MONTECARLO METHOD APPLIED TO COMPARISON OF TENDERS IN CONSTRUCTION PROJECTS

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The comparison of the different bids in the tender for a project, with the traditional contract system based on unit rates open to and re-measurement, requires analysis tools that are able to discriminate proposals having a similar overall economic impact, but that might show a very different behaviour during the execution of the works.

One situation not easily detected by traditional methods is the reaction of the actual cost to the changes in the exact quantity of works finally executed respect of the work estimated in the project.

This paper intends to address this situation through the Monte Carlo method, a system of quantitative risk analysis. This procedure, as is known, is allows the input data defining the problem to vary some within well defined probability functions, generating a large number of test cases, the results being statistically treated to obtain the most probable final values, with the rest of the parameters needed to measure the reliability of the estimate.

We present a model for the comparison of bids, designed in a way that it can be applied in real cases, based on data and assumptions that are easy to understand and set up by professionals who wish to perform these tasks.

Keywords: Montecarlo; Tender; Comparison; Bidding; Probabilistic; Risk

APLICACIÓN DEL MÉTODO MONTECARLO A LA COMPARACIÓN DE OFERTAS

La comparación de las diferentes ofertas presentadas en la licitación de un proyecto, con el sistema de contratación tradicional de medición abierta y precio unitario cerrado, requiere herramientas de análisis que sean capaces de discriminar propuestas que teniendo un importe global parecido pueden presentar un impacto económico muy diferente durante la ejecución.

Una de las situaciones que no se detecta fácilmente con los métodos tradicionales es el comportamiento del coste real frente a las variaciones de las cantidades realmente ejecutadas en obra respecto de las estimadas en el proyecto.

Este texto propone abordar esta situación mediante un sistema de análisis cuantitativo del riesgo como el método de Montecarlo. Este procedimiento, como es sabido, consiste en permitir que los datos de entrada que definen el problema varíen unas funciones de probabilidad definidas, generar un gran número de casos de prueba y tratar los resultados estadísticamente para obtener los valores finales más probables, con los parámetros necesarios para medir la fiabilidad de la estimación.

Se presenta un modelo para la comparación de ofertas, desarrollado de manera que puede aplicarse en casos reales aplicando a los datos conocidos unas condiciones de variación que sean fáciles de establecer por los profesionales que realizan estas tareas.

Palabras clave: MonteCarlo; Comparación; Ofertas; Probabilista; Riesgo

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1. Introduction

The comparison of tenders for the execution of works tendered within procurement systems based on unit contract prices, open to re-measurement, requires analysis tools capable of discriminating between proposals that having a similar overall amount may represent a very different final economic impact.

The *horizontal* averages and standard deviations, in work units, detect the elements of the work with more differences, which may be due to incorrect specifications or risks.

The *vertical* averages, comparing the difference between each bidder and the average of all of them, or in respect to the project estimate, may detect bidder's strategies, which can range from the global low to the careful analysis of every operation.

The bidder financing strategies is this project stage, in which there is still no detailed planning, can be compared using fast, simplified models. For example, it can be assumed that the work units are executed sequentially, as they appear in the estimate. Work units cost may have a fixed duration, or proportional to the estimate, applying a predefined cost schedule based in "S" or Gaussian bell curves (Guadalupe and Valderrama, 2013). Based on these amounts, the curves of cumulative spending for all bidders may be obtained. Their overlap indicates at a glance, the offers presenting by the phenomenon known as "front-loading", i.e., higher prices in the early stages to increase funding at the expense of the customer. The numerical indicators such as net present value and internal return rate are also easily obtained with this system.

However, when the contract is open to re-measurement a more specific analysis is needed. Since the client has to pay the amounts related to the quantities actually executed, it is possible that some bidders increase the price of work units where it is most likely that an excess of measurement appears. These units are balanced by a lower price in other work units that rarely grow during execution. This strategy is not detected by the above mentioned methods and it is very difficult to observe in a manual data revision, even by highly experienced professionals.

This paper proposes the use of Monte Carlo analysis to detect this strategy and to assess and compare the offers accordingly.

2. Test case

As is well known (Sóbol, M., 1976), the Monte Carlo method is based on calculating a case several times, each time using random input variables with values whose probability of occurrence is similar to the real phenomenon.

To describe the methodology in a way that can be analyzed and reviewed manually, we propose a project with three work units, which have the same quantity and unit price, but different variability characteristics.

For the execution of this work three bids have been received, with different unit prices for each work unit, but in such a way that the three total amounts are equal.

The values have been selected, so that the results are easily comparable and the conclusions of the analysis might be directly observed, but the methodology is identical to a real case with any number of values, where this quick analysis would not be possible.

Table 1. Bidding example

	Summary	QtyEst	Unit Price	Total Bid 1	Unit Price	Total Bid 2	Unit Price	Total Bid 3
E01	Earthwork	100,00	9,00	900,00	11,00	1.100,00	10,00	1.000,00
E02	Partition walls	100,00	10,00	1.000,00	9,00	900,00	11,00	1.100,00
E03	Lighting fixtures	100,00	11,00	1.100,00	10,00	1.000,00	9,00	900,00
	Total			3.000,00		3.000,00		3.000,00

Given this coincidence of total amounts, a better comparison method is needed for discriminating the bids and makes the decision.

3. Variation range and probability density function

The Monte Carlo method requires defining two aspects:

- The selection of the probability density function, PDF, for the variation of the actually executed quantities during the execution of the project.
- The values for the parameters defining the selected function, at least for the economically relevant work units.

The practical application feasibility of the system requires that both data and assumptions can easily be understood and defined by professionals who wish to use it.

Regarding the variation form, several authors have proposed different PDFs, specific for construction except where mentioned:

- Rectangular, triangular and trapezoidal (Seely, 1996).
- Triangular (Ashworht & Hogg, 2002) and several documents published by the American Association of Cost Engineering, as (AACE, 2008; AACE, 2011).
- Discontinuous double triangular, with different left and right most likely values; beta and triangular, mentioned as probably biased in some cases (AACE, 2009).
- Triangular and beta o betaPert (AACE, 2012).
- Uniform, triangular, normal and beta (Flanagan, 1999).
- Gumbeal and exponential, related to road construction (Sato, Miyamoto & Kitazumi, 2005).
- Bernouilli, beta, lognormal, normal, Poisson, triangular, uniform, Weibull, not specific for construction projects, in (GAO, 2007).
- Normal, lognormal, beta with specific recommendation for triangular (Altez, 2009).

The triangular PDF appears as the most common suggestion, appearing in every reference, except the case study related to road construction. Altez writes:

> "Se pueden definir diferentes distribuciones de probabilidad para las variables escogidas: normal, lognormal, beta, etc., siempre y cuando se cuente con registros históricos o se sepa con certeza su distribución. Sin embargo, como se sabe, en la

industria de la construcción a veces la información disponible es insuficiente o no es aplicable, en cuyo caso lo recomendable es usar la distribución triangular".

"Different probability distributions may be defined for selected variables: normal, lognormal, beta, etc., as long as you count on historical records or the distribution is known for certain. However, as is known, in the construction industry available information is sometimes insufficient or not applicable, in which case it is advisable to use the triangular distribution".

It must be remembered that these recommendations are mostly oriented to fluctuations in prices, not in quantities. Triangular distribution has three values, minimum, most likely and maximum, which is well suited to price variations. This is not the case for quantities, which rarely decrease in contracts open to re-measurement. Accordingly, we propose a single triangular probability density function, with the estimated quantity acting both as a minimum and a most likely value. Later we will comment the difference between the mode and the average values, and the apparent contradiction between the two amounts.

In a graph with the amounts in the horizontal axis, this PDF leads to a triangle of unit area, where the hypotenuse decreases from its peak on the reference amount to zero, where it reaches the maximum allowable variation. It is important to note that the probability of reaching each x-value is represented by the area of the triangle on the left, rather than the hypotenuse height, which would correspond to a rectangular probability distribution.

Figure 1. Probability function
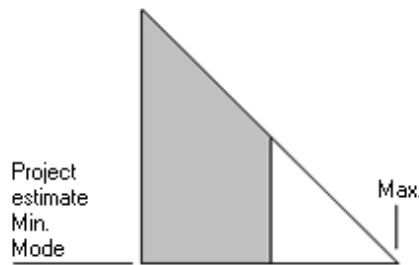
Regarding the parameters values, most likely and maximum, and despite the large number of construction price databases published in Spain (Valderrama, 2010), none of them publishes final variations on the project estimate quantities that could be used as reference values, let alone probability functions.

A proposal for practical application of the methodology, according to regular observations in the construction industry, and in the absence of better data, is shown in the table below, taking as parameters the unit of measurement and the type of customer:

Table 2. Proposal for maximum variation values (%)

Unit of measurement	Volume	Area	Other
Contract under the LCSP	25	15	10
Occasional promoter	20	10	5
Professional promoter	10	5	2

LCSP: Spanish regulations for public works contracting

The maximum rates, as is usual in these cases, refer to extreme situations, which would only be achieved in 1 % of the actual cases.

According to the proposal, we will adopt a large variation for earthwork, smaller for partition walls and very small for the facilities elements.

Table 3. Variations for the test case

	Summary	Unit	Variation (%)
E01	Earthwork	m3	20
E02	Partition walls	m2	10
E03	Lighting fixtures	Each	5

4. Numerical application

Since the triangle area, which measures the probability of reaching the maximum, is the unity, and the variation being a percentage of the estimated quantity, the probability that the actual quantity exceeds a given quantity QtyEst is:

$$Prob\ (Quantity) = ((QtyEst + Variation - Quantity) / Variation) \wedge 2 \qquad (1)$$

For example, the probability of exceeding the quantity which is right in the middle of the range of variation is:

$$Prob\ (QtyEst + Variation / 2) =$$
$$((QtyEst + Variation - QtyEst - Variation / 2)/Variation) \wedge 2 = 0.25 \qquad (2)$$

Solving the quantity, we obtain:

$$Quantity = QtyEst + Variation - Variation * Prob\ (Quantity) \wedge 0.5 \qquad (3)$$

The quantity having a 50% probability of being exceeded is:

$$Quantity = QtyEst + Variation - Variation* 0.5 \wedge 0.5 = QtyEst + Variation *(1 - 0.5 \wedge 0.5)\ (4)$$

That for the first unit of work in the example is:

$$Quantity = 100 + 20 * (1 - 0.5 \wedge 0.5) = 105.86 \qquad (5)$$

For each test case three random numbers are generated, independent of each other. Each calculation is applied to calculate the measurement of the three work units, using the expression above. This measurement is multiplied by the unit price bid of each bidder, obtaining the amount to be paid if this were the actual case work.

Table 4. Test case

	Prob	Bid 1	Bid 2	Bid 3
E01	0.48	954.78	1060.87	1248.53
E02	0.67	1017.87	1119.65	916.08
E03	0.11	1137.13	930.38	1033.76
Total		3109.78	3110.90	3198.37

The probabilistic nature of Monte Carlo method is based on applying the same system repeatedly, using new random numbers, and getting the average. In the example where calculation was performed one thousand times.

Table 5. Summary of test cases (I)

N	E01	E02	E03	Bid 1	Bid 2	Bid 3
1	0.48	0.67	0.11	3109.78	3110.90	3198.37
2	0.00	0.41	0.75	3214.86	3236.16	3068.72
3	0.41	0.06	0.51	3156.08	3167.98	3145.14
4	0.19	0.13	0.26	3193.27	3206.20	3189.34

Table 6. Summary of test cases (II)

| E01	E02	E03	E01	E02	E03	E01	E02	E03
954.78	1017.87	1137.13	1060.87	1119.65	930.38	1248.53	916.08	1033.76
1071.41	1036.06	1107.39	1190.45	1139.67	906.05	1129.55	932.45	1006.72
964.92	1075.43	1115.74	1072.13	1182.97	912.88	1162.95	967.89	1014.31
1001.97	1064.57	1126.73	1113.30	1171.03	921.87	1206.92	958.12	1024.30

It is convenient to check that figures are meeting desired probability functions. For this purpose, we generated a series of equally spaced variations between the reference and the quantity and maximum, counting the number of cases where obtained quantity falls within each range.

Table 7. Checking the probability function

N	Range 1	Range 2	Range 3	Count 1	Count 2	Count 3
0	100	100	100	187.00	193.00	198.00
1	102	101	100.5	182.00	154.00	169.00
2	104	102	101	159.00	138.00	151.00
3	106	103	101.5	128.00	140.00	116.00
4	108	104	102	97.00	121.00	105.00
5	110	105	102.5	91.00	95.00	92.00
6	112	106	103	75.00	82.00	73.00
7	114	107	103.5	40.00	35.00	50.00
8	116	108	104	33.00	37.00	34.00
9	118	109	104.5	8.00	5.00	12.00
10	120	110	105	0.00	0.00	0.00

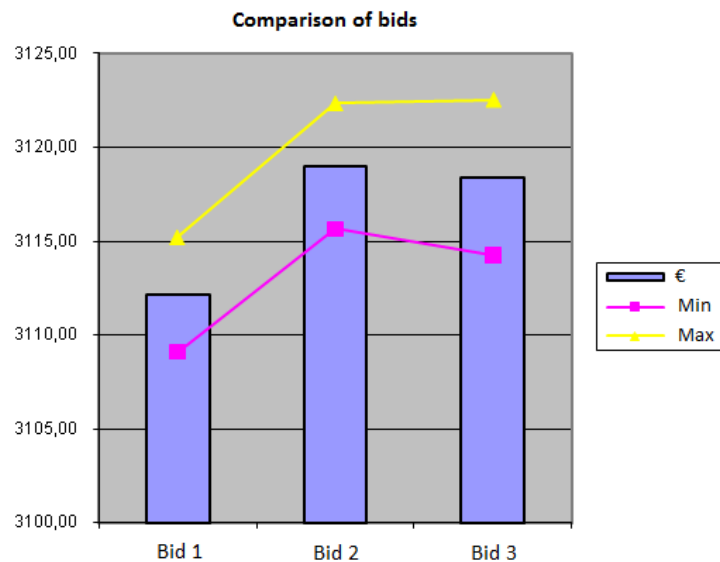The graphic displays visually the resulting triangular function.

Figure 2. Checking the probability function
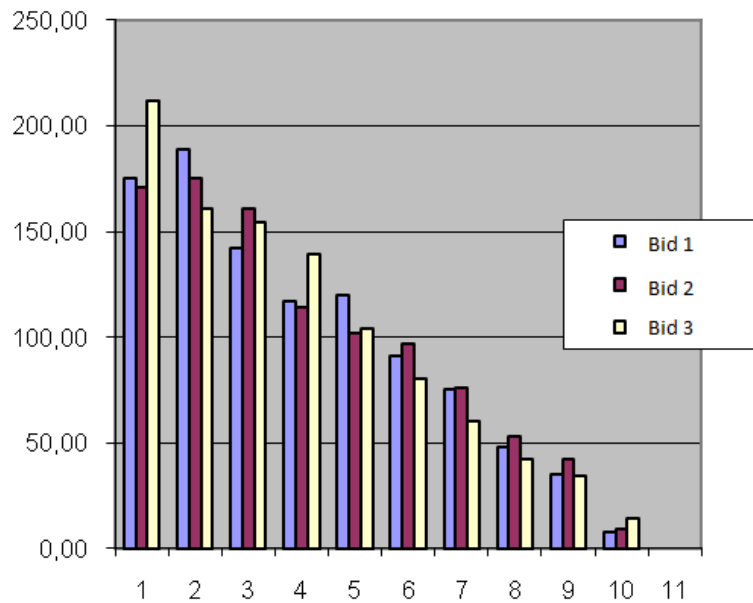
5. Results

Average amounts now are obtained for each bidder the average amounts. Each new generation of random numbers results in different averages and variation is reflected visually in the graph.

Figure 3. Averages and confidence intervals

After a series of iterations is found that the first offer results in a lower expected cost of the works. The other two tenders are closer, with a slight advantage of the third.

Since the sample is a small subset of the unlimited population possible cases, it is convenient to calculate the confidence interval to ensure a certain security, for which we will take 95%, that the average obtained is meaningful.

Table 8. Confidence interval

	Bid 1	Bid 2	Bid 3
Average	3,112.57	3,119.11	3,123.70
Standard deviation	49.57	54.11	67.22
Standard deviation (%)	1.59%	1.73%	2.15%
95% confidence interval (+ / -)	3.07	3.35	4.17
Minimum	3,109.50	3,115.75	3,119.53
Maximum	3,115.64	3,122.46	3,127.87

This interval, added and subtracted to the average value, provides the variation range of each offer. If the difference between two offers is smaller than the confidence interval, it can´t be assured that there is a real advantage of one over the other. In this case happens between second and third supplier. This range of variation is also shown in Figure 3.

In this simple example, with learning objectives, the result could be deduced from the data by a trained user, as the first bidder clearly proposes a lower unit price for work units with a greater risk of increasing. In a real project, with hundreds of work units, this behavior cannot be identified without the numerical analysis.

Figure 4. The final spreadsheet
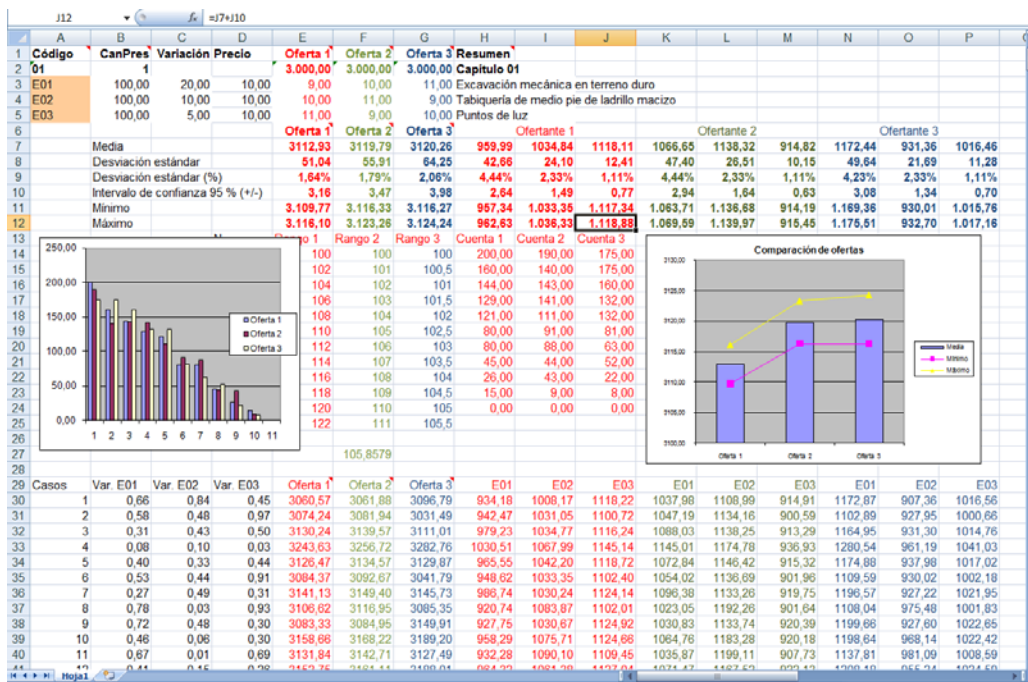
This spreadsheet can be completely built from the data, expressions and formulas published in this text.

6. Alternatives and variations of the method

6.1 The paradox of the two averages

In a triangular distribution the most likely value, or mode, i.e., the highest point, is the estimated value of the project, which from the author or the cost manager`s point of view, would probably be considered instead as an average value. However, the average value obtained in this distribution is a different and higher value.

This paradox is confirmed by the fact that during the works execution, built quantities tend to increase over the estimated quantities, not to decrease, suggesting that project estimates are low. The estimator value for the average is otherwise the minimum value for the reality.

As for its deeper causes, must be sought in two common elements in human behavior:

- The fallacy of *voluntarism* or *control*, by which we believe we can alter reality simply writing a convenient figure.

- The self-fulfilling prophecy, whereby quantities in a procurement contract open to re-measurement tend to increase much more than quantities in other contracting systems in which this risk is transferred to the main contractor.

Not to forget all the interests of involved stakeholders to get the project into practice, giving rise to widespread underestimation of costs and its uncritical assumption by the promoters themselves.

6.2 Other probability functions

It is possible to choose other probability functions. It has sometimes been proposed in construction the double triangular function, including the possibility that the quantity also decreases. Gaussian bell curves may also be used, symmetrical or asymmetrical, and all kinds of functions. However, the model defined in this text can be performed using a spreadsheet, as has been described, while other cases require specialized programs in quantitative risk analysis, as those mentioned in the reference section.

It should be remembered, that the results accuracy is never greater than the input data, so a more complicated model generally only give rise to a greater complexity of the mathematical expressions, increasing the work of definition and calculation and hiding relationship between data and results, without relevantly improving delivered values.

6.3 Correlation between variables

The possible existence of dependence or correlation between variables was not taken into account in the test case. When quantities vary, as in this model, it can be assumed that there is not a relationship between them, if the underlying causes of the increases or decreases are unsystematic. But it may be otherwise, for deficiencies in drafting the estimate, bad faith or mismanagement during the construction stage.

If desired, the dependence between variables may be simulated using scenarios, defining possible global situations in which a set of variables behaves in a coordinated way. The results of the scenarios, in turn, may be combined by assigning weights.

We must distinguish, however, the existence of correlation between variables, which requires the specific methodology incorporated in specialized programs already mentioned, from the situations in which ranges of variation of many or all of the variables are greater that the generic reference values.

6.4 Other points of view

The same method can be applied from other building agent's point of view.

The contractor can perform the same analysis in contracts with close measurement, where he has to bear the impact of changes. From the estimates measurements, using the bid price and the estimated cost, the contractor can get the expected static margin. However, to understand the impact of changes in the margin a Monte Carlo analysis similar to that described is required. Estimates of costs and sales are calculated with the same unit prices, applying variation of quantities and calculating the margin average for a large number of cases.

The same method can be applied to analyze the behavior of a bid in front of future changes in prices, although this application is sufficiently well-known.

7. Future work

The proposed method requires simple data and assumptions. However, widespread implementation requires obtaining and publishing approximate variation ranges between the original project and the final quantity in actual works, for the work units with greater economic impact.

This data is available both for project management professionals and for the construction companies, since by their very nature are always known to both parties. It would be feasible to compile cases within future collaborations, and could also be interesting as supplement information to that provided in construction price databases, until now very little innovative in this regard.

8. Conclusions

The Monte Carlo method, though little known by construction professionals and Spanish companies, is easy to apply, having adequate tools. However, it must be based on assumptions and data that can be readily determined by those who wish to use it.

The method described, based on a triangular function with a maximum probability of variation, meets these requirements and can be applied directly to the analysis and comparison of actual offers.

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Software
IBM, SPSS.
MathWorks, Monte Carlo Simulation
Oracle, Crystal Ball.
Oracle, Primavera Risk Analysis
Palisade, @RISK Monte Carlo Simulation Software for Excel.
Soft, Presto.