence among the population values is assumed in the form of the following function:

$$(2.2) \quad D_n = A \cdot \exp(b \cdot lr_n) \; ,$$

where

D_n is the population density at node n, i.e., the population (P_n) per unit of total neighborhood area $(\bar{\Omega}_n)$:

$$D_n = P_n / \bar{\Omega}_n$$

A and b are two parameters.

lr_n is the land rent value at node n. The computation of the land rent values is described in Section 2.9.

Equation (2.2) is an extension of the empirical function given earlier by Clark (1951). Muth (1969) and Mills (1972). The value of A is obtained from an equilibrium condition for the city, i.e., that the total city population must be provided with residences within the city limits.

2.4 Model for housing

Once the population has been allocated over the city nodes, it may become apparent that new residences have to be constructed. The solution of this problem involves three major steps:

1. Determination of average household size for every node (nf_n). The average household size for the whole city is known from statistics. Household size at each node and intersection time is determined using exogenously given rules.

2. Determination of the number of apartments at each node (na_n). The following simple formula is used for determining the number of residences required.

$$(2.3) \quad na_n = \frac{P_n}{nf_n}$$

where

P_n is the population at node n

nf_n is the household size at node n.

3. Determination of the number of storeys at each node (α_{1n}).

These values may be found by introducing the concept of the neighborhood area Ω. The neighborhood area of a node is the fraction of the total area of the node required for residences, open space, local roads, etc. The proportion between the neighborhood area and the total area is called the exploitation factor (e_n) and is given as data. The following relationship holds:

$$(2.4) \quad \overline{\Omega}_n = e_n \cdot \Delta x_n \Delta y_n .$$

In general, the neighborhood area, $\overline{\Omega}_n$ required per apartment is a function of the number of storeys, i.e., $\overline{\omega}_n = \overline{\omega}_n(\alpha_1)$. This function is given as data.

The number of storeys at each node is obtained by using the above concepts.

2.5 Model for transportation

The only form of transportation explicitly considered in the model is from the residential nodes to the working place centers. Four possible modes of commuting are included: walking, riding a bicycle, driving a car and commuting by bus.

The transportation layout is given as data for the existing city at the initial point in time and for the different intersecting points of time considered in the span of the study. All the possible routes for commuting to the working place centers in the district are also given as data.

From among the possible routes and modes of commuting, each worker living at node n and working at a working place center p will choose the route and mode which minimizes his individual commuting costs, i.e.,

$$(2.7) \quad c_{bj} = \min_{r} c_{bj}^{r}(n,p)$$

and

$$(2.8) \quad c_{b} = \min_{n} c_{bj} \quad ,$$

where $c_{bj}^{r}(n,p)$ are the commuting costs from node n to working place center p using route r and comuting mode j.

These costs include such items as time, gasoline, maintenance, deprecia-tion, parking fees, bus fares, etc., depending on the particular commuting mode. Traffic congestion and the costs due to its external effects are determined endogenously by the model; see Solow (1973).

2.6 Model for assignement of working place centers

The process of finding the most efficient choice of working place center for each residential node is regarded as an assignment problem. In order to obtain a "reasonable" solution to this problem, a simplify-ing assumption, known as a gravity rule, is introduced.

The main purpose of the gravity rule is to define some "attraction value", AV_{np}, between a residential node n and a working place center p:

$$(2.9) \quad AV_{np} = \frac{WP_{n} \cdot \overline{WP}^{P}}{c_{b}(n,p)}$$

where

$\overline{WP}^{P}$ is the number of employment opportunities at working place center p,

WP_{n} is the number of workers living at node n, and

$c_{b}(n,p)$ is the individual costs related to commuting from node n to working place center p.

The attraction value given by (2.9) permits workers to be assigned to the working place centers according to the following conditions:

1. A working place center p is preferred to p' by the workers living at residential node n if AV_{np} is greater than $AV_{np'}$.

2. Workers living at node n are more likely to work at working place center p than those living at node n', if AV_{np} is greater than $AV_{n'p}$.

3. For all nodes n with workers who choose working place center p, the following equilibrium condition must hold:

(2.10) $\sum_{n} WP_n \leq \overline{WP}^P$.

2.7 Model for heating

This model is similar to the transportation model. The heating mode chosen by the residents living at node n is such that their own individual heating costs are minimized:

(2.11) $c_h = \min_{j} c_{hj}$,

where c_{hj} are individual heating costs corresponding to the heating mode j.

2.8 Model for schools

This model amounts to a simplified combination of the model for transportation and the model for assignment of working place centers. Since the model for schools is the focal point of this study, it is described in detail in Section 3 below.

2.9 Model for land rent distribution

Land rent distribution corresponds to a set of shadow values which provide an indication of the values of one particular allocation of the inhabitants over the city area with respect to the locations of relevant city activities. In order to find the land rent distribution, the following condition is assumed to hold for all individuals in a given income class regardless of where in the city they live; see e.g. Mohring (1961) and Alsonso (1964).

(2.12)   apartment rents + heating costs + commuting costs = constant = K.

The constant is the same throughout the city for a given income class.

Equation (2.12) was given in Andersson & Samartin (1983b). It reflects the indifference of an individual of a given income class to living at

one node or another when his individual costs for housing, heating and commuting are the only factors taken into account.

The expression used to evaluate individual commuting and heating costs were presented in Sections 2.5 and 2.7. The apartment rent, ar, for an individual is determined by the following equation:

$$(2.13) \quad ar = (\frac{lr}{\alpha_1} + bc) \, \alpha \, ,$$

where

lr	is the land rent per unit area per unit of time
α_1	is the number of storeys
bc	is building costs (construction, maintenance) per unit of area and per unit of time.[1]
α	is the amount of habitable space demanded per person, and is assumed to depend on income and apartment rents:

$$(2.14) \quad \alpha = h(ic)^{\theta_1} (ar)^{\theta_2} \geq \bar{\alpha}_{min} \, ,$$

where

h	is a constant
ic	is the income class assumed for the individual
θ_1 and θ_2	are given elasticity coefficients
α_{min}	is a given minimum prescribed value for α.

The value of the constant in (2.12) can be obtained for each income class by solving this equation at the city limits for each income class. The land rent at the city limits is given by the value of the land in agricultural use, which is quite low. The city limits are defined by the set of nodes at which the land rent is a minimum, i.e.,

$$(2.15) \quad lr = minimum.$$

Once the value of the constant of (2.12) is known, that equation can

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[1] Building costs are obtained in the model as a function of the number of storeys and apartment size.

be used to determine the value of the land rent for each income class at each city node.

The land rent distribution determined using this method reflects the scarcity of land in urban use, so that the costs of land are greater near the city center than at the city limits. Therefore, when residences are built at the city node, n, the land costs per unit of time are evaluated using the following equation

$$(2.16) \quad 1r = \frac{\alpha_1}{\alpha} K - \frac{\alpha_1}{\alpha} (bc \cdot \alpha + \text{commuting costs} + \text{heating costs}).$$

2.10 Model for economic evaluation

The models described in Sections 2.2-2.9 are used to simulate the various city activities. The computational procedure is iterative. After a solution has been obtained for all of the city activities, the model for economic evaluation can be applied.

The following cost items are included in the economic evaluation:

TC^A costs for land

TC^B costs for residences

TC^C costs for the transportation system

TC^D costs for heating

TC^E costs for schools

TC^F revision costs.

All these costs are obtained for each intersection point in time, t_a. At a particular point in time, the simulated path of employment opportunities, η, can be reviewed and contrasted with the actual path, η_{actual}. The difference in costs for the expansion and contraction cases can be calculated and designated by IC^+ and IC^-, respectively. The probabilities that these incremental costs (or savings) will be realized are π^e and π^c, respectively. Revision costs can then be formulated as follows:

$$(2.17) \quad TC = \mu_1 IC^+ \pi^e + \mu_2 IC^- \pi^c \; ,$$

where μ_1 and μ_2 are two risk aversion factors. These factors will usually be equal to unity, thus reflecting risk neutrality.

The total costs for each period of time are calculated as:

$$(2.18) \quad TC(t_a) = TC^A + TC^B + TC^C + TC^D + TC^E$$

All these costs are discounted to present values for each period of time using the given real interest rate for the time horizon chosen.

An objective function may now be defined in order to compare different plans, where following is taken into consideration:

The total costs, u, incurred due to the simulated path, η, are given by the expression

$$(2.19) \quad u = \int_0^T TC(t_a) \, dt_a \equiv u_\eta .$$

The variable, u, is a random variable which depends on the working population path, η, and the probability of its occurrence is equal to the probability of η, i.e., $\pi(\eta)$. The mean value of the variance of u can be defined as:

$$(2.20) \quad \text{mean value} = \bar{u} = E[u]$$

$$(2.21) \quad \text{variance} = \sigma^2 = E[(u-\bar{u})^2].$$

The objective function, OF, is then defined as follows; see Arrow (1970):

$$(2.22) \quad OF = \bar{u} + \mu_3 \sigma^2 ,$$

where μ_3 is a risk aversion factor.

The objective function of (2.22) allows for comparison of two master city plans. In general, the plan which gives a lower value of the objective function is preferred to the plan which gives a higher value.

3. <u>Model for schools</u>

3.1 <u>Introduction</u>

The main objective of this model is to assign the children living at
each residential node to the different existing or planned school
centers in an economically efficient way. In some respects this problem
is very similar to two models that have already been described, i.e.,
the assignment of working place centers to the working population and
the selection of modes and routes for commuting. The model for schools
is developed on the basis of these two models, with some strong simpli-
fications.

The school model may be divided into three parts. First, the distribu-
tion of the school children over the city area and over time has to be
forecasted. Second, once this distribution is known, a simulation of
the assignment of school children to the different schools has to be
carried out. Third, after the assignment of school children has taken
place at each intersecting time, the corresponding costs can be computed.

These three computational steps may be summarized as follows:

1. Distribution of the total number of school children among the dif-
   ferent residential nodes. The number of children at the initial
   point in time and throughout the time span of the study are given
   as data. The distribution of school children is also known for the
   initial point in time. Starting from this distribution, specific
   rules are applied to determine the distribution of school children
   over the city area for the total time of the study.

2. Assignment of school children to the different schools. The con-
   cept of minimum commuting costs is used to determine the choice of
   school and transportation mode for children residing at each resi-
   dential node.

   The following constraints apply in this assignment process:

   - Walking and busing are the only means of transporation considered
     for school children.

   - Maximum distances for commuting are specified. For Västerås, the
     maximum walking distance is set at two kilometers and the
     maximum busing distance at ten kilometers.

   - The capacities of the schools are given as data and the number
     of children attending a school may not exceed the capacity of
     the school.

3. Calculation of school costs such as investment costs for new schools, maintenance costs, transportation costs, etc.

3.2 Forecast for the distribution of school children over the city nodes

The total number of school children in the city for each point in time is assumed to be exogenously given when a mean forcast of the total population is assumed. In order to consider different levels of the total population over time, and subsequently changes in the total number of school children, the following data are assumed given for each point in time:

- The ratio of the total number of school children to the total population.

- The ratio of the number of children in each residential zone (a set of residential nodes) to the total population of the zone.

In addition, the following information is given for the initial point in time:

- The ratio of the number of children at each residential node to the total population at the node.

These data permit calculation of the ratio of the number of children in each residential node to the population at the node for each point in time throughout the simulation, if the following simplifications are introduced:

(a) Nodes which belong to the same zone of the city retain the same value of this ratio throughout the time period covered by the simulation.

(b) New residential nodes, including such nodes in the inner city where old residences have been demolished and replaced by new ones, are given a value of this ratio equal to the mean value of the ratio for the entire city at the point in time they come into being.

(c) Nodes which exist in the initial period retain a value of this ratio that is proportional to the value in the initial period.

The proportionality factor for existing nodes is obtained from the condition that the total number of children must equal the forecasted total.

This can be expressed in mathematical terms as follows:

It is assumed that the values of the ratio $\overline{sr}^a$ between the total number of school chilren (SC^a) and the total population (p^a), given as data for each point in time, t_a, are independent of the level of population, i.e., of the value of the simulated population path at time t_a.

The given values of the ratio $\overline{sr}_z^a$ between the number of children living at a residential zone (z) and the total population there p_z^a can be used to find the distribution of the total school population SC^a among the residential city nodes (n), i.e., the values of sr_n^a defined according to the expression

(3.1) $sr_n^a = \dfrac{\text{school children living at node n}}{\text{total population living at node n}}$ (at time t_a)

The values of sr_n^o (initial time) are also known as data.

The hypotheses presented may then be expressed in the following way:

(a) Each zone z is homogeneous, i.e., the sr_z^a for the zone is equal to sr_n^a for every node n belonging to z.

(3.2) $sr_n^a = sr_z^a$ for every $n \in z$

(b) For new residential nodes, the value of sr_n^a is equal to the mean (average) value given as data for the total city, i.e.,

(3.3) $sr_n^a = \overline{sr}^a$,

   where n is a new residential node or a residential node in the inner city where houses have been demolished and replaced by new residences.

(c) For each zone z, where no new residences will be built, the value of sr^a is scaled by a factor λ^a that is the same for all zones. The value of sr_z^a is obtained from the condition that the number of school children in the city at each point in time must be equal to the forecased number; thus:

$$(3.4) \quad \lambda_a \sum_{z \in Z} sr_z^a p_z^a + \overline{sr}^a \sum_{z \in Z'} p_z^a = \overline{sr}^a \sum_{z \in Z+Z'} p_z^a \, ,$$

where Z is the set of city zones in which no new residences are built at time t_a and Z' is the set of remaining zones. The total city population is

$$P_a = \sum_{z \in Z+Z'} p_z^a$$

and p_z^a is the total population living at zone z.

Thus, the following equations are obtained:

If p_n^a is the population at node n at time t_a, the number of school chilen in zone z is

$$(3.5) \quad SC_z^a = sr_z^a \sum_{n \in z} p_n^a$$

and at node n

$$(3.6) \quad SC_n^a = SC_z^a \frac{SC_n^0}{\sum_{n \in z} SC_n^0} = sr_z^a \, SC_n^0 \frac{\sum_{n \in z} p_n^a}{\sum_{n \in z} SC_n^0}$$

where SC_n^0 corresponds to the number of school children at node n at initial time t_o.

Equation (3.6) applied to all existing nodes. For new nodes, the number of school children is simply

$$(3.7) \quad SC_n^a = \overline{sr}_n^a p_n^a \, .$$

3.3 School assignment

The school children choose their commuting mode (bus or walking) and school according to the following procedure, to be applied to very city node:

1. The commuting costs c_b from a particular residential node to each of the school centers are calculated for the two modes of transportation allowed (walking and busing), i.e., costs c_{bw}^{nM} and c_{bB}^{nM} from node n to school center M.

The minimum of these two values of c_b is choosen for each school and the mode of transportation is also determined implicitly. This minimum is called c_b^{nM}:

(3.8) $\quad c_b^{nM} = \min_{W,B}(c_{bW}^{nM}, c_{bB}^{nM})$.

2. Maximum walking and busing distances are specified. If the distance to the school exceeds the maximum walking distance, then busing is compulsory. However, if the distance also exceeds the maximum busing distance, then the school is not a feasible one for the particular city node.

3. If there is a feasible school relative to a given residential node, then the school children living at the node will be sent to the school with the least commuting costs if there is sufficient capacity at that school. If not, then the next cheapest school is selected, if it has enough capacity. Otherwise, the process is continued until a school with sufficient capacity is obtained.

In other words, if the minimum value of $c_b^{nM'}$ over all school centers M', is c_b^n and it corresponds to a particular school center M, this indicates that the children living at n will attend school center M, providing there exists enough free capacity in this school. Otherwise, the next cheapest school center M (producing the possible minimum value of $c_b^{nM'}$) with sufficient capacity for the school children living at node n has to be found.

4. If there is no school within the maximum busing distance from the residential node, then a new school center near the node is required. This situation is revealed by the model.

3.4 Calculation of school costs

School costs are calculated within the model for economic evaluation in Section 2.10. The details of this calculation procedure may be summarized as follows.

The costs of schools are calculated according to the type of school, i.e., existing or new. The following cost items are considered for each case.

Existing schools

- Costs due to alternative use of the schools
- Maintenance costs
- Transportation costs

New schools

- Investment costs
- Maintenance costs
- Transportation costs.

The computation of these different cost items may be commented on as follows.

- The costs due to alternative use of existing schools are the estimated remaining value RV (value for the actual alternative use) per student place in each school given as data, multiplied by the total number of existing students places. These costs are annualized for the remaining lifetime and summed up to a present value.

- The investment costs for new schools are computed in a similar way to the above costs, in the sense that the estimated remaining value (RV) of the existing school for an alternative use is replaced by the investment costs per student. These costs are put in costs per annum for the remaining economic lifetime and summed up to a present value.

- The given data for maintenance costs correspond to the annual maintenance costs per student. Once the number of students attending the school is known, these maintenance costs are obtained by multiplication and are discounted and summed to present values.

- Transportation costs are the minimum of the costs for the two possible commuting modes between residence and school.

  The individuals' walking costs are simply

$$(3.9) \quad c_{tl} = \frac{d_1}{v_1} P_{sl}$$

  where

  d_1 is the straight-line distance between residence and school. If this distance is greater than some given limit (2 km) then walking will be excluded, i.e., these costs are assumed to be very large relative to the costs for busing;

v_1 is the walking speed;

P_{s1} is the value of time spent in walking to school. It is assumed to be half the value of time spent in walking to work (by workers).

The individuals' busing costs are

$$(3.10) \quad c_{t2} = t_4 P_{s4} + \frac{NBUS}{SC_n}(d_4 \cdot p_{s4}^{bus} + t_4 \cdot P_{driv} + \frac{p_4'}{2 \cdot NDAY}) \, ,$$

where

$t_4 \quad _4/v_{40}$ is the time required for traveling by bus from the residential node to school

d_4 is the busing distance between the residential node and school (not the straight-line distance)

v_{40} is the bus velocity without congestion[1]

P_{s4} is the value of time spent in traveling to school by bus

p^{bus} is the costs for fuel, oil and tires, and depreciation and maintenance per kilometer

NBUS   is the number of bus trips from the residential node to school and is equal to the number of students living at the residential node divided by the capacity of a bus (q_4)

P_{driv} is the cost of manning the bus

p_4' is the yearly maintenance costs

NDAY   is the number of school days, assumed to be the same as the number of working days

SC_n is the number of school children living at residential node   n

These costs are discounted and summed to a present value in a way similar to that for other cost items in the master city planning model.

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[1] The noncongestion is introduced in this particular model as a simplification.

4. Results

We now turn to the main results from the evaluation of two master city plans for Västerås. First, the calculated total costs are presented and interpreted. The school costs are then given in some detail. A tentative sensitivity analysis is also shown. Some complementary results for the assignment of school children to different schools are given in the form of computer-drawn maps.

It should be emphasized that these results are preliminary. Therefore, only the results corresponding to one path for the working population are reported. The same simulated path is used for all the plans to facilitate comparison.

4.1 Total costs

Two different master city plans for Västerås are studied. Alternative B is a master city plan with an emphasis on building in the outskirts and satellites of Västerås. Alternative D is an urban renewal plan with demolition and concentration of new residences in the inner city.

The calculated total costs for the two plans are shown in Table 4.1 (SEK in present values). The table also shows the differences in costs between alternatives D and B. The costs are shown for six main cost items: land, residences, roads, commuting, heating and schools.

First of all, the ranking of the alternatives with respect to total costs turns out to be as expected. Master city plan D, which emphasizes urban renewal, is the most expensive and costs SEK 632 million more than B. The major part of the difference is attributed to the costs for resi-dences. This reflects the fact that plan D includes costs for the demo-lition of these additional apartments. But an even more important factor is that 3000 additional apartments have to be built if plan D is adhered to rather than any of the other plans. The conclusion that can be drawn from this is clear: if an urban renewal alternative is to be of interest, it cannot include such extensive premature demolition of residences as has been assumed for alternative D.

Table 4.1: Total costs for four master city plans for Västerås
(millions of SEK)

Cost items	Alt. B	Alt. D	Alt.D–Alt.B
Costs for land	19	38	+19
Costs for residences	1025	1801	+776
Costs for roads	106	52	–54
Costs for commuting	1179	1115	–64
Costs for heating	1430	1408	–22
Costs for schools	237	214	–23
Total costs	3996	4628	+632

As would be expected, land costs are also greater in the urban renewal alternative owing to the higher level of land rents in the inner city. On the other hand, the remaining costs are – as expected – lower in the urban renewal alternative. The reasons for this are:

- Fewer new roads are necessary in the urban renewal alternative (D); the existing road system in the inner city can be used more intensively.

- The average commuting distance is less in alternative D.

- A larger, more efficient boiler can be used for the district heating system and fewer new heating pipes have to be installed.

- The capacity of the existing inner city schools can be used to a greater extent, so that fewer new schools have to be built.

Inspection of the results indicates where it may be possible to improve a plan by making marginal changes. For example, the urban renewal alternative may be improved by decreasing the number of apartments to be demolished, thus significantly reducing the costs for residences.

Generally, proximity to the CBD and other city centers is reflected in a higher willingness to pay for housing, heating, etc. in the inner city than at the city limitjs. The willingness to pay for such benefits not reflected in our cost calculations should be at least as great as the difference in costs to justify selecting the more expensive alternative.

4.2 Results for schools

The distribution of school costs among different cost items is shown
in Table 4.2 for alternatives B and D.

Table 4.2: School costs (millions of SEK in present values)

Cost items	Alt. B	Alt. D	Alt.B-Alt.D
Costs for busing	12	12	0
Costs for school buildings	67	44	23
Maintenance costs	158	158	0
Total costs	237	214	23

The difference in school costs between the two alternatives is due to
the fact that six new schools are planned to be built in the satellite
alternative B and only two new ones in the urban renewal alternative D.

The assignment of school children from the different residential nodes
in Västerås to the various schools according to the rule of least indi-
vidual "commuting" costs is shown in Figures 4.1 through 4.3. The assign-
ment of children to the existing 43 schools at the initial point in
time is shown in Figure 4.1. The capacities available and used for the
different schools is shown in Table 4.5. Ten of these schools are closed
during the period covered by the simulation due to assumed expiring
lifetimes. Figures 4.2 and 4.3 show the assignment of school children
to the remaining schools and to the newly built schools for alternatives
B and D, respectively, in the final period (1996-2000). The maps illustrate
the consequences of the different plans and may provide suggestions for
iterative changes in the plans which could be worthwhile. The results
should not be interpreted to imply that the schools which are scheduled
to be closed should actually be closed.

The value of a particular school does not necessarily depend only on
the age of the school building. It might also depend on its location
relative to other schools. Also, the quality of teachers at a school
and the ability of the principal to stimulate the teachers to engage
in fruitful educational activities might differ substantially from one

school to the next. Therefore, some of the schools assumed to be closed in accordance with an expiring lifetime might be remodelled at some cost and be allowed to continue, while others on the list might be closed on schedule.

4.3 A sensitivity analysis

A sensitivity analysisis for alternative B with respect to some assumed values is shown in Tables 4.3 and 4.4. The following variations are studied:

- A reduced maximum busing distance (from 10 to 5 km)

- A reduced remaining economic lifetime for already existing schools (- 8 years)

- A decrease in the number of school children per class (from 25 to 20)

- An increase in the number of school children per class (from 25 to 30)

Table 4.3: Sensitivity analysis fro alt. B with respect to school costs (millions of SEK)

Cost items	Reduced busing distance (from 10 to 5 km)	Reduced econ. life-time for the school build-ings (-8 years)	No of school children per class decreased from 25 to 20)	No of school children per class increased (from 25 to 30)
Costs for busing	9	17	15	9
Costs for school buildings	66	68	69	64
Maintenance costs	151	152	186	135
Total costs	227	237	270	208

The following observations can be made concerning the results for the main case of alternative B presented in Table 4.3:

- A reduced busing distance will decrease the costs for busing and maintenance. As an overcapacity of student places exists in the main case (see Table 4.4), there is no increase in the costs for school buildings.

- Busing costs increase when the assumed economic lifetime of the school building is reduced, as the average "commuting" distance to the schools increases. The capacity of student places will not be sufficient at the end of the time horizon under study (see Figure 4.4 and Table 4.4)

- Maintenance costs (and busing costs) will increase when the number of school children per class is decreased and vice versa. The capacity of student places will not be sufficient at the beginning and the end of the time horizon. On the other hand, there will be a considerable overcapacity during the whole period when the number of students per class is increased.

Table 4.4: Total number of students and schools in Västerås, 1980-2000; sensitivity analysis of alternative B

| | No of students[1] | Number of school places | | | |
		Main case	Reduced econ. lifetime[2]	Reduced no of students/ class[3]	Increased no of students/ class
1980-1981	9432	10538	10538	8436	12638
1982-1985	8003	11078	9061	8868	13286
1986-1990	7089	11888	9871	9516	14258
1991-1995	7716	10411	7983	8333	12488
1996-2000	8802	10411	5757	8333	12488

[1] It should be kept in mind that the forecast number of school children depends to a large extent on the total population forcast only for the particular path simulated here, i.e., the path simulated is not necessarily the most probable one and it is given below.

Period	1980-81	1982-1985	1986-1990	1991-1995	1996-2000
Population	114 160	112 680	116 555	111 675	118 291

[2] Assumed economic lifetime for existing schools is reduced by 8 years.

[3] Number of school children per class is reduced from 25 to 20.

[4] Number of school children per class is increased from 25 to 30.

5. Conclusions

In view of the strong interdependence among various sectors, a local
government cannot achieve efficient solutions for its planning problems
unless it has access to a tool for drafting school plans within the
framework of master city planning. The model developed in this study
is an attempt to create such a tool. Although only the cost side of
the problem is dealt with explicitly in the model, it may still be
helpful in an iterative procedure, where plans are successively revised
in the light of information obtained from previous evaluations. The
model can provide planners with information as to e.g. how great addi-
tional benefits of a more costly alternative have to be for that alter-
native to be preferable.

This model is being used for a pilot study in the Swedish city of
Västerås. Different master city plans such as a satellite alternative
and an urban renewal alternative have been studied using the same model
with different input data.

Ackknowledgements

Figure 4.1: Assignment of school children to schools. Initial time.
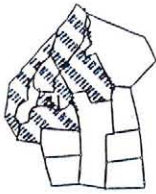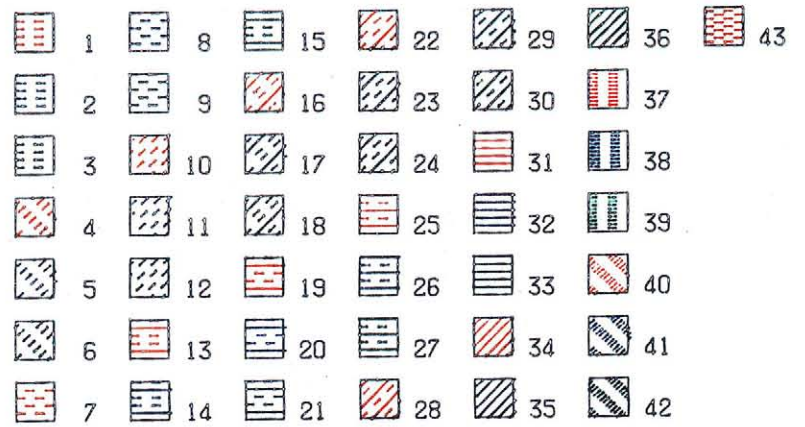
Table 4.5: Assignment of school children to schools; initial time

| School (number)	School places available	used	School (number)	School places available	used
1	382	313	26	337	270
2	180	179	27	157	157
3	202	152	28	157	130
4	360	140	29	202	202
5	247	183	30	157	143
6	225	116	31	270	246
7	157	134	32	270	240
8	450	316	33	270	210
9	450	430	34	270	265
10	180	158	35	450	435
11	270	236	36	270	193
12	180	146	37	157	135
13	472	472	38	40	9
14	202	110	39	382	380
15	225	116	40	180	0
16	225	220	41	292	249
17	67	67	42	247	238
18	138	38	43	225	158
19	228	214			
20	202	131			
21/	157	148			
22	405	381			
23	157	112			
24	270	242			
25	180	154			

Figure 4.2: Assignment of school children to schools. Alternative B 1996-2000

Figure 4.3: Assignment of school children to schools. Alternative D
             1996-2000

Figure 4.4:   Residential areas without assigned schools. Alternative B
              1995-2000 (Reduced economic lifetime for the schools)

 NODES WITHOUT SCHOOL

 NODES WITH SCHOOL
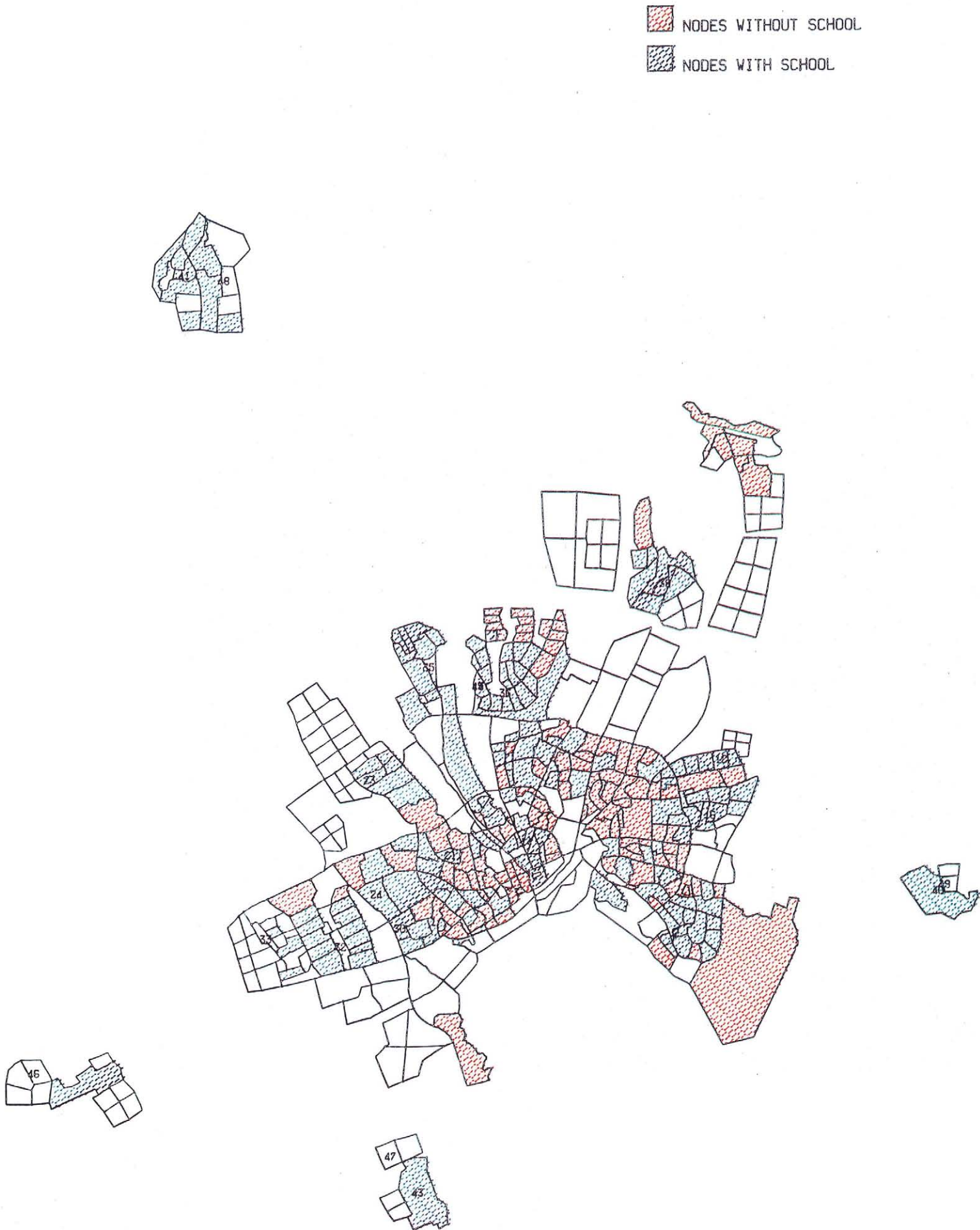
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