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Assessment of the influence of the elastic properties of rail vehicle suspensions on safety, ride quality and track fatigue

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ABSTRACT

A sensitivity analysis has been performed to assess the influence of the elastic properties of railway vehicle suspensions on the vehicle dynamic behaviour. To do this, 144 dynamic simulations were performed modifying, one at a time, the stiffness and damping coefficients, of the primary and secondary suspensions. Three values were assigned to each parameter, corresponding to the percentiles 10, 50 and 90 of a data set stored in a database of railway vehicles.

After processing the results of these simulations, the analyzed parameters were sorted by increasing influence. It was also found which of these parameters could be estimated with a lesser degree of accuracy in future simulations without appreciably affecting the simulation results. In general terms, it was concluded that the highest influences were found for the longitudinal stiffness and the lateral stiffness of the primary suspension, and the lowest influences for the vertical stiffness and the vertical damping of the primary suspension, with the parameters of the secondary suspension showing intermediate influences between them.

Keywords: railway dynamics, sensitivity analysis, dynamic behaviour, stiffness, damping, primary suspension, secondary suspension, safety, track fatigue, ride quality, EN 14363 standard, UIC 518 leaflet

1 INTRODUCTION

Usually, when building any model to simulate the dynamic behaviour of a railway vehicle, not all the required data are known. Sometimes, the value of some vehicle parameters has a related uncertainty due to the manufacturing process or even to component degradation during service. Other times, the value of these parameters is completely unknown. This usually happens when the components to be modelled were manufactured by third parties or even when the vehicle to be analyzed was manufactured many years ago.

In this work, a sensitivity study was performed in order to find the degree of accuracy required in the definition of each vehicle parameter (1). In particular, it was intended to analyze the sensitivity of the vehicle dynamic behaviour to the elastic properties of the primary and secondary suspensions, under different running conditions. This study was complemented by two others, where the influences of the inertial properties of the main bodies of the vehicle (2), as well as that of the rolling features, were also analyzed (3).

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Both the simulation models and the working methodology used in this work are described in detail in a previous work (2). Nevertheless, they are summarized here, in paragraph 3, in order to provide some continuity to the present text.

To consider the influence of the model uncertainties on the vehicle dynamics, a probabilistic approach should be used, as it would predict how the uncertainty of input parameters would be propagated to the model output. A probabilistic method commonly used in such situations is the Monte Carlo simulation, but it requires extremely high computation costs when many uncertain input parameters have to be considered. Other more effective probabilistic methods are sometimes used as, for example, the combination of Monte Carlo simulation technique and the design of experiments theory (4) or the generalized polynomial chaos theory (5), though they are also time consuming.

Despite their high computational cost, probabilistic methods should be used whenever quantitative results are required. However, for preliminary research studies, where qualitative results showing the relative importance of the parameters being considered would suffice, simpler methods could be applied. The simplest method consists in modifying input parameters one at a time, thus neglecting any possible relationship that could exist between them. These simplifications make the precision of this method lower than the precision of the previously mentioned probabilistic methods. However, it is very helpful due to its simplicity, which allows analyzing many parameters with a relatively low computational cost, in comparison with the probabilistic methods.

As stated before, the work here exposed is part of a wider study, where the influences of 24 input parameters of the vehicle model were considered: 12 inertial properties of the vehicle bodies, 8 elastic properties of the suspension components and 4 parameters related with the rolling contact. For each parameter, several track layouts and vehicle speeds were also considered. Due to the large number of uncertain input parameters and external conditions to be considered, and having in mind the above mentioned considerations, the simplest approach was chosen for this study. Therefore, the input parameters were modified one at a time, with just three values in each variation, even for parameters with large variation ranges, so assuming that the output quantities are smooth functions of the input parameters.

In view of the above mentioned limiting conditions, the present work could be considered as a starting point, as it would provide a qualitative idea about which influence quantities need to be addressed with particular care when performing simulations addressing a specific problem. From the results obtained, the number of parameters to be considered to undertake in the future a probabilistic approach could be reduced. This way, quantitative and more accurate results could be obtained with a considerable lower computational cost than considering all the uncertain input parameters.

To undertake this work, a reference vehicle model was defined. From this reference model, the values of the elastic properties to be analyzed were independently modified, one at a time. In principle, the elastic properties to be considered, both for the primary and secondary suspensions, are the longitudinal, the lateral and the vertical stiffness and damping. However, the longitudinal and the lateral damping of the primary suspension, as well as the longitudinal stiffness and damping of the secondary suspension were discarded for this analysis. This decision was taken due to the scarcity of data available for these parameters, which would lead to unrealistic estimations for both their median values and their variation ranges.

Both the reference model characteristics and the variation ranges assigned to each parameter (6), (7), were assessed from the information stored in a database of railway vehicles (8), which was specifically built for this purpose.

A methodology was also developed, to allow a systematic analysis of vehicle dynamic response, thus avoiding to focus on extremely specific cases. With this aim, entirely generic simulation scenarios were defined. In the same way, a systematic statistical treatment was carried out on the simulation results. To define the track layouts and the track qualities to be used in the simulations, the specifications stated in the UIC-518 leaflet (9) and in the European standard EN-14363 (10), generally used for railway vehicle authorisation by means of on-track tests, were applied in a virtual environment. The same specifications were also used to post-process the results of the simulations. This procedure was chosen for this project because it is well established, supported by many years of experience, and allows the assessment of vehicle dynamics by means of only a few indexes. These indexes can be compared with some limit values, defined in these standards, in order to find whether the vehicle behaviour is suitable or not from a safety, track fatigue and ride quality point of view.

This methodology allows the identification of the critical parameters of the simulation models. It also allowed the identification of those elastic properties which could be estimated with lesser accuracy due to their low impact on the accuracy of the simulation results.

2 BACKGROUND

Sensitivity analyses which directly involve the parameters of the two stages of the vehicle suspension are by far the most common, especially those regarding the stiffness of primary and secondary suspensions. In this context, this paragraph shows a literature review of various publications presenting the results of several dynamic simulations in which the value of some parameters related with the elastic properties of the primary and the secondary suspensions were modified. The parameters analyzed in each reference are listed below. These bibliographic references were classified according to the type of dynamic study described in each of them. Three types of studies were considered: safety, track fatigue and ride quality. The former, in turn, was also divided into linear stability, non-linear stability and derailment risk studies.

A review of the state of the art in suspension component modelling can be found in (11), (12) and (13). Before presenting the above mentioned literature review, it is worth mentioning some recent works including a thorough description of advanced simulation techniques used to model non linear suspension components, such as the air springs commonly used in the secondary suspension of passenger vehicles (14), (15), (16), (17), the rubber springs used in some primary suspensions (18), (19), or even the link suspensions still used in some freight wagons (20), (21). Despite the interest of these modelling techniques, they are out of the scope of the present study, as the amount of numerical data related to nonlinear suspension models is too small to perform a statistical analysis such as the one made here with linear suspension models and briefly described in paragraph 3.3.

2.1 SAFETY STUDIES

Table 1, Table 2 and Table 3 respectively gather some bibliographic references on linear stability, non-linear stability and derailment risk studies.

Reference	Modified parameter^1	Method of analysis^2	Vehicle model^3	Variables analyzed^4
(22)	k1x/y, d1x/y, k2x/y	Own formulation	Car	V_C
(23)	k2x/y/z/α/γ, d2y/z	MBS (ADAMS)	Car	eigenvalues
(24)	k1z, k2z, kw, dw	FEM/MBS	1/4 Car	V_C
(25)	k1y/γ	Own formulation	Wheelset	V_C
(25)	k1y	Own formulation	Bogie	V_C
(26)	k1b/s	Own formulation	Bogie	V_C
(27)	k1b, d1x	Own formulation	Bogie	V_C
(28)	k1x/y, d1y, k2γ, d2y/γ	Own formulation	Car	V_C
(29)	k1b/s	MBS (SIDIVE)	Bogie	eigenvalues
(29)	k1x/y, k2x/y, d2y	MBS (SIDIVE)	Car	V_C
(30)	k1x/y/z, d1x/y/z, k2y/z, d2y/z	MBS (A'GEM)	Car	eigenvalues, V_C
(31)	k2x	Own formulation	NA Train	V_C
(32)	k1x/y/z, d1z, d2x/y	Own formulation	Bogie	V_C
(33)	k1x/y	MBS (ADAMS)	Car	V_C
(34)	k1y, d1y	MBS (A'GEM)	Car	V_C
(35)	k1x/y	-	-	V_C
(6)	k1b/s	Own formulation	Bogie	V_C
(36)	k1x/y	Own formulation	1/2 Car	V_C
(37)	k1x/y/s/b	Own formulation	Bogie	V_C
(11)	k1x/y	-	-	V_C
(38)	k1x/y	Own formulation	Bogie	V_C
(39)	k1x/y	Own formulation	Bogie	V_C
(40)	k1x/y/z, d1yz, k2x/y/z, d2x/y/z	Own formulation	NA Train	vehicle stability
(41)	k1y	Own formulation	Wheelset	V_C

Note (1: k: stiffness; d: damping; 1(2): primary (secondary) suspension; x: longitudinal; y: lateral; z: vertical; α: roll; γ: yaw; b: bending; s: shear; w: wheel.

Note (2: FEM: 'Finite Element Method'; MBS: 'Multibody System'; Own formulation: the authors formulate the equations of motion.

Note (3: A: articulated (with Jacobs bogies); NA: not articulated (with two bogies per carbody).

Note (4: V_C: critical velocity.

TABLE 1 LITERATURE REVIEW: LINEAR STABILITY

Reference	Modified parameter^1	Method of analysis^2	Vehicle model^3	Variables analyzed^4
(42)	k2y/γ	MBS (VAMPIRE)	Car	y wheelset
(43)	k1y/γ, d1y, k2y, d2x/y	Own formulation	Bogie	V_C
(44)	k1x, k1y	Own formulation	1/2 Car	V_C
(45)	k2x	Own formulation	1/2 Car	V_C
(46)	k1x/y	MBS (SIMPACK)	A Train	V_C
(47)	k yaw damper	Own formulation	Car	ay bogie, ay carbody
(48)	k1y, d1y/z, d2y/γ	MBS (NUCARS)	Car	y wheelset
(49)	k1x/y, d1x/y, k2y	Own formulation	Car	y wheelset/bogie/carbody, V_C
(50)	k1t	Own formulation	Car	V_C
(51)	k1x/y/z	Own formulation	Car	y wheelset, y bogie, y carbody
(52)	k1b	MBS (SIMPACK)	Car	Y, V_C
(53)	k1y/γ/b/s, d1y, k2y/γ, d2y/γ	Own formulation	Car	V_C
(41)	k1y	Own formulation	Wheelset	y wheelset
(54)	k1y/γ, d1y/γ, k2x/y, d2y	Own formulation	Bogie	y wheelset, y bogie
(55)	k1x/y	Own formulation	Car	y wheelset, y bogie
(56)	k1x, d1y	Own formulation	Car	y wheelset, Δy wheelset
(57)	k1x/y/z, d1x/y	Own formulation	Car	y wheelset, Δy wheelset
(58)	k1x	-	-	V_C
(59)	k, d yaw damper	MBS	-	V_C

Note (1: k: stiffness; d: damping; 1(2): primary (secondary) suspension; x: longitudinal; y: lateral; z: vertical; γ: yaw; b: bending; s: shear; w: wheel; t: bump-stop.

Note (2: MBS: 'Multibody System'; Own formulation: the authors formulate the equations of motion.

Note (3: A: articulated (with Jacobs bogies); NA: not articulated (with two bogies per carbody).

Note (4: V_C: critical velocity; a: acceleration; Y: lateral wheel-rail force.

TABLE 2 LITERATURE REVIEW: NON-LINEAR STABILITY

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Reference	Modified parameter¹	Method of analysis²	Vehicle model	Variables analyzed³
(45)	k2z	Own formulation	Car	Y/Q
(60)	k1x/y, Δ1z	MBS (GENSYS)	Car	derailment risk
(27)	k1b, d1x	Own formulation	Bogie	Y/Q
(29)	k1x	MBS (SIDIVE)	Car	ΣY, Y/Q
(61)	k1x/y/z, k2y	MBS (A'GEM)	Car	Y/Q
(30)	k1x/y/z, d1x/y/z, k2y/z, d2y/z	MBS (A'GEM)	Car	Y/Q
(48)	k1b/s, Δ1x/y	MBS (NUCARS)	Car	Y/Q
(38)	k1x/y	Own formulation	Bogie	Y/Q

Note (1: k: stiffness; d: damping; f: friction; 1(2): primary (secondary) suspension; x: longitudinal; y: lateral; z: vertical; γ: yaw; b: bending; s: shear; Δx(z): longitudinal (vertical) clearance.

Note (2: MBS: 'Multibody System'; Own formulation: the authors formulate the equations of motion.

Note (3: Y(Q): lateral (vertical) wheel-rail force.

TABLE 3 LITERATURE REVIEW: DERAILMENT RISK

2.2 TRACK FATIGUE STUDIES

Table 4 gathers some bibliographic references on track fatigue.

Reference	Modified parameter¹	Method of analysis²	Vehicle model³	Variables analyzed⁴
(22)	k1x/y, d1x/y, k2x/y	Own formulation	Car	Y
(62)	d1z	Own formulation	Bogie	Q
(28)	Δ1y, k1y	Own formulation	Car	Y
(34)	k1y, d1y	MBS (A'GEM)	Car	Y
(11)	k1b	-	-	Y
(63)	k2z, d2z	Own formulation	NA Train	z track

Note (1: k: stiffness; d: damping; f: friction; 1(2): primary (secondary) suspension; x: longitudinal; y: lateral; z: vertical; b: bending; s: shear; Δy: lateral clearance.

Note (2: MBS: 'Multibody System'; Own formulation: the authors formulate the equations of motion.

Note (3: A: articulated (with Jacobs bogies); NA: not articulated (with two bogies per carbody).

Note (4: Y(Q): lateral (vertical) wheel-rail force.

TABLE 4 LITERATURE REVIEW: TRACK FATIGUE

2.3 RIDE QUALITY STUDIES

Table 5 gathers some bibliographic references on ride quality.

Reference	Modified parameter¹	Method of analysis²	Vehicle model³	Variables analyzed⁴
(42)	k2y	MBS (VAMPIRE)	Car	ay carbody, az carbody
(23)	k2x/y/z/α/γ, d2y/z	MBS (ADAMS)	Car	Wz
(64)	k2y/γ, d2y/γ	Own formulation	A Train	ay carbody
(65)	k1x/y/z, d1z, k2y/z, d2y/z/α/γ	MBS (GENSYS)	Car	ay carbody
(66)	k1z, d1z, k2z, d2z	Own formulation	Car	az carbody
(67)	k2x/y/z, d2	MBS (ADAMS)	Car	az carbody
(30)	k1x/y/z, d1x/y/z, k2y/z, d2y/z	MBS (A'GEM)	Car	comfort level
(20)	k1x, k1y	MBS (GENSYS)	Car	ay carbody
(68)	k2z, d2z	Own formulation	Car	az carbody, comfort level
(33)	k2y/t, d2y/z, Δ2y	MBS (ADAMS)	Car	Wz
(69)	k1z, k2z, d2z	Own formulation	1/8 Car	Q, az carbody, Δz2
(70)	k2y, d2y	Own formulation	Car	y carbody
(36)	k1z, k2z	Own formulation	1/8 Car	az carbody
(63)	k2z, d2z	Own formulation	NA Train	az carbody

Note (1: k: stiffness; d: damping; f: friction; 1(2): primary (secondary) suspension; x: longitudinal; y: lateral; z: vertical; α: roll; γ: yaw; Δy: lateral clearance; t: bump-stop.

Note (2: MBS: 'Multibody System'; Own formulation: the authors formulate the equations of motion.

Note (3: A: articulated (with Jacobs bogies); NA: not articulated (with two bogies per carbody).

Note (4: Q: vertical wheel-rail force; a: acceleration; Wz: Sperling index; Δz2: deflection of the secondary suspension.

TABLE 5 LITERATURE REVIEW: RIDE QUALITY

2.4 COMPENSATION OF THE NOTED DEFICIENCIES

As can be seen, the sensitivity to the elastic properties of the vehicle suspensions has been extensively treated before. However, all these references generally show very specific applications, focused on the study of a given vehicle, running over small track sections with simple geometry. They are also generally focused either on stability, on curve negotiation or on comfort studies, but seldom on the three types of study simultaneously. On the other hand, the range of values considered for the variability of each parameter is generally arbitrary, showing high variability from one study to another.

This study intends to extend the scope of the previous works, trying to cover as many parameters of interest as possible. For each parameter it is also intended to cover a range of values wide enough to consider many of the possible values that could be found in different railway vehicles.

Unlike the previous works, this study presents a more comprehensive approach, trying to simultaneously analyze the influence of all the different elastic properties of the vehicle in order to assess their impact on safety, track fatigue and ride quality, thus making it possible to determine which of these studies is more critical.

The same reference vehicle was employed to analyze the influence of all vehicle parameters considered in the above mentioned wider study: inertial properties, elastic properties and wheel-rail rolling properties. Three realistic track layouts were also used, with a cumulative length of 35 km, covering a wide range of curve radii. In the same way, three different running speeds were considered for each track layout. Both the vehicle and track models are described in the next section.

In this way, it was intended to provide a wider view when analyzing the influence of the elastic properties of vehicle suspensions so as to assess their impact on vehicle dynamics.

3 WORKING METHODOLOGY

3.1 SET-UP OF THE REFERENCE MODEL

To perform the sensitivity analysis, multibody system (MBS) simulation techniques were employed. In particular, the SIMPACK commercial program was used. It allows simulating multi-body systems with especial features related with railway vehicle models, as the longitudinal guidance and the wheel–rail contact, which involves great forces transmitted through a small surface. SIMPACK has been tested in several benchmarks, as the Manchester benchmarks for railway vehicle dynamics (71), the ERRI Benchmark (72) or the Volpe LD benchmark (73).

The vehicle model represents a passenger car with two bogies, with the carbody resting on the elastic elements of the secondary suspension without any pivot or centre plate. The main bodies of the vehicle (carbody, bogie frames and wheelsets) were modeled as rigid bodies, connected to each other by means of linear springs and dampers that characterize the primary and secondary suspensions.

The vehicle model was parameterized with the aim of facilitating the variation of its features during the subsequent sensitivity analysis. Over 160 parameters were used. The vehicle model was built from smaller models of the individual components of the vehicle (carbody, bogie, primary suspension and secondary suspension). These sub-models are reusable and are assembled into the whole vehicle model. Numerical values were assigned to the different model parameters. These values were obtained from the median values of the data stored in the RVDynDB database, specifically made for this purpose (74), (7).

Three track models were also built, following the indications stated in the standard EN-14363. These models include curves with large (R > 600 m), medium (400 m ≤ R ≤ 600 m) and small (250 m ≤ R<400 m) radii, respectively called RL, RM and RS. The specifications of this standard were also followed in the definition of track defects, specifically the alignment and longitudinal level, having chosen a track quality inside level QN1.

Vehicle and track models were coupled through the wheel-rail contact properties, defined by the Hertz theory for the normal forces, and by the Kalker's simplified theory for the tangential forces.

To consider the track elasticity in the model, track pieces were included under each wheelset. Each piece of track is directly supported by a pair of spring-damper elements.

The operating conditions were also set, following the specifications of the standard EN-14363, taking into account that the maximum operating speed of the reference vehicle was 160 km/h. Table 6 gathers the speeds used in the models for each of the three track layouts.

Track layout	R_{min} [m]	R_{max} [m]	V_{max} [km/h]
RL	1620	1950	175
RM	570	600	105
RS	290	375	75

TABLE 6 CURVE RADII AND RUNNING SPEEDS

3.2 INITIAL CONDITIONS FOR THE SENSITIVITY ANALYSIS

In order to determine the velocity ranges to be used in the sensitivity analysis, the dynamics of the reference vehicle were simulated at several speeds, starting with the lower speed and progressively increasing it until it was detected that at least one wheel completely left the track, this indicating a derailment by excessive speed. This way, the instability critical speed was identified for the track layout RL, as well as the minimum derailment speeds for each track model (Table 7).

For the RL track, the maximum speed, Vmax, was chosen just below the instability critical speed. The minimum speed, Vmin, was chosen so that the cant deficiency for the reference speed (Vref) would lie at the midpoint between the cant deficiencies for the extreme speeds Vmin and Vmax. The speed variation ranges for the track models RM and RS were defined so that they would have the same cant deficiency range as the first track model, RL. Table 7 shows the speed ranges so obtained for each track layout, together with their related cant deficiencies.

Track layout	Cant deficiency [mm]	V [km/h]	Observations
RS	100	65	Lower end
	153	75	Reference speed
	201	83	Upper end
RM	100	93	Lower end
	148	105	Reference speed
	202	117	Upper end
RL	99	160	Lower end
	134	175	Reference speed
	201	201	Upper end

TABLE 7 RUNNING SPEEDS FOR THE SENSITIVITY STUDY

A more detailed description of the process followed to determine the speed ranges to be applied in the sensitivity study can be found in (2).

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3.3 DEFINITION OF SCENARIOS FOR THE SENSITIVITY ANALYSIS

To perform the sensitivity analysis, the vehicle dynamics were simulated in different scenarios, which were built taking the vehicle reference model as starting point.

The first step in the definition of these scenarios was to modify independently, one at a time, the value of each of the parameters to be analyzed. Particularly, variations on the following parameters of the primary and the secondary suspensions were performed:

- Primary suspension: longitudinal, lateral and vertical stiffness (kx, ky, kz) and vertical damping (dz).
- Secondary suspension: lateral and vertical stiffness (ky, kz) and lateral and vertical damping (dy, dz).

Apart from the reference value, two additional values were considered for each parameter: a higher value and a lower value, corresponding to the percentiles 10 and 90 of the data set stored in the RVDynDB database (7), as indicated in Table 8. This table shows the values obtained for single suspension components.

	Parameter (1	Unit	Median (Pctl. 50)	Variation range			
				Actual value		Percentage variation (2	
				Lower (Pctl. 10)	Upper (Pctl. 90)	Lower (Pctl. 10)	Upper (Pctl. 90)
primary	kx	[N/m]	$5.20\cdot10^{6}$	$0.750\cdot10^{6}$	$44.0\cdot10^{6}$	13 %	748 %
	ky	[N/m]	$4.55\cdot10^{6}$	$0.580\cdot10^{6}$	$17.7\cdot10^{6}$	12 %	373 %
	kz	[N/m]	$1.17\cdot10^{6}$	$0.330\cdot10^{6}$	$3.24\cdot10^{6}$	27 %	268 %
	dz	[N·s/m]	$12.0\cdot10^{3}$	$2.28\cdot10^{3}$	$37.0\cdot10^{3}$	19 %	318 %
secondary	ky	[N/m]	$250\cdot10^{3}$	$90.0\cdot10^{3}$	$940\cdot10^{3}$	35 %	370 %
	kz	[N/m]	$0.560\cdot10^{6}$	$0.250\cdot10^{6}$	$1.69\cdot10^{6}$	42 %	288 %
	dy	[N·s/m]	$22.5\cdot10^{3}$	$9.02\cdot10^{3}$	$64.5\cdot10^{3}$	40 %	287 %
	dz	[N·s/m]	$27.0\cdot10^{3}$	$8.00\cdot10^{3}$	$90.0\cdot10^{3}$	30 %	334 %

Note (1: kj: stiffness along the j axis; dj: damping along the j axis.
Note (2: Relative variation of lower and upper ends as a percentage of the median value, being Pctl. 50 = 100 %

TABLE 8 MEDIAN AND VARIATION RANGE OF SUSPENSION PARAMETERS

From Table 8, it can be observed that these parameters present both wide and skewed variation ranges. Although both the amplitude and asymmetry of the variation ranges were taken into account when assessing the influence of each parameter, they could affect the sensitivity study, probably making the results somewhat skewed towards parameters that naturally have a wide variation. However, it should be remembered that the present study intends to obtain qualitative, but not quantitative, results.

After modifying any vertical stiffness, a nominal force calculation was performed to recalculate the vertical reaction forces of the suspension springs, so that the vehicle could recover its initial equilibrium state, avoiding unrealistic vertical displacements of the carbody or even of the bogie.

Several operating conditions were considered for each of these values. Particularly, each vehicle model was combined with the track layouts RS, RM and RL, all of them with track quality QN1. In turn, each track layout was run through at the running speeds Vmin, Vref and Vmax.

8 different scenarios were considered: 4 to analyze the influence of the elastic properties of the primary suspension and 4 to analyze the influence of the elastic properties of the secondary suspension. Once the models were ready, they were simulated, with a total of

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144 simulations (8 parameters · 2 variations · 3 speeds · 3 track layouts), apart from the 3 simulations needed to analyze the reference case.

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3.4 POST-PROCESSING METHODOLOGY

To make the comparison between the results of different simulations easier, the post-processing methodology was systematized, reducing all the results of each simulation to a small set of indexes. To this end, the indications of the standard EN-14363 were followed. This standard proposes a statistical evaluation which allows the assessment of the vehicle dynamics from the safety, track fatigue and ride quality points of view.

As a whole, 5 assessment quantities were considered to evaluate running safety, 3 for track fatigue and another 5 for ride quality (Table 9).

	Index	Assessment quantity	Symbol
Safety	SAF-1	Sum of wheelset guiding forces	$(\Sigma \mathrm{Y})_{2\mathrm{m}}$
	SAF-2	Ratio of guiding force and wheel load	(Y/Q)
	SAF-3	Lateral acceleration of the bogie frame	$\ddot{y}_s^+$
	SAF-4	Lateral acceleration of the carbody	$\ddot{y}_s^*$
	SAF-5	Root mean square of the sum of wheelset guiding forces	$\mathrm{s}\Sigma\mathrm{Y}$
Track fatig.	FAT-1	Vertical wheel load	Q
	FAT-2	Quasi-static lateral wheel force	Y_{qst}
	FAT-3	Quasi-static vertical wheel force	Q_{qst}
Ride quality	COM-1	Lateral acceleration of the carbody	$\ddot{y}_q^*$
	COM-2	Vertical acceleration of the carbody	$\ddot{z}_q^*$
	COM-3	Root mean square of lateral acceleration of the carbody	$\mathrm{s}\ddot{y}_q^*$
	COM-4	Root mean square of vertical acceleration of the carbody	$s\ddot{z}_q^*$
	COM-5	Quasi-static lateral acceleration of the carbody	$\ddot{y}_{qst}^*$

TABLE 9 ASSESSMENT QUANTITIES FOR SAFETY, TRACK FATIGUE AND RIDE QUALITY

The simulation results for each assessment quantity were post-processed following the indications of the standard EN-14363, in order to calculate their maximum estimated values.

The standard provides a limit value for each of the assessment quantities used to evaluate the vehicle dynamics, considering that the vehicle exhibits a suitable dynamic behaviour if the maximum estimated value for each assessment quantity is less than its related limit value.

4 INDIVIDUAL RESULTS OF THE SENSITIVITY ANALYSIS

After finishing the models and performing the appropriate simulations, the next step is to process the results obtained, following the indications of the standard EN-14363. The influence of a given parameter can then be determined by comparing each assessment index for all the simulations related to that parameter. These influences were independently evaluated for the different assessment quantities related to safety, track fatigue and ride quality studies.

4.1 DESCRIPTION OF THE TABLES USED

To make comparisons easier, instead of using the safety (SAF), track fatigue (FAT) and ride quality (COM) evaluation indexes directly obtained when processing the simulations results, a λ ratio was computed for each index. This factor represents the ratio between a given index and its related limit value and it is expressed as a percentage, so that the values under 100% represent standard-compliant situations, while those over 100 % represent non-compliant ones.

An influence indicator was then computed, combining the λ ratios obtained for either the percentiles P10 or P90 and for the percentile P50 of the parameter being modified:

$$Inf_n = \frac{\lambda_n - \lambda_{50}}{(Pn - P50)/P50}, \quad n = 10, 90$$

The influence indicator represents the ratio between the relative variation of the output and the relative variation of the modified parameter. Its denominator would be 1 if the modified parameter would increase a 100% from P50 to P90, or -1 if it would decrease a 100% from P50 to P10. Therefore, if the variation of the output is supposed to be linear, an influence indicator of r% means that the maximum estimated value of the assessment quantity being analyzed increases/decreases an r% of its related limit value when the modified parameter increases/decreases a 100%. According to this interpretation, 5 different levels were set for the influence of a given parameter: low (|Inf| < 10%); moderate (10% ≤ |Inf| < 25%); noticeable (25% ≤ |Inf| < 50%); high (50% ≤ |Inf| < 75%) and very high (|Inf| ≥ 75%).

The highest influence was then computed from the influence indicators obtained for the results of simulations with similar scenarios (same evaluation index, track layout, type of track section and speed) that only differ in the value assigned to the parameter being analyzed:

$$\widehat{Inf} = \begin{cases} Inf_{10}, & if \ \ |Inf_{10}| \geq |Inf_{90}| \\ Inf_{90}, & if \ \ |Inf_{90}| \geq |Inf_{10}| \end{cases}$$

The highest influences were grouped by type of track section (Cv, Tr, St) (seeTable 10). They were also grouped by track layout (RS, RM o RL) and by speed (Vmin, Vref and Vmax). Then, the global influence was calculated as the highest absolute value obtained inside each group.

The global influences were put together in a table (see Table 11), where columns 3-5 show the global influence found for each type of track section: curve, Cv, transition curve, Tr, and straight track, St; columns 6-8 show the global influence found for each track layout: RS, RM and RL; and columns 9-11 show the global influence found for each speed category: Vmin, Vref and Vmax. The last column shows the highest global influence obtained in all these categories. This table allows to quickly determine which kind of dynamic behaviour leads to the most critical situations. The least critical results (below 10 %) were identified with an empty circle, ○, the most critical (over 75 %) with a full black circle, ●, and the intermediate ones with partially-filled circles: ◔ for influences between 10 % and 25 %, ◑ for influences between 25 % and 50 %, and ◕ for influences between 50 % and 75 %.

4.1.1 Numerical example

As an example, for the variations performed on the lateral stiffness of the secondary suspension, the stiffness takes the values: P10 = 0.09 kN/mm; P50 = 0.25 kN/mm; P90 = 0.94 kN/mm. For the evaluation index SAF-1, the corresponding λ ratios obtained for the simulations performed for RS track layout at Vmin in curved track sections, Cv, are: λ_{10} = 30.1 %; λ_{50} = 29.2 %; λ_{90} = 29.1 %. From these values, the related influence indicators result in: Inf_{10} = 0.9 %/(-0.65) = -1.39 % and Inf_{90} = -0.1 %/2.70 = -0.04 %, the highest influence being -1.39%. This value is collected in the top-left cells of the three blocks of Table 10.

Table 10 shows the highest influences computed for the evaluation index SAF-1 when modifying the lateral stiffness of the secondary suspension. The left, central and right blocks respectively show the highest influences grouped by type of track section (Cv, Tr, St), by track layout (RS, RM, RL) and by speed (Vmin, Vref, Vmax). The value with highest absolute value of each column was highlighted in bold.

	(Cv)	(Tr)	(St)		RS	RM	RL		Vmin	Vref	Vmax
RS-Vmin	-1.39	-2.63	2.07	Cv-Vmin	-1.39	1.24	3.22	RS-Cv	-1.39	-1.39	-0.93
RS-Vref	-1.39	-4.18	2.18	Cv-Vref	-1.39	6.04	0.62	RS-Tr	-2.63	-4.18	-2.94
RS-Vmax	-0.93	-2.94	1.67	Cv-Vmax	-0.93	**10.38**	15.02	RS-St	2.07	2.18	1.67
RM-Vmin	1.24	0.31	2.04	Tr-Vmin	-2.63	0.31	-2.79	RM-Cv	1.24	**6.04**	10.38
RM-Vref	6.04	3.25	4.55	Tr-Vref	**-4.18**	3.25	-1.63	RM-Tr	0.31	3.25	8.52
RM-Vmax	10.38	**8.52**	8.96	Tr-Vmax	-2.94	8.52	-4.49	RM-St	2.04	4.55	8.96
RL-Vmin	3.22	-2.79	-4.65	St-Vmin	2.07	2.04	-4.65	RL-Cv	3.22	0.62	15.02
RL-Vref	0.62	-1.63	-5.89	St-Vref	2.18	4.55	-5.89	RL-Tr	-2.79	-1.63	-4.49
RL-Vmax	**15.02**	-4.49	**-15.95**	St-Vmax	1.67	8.96	**-15.95**	RL-St	**-4.65**	-5.89	**-15.95**

TABLE 10 TABLE OF HIGHEST INFLUENCES: LATERAL STIFFNESS OF THE SECONDARY SUSPENSION: SAF-1

Finally, Table 11 shows the global influence for the variations in the lateral stiffness of the secondary suspension, k2Y, for the five safety criteria. Note that the values gathered in the second row are the highlighted values of Table 10 rounded to the nearest integer.

Index	Assess. quantity	Type of track section						Track layout						Speed						MAX	
		Cv		Tr		St		RS		RM		RL		Vmin		Vref		Vmax			
SAF-1	$(\Sigma Y)_{2m}$	◔	15	○	9	◔	-16	○	-4	◔	10	◔	-16	○	-5	○	6	◔	-16	◔	**-16**
SAF-2	$(Y/Q)_{2m}$	◔	12	◔	17		-	◔	**17**	○	4	◔	-13	◔	11	◔	14	◔	17	◔	**17**
SAF-3	$\ddot{y}_s^+$	◔	18	◔	-20	◔	-15	○	0	○	-1	◔	-20	○	-3	○	-5	◔	-20	◔	**-20**
SAF-4	$\ddot{y}_s^*$	◔	**20**	◔	13	◔	**17**	◔	11	◔	**17**	◔	20	◔	**20**	◔	**16**	◔	17	◔	**20**
SAF-5	$s\Sigma Y$	◔	15	◔	**-25**	◔	-11	○	0	○	-1	◔	**-25**	○	-3	○	-2	◔	**-25**	◔	**-25**
Variation range (P10; P90)from				*35%*		*to*	*370%*	*of the reference value (P50).*													
Legend:		○	Low (Influence < 10%);					◔	Moderate (10% ≤ Influence < 25%)					◑	Noticeable (25% ≤ Influence < 50%						
		◕	High (50% ≤ Influence < 75%);					●	Very high (Influence ≥ 75%).												

TABLE 11 TABLE OF GLOBAL INFLUENCES: LATERAL STIFFNESS OF THE SECONDARY SUSPENSION

Whether the differences were classified by type of track section, by track layout or by speed, the maximum difference found is always the same (-25 % in this example). In this way, it is easy to identify which is the type of track section, the track layout or even the vehicle speed that show the largest influence. On the other hand, by looking at the values in the last column, it is also possible to identify which evaluation index provides the worst results.

In the following paragraphs, the global influence obtained when comparing the results of the different simulations performed is presented. Hereafter, for simplicity, global influences will be called just influences.

4.2 INFLUENCE OF THE ELASTIC PROPERTIES OF THE PRIMARY SUSPENSION

The following tables show the influence obtained when analyzing the following elastic properties of the primary suspension:

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- Longitudinal stiffness of the primary suspension, k1X (Table 12)
- Lateral stiffness of the primary suspension, k1Y (Table 13)
- Vertical stiffness of the primary suspension, k1Z (Table 14)
- Vertical damping of the primary suspension, d1Z

Those results related to d1Z were omitted, as they present low influence in all the analyzed scenarios.

Index	Assess. quantity	Type of track section			Track layout			Speed			MAX
		Cv	Tr	St	RS	RM	RL	Vmin	Vref	Vmax	
SAF-1	$(\Sigma Y)_{2m}$	● **-2203**	● -2001	● **-1625**	◔ -11	● -125	● -2203	● **-1179**	● **-1174**	● -2203	● **-2203**
SAF-2	$(Y/Q)_{2m}$	● -1568	● **-19714**	-	◕ **53**	◑ -46	● **-19714**	● -372	● -803	● **-19714**	● **-19714**
SAF-3	$\ddot{y}_s^+$	● -335	● -335	● -520	◑ -38	● **-254**	● -520	● -476	● -520	● -461	● **-520**
SAF-4	$\ddot{y}_s^*$	◑ -36	◑ -32	◑ -50	○ -8	◑ -48	◑ -50	◑ -50	◑ -48	◑ -41	◑ **-50**
SAF-5	$s\Sigma Y$	● -845	● -881	● -490	○ -1	● -183	● -881	● -773	● -845	● -881	● **-881**

Index	Assess. quantity	Type of track section			Track layout			Speed			MAX
		Cv	Tr	St	RS	RM	RL	Vmin	Vref	Vmax	
FAT-1	Q	● **-640**	● **-596**	● **-704**	○ -2	◑ -35	● **-704**	● **-704**	● **-686**	● **-681**	● **-704**
FAT-2	Y_{qst}	● -82	-	-	◑ **29**	● **-82**	-	◑ 29	◕ -51	● -82	● **-82**
FAT-3	Q_{qst}	○ 4	-	-	○ 1	○ 4	-	○ 2	○ 3	○ 4	○ **4**

Index	Assess. quantity	Type of track section			Track layout			Speed			MAX
		Cv	Tr	St	RS	RM	RL	Vmin	Vref	Vmax	
COM-1	$\ddot{y}_q^*$	● -79	● -80	● -101	○ -8	◑ -49	● -101	● -101	● -100	◕ -66	● **-101**
COM-2	$\ddot{z}_q^*$	○ -3	○ -3	○ -1	○ -1	○ -1	○ -3	○ -1	○ -1	○ -3	○ **-3**
COM-3	$s\ddot{y}_q^*$	● **-279**	● **-283**	● **-356**	◔ **-17**	● **-150**	● **-356**	● **-356**	● **-352**	● **-249**	● **-356**
COM-4	$s\ddot{z}_q^*$	○ -1	○ -2	○ 0	○ 0	○ 0	○ -2	○ 0	○ -1	○ -2	○ **-2**
COM-5	$\ddot{y}_{qst}^*$	◔ -18	-	-	○ 0	○ -4	◔ -18	◔ -12	◔ -13	◔ -18	◔ **-18**

Variation range (P10; P90)from 13% to 748% of the reference value (P50).

Legend: ○ Low (Influence < 10%); ◔ Moderate (10% ≤ Influence < 25%) ◑ Noticeable (25% ≤ Influence < 50%
◕ High (50% ≤ Influence < 75%); ● Very high (Influence ≥ 75%).

TABLE 12 GLOBAL INFLUENCE: LONGITUDINAL STIFFNESS OF THE PRIMARY SUSPENSION

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Index	Assess. quantity	Type of track section			Track layout			Speed			MAX
		Cv	Tr	St	RS	RM	RL	Vmin	Vref	Vmax	
SAF-1	$(\Sigma Y)_{2m}$	● -185	● **-504**	● -412	○ 7	○ 8	● -504	● -83	● -85	● **-504**	● **-504**
SAF-2	$(Y/Q)_{2m}$	● -200	● -453	-	○ 2	◔ -11	● -453	● -111	● -103	● -453	● **-453**
SAF-3	$\ddot{y}_s^{+}$	● **-294**	● -359	● **-509**	◔ **-10**	◕ **-53**	● **-509**	● **-491**	● **-509**	● -430	● **-509**
SAF-4	$\ddot{y}_s^{*}$	◑ -34	◑ -29	◑ -26	○ -5	○ -6	◑ -34	◑ -34	◔ -23	◔ -21	◑ **-34**
SAF-5	$s\Sigma Y$	● -88	● -331	● -88	○ 0	○ -4	● -331	● -88	● -80	● -331	● **-331**
Index	**Assess. quantity**	**Type of track section**			**Track layout**			**Speed**			**MAX**
		Cv	**Tr**	**St**	**RS**	**RM**	**RL**	**Vmin**	**Vref**	**Vmax**	
FAT-1	Q	◑ **-34**	◕ **-55**	● **-92**	○ **-2**	○ 3	● **-92**	◑ **-50**	◑ **-44**	● **-92**	● **-92**
FAT-2	Y_{qst}	○ -7	-	-	○ 1	○ **-7**	-	○ 1	○ 2	○ -7	○ **-7**
FAT-3	Q_{qst}	○ 1	-	-	○ -1	○ 1	-	○ 1	○ 1	○ 1	○ **1**
Index	**Assess. quantity**	**Type of track section**			**Track layout**			**Speed**			**MAX**
		Cv	**Tr**	**St**	**RS**	**RM**	**RL**	**Vmin**	**Vref**	**Vmax**	
COM-1	$\ddot{y}_q^{*}$	◑ -42	◑ -40	◑ -46	○ -4	○ -5	◑ -46	◑ -42	◑ -46	◑ -25	◑ **-46**
COM-2	$\ddot{z}_q^{*}$	○ -1	○ 0	○ 0	○ 0	○ 0	○ -1	○ 0	○ -1	○ 0	○ **-1**
COM-3	$s\ddot{y}_q^{*}$	● **-136**	● **-126**	● **-147**	○ **-9**	◔ **-12**	● **-147**	● **-136**	● **-147**	● **-89**	● **-147**
COM-4	$s\ddot{z}_q^{*}$	○ 0	○ 0	○ 0	○ 0	○ 0	○ 0	○ 0	○ 0	○ 0	○ **0**
COM-5	$\ddot{y}_{qst}^{*}$	○ 3	-	-	○ 0	○ 0	○ 3	○ 1	○ 2	○ 3	○ **3**

Variation range (P10; P90)from *12%* *to* *373%* *of the reference value (P50).*

Legend: ○ Low (Influence < 10%); ◔ Moderate (10% ≤ Influence < 25%) ◑ Noticeable (25% ≤ Influence < 50%

◕ High (50% ≤ Influence < 75%); ● Very high (Influence ≥ 75%).

Table 13 Global influence: Lateral stiffness of the primary suspension

Index	Assess. quantity	Type of track section			Track layout			Speed			MAX
		Cv	Tr	St	RS	RM	RL	Vmin	Vref	Vmax	
SAF-1	$(\Sigma Y)_{2m}$	○ 6	○ 3	○ -2	○ -2	○ 3	○ 6	○ -2	○ -2	○ 6	○ **6**
SAF-2	$(Y/Q)_{2m}$	○ 4	○ **5**	-	○ **5**	○ 2	○ 2	○ **4**	○ **4**	○ 5	○ **5**
SAF-3	$\ddot{y}_s^{+}$	○ 7	○ -2	○ 1	○ 0	○ 0	○ 7	○ 1	○ 2	○ 7	○ **7**
SAF-4	$\ddot{y}_s^{*}$	○ -3	○ -3	○ **4**	○ 4	○ **-3**	○ -3	○ **4**	○ -3	○ -3	○ **4**
SAF-5	$s\Sigma Y$	○ **8**	○ -1	○ -3	○ 0	○ 0	○ **8**	○ 0	○ -1	○ **8**	○ **8**
Index	**Assess. quantity**	**Type of track section**			**Track layout**			**Speed**			**MAX**
		Cv	**Tr**	**St**	**RS**	**RM**	**RL**	**Vmin**	**Vref**	**Vmax**	
FAT-1	Q	○ **-8**	○ **-7**	○ **4**	○ **-5**	○ **-7**	○ **-8**	○ **-4**	○ **-6**	○ **-8**	○ **-8**
FAT-2	Y_{qst}	○ 1	-	-	○ 1	○ 1	-	○ 0	○ 1	○ 1	○ **1**
FAT-3	Q_{qst}	○ -4	-	-	○ -4	○ -4	-	○ -2	○ -3	○ -4	○ **-4**
Index	**Assess. quantity**	**Type of track section**			**Track layout**			**Speed**			**MAX**
		Cv	**Tr**	**St**	**RS**	**RM**	**RL**	**Vmin**	**Vref**	**Vmax**	
COM-1	$\ddot{y}_q^{*}$	○ 3	○ -3	○ 3	○ 3	○ -3	○ 1	○ 3	○ -2	○ 2	○ **3**
COM-2	$\ddot{z}_q^{*}$	◔ 21	◔ 22	◑ 25	○ -6	◑ 25	○ 10	○ 10	◔ 18	◑ 25	◑ **25**
COM-3	$s\ddot{y}_q^{*}$	○ 10	○ -9	◔ 10	◔ 10	○ -9	○ 1	◔ 10	○ 8	○ 7	◔ **10**
COM-4	$s\ddot{z}_q^{*}$	◑ **42**	◑ **39**	◑ **48**	◔ **-12**	◑ **48**	◔ **19**	◔ **19**	◑ **31**	◑ **48**	◑ **48**
COM-5	$\ddot{y}_{qst}^{*}$	○ 0	-	-	○ 0	○ 0	○ 0	○ 0	○ 0	○ 0	○ **0**

Variation range (P10; P90)from *27%* *to* *268%* *of the reference value (P50).*

Legend: ○ Low (Influence < 10%); ◔ Moderate (10% ≤ Influence < 25%) ◑ Noticeable (25% ≤ Influence < 50%

◕ High (50% ≤ Influence < 75%); ● Very high (Influence ≥ 75%).

Table 14 Global influence: Vertical stiffness of the primary suspension

From these results, it can be concluded that:

- k1X shows a very high influence for any study (safety, track fatigue and ride quality).

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- k1Y shows a very high influence for any study. However, it shows low influence for track fatigue studies with RS or RM track layouts at any speed, as well as for ride quality studies with RS track layout at any speed.
- k1Z shows low influence for safety and track fatigue studies and noticeable influence for ride quality studies.
- d1Z shows low influence for any study.

4.3 INFLUENCE OF THE ELASTIC PROPERTIES OF THE SECONDARY SUSPENSION

The following tables show the influence obtained when analyzing the following elastic properties of the secondary suspension:

- Lateral stiffness of the secondary suspension, k2Y (Table 15)
- Vertical stiffness of the secondary suspension, k2Z (Table 16)
- Lateral damping of the secondary suspension, d2Y (Table 17)
- Vertical damping of the secondary suspension, d2Z (Table 18)

Index	Assess. quantity	Type of track section			Track layout			Speed			MAX
		Cv	Tr	St	RS	RM	RL	Vmin	Vref	Vmax	
SAF-1	$(\Sigma Y)_{2m}$	◔ 15	○ 9	◔ -16	○ -4	◔ 10	◔ -16	○ -5	○ 6	◔ -16	◔ **-16**
SAF-2	$(Y/Q)_{2m}$	◔ 12	◔ 17	-	◔ **17**	○ 4	◔ -13	◔ 11	◔ 14	◔ 17	◔ **17**
SAF-3	$\ddot{y}_s^{+}$	◔ 18	◔ -20	◔ -15	○ 0	○ -1	◔ -20	○ -3	○ -5	◔ -20	◔ **-20**
SAF-4	$\ddot{y}_s^{*}$	◔ **20**	◔ 13	◔ **17**	◔ 11	◔ **17**	◔ 20	◔ **20**	◔ **16**	◔ 17	◔ **20**
SAF-5	$s\Sigma Y$	◔ 15	◔ **-25**	◔ -11	○ 0	○ -1	◔ **-25**	○ -3	○ -2	◔ **-25**	◔ **-25**

Index	Assess. quantity	Type of track section			Track layout			Speed			MAX
		Cv	Tr	St	RS	RM	RL	Vmin	Vref	Vmax	
FAT-1	Q	◑ **-28**	◔ **-19**	○ **-5**	◑ **-28**	◑ **-25**	◑ **-26**	◔ **-14**	◔ **-22**	◑ **-28**	◑ **-28**
FAT-2	Y_{qst}	○ 5	-	-	○ 5	○ 2	-	○ 4	○ 4	○ 5	○ **5**
FAT-3	Q_{qst}	◔ -13	-	-	◔ -13	◔ -11	-	○ -7	◔ -10	◔ -13	◔ **-13**

Index	Assess. quantity	Type of track section			Track layout			Speed			MAX
		Cv	Tr	St	RS	RM	RL	Vmin	Vref	Vmax	
COM-1	$\ddot{y}_q^{*}$	◔ 20	◔ 20	◔ 17	◔ 10	◔ 16	◔ 20	◔ 20	◔ 17	◔ 16	◔ **20**
COM-2	$\ddot{z}_q^{*}$	○ -1	○ 0	○ 0	○ -1	○ 0	○ 0	○ -1	○ 0	○ 0	○ **-1**
COM-3	$s\ddot{y}_q^{*}$	◕ **54**	◑ **49**	◑ **45**	◑ **26**	◑ **42**	◕ **54**	◕ **54**	◑ **45**	◑ **42**	◕ **54**
COM-4	$s\ddot{z}_q^{*}$	○ 2	○ 2	○ 0	○ -1	○ 2	○ 0	○ -1	○ 2	○ 2	○ **2**
COM-5	$\ddot{y}_{qst}^{*}$	○ 0	-	-	○ 0	○ 0	○ 0	○ 0	○ 0	○ 0	○ **0**

Variation range (P10; P90)from 35% to 370% of the reference value (P50).

Legend: ○ Low (Influence < 10%); ◔ Moderate (10% ≤ Influence < 25%) ◑ Noticeable (25% ≤ Influence < 50%
◕ High (50% ≤ Influence < 75%); ● Very high (Influence ≥ 75%).

TABLE 15 GLOBAL INFLUENCE: LATERAL STIFFNESS OF THE SECONDARY SUSPENSION

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Index	Assess. quantity	Type of track section			Track layout			Speed			MAX
		Cv	Tr	St	RS	RM	RL	Vmin	Vref	Vmax	
SAF-1	$(\Sigma Y)_{2m}$	◔ 17	○ 9	○ -3	○ -5	○ **10**	◔ 17	○ -3	○ 6	◔ 17	◔ **17**
SAF-2	$(Y/Q)_{2m}$	◔ 12	◔ **15**	-	◔ **15**	○ 6	○ 3	○ **9**	◔ **12**	◔ 15	◔ **15**
SAF-3	$\ddot{y}_s^{+}$	◔ 17	○ 4	○ -1	○ 0	○ -1	◔ 17	○ 1	○ 3	◔ 17	◔ **17**
SAF-4	$\ddot{y}_s^{*}$	○ -5	○ -4	○ **6**	○ 6	○ -5	○ -4	○ 6	○ -5	○ -4	○ **6**
SAF-5	$s\Sigma Y$	◔ **18**	○ -1	○ 1	○ 0	○ 0	◔ **18**	○ 0	○ -1	◔ **18**	◔ **18**
Index	Assess. quantity	Type of track section			Track layout			Speed			MAX
		Cv	Tr	St	RS	RM	RL	Vmin	Vref	Vmax	
FAT-1	Q	◔ **-25**	◔ **-16**	○ **-4**	◔ **-25**	◔ **-21**	◔ **-21**	◔ **-11**	◔ **-18**	◔ **-25**	◔ **-25**
FAT-2	Y_{qst}	○ 3	-	-	○ 3	○ -1	-	○ 2	○ 3	○ 3	○ **3**
FAT-3	Q_{qst}	◔ -11	-	-	◔ -11	○ -9	-	○ -6	○ -8	◔ -11	◔ **-11**
Index	Assess. quantity	Type of track section			Track layout			Speed			MAX
		Cv	Tr	St	RS	RM	RL	Vmin	Vref	Vmax	
COM-1	$\ddot{y}_q^{*}$	○ 5	○ 4	○ 6	○ 6	○ -5	○ 3	○ 6	○ 5	○ -5	○ **6**
COM-2	$\ddot{z}_q^{*}$	◑ 32	◑ 34	◑ 40	◔ 15	◑ 40	◑ 31	◑ 30	◑ 36	◑ 40	◑ **40**
COM-3	$s\ddot{y}_q^{*}$	◔ 13	◔ -14	◔ -16	◔ 14	◔ -16	○ 3	◔ -16	◔ -12	◔ 11	◔ **-16**
COM-4	$s\ddot{z}_q^{*}$	◕ **65**	◕ **63**	● **77**	◔ **25**	● **77**	◕ **63**	◕ **59**	◕ **68**	● **77**	● **77**
COM-5	$\ddot{y}_{qst}^{*}$	○ 1	-	-	○ 0	○ 0	○ 1	○ 0	○ 0	○ 1	○ **1**

Variation range (P10; P90)from 42% to 288% of the reference value (P50).

Legend: ○ Low (Influence < 10%); ◔ Moderate (10% ≤ Influence < 25%) ◑ Noticeable (25% ≤ Influence < 50%
◕ High (50% ≤ Influence < 75%); ● Very high (Influence ≥ 75%).

TABLE 16 GLOBAL INFLUENCE: VERTICAL STIFFNESS OF THE SECONDARY SUSPENSION

Index	Assess. quantity	Type of track section			Track layout			Speed			MAX
		Cv	Tr	St	RS	RM	RL	Vmin	Vref	Vmax	
SAF-1	$(\Sigma Y)_{2m}$	◑ -36	● -168	● -180	○ 2	○ 8	● -180	○ 6	○ 8	● -180	● **-180**
SAF-2	$(Y/Q)_{2m}$	◑ -44	● -154	-	○ 1	○ 1	● -154	○ 1	○ -2	● -154	● **-154**
SAF-3	$\ddot{y}_s^{+}$	◕ -52	● -296	● **-283**	○ 0	○ 1	● -296	○ 1	○ -5	● -296	● **-296**
SAF-4	$\ddot{y}_s^{*}$	○ 10	◔ -11	◔ -12	◔ **-12**	◔ **-11**	○ 10	◔ **-11**	◔ **-12**	◔ -12	◔ **-12**
SAF-5	$s\Sigma Y$	● **-76**	● **-350**	● -235	○ 0	○ -1	● **-350**	○ -4	○ -10	● **-350**	● **-350**
Index	Assess. quantity	Type of track section			Track layout			Speed			MAX
		Cv	Tr	St	RS	RM	RL	Vmin	Vref	Vmax	
FAT-1	Q	○ **-3**	○ **-10**	◑ **-31**	○ 0	○ -1	◑ **-31**	○ -1	○ -1	◑ **-31**	◑ **-31**
FAT-2	Y_{qst}	○ -2	-	-	○ **0**	○ **-2**	-	○ **-2**	○ **-2**	○ -2	○ **-2**
FAT-3	Q_{qst}	○ 0	-	-	○ 0	○ 0	-	○ 0	○ 0	○ 0	○ **0**
Index	Assess. quantity	Type of track section			Track layout			Speed			MAX
		Cv	Tr	St	RS	RM	RL	Vmin	Vref	Vmax	
COM-1	$\ddot{y}_q^{*}$	◔ -12	◔ 13	◔ -13	◔ -13	○ -10	◔ 13	◔ 11	◔ 13	◔ 13	◔ **13**
COM-2	$\ddot{z}_q^{*}$	○ 2	○ 0	○ 0	○ 0	○ 0	○ 2	○ 0	○ 0	○ 2	○ **2**
COM-3	$s\ddot{y}_q^{*}$	◑ **-32**	◑ **-31**	◑ **-34**	◑ **-34**	◑ **-27**	◑ **28**	◑ **-27**	◑ **-34**	◑ **-32**	◑ **-34**
COM-4	$s\ddot{z}_q^{*}$	○ 0	○ 0	○ 0	○ 0	○ 0	○ 0	○ 0	○ 0	○ 0	○ **0**
COM-5	$\ddot{y}_{qst}^{*}$	○ 1	-	-	○ 0	○ 0	○ 1	○ 0	○ 0	○ 1	○ **1**

Variation range (P10; P90)from 40% to 287% of the reference value (P50).

Legend: ○ Low (Influence < 10%); ◔ Moderate (10% ≤ Influence < 25%) ◑ Noticeable (25% ≤ Influence < 50%
◕ High (50% ≤ Influence < 75%); ● Very high (Influence ≥ 75%).

TABLE 17 GLOBAL INFLUENCE: LATERAL DAMPING OF THE SECONDARY SUSPENSION

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Index	Assess. quantity	Type of track section			Track layout			Speed			MAX
		Cv	Tr	St	RS	RM	RL	Vmin	Vref	Vmax	
SAF-1	$(\Sigma Y)_{2m}$	○ -1	○ -1	○ -2	○ -2	○ **-2**	○ -1	○ **-2**	○ -2	○ -2	○ **-2**
SAF-2	$(Y/Q)_{2m}$	○ -2	○ -1	-	○ 0	○ -2	○ -1	○ -2	○ -1	○ -2	○ **-2**
SAF-3	$\ddot{y}_s^+$	○ 0	○ -1	○ -1	○ 0	○ 0	○ -1	○ 0	○ -1	○ -1	○ **-1**
SAF-4	$\ddot{y}_s^*$	○ **-2**	○ **-1**	○ **2**	○ **2**	○ **-2**	○ **-1**	○ 2	○ **2**	○ **2**	○ **2**
SAF-5	$s\Sigma Y$	○ 0	○ -1	○ 0	○ 0	○ 0	○ -1	○ 0	○ 0	○ -1	○ **-1**
Index	Assess. quantity	Type of track section			Track layout			Speed			MAX
		Cv	Tr	St	RS	RM	RL	Vmin	Vref	Vmax	
FAT-1	Q	○ **-1**	○ **-2**	○ **-1**	○ **1**	○ **-2**	○ **-1**	○ **-1**	○ **-1**	○ **-2**	○ **-2**
FAT-2	Y_{qst}	○ 0	-	-	○ 0	○ 0	-	○ 0	○ 0	○ 0	○ **0**
FAT-3	Q_{qst}	○ 0	-	-	○ 0	○ 0	-	○ 0	○ 0	○ 0	○ **0**
Index	Assess. quantity	Type of track section			Track layout			Speed			MAX
		Cv	Tr	St	RS	RM	RL	Vmin	Vref	Vmax	
COM-1	$\ddot{y}_q^*$	○ -2	○ -2	○ -2	○ 2	○ -2	○ -1	○ 2	○ -2	○ -2	○ **-2**
COM-2	$\ddot{z}_q^*$	◑ -27	◑ -28	◑ -34	◔ -21	◑ -34	○ 5	◑ -29	◑ -34	◑ -32	◑ **-34**
COM-3	$s\ddot{y}_q^*$	○ -6	○ -6	○ -6	○ 5	○ -6	○ -2	○ -5	○ -6	○ -6	○ **-6**
COM-4	$s\ddot{z}_q^*$	◕ **-62**	◕ **-58**	◕ **-67**	◑ **-46**	◕ **-67**	○ **9**	◕ **-55**	◕ **-67**	◕ **-67**	◕ **-67**
COM-5	$\ddot{y}_{qst}^*$	○ 0	-	-	○ 0	○ 0	○ 0	○ 0	○ 0	○ 0	○ **0**
Variation range (P10; P90)from			*30%*	*to*	*334%*	*of the reference value (P50).*					
Legend:		○ Low (Influence < 10%);			◔ Moderate (10% ≤ Influence < 25%)			◑ Noticeable (25% ≤ Influence < 50%			
		◕ High (50% ≤ Influence < 75%);			● Very high (Influence ≥ 75%).						

TABLE 18 GLOBAL INFLUENCE: VERTICAL DAMPING OF THE SECONDARY SUSPENSION

From these results it can be concluded that:

- k2Y shows moderate influence for safety studies, noticeable influence for track fatigue studies and high influence for ride quality studies.
- k2Z shows moderate influence for safety and track fatigue studies and very high influence for ride quality studies. However, it shows low influence for safety studies with RM track layouts at any speed, as well as at Vmin with any track layout.
- d2Y shows noticeable influence for ride quality studies and very high influence for safety studies. For track fatigue studies, it also shows noticeable influence for RL track layouts at Vmax, showing low influence in any other condition.
- d2Z shows low influence for safety and track fatigue studies, and high influence for ride quality studies. However, it can show a low influence for ride quality studies with RL track layouts at any speed.

4.4 FURTHER COMMENTS

Influences of hundreds and even thousands of percent respect the related limit values were obtained for k1X, k1Y and d2Y. For all these parameters, the safety index SAF-5 was also over 100 % for RL track layouts at Vmax, thus indicating that the vehicle become unstable under such conditions, so conditioning the high influences found. However, lower influences may be found for these parameters if they had not been varied independently from the other elastic properties, or even if they had been varied in lower steps.

5 COMPARISON OF RESULTS OF THE SENSITIVITY ANALYSIS

In this paragraph, the results obtained in the sensitivity analysis are compared. The conclusions obtained from this analysis are also presented here. To have a more

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comprehensive view, results are grouped into two different categories: elastic properties of the primary suspension and elastic properties of the secondary suspension.

Within each group, the results are gathered in the same table, showing the characteristics of the most critical scenarios found when analyzing the influence of each parameter. The notations used for the output quantities shown in these tables were explained in Table 9.

5.1 INFLUENCE OF THE ELASTIC PROPERTIES OF THE PRIMARY SUSPENSION

Table 19 summarizes the characteristics of the most critical scenarios found when analyzing the influence of the elastic properties of the primary suspension.

		k1X	**k1Y**	**k1Z**	**d1Z**
Safety:					
	Index:	SAF-2, $\left(\frac{\mathrm{Y}}{\mathrm{Q}}\right)_{2m}$	SAF-3, $\ddot{\mathbf{y}}_s^+$	SAF-5, $\boldsymbol{s\Sigma Y}$	SAF-5, $\boldsymbol{s\Sigma Y}$
	Layout:	RL	RL	RL	RL
	Speed:	Vmax	Vref	Vmax	Vmax
	Influence:	● (-19714 %)	● (-509 %)	○ (8 %)	○ (-1 %)
Track fatigue:					
	Index:	FAT-1, Q	FAT-1, Q	FAT-1, Q	FAT-1, Q
	Layout:	RL	RL	RL	RL
	Speed:	Vmin	Vmax	Vmax	Vmax
	Influence:	● (-704 %)	● (-92 %)	○ (-8 %)	○ (-1 %)
Ride quality:					
	Index:	COM-3, $\boldsymbol{s\ddot{y}_q^*}$	COM-3, $\boldsymbol{s\ddot{y}_q^*}$	COM-4, $\boldsymbol{s\ddot{z}_q^*}$	COM-4, $\boldsymbol{s\ddot{z}_q^*}$
	Layout:	RL	RL	RM	RM
	Speed:	Vmin	Vref	Vmax	Vmax
	Influence:	● (-356 %)	● (-147 %)	◑ (48 %)	○ (-3 %)

Influence (I): ○ Low (I < 10%); ◔ Moderate (10% ≤ I < 25%); ◑ Noticeable (25% ≤ I < 50%); ◕ High (50% ≤ I < 75%); ● Very high (I ≥ 75%).

TABLE 19 SUMMARY TABLE: ELASTIC PROPERTIES OF THE PRIMARY SUSPENSION

In view of these results, the parameter to which vehicle dynamic behaviour is most sensitive is k1X, followed by k1Y, both showing very high Influence for safety, track fatigue and ride quality studies, the highest influences being found when running on the RL track layout; they are followed by k1Z, showing noticeable influence for ride quality studies and low influence for safety and track fatigue studies, the highest influences being found when running at Vmax. The only parameter that shows low influence for safety, track fatigue and ride quality studies is d1Z.

5.2 INFLUENCE OF THE ELASTIC PROPERTIES OF THE SECONDARY SUSPENSION

Table 20 summarizes the characteristics of the most critical scenarios found when analyzing the influence of the elastic properties of the secondary suspension.

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		k2Y	k2Z	d2Y	d2Z
Safety:					
	Index:	SAF-5, $\mathbf{s\Sigma Y}$	SAF-5, $\mathbf{s\Sigma Y}$	SAF-5, $\mathbf{s\Sigma Y}$	SAF-4, $\ddot{\mathbf{y}}_s^*$
	Layout:	RL	RL	RL	RS
	Speed:	Vmax	Vmax	Vmax	Vmax
	Influence:	◔ (-25 %)	◔ (18 %)	● (-350 %)	○ (2 %)
Track fatigue:					
	Index:	FAT-1, Q	FAT-1, Q	FAT-1, Q	FAT-1, Q
	Layout:	RS	RS	RL	RM
	Speed:	Vmax	Vmax	Vmax	Vmax
	Influence:	◑ (-28 %)	◔ (-25 %)	◑ (-31 %)	○ (-2 %)
Ride quality:					
	Index:	COM-3, $s\ddot{y}_q^*$	COM-4, $s\ddot{z}_q^*$	COM-3, $s\ddot{y}_q^*$	COM-4, $s\ddot{z}_q^*$
	Layout:	RL	RM	RS	RM
	Speed:	Vmin	Vmax	Vref	Vref
	Influence:	◕ (54 %)	● (77 %)	◑ (-34 %)	◕ (-67 %)

Influence (I): ○ Low (I < 10%); ◔ Moderate (10% ≤ I < 25%); ◑ Noticeable (25% ≤ I < 50%); ◕ High (50% ≤ I < 75%); ● Very high (I ≥ 75%).

TABLE 20 SUMMARY TABLE: ELASTIC PROPERTIES OF THE SECONDARY SUSPENSION

In view of these results, the parameters to which vehicle dynamic behaviour is most sensitive are d2Y and k2Z. The former shows very high influence for safety studies and noticeable influence for both track fatigue and ride quality studies; the second one shows very high influence for ride quality studies and moderate influence for both safety and track fatigue studies, the highest influences being found when running at Vmax. They are followed by k2Y, with high influence for ride quality studies, noticeable influence for track fatigue studies and moderate influence for safety studies. The least sensitive parameter is d2Z, showing high influence for ride quality studies and low influence for both safety and track fatigue studies.

5.3 ORDERING ELASTIC PROPERTIES BY INCREASING INFLUENCE

In accordance with the previous results, generally speaking, it could be said that the highest influences were found for the longitudinal stiffness and the lateral stiffness of the primary suspension, and the lowest influences for the vertical stiffness and the vertical damping of the primary suspension, with the parameters of the secondary suspension showing intermediate influences between them.

Finally, the elastic properties of the vehicle were grouped considering the operating conditions under which their value could be estimated with a lesser degree of accuracy for future simulations, without significantly affecting the simulation results:

- d1Z: for any condition.
- k1Z, d2Z: for almost any condition, except for some ride quality indexes.
- k1Y, k2Z, d2Y: just for some particular combinations of study, track layout and speed.
- k1X, k2Y: for no condition.

However, as already seen, those conditions in which it is admissible to assess the numerical value of a given parameter differ according to the study to be performed: safety, track fatigue or ride quality. Consequently, the value of the above parameters could be estimated with a lesser degree of accuracy for some other operating conditions.

6 CONCLUDING REMARKS

In this work, the influence of the elastic properties of the primary and secondary suspensions was analyzed to assess their impact on the vehicle's dynamic behavior. As a whole, 8 different elastic properties were considered:

- longitudinal, lateral and vertical stiffness of the primary suspension;
- lateral and vertical stiffness of the secondary suspension;
- vertical damping of the primary suspension;
- lateral and vertical damping of the secondary suspension.

Due to the great number of uncertain parameters and external conditions (vehicle speed and track layout) to be considered, a simple approach, consisting in modifying the input parameters one at a time, was chosen to perform this study.

To undertake the study, a reference value, extracted from the railway vehicles database RVDynDB, was considered for each elastic property, thus defining an initial point in the space of input parameters:

Two additional values were assigned to each parameter, corresponding to the percentiles 10 and 90 of the data set stored in the vehicles database, hence exploring the 8-dimensional space by looking to both a lower and an upper value in each direction. These represent just $2 \cdot 8 = 16$ vertices of a hypercube with $2^8 = 256$ vertices.

From these figures, it can be seen that the number of cases possible to test was necessary limited, thus conditioning the simplicity of the methodology to be used. This way, the study was focused on passenger vehicles with linear suspension models, and the input parameters were modified one at a time, with just three values in each variation, even for parameters with large variation ranges, so assuming that the output quantities are smooth functions of the input parameters. Therefore, as previously stated, the conclusions that can be drawn are also limited from a quantitative point of view, but they can provide a qualitative idea about which influence quantities need to be addressed with particular care when performing simulations addressing a specific problem.

After processing the results of the simulations, the elastic properties of the vehicle suspension were ordered by increasing influence. It was concluded that the highest influences were found for the longitudinal stiffness and the lateral stiffness of the primary suspension, and the lowest influences for the vertical stiffness and the vertical damping of the primary suspension, with the parameters of the secondary suspension showing intermediate influences between them.

The elastic properties of the vehicle suspensions were also grouped considering the operating conditions under which their value could be estimated with a lesser degree of accuracy for future simulations, without significantly affecting the simulation results:

- d1Z: for any condition.
- k1Z, d2Z: for almost any condition, except for some ride quality indexes.
- k1Y, k2Z, d2Y: just for some particular combinations of study, track layout and speed.
- k1X, k2Y: for no condition.

Though it was not considered in this analysis, it should be pointed out that, in fact, some elastic parameters are interconnected to each other. So, if certain stiffness is well known, some other suspension characteristics would have a lower initial uncertainty. Nevertheless, the results found in this study could be useful when not all the data required to undertake a future simulation job are initially known, and there are no possibilities of testing. Such

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situations may arise, for instance, when the components to be modelled were manufactured by third parties or even when the vehicle to be analyzed was manufactured many years ago, as sometimes happens when dealing with derailment reconstructions. In such situations, the previous results could help to decide whether to accept or not any possible request to undertake a dynamic analysis for a vehicle with some unknown parameters.

Further development of the work proposed here might consist in varying the parameters found to be most important in smaller steps or, even further, in undertaking a probabilistic approach to consider simultaneous variations of the uncertain input parameters. In fact, from the results obtained, the number of parameters to be considered to undertake such probabilistic approach could be reduced. This way, quantitative and more accurate results could be obtained with a considerable lower computational cost than considering all the uncertain input parameters.

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