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Computation of nonlinear streaky structures in boundary layer flow

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Abstract. *In this work, the Reduced Navier Stokes (RNS) are numerically integrated, and used to calculate nonlinear finite amplitude streaks. These structures are interesting since they can have a stabilizing effect and delay the transition to the turbulent regime. RNS formulation is also used to compute the family of nonlinear intrinsic streaks that emerge from the leading edge in absence of any external perturbation. Finally, this formulation is generalized to include the possibility of having a curved bottom wall*

Keywords: streaks; nonlinear; boundary layer; intrinsic

1 INTRODUCTION

Streaks are three dimensional boundary layer flow structures that take the form of spanwise thin and streamwise elongated regions of high and low speed flow that alternate in the spanwise direction. The resulting streamwise velocity profile exhibits a strong modulation in the spanwise direction, with a characteristic scale of the order of the boundary layer width. The slow transversal (wall-normal and spanwise) motion points downwards in the high speed region and upwards in the low speed region (see Figure 1).

This work is focused on the streaks that are forced from inside the boundary layer, with no free stream perturbations, by means of a spanwise periodic array of small cylindrical roughness elements attached to the plate near the leading edge. (see, e.g., [1, 2, 4]), and they are interesting for their potential to extend downstream the laminar regime of the flow, moving further downstream the location of the transition point. The stability of the boundary layer with finite amplitude streaky distortions has been analyzed in [3, 4, 7], and experimentally tested [1, 2], detecting that the increase of the amplitude of the streaks can have an stabilizing effect in the Blasius boundary layer, delaying the onset of turbulence. The linear inviscid stability depends on the amplitude of the streak. In [5], it was found that there is a maximum streak amplitude ($\sim 26\%$ of the free stream velocity) beyond which transition is promoted in the boundary layer.

Nevertheless, some recent experimental results [6] have reported the possibility of inducing stable, robust steady streaks with amplitude significantly above the 'critical one' ($\sim 26\%$), without having any secondary instability acting on them. So, the streak stability characteristics are definitely far from clear, and that they probably depend not only on the magnitude of its spanwise modulation but also on how the streaks are generated. Thus, there is a clear motivation to compute high amplitude streaks.

2 RNS FORMULATION

High intensity streaks are nonlinear, and are typically computed using either direct numerical simulation (DNS) or nonlinear parabolized stability equations (PSE). The DNS for the streak computations is very CPU costly since this is a three dimensional problem that, in order to correctly reproduce the boundary layer conditions, has to be performed at a very high Reynolds number [3, 4]. On the other hand, the nonlinear PSE has strong consistency problems that require the addition of extra stabilization terms, and it simply fails to converge when the amplitude of

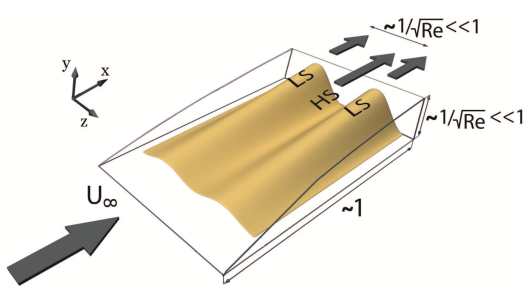

Figure 1: Sketch of the development of a spanwise periodic array of streaks on a flat plate boundary layer, with the asymptotic scaling for $Re \gg 1$ indicated. HS (LS) stands for high (low) streamwise velocity.

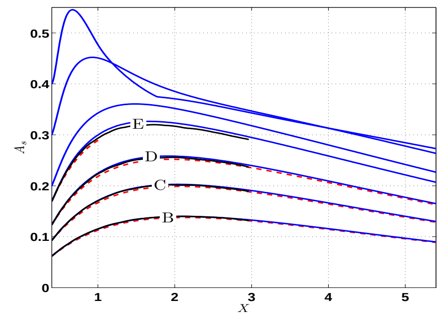

Figure 2: Nonlinear streamwise streak amplitude evolution, for different initial amplitudes: RNS (blue line), PSE from [7] (red dashed line), and DNS from [3, 4] (black line).

the streak is not small (see [7]). So, in this work, a much more efficient way is used to compute nonlinear streaks, the reduced Navier-Stokes (RNS) equations.

A flat plate boundary layer at zero angle of incidence with a spanwise periodic array of counter-rotating steady streaks developing in the streamwise direction is considered, see Figure 1. The velocities are made nondimensional with the reference velocity the free stream flow, U_∞, the spatial scales with a characteristic length L, and the resulting Reynolds number is defined in the usual form, $Re = U_\infty L/\nu$, where ν is the kinematic viscosity.

The RNS are obtained from the full 3D steady incompressible Navier Stokes equations in the limit of large Reynolds number. The asymptotic structure of the streaks for $Re \gg 1$ exhibits (Figure 1) a slow spatial dependence in the streamwise direction, and two short spatial scales, in the normal and spanwise direction. This scaling is similar to that of the standard 3D Boundary Layer equations (BL) [11] but with two short scales instead of just one.

The appropriate expansions for the flow variables are of the form

$$\hat{x} = x,\ (\hat{y}, \hat{z}) = (y, z)/\sqrt{Re}, \quad \hat{u} = u + \ldots,\ (\hat{v}, \hat{w}) = ((v, w) + \ldots)/\sqrt{Re},\ \hat{p} = p_0 + p_1/Re + \ldots,$$

which, once inserted into the Navier Stokes equations yields, at first order,

$$p_{0_y} = 0 \quad \text{and} \quad p_{0_z} = 0.$$

The pressure is then $p_0 = p_0(x)$, given by the prescribed inviscid pressure at the upper edge of the boundary layer (as in the standard BL formulation). And, at next order, the full set of RNS is obtained:

$$\begin{aligned} u_x + v_y + w_z &= 0, \\ uu_x + vu_y + wu_z &= -p_{0_x} + u_{yy} + u_{zz}, \\ uv_x + vv_y + wv_z &= -p_{1_y} + v_{yy} + v_{zz}, \\ uw_x + vw_y + ww_z &= -p_{1_z} + w_{yy} + w_{zz}. \end{aligned} \tag{1}$$

The RNS are a truly parabolic system in x that can be solved by marching techniques [8, 9]. In contrast with the standard 3D BL formulation, now the second order y and z momentum equations are required to complete the system, and the pressure correction term, p_1, has to be computed in order to solve problem.

The RNS equations are well known [9, 10]. They have been previously used to compute high Re number micro channel and micro tube flow [12], the nonlinear development of Görtler vortices [13], the transient growth of small amplitude streaks [14, 15, 16], or the streak excitation by vortical structures on the free stream, both in steady and unsteady conditions. [17, 18],

The full 3D RNS system (1) is integrated to compute the nonlinear spatial evolution of streaks on a flat plate boundary layer. The appropriate boundary conditions for this case are periodicity in z (i.e., a spanwise periodic array of streaks), together with no slip at the bottom wall, and, at the upper edge of the boundary layer, all components of v must vanish at $y \to \infty$ except for its mean value $< v >_z$ computed as part of the solution [16].

The RNS formulation allows to perform 3D streaky BL computations with much less CPU cost than previous DNS computations[3], and avoid the numerical problems of the PSE simulations [7].

3 STREAK SIMULATIONS

In Figure 2 are presented the RNS results for the simulation of fully nonlinear streaks. The streamwise evolution of the amplitude of the streak,

$$A_s(x) = (\max_{y,z}(u) - \min_{y,z}(u))/2,$$

is plotted for the same four streaks that were computed using DNS in [3], and PSE in [7]. The integration is started at the station $x_0 = 0.4$, and the initial profile data is taken from [7]. The results show that the agreement with the DNS and PSE data is quite good: the difference between both results grows with the amplitude of the streak, but never exceeds 3%. Note that the RNS results are always closer to the DNS than the PSE, and that the PSE was simply not able to complete the computation of the largest streak (labeled E).
An interesting way to visualize the tree dimensional streak flow dynamics is to follow the downstream evolution of the particle trajectories departing from a horizontal line located at the beginning of the computational box: $x = 0.4, \ y = 3$. The resulting surfaces of particle trajectories are shown in Figure 3. The streaks induce a counter-rotating motion in the (z, y) plane that gives these trajectory surfaces a characteristic mushroom-like shape in the transversal section; very evident in the case of a high intensity streak $(A_{s0} = 0.4)$ depicted in the right plot of Figure 3. Analyzing these particle trajectories could be helpful to understand a strong quantitative discrepancy between the streak spanwise period measured in some experiments using smoke visualizations [19], and that predicted by the linear optimal theory [5, 15].

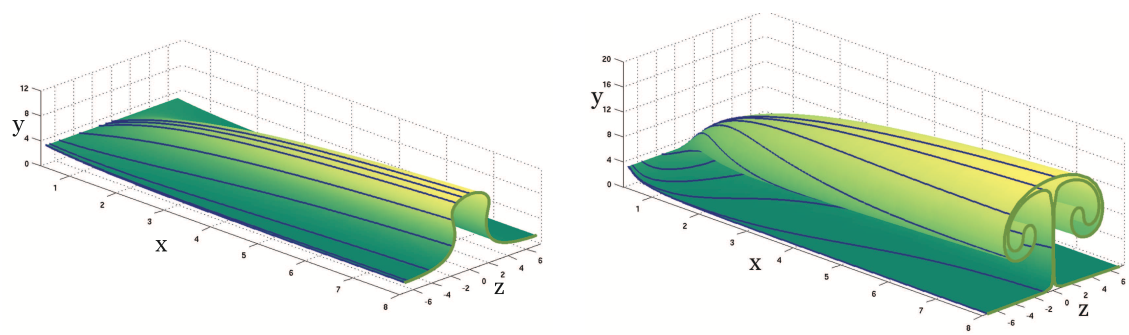

Figure 3: Surfaces of particle trajectories departing from the line $x = 0.4, \ y = 3$, for the streaks with initial amplitudes $A_{s0} = 0.1$ (left) and $A_{s0} = 0.4$ (right). Several representative particle trajectories are also plotted with blue lines.

4 INTRINSIC STREAKS

In [14] the flow near the leading edge of a flat plate boundary layer is analyzed using the linearized problem around the Blasius solution. It is found that there is just one single streaky mode (periodic in the spanwise direction) that grows downstream from the leading edge. The existence of this growing mode indicates that there is a one parameter family of 3D steady streak solutions that emerge from the leading edge of the boundary layer. The resulting expansion of the solution for the growing mode of the velocity profile near the leading edge can be expressed then as

$$\begin{aligned} u &= U_{Blasius} + a x^{1-\lambda} \tilde{U}(\eta) \cos(z) + \cdots, \\ v &= V_{Blasius} + a x^{1/2-\lambda} \tilde{V}(\eta) \cos(z) + \cdots, \\ w &= -a x^{-\lambda} \left(h(\eta) - \sqrt{x} \tilde{V}(\eta) + \cdots \right) \sin(z), \end{aligned} \tag{2}$$

where $\lambda = 0.7865\ldots$, a is a free parameter, and the profiles for $h(\eta)$, $\tilde{U}(\eta)$, and $\tilde{V}(\eta)$ are obtained in [14] and [16]. The near leading edge expressions (2) obtained are used here as initial conditions for the RNS equations in order to extend downstream the computation this family of intrinsic streaks (intrinsic because they appear in complete absence of any free stream perturbations) using the RNS formulation.
The evolution of the streamwise and spanwise velocity component is represented in Figure 4 for different values of the a parameter. As it grows, nonlinear effects appear and the evolution of the maximum moves away from the linear theory predictions.

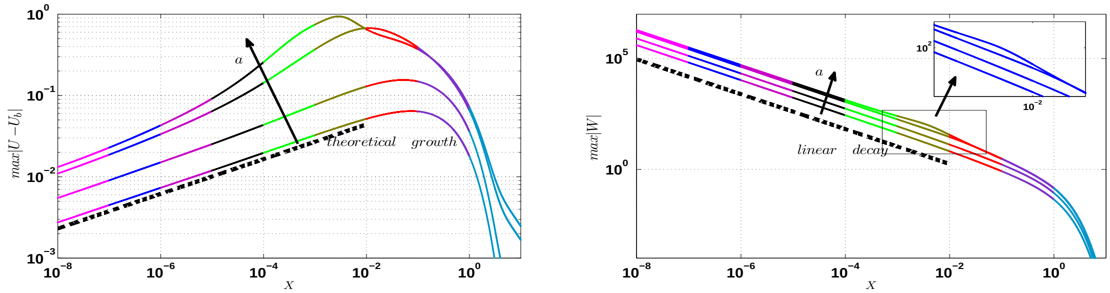

Figure 4: Downstream evolution of maximum deviation from Blasius profile of the streamwise velocity (left) and the maximum of the spanwise velocity component (right). Solid lines: computations for $a = 0.125, 0.25, 0.5$ and 0.6, arrow indicates increasing values of a. Dashed line: asymptotic behavior of the solution near leading edge.

5 CONCLUSIONS

RNS formulation has been used to describe the downstream evolution of a spanwise periodic array of fully nonlinear streaks in a flat plate boundary layer, and the intrinsic nonlinear streaks that emerge from its leading edge, with a much lower computational cost than DNS, and also more robust than nonlinear PSE. When the amplitude of the streak increases, the nonlinear terms come into play and the flow configuration changes substantially, and the transversal motion is important to understand the flow pattern.

The RNS formulation can be used in other fluid configurations provided that, at high Reynolds number, they exhibit two short and one long length scale. For example, the development of cross-flow vortices in a swept wing [20].

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