PIC Computation Of Electron Current Collection To a Moving Bare Tether In the Mesothermal Condition

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In tethered satellite technology, it is important to estimate how many electrons a spacecraft can collect from its ambient plasma by a *bare electrodynamic tether*. The analysis is however very difficult because of the small but significant Geomagnetic field and the spacecraft's relative motion to both ions and electrons. The object of our work is the development of a numerical method, for this purpose. Particle-In-Cell (PIC) method, for the calculation of electron current to a positive bare tether moving at orbital velocity in the ionosphere, i.e. in a flowing magnetized plasma under Maxwellian collisionless conditions. In a PIC code, a number of particles are distributed in phase space and the computational domain has a grid on which Poisson equation is solved for field quantities. The code uses the *quasi-neutrality condition* to solve for the local potential at points in the plasma which coincide with the computational outside boundary. The *quasi-neutrality condition* imposes $n_e = n_i$ on the boundary. The Poisson equation is solved in such a way that the presheath region can be captured in the computation. Results show that the collected current is higher than the Orbital Motion Limit (OML) theory. The OML current is the upper limit of current collection under steady collisionless unmagnetized conditions. In this work, we focus on the flowing effects of plasma as a possible cause of the current enhancement. A deficit electron density due to the flowing effects has been worked and removed by introducing adiabatic electron trapping into our model.

Introduction

Sanmartin, Martinez-Sánchez and Ahedo [1] proposed a thin bare electrodynamic tether to collect currents in the Orbital-Motion-Limit (OML) regime. The OML current is derived under steady isotropic Maxwellian conditions by calculating the current contribution of all particles whose trajectories can be traced back to infinity from the surface of a collector [11] Therefore we know that the distribution function of electrons on the collector's surface is Maxwellian except that particles

corresponding to negative total energy are excluded. This gives rise to the upper limit of current collection in a steady state. However this prediction may in practice be violated, as seen in the space experiment using a spherical collector (TSS-1R), where a different "upper bound" was seen to be broken.

The TSS-1R space experiment which took place in 1996 brought about unexpected results. The experiment used a spherical collector, whose radius is much larger than the Debye length of unperturbed plasma. Previously it was expected that the current collection would have an upper limit which was derived from the canonical angular momentum conservation–Parker-Murphy model [2] The result was

Table 1: Plasma parameters

Ion mass (O^+)	2.67×10^{-26} kg
Electron temperature	0.1 eV
Ion temperature	0.1 eV
Electron thermal velocity	212 km/sec
Ion thermal velocity	1 km/sec
Satellite speed	4895.9 m/sec ($\sim 2eV$)
Electron (Ion) density	$10^{11}/m^3$
Debye length at infinity	0.74 cm

that TSS-1R collected more current than the Parker-Murphy predictions. An electron temperature increase in the near presheath was also observed. In order to explain the current enhancement, *Cooke and Katz* used a fluid model to relate the potential increase to the temperature increase, assuming that there are trapped electrons in the presheath, being trapped for long enough to use the fluid approximation [3]. *Laframboise* introduced the concept of magnetic presheath to modify (enlarge) the Parker-Murphy collection "tube" [7]. These theories still await experimental and/or computational results for its verification. For a thin tether, the Parker-Murphy limit may be superseded by the OML limit as the upper bound of collected current. However, some of the same phenomena discussed in [3] and [7] may be still involved

In order to predict current collection to a bare electrodynamic tether, we have developed a numerical code using Particle-In-Cell (PIC) method. The PIC method has been established to analyze collisionless plasmas. Especially it works well to simulate particle-field interactions [12]. We incorporate the quasi-neutrality condition at the boundary developed by the authors, to capture the presheath region. This scheme is shown to give good quantitative approximations in the prediction of current collection to a cylindrical probe in and out of the OML regime, in the case of a quiescent unmagnetized plasma [8]. In the ionosphere, at orbital speed where the bare tether will be put in practice, the so-called "mesothermal condition" applies, meaning that the tether's orbital velocity, U_{tether}, is much faster than the ion thermal speed, $v_{t,i}$, and much slower than the electron thermal speed, $v_{t,e}$.

$$v_{t,i} \ll U_{tether} \ll v_{t,e} \quad (1)$$

In this work, we presents recent results from our PIC computations and discuss the effects of the mesothermal condition on the current collection to a bare tether in an unmagnetized plasma. We have developed and incorporated a new and larger grid system to reduce the boundary effects.

A Paradox

When calculated in steady conditions, ion density and electron density in a flowing plasma appear to be incapable of satisfying the quasi-neutrality condition, as explained below. In the mesothermal condition, approximately electrons have a shifted Maxwellian distribution.

$$f_e(w) = A_e \exp\left(-\frac{m_e(w_x - U_{tether})^2 + w_y^2 + w_z^2}{2\kappa T_e} \right) \quad (2)$$

where $A_e = n_\infty (m_e/2\pi\kappa T_e)^{3/2} \exp(e\phi/\kappa T_e)$, n_∞ electron density at infinity, m_e electron mass, T_e electron temperature, e electric charge of electron, ϕ local potential, κ Boltzmann constant. Since the electron thermal speed is much faster than the tether's orbital speed (See. Table 1), the effect of plasma flow on electrons' behavior is negligible. On the other hand, the ion's thermal speed is much smaller than the orbital velocity. Therefore, in the first order approximation, we can assume that ions move one-dimensionally toward the tether in the frame of reference moving with the tether. Using the fluid model for ions, ion density is obtained as

$$n_i = \frac{n_\infty}{\sqrt{1 - \frac{2e\phi}{m_i U^2}}} \quad (3)$$

where m_i is ion mass. This is clearly the function of potential ϕ, and it increases as ϕ increases.

The electron density is obtained by taking the integral of Equation (2) over the range of velocity which is energetically possible. When the local potential ϕ is positive, the minimum velocity of electrons coming from infinity where $\phi = 0$ without collisions, is $\sqrt{2e\phi/m_e}$, corresponding to $w = 0$ at infinity. In the two dimensional unmagnetized plasma, this restriction applies only to the projection of velocity components on the plane of interest. In our case, taking the z-axis along the tether, only electrons with $w_x^2 + w_y^2 > 2e\phi/m_e$ are taken in the integral. Neglecting the flow effects for the aforementioned reason, the local electron density is approximated as

$$n_e \sim A_e \int_{w_x^2 + w_y^2 > 2e\phi/m_e} \exp\left(-\frac{m_e(w_x^2 + w_y^2 + w_z^2)}{2\kappa T_e} \right) dw \quad (4)$$

$$= n_\infty \quad (5)$$

In the two dimensional collisionless plasma case, for $\phi > 0$, electron density is no larger than electron density at infinity. If there is a sink such as a EDtether, the density should be smaller.

This result ($n_e < n_i$ for $\phi > 0$) contradicts the plasma's tendency to keep the overall charge neutral (quasi-neutrality). This "paradox" prevails all over the presheath region, where

local potential is slightly positive (with respect to infinity). The presheath region extends as far as the mean free path. None of the near field explanations of the increased electron density [3, 7] seems appropriate. *Sanmartin* [6] pointed out that the adiabatic electron trapping by a slowly moving potential well in a collisionless plasma, analyzed by *Gurevich* may account for the needed increased electron density in the presheath and satisfy the quasi-neutrality condition. He also derived the relation between trapped electron density and local potential in the 2D case (Gurevich's original paper deals with a 1D problem).

Electron trapping in the presheath

Adiabatic electron trapping in the presheath may occur when a pontential well $(q\phi)$ * is moving more slowly than electrons. In space, a pontential hump created by the highly biased tether is moving at the orbital speed, which is much slower than the electron thermal speed. When the local potential is much lower than the plasma energy, $\frac{q\phi}{\kappa T_e} \ll 1$, the trapped electron density, n_{tr}, is given by

$$\frac{n_{tr}}{n_\infty} = \frac{q\phi}{\kappa T_e} \qquad (6)$$

The brief explanation of the above expression is the following, which is based on the unpublished work by *Sanmartin*.

At given r, the trapped electron density is

$$\frac{n_{tr}}{n_\infty} = \int f_{tr}(\vec{w})\, d\vec{w} \qquad (7)$$

where f_{tr} is the distribution function of trapped electrons (i.e. electrons with negative total energy). Changing variables from velocities (w_x, w_y) to total energy and angular momentum (E, J), we have

$$\frac{n_{tr}(r)}{n_\infty} = \int\int \frac{f_{tr}(E,J)\, dE\, dJ \times 2}{\sqrt{J_r^2(E) - J^2}} \qquad (8)$$

where $\times 2$ accounts for the integration over E from 0 to ∞ once for $v_r < 0$ and again for $w_r > 0$. $J_r(E)$ is defined as

$$J_r^2(E) \equiv 2m_e r^2 [E + e\phi(r)] \qquad (9)$$

If time variations of $\phi(r,t)$ are controlled by the slow ion motion, adiabatic trapping occurs. The trapped electron distribution function, time-averaged on the slow scale of $\phi(r,t)$ is governed by a time-averaged Maxwell-Boltzmann equation

$$\frac{\partial f_{tr}}{\partial t} + <\dot{E}> \frac{\partial f_{tr}}{\partial E} = 0 \qquad (10)$$

* For electron trapping, the actual potential is a hump $(\phi > 0)$

where

$$<\dot{E}> = \left\langle -e\frac{\partial\phi}{\partial t} \right\rangle = \frac{\int dt(-e\partial\phi/\partial t)}{dt} \qquad (11)$$

The solution to the kinetic equation (10) is

$$f_{tr} = f_{tr}[I(t,E,J)] \qquad (12)$$

from which we know

$$\frac{df}{dI}\left[\frac{\partial I}{\partial t} + <\dot{E}> \frac{\partial I}{\partial E} + 0 \times \frac{\partial I}{\partial J}\right] = 0 \qquad (13)$$

where

$$I = \text{constant} \times \int_{r_m}^{r_M} dr\, \sqrt{E + e\phi - J^2/2m_e r^2} \qquad (14)$$

is an integral of the motion. r_m and r_M are two ends of bounded orbits in which electrons may be trapped, given by $v_r = 0$

$$v_r = \frac{dr}{dt} = \sqrt{\frac{2}{m_e}}\sqrt{E + e\phi - J^2/2m_e r^2} \qquad (15)$$

When $J > J_r(0)$, there exist bounded orbits to trap electrons. The limiting energy for trapped electrons, separating bounded and unbounded orbits, is a fixed value, $E = 0$. The distribution function should be continuous at $E = 0$.

$$f_{tr}[I(t,0,J)] = f_{ntr}(E = 0) = f_\infty(E = 0) \qquad (16)$$

since I depends on t and J,

$$f_{tr} = \text{constant (independent of } I) = f_\infty(E = 0) \qquad (17)$$

Substituting equation (17) into equation (8), we have

$$
\begin{aligned}
\frac{n_{tr}}{n_\infty} &= \int_{-e\phi}^{0} \frac{dE}{\kappa T}\left[e^{\frac{0}{\kappa T}}\frac{2}{\pi}\int_{J_R(0)}^{J_r(E)} \frac{dJ}{\sqrt{J_r^2(E) - J^2}}\right] \\
&= \int_{-e\phi}^{0} \frac{dE}{\kappa T}\left[1 - \frac{2}{\pi}\sin^{-1}\frac{J_R(0)}{J_r(E)}\right] \\
&\sim \frac{e\phi}{\kappa T}\left[1 - \frac{2}{\pi}\int_{-e\phi}^{0} \frac{dE}{e\phi}\sqrt{\frac{R^2 e\phi_p}{r^2(e\phi + E)}}\right] \\
&\sim \frac{e\phi}{\kappa T}\left[1 - \frac{4}{\pi}\sqrt{\frac{\phi_p R^2}{\phi(r)r^2}}\right] \qquad (18)
\end{aligned}
$$

The last term is a negligible correction for $r \gg r_0$, arising from low enough J electrons missing from the trapped population. r_0 is the point where $r^2\phi(r)$ has a minimum. This population of $E < 0$ electrons is added at the boundaries, and tracked numerically afterwards. It provides the extra negative charge to break the apparent paradox raised earlier.

Computation

The major difficulty of a PIC method applied to an infinitely large plasma appears in the specification of the computational outside boundary condition, namely the velocity distribution function at a boundary point. In order to treat the computational boundary, electrons are assumed to have a shifted Maxwellian distribution given by equation (2) and ions are assumed to have a distribution function given by

$$f_i = \xi \exp \left\{ -\frac{\left(\sqrt{w_i^2 + \frac{2e\phi}{m_i}} - U_{tether}\right)^2 + (w_y - U_y)^2}{v_{T_i}^2} \right\}$$

(19)

where $\xi = n_\infty \left(\frac{m_i}{2\pi\kappa T_i}\right)$. The ion flow velocity U_y is locally determined computationally.

The density of incoming electrons at the boundary is calculated from equation (2) and the trapped electrons, and that of all ions at the boundary is from equation (19). The density of outgoing electrons is obtained from the numerical technique developed elsewhere[9]. Using these densities, the quasi-neutrality equation is solved to give a local potential at the boundary. This potential is used in the Poisson solver. When the quasi-neutrality equation is not soluble, $\nabla\phi = 0$ is used instead.

To avoid an error associated with the very large velocity of electrons near the high potential tether, sub-iterations are used for fast electrons. Tether charge is also kept at $7eV$, which is above the ion ram energy, reducing the number of very fast particles near the tether. Higher potential cases can be computed without any practical problems. We are in the process of incorporating analytically the movement of fast particles in the immediate vicinity of a tether. We expect the analytical treatment of particles to enable a faster computation and better accuracy.

Results

In Figure 1, instantaneous maps of electron densities (non-trapped, trapped and overall), ion density, electric charge density and potential are shown for unmagnetized plasma. Plasma is flowing from left to right.

The ion density map (middle right) clearly indicates the wake region behind the tether. Due to the high positive charge on the tether, all ions are deflected from their quasi one-dimensional motion as they approach the tether. Since the potential in the presheath is mostly of the order of $0.1eV$, ion density increases very gradually ($\sim 1.1 \times n_\infty$). The peak density is around $3.6 \times n_\infty$.

In the figure for non-trapped electron density (upper right), there are some region where the density is lower than that at

infinity. This is considered to be due to the electron sink at the tether. Even without the sink, as mentioned earlier, non-trapped electron density can be no more than that of infinity in a steady state. However this figure also shows the electron density increase where ion density also increases. The same tendency of electron behavior is recognized in the figure for trapped electron density (upper left).

Overall electron density (middle left) is obtained so that plasma can maintain the quasi-neutrality in the pre-sheath. Due to the high mobility of electrons, increased electron density spreads whereas ion density increase is concentrated in a narrow region. The very high positive potential on the tether does not allow any ions inside the sheath, and only electrons reside there. The density peaks of electrons near the tether are typically from $1.3 \sim 1.7 n_\infty$.

Current collection was found to be around 1.3 times more than the OML current. Since ϕ_p is relatively low, maybe the simple formula for OML is not enough. On second thought, $\frac{e\phi_p}{\kappa T} \sim 70$ is large enough.

In Figure 2, we check the distribution of current collection on the tether surface. The X-axis is the angle from the wake side, increasing counterclockwise. The distribution shows that there are more electrons absorbed on the ram side of the tether than the wake side. The distribution of electron collection on the tether surface is plotted. The current collection is normalized by the OML current. The distribution of current collection on the tether surface shows that more electrons are collected on the ram side than the wake side. This is a clear indication of the violation of the OML theory. In the OML regime, regardless the shape of a collector, the distribution of current collection must be uniform. This also suggests that the increased electron density (non symmetric) may be responsible for the current collection.

Conclusion and Future Work

A Particle-In-Cell method has been developed for the calculation of current collection by a moving bare tether. Current collection enhancement has been recognized in the computation. Deficit electron density in the steady state solution brought about "paradox" in the case of unmagnetized flowing plasma, if only free ($E > 0$) electrons were included. Due to the mesothermal condition and the very high positive potential, ion density increases as ions approach the tether, whereas electron density remains no more than that of infinity as stated in Laframboise and Parker theory. In order to resolve this, adiabatic electron trapping has been considered to fill the deficiency of electrons. Adiabatic electron trapping occurs when the motion of a potential well, $q\phi$, is much slower (or faster) than that of electrons. In space, a potential well created by an EDtether is moving at the orbital

speed which is much slower than the electron thermal speed. Therefore we may introduce the phenomenon into our model. As a result, the abrupt potential increase seen previously near outside computational boundary due to the deficiency of electrons disappeared.

In order to maintain the quasi-neutrality, electrons, trapped or non-trapped, increase their density where ions are accumulated due to the tether potential. The peak electron density in the region is found to be $1.3 \sim 1.7n_{\infty}$. Since the current collection is about $1.3 \times I_{OML}$, the increased electron density potentially provides a good source for the enhanced current. The mechanism of the increase of electron density is still unclear. More detailed analysis should be followed after completing a 2-D PIC code. Work in process includes the analytical movement of fast particles near the tether, and wave damping at the boundary by introducing a virtual vacuum. Detailed anaylyses of velocity distribution functions and particle trajectories are to be performed as well.

Acknowledgments

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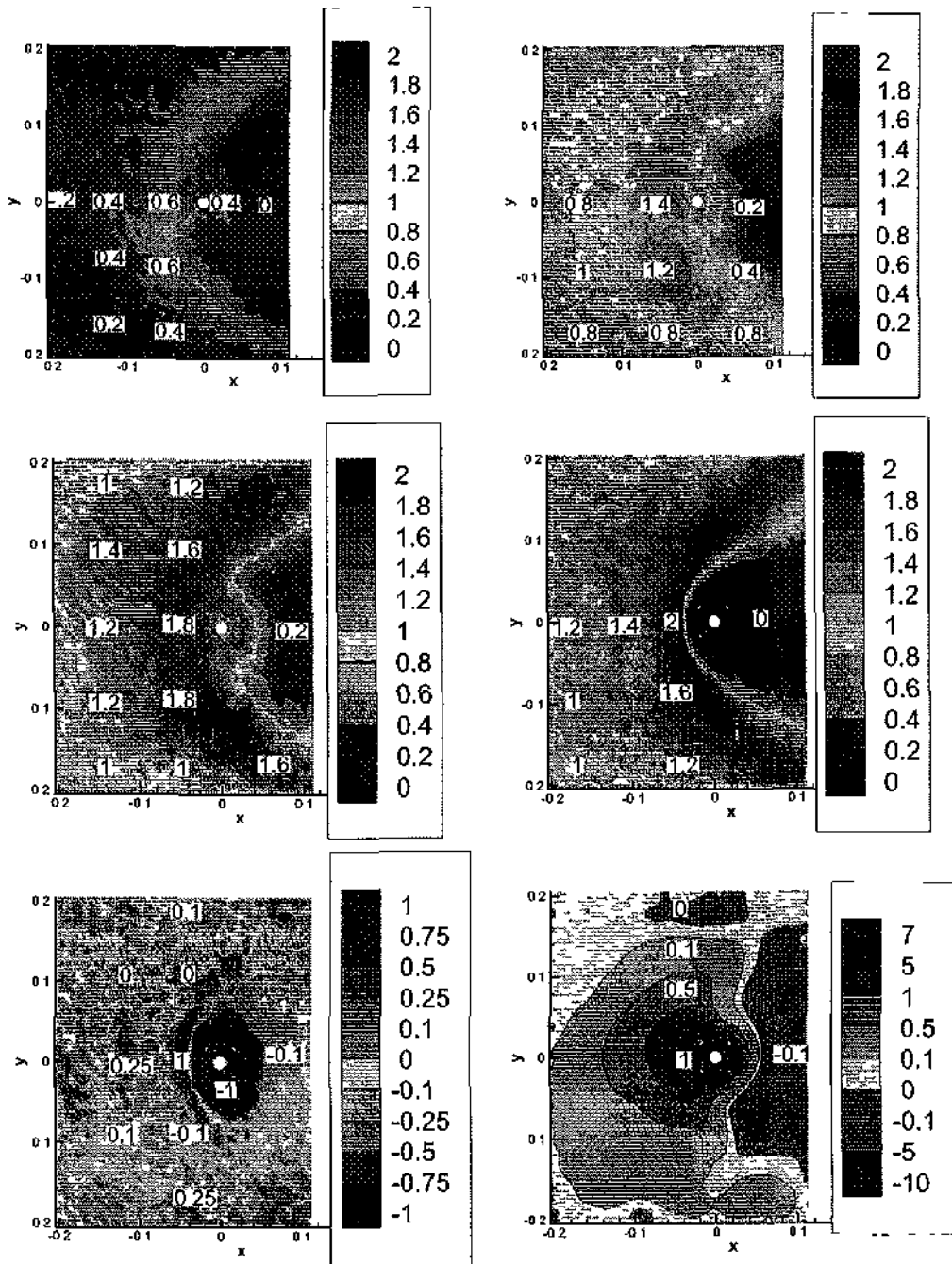
Figure 1: Instantaneous maps of trapped electron density (top left) , non-trapped electron density (top right), overall electron density (middle left), ion density (middle right), electric charge density (bottom left) and potential (bottom right). All densities are normalized by n_∞. Electric charge density is normalized by $e n_\infty$. Potential is in eV. ($\kappa T_e = 0.1 eV$)
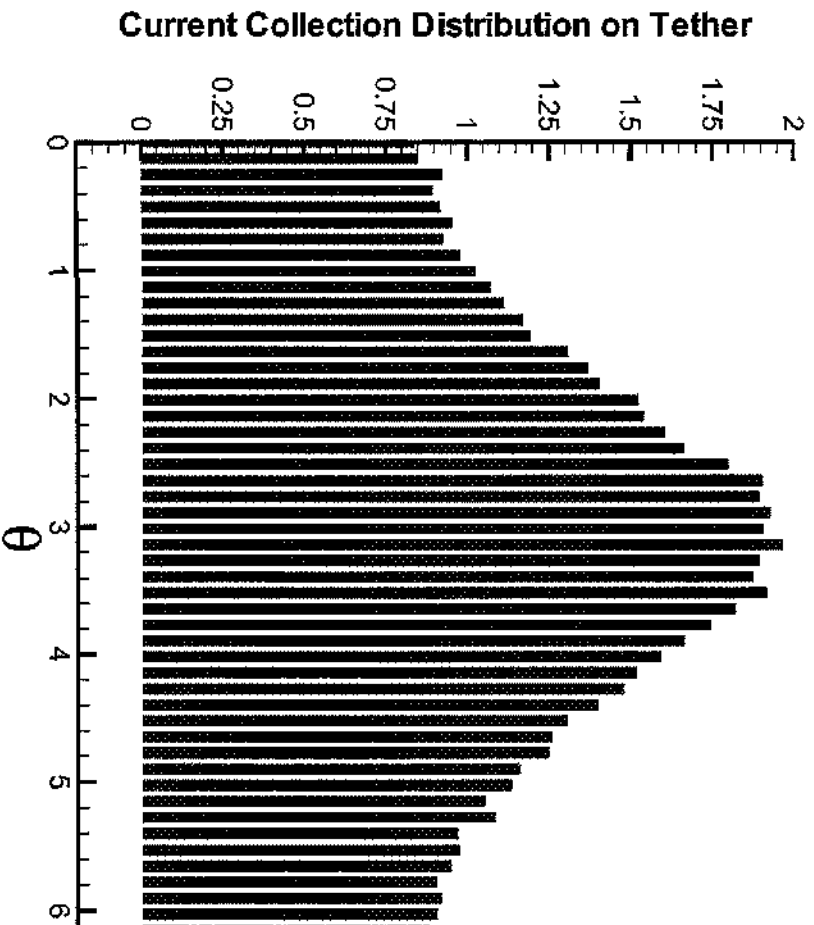
Figure 2: The distribution of electron current collection on the tether surface. θ(rad.) is taken from the wake side and increases counterclockwise.