Enhanced thermionic currents by non equilibrium electron populations of metals

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Abstract. An analytical expression is derived for the electron thermionic current from heated metals by using a non equilibrium, modified Kappa energy distribution for electrons. This isotropic distribution characterizes the long high energy tails in the electron energy spectrum for low values of the index κ and also accounts for the Fermi energy for the metal electrons. The limit for large κ recovers the classical equilibrium Fermi-Dirac distribution. The predicted electron thermionic current for low κ increases between four and five orders of magnitude with respect to the predictions of the equilibrium Richardson-Dushmann current. The observed departures from this classical expression, also recovered for large κ, would correspond to moderate values of this index. The strong increments predicted by the thermionic emission currents suggest that, under appropriate conditions, materials with non equilibrium electron populations would become more efficient electron emitters at low temperatures.

1 Introduction

The hot metallic surfaces with a uniform temperature T emit a thermionic electron current density $J_{rd}(T)$ predicted by the classical Richardson-Dushmann (RD) expression. This model considers the Fermi-Dirac (FD) distribution for the energy spectrum of electrons in equilibrium at the metal temperature. Under these ideal conditions $J_{rd}(T)$ essentially relies on both, the metal work function, W_f and its temperature T. This simple approach disregards other physical effects such as the geometry of the emitting surface or the presence of eventual contaminants. However, materials with irregular shapes, complex geometries and/or contaminated surfaces are frequently employed as thermionic electron emitters. Then, the assumption of thermal equilibrium is scarcely found in practice. This results in irregular emitted electron fluxes and the emitted thermionic current varies according to the facial orientation of pure tungsten crystals [1,2]. Additionally, traces of surface contaminants also strongly affect the thermionic electron emission properties [3], as well as the surface degradation during long times of operation [4,5].

The experiments with ultrashort laser pulse irradiation of metals made apparent the different time scales involved in the electron energy relaxation. The excitation by femtosecond laser pulses produces the thermal decoupling between the electrons and the metal lattice. The theoretical models consider a two temperature system where the excited electron population has a higher temperature than the metal lattice [6–8]. Therefore, the electrons within hot metals might have a non equilibrium energy distribution function that deviates from the equilibrium FD statistics, as the high energy tails in the experiments indicate [9–11]. For ion and electron plasmas, the Kappa distribution for electrons corresponds to the solution of the Fokker-Planck equation accounting for collective effects and Coulomb collisional processes [12,13]. The non equilibrium electron energy spectrum of excited electrons interacting with the metal lattice might be approximated by this distribution function. The classical derivation for $J_{rd}(T)$ does not account for the departure from thermal equilibrium of metal electrons.

In this paper we derive an analytical expression for the thermionic electron current density $J_\kappa(T)$ by using a modified Kappa energy distribution. This latter accounts for the Fermi energy of the metal electrons and is the low temperature approximation of a more general isotropic κ distribution as those discussed in references [14,15]. The non equilibrium electron energy spectrum leads to a more involved equation for $J_\kappa(T)$ which recovers the expression for $J_{rd}(T)$ in the limit for large κ. To the best of our knowledge, this analytical expression for $J_\kappa(T)$ is not currently available.

As we shall see, the departures from the FD statistics found in the experiments, as those reported in references [2,9,10], could be explained by values of κ corresponding to moderate deviations from the thermal equilibrium. Besides, the effect of the long electron energy tails for low κ increases the non equilibrium thermionic current $J_\kappa(T)$ by several orders of magnitude with respect to the classical RD expression.

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2 The modified κ distribution and the electron thermionic emission

In the first place we modify the isotropic κ distribution function to characterize the non equilibrium energy spectrum of metal electrons with temperature T_e. This latter corresponds to the average kinetic energy of the electron population, and the usual Kappa distribution is currently defined by $\kappa > 3/2$ as [14,15],

$$g_\kappa(T_e, E) = B_\kappa(T_e) \left(1 + \frac{E}{\kappa E_c}\right)^{-(\kappa+1)},$$

where $E = m_e v^2/2$ is the electron energy, and $B_\kappa(T_e)$ is the normalization constant. Besides, $E_c = m_e v_c^2/2$ is a characteristic kinetic energy related to the electron gas temperature T_e through the average kinetic energy $\langle m_e v^2/2 \rangle$. This gives [14],

$$\kappa E_c = \left(\kappa - \frac{3}{2}\right) k_B T_e,$$

where k_B is the Boltzmann constant. The long high energy tails in the electron energy spectrum develop for low κ while for large values $g_\kappa(T_e, E)$ reduces to the usual Maxwell-Boltzmann velocity distribution.

As in the classical FD distribution function, we introduce the Fermi energy $\epsilon_F = m_e v_1^2/2$ for the electron gas [16]. This transforms $g_\kappa(T_e, E)$ into,

$$g_\kappa(T_e, E) = B_\kappa(T_e) \left(1 + \frac{E - \epsilon_F}{\kappa E_c}\right)^{-(\kappa+1)}.$$

The Fermi level energy modifies the average kinetic energy E_c because the electron energies are now calculated with respect to ϵ_F. The average kinetic energy using the above distribution leads to,

$$\kappa E_c = \epsilon_F + \left(\kappa - \frac{3}{2}\right) k_B T_e.$$

The modified κ distribution for the electron kinetics becomes,

$$f_\kappa(T_e, E) = C_\kappa(T_e) \left(1 + \frac{E - \epsilon_F}{k_B T_\kappa}\right)^{-(\kappa+1)}, \qquad (1)$$

where we introduced for short $T_\kappa = (\kappa - 3/2 + \gamma_1)\, T_e$ with $\gamma_1 = \epsilon_F / k_B T_e$. This energy distribution is normalized to the electron density n_{eo} as,

$$n_{eo} = \frac{\pi\, 2^{5/2}}{m_e^{3/2}} C_\kappa(T_e) \int_0^\infty \left(1 + \frac{E - \epsilon_F}{k_B T_\kappa}\right)^{-(\kappa+1)} \sqrt{E}\, dE.$$

The constant $C_\kappa(T_e)$ in equation (1) becomes,

$$C_\kappa(T_e) = n_{eo} \frac{\Gamma(\kappa+1)}{\Gamma(\kappa - \frac{1}{2})} \times \left(\frac{m_e}{(\kappa - 3/2)\, 2\pi\, k_B T_e}\right)^{3/2} \left(\frac{\kappa - 3/2}{\kappa - 3/2 + \gamma_1}\right)^{\kappa+1}.$$

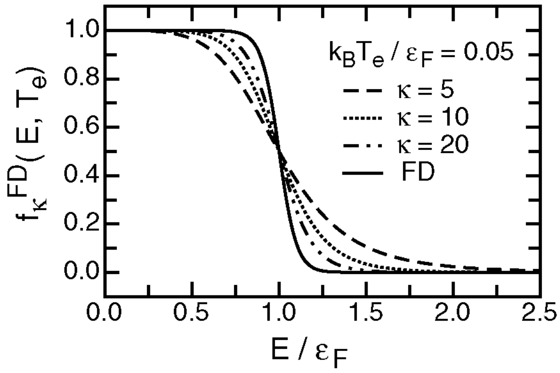

Fig. 1. Comparison of the classical equilibrium Fermi-Dirac distribution (solid line) with the modified κ-FD distribution of equation (2) for different values of κ.

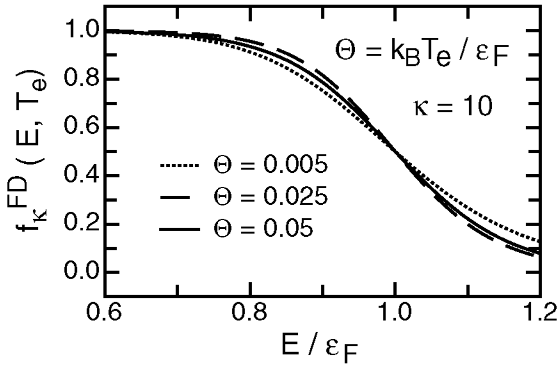

Fig. 2. The effect of the temperature in the κ-FD distribution. Solid line: $\Theta = 0.05$; dashed line: $\Theta = 0.025$; dotted line: $\Theta = 0.005$.

Notice that $C_\kappa(T_e)$ with vanishing γ_1 recovers the normalization for the usual Kappa distribution.

We suggest that the equation (1) derived for the electron kinetics corresponds to the limit of the following generalized non equilibrium κ Fermi-Dirac (κ-FD) distribution for electrons,

$$f_\kappa^{FD}(T_e, E) = \frac{D_\kappa(T_e)}{1 + (1 + (E - \epsilon_F)/k_B T_\kappa)^{(\kappa+1)}}. \qquad (2)$$

Here $D_\kappa(T_e)$ is a normalization constant and this κ-FD distribution reduces to the equilibrium Fermi-Dirac statistics for large κ. Similar expressions for the generalized energy distributions are considered in references [15,17] for non equilibrium systems.

Figure 1 compares the κ-FD distribution of equation (2) for different values of κ with the classical Fermi-Dirac statistics. The energies and temperatures are scaled with ϵ_F and $f_\kappa^{FD}(T_e, \epsilon_F) = 1/2$ for all κ. The κ-FD becomes smoothed for low κ and $f_\kappa(T_e, E)$ of equation (1) is recovered when $(E - \epsilon_F) \gg k_B T_e$ or $(E - \epsilon_F) \ll k_B T_e$. The latter is our case, since the Fermi energies always lie in the order of few electron volts and are therefore much higher that the melting temperature of metals. Figure 2 shows the small effect of the electron temperature T_e for $E \sim \epsilon_F$ for fixed $\kappa = 10$. Setting $\epsilon_F \sim 4.5$ eV as for

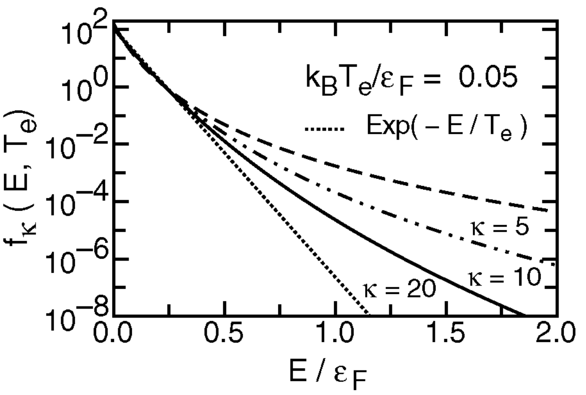

Fig. 3. The high energy tails for $E > \epsilon_F$ for different values of κ of the low temperature approximation for the κ-FD distribution $f_\kappa(E, T_e)$ (Eq. (1)).

tungsten the scaled temperatures $\Theta = T_e/\epsilon_F$ of Figure 2 corresponds to $T_e \sim$ 2600–3900 K. These are the typical metal temperatures T for appreciable thermionic emission where $T_e = T$ is also currently assumed.

Equation (1) is represented in Figure 3 which is recovered for low temperature approximation $f_\kappa^{FD}(T_e, E) \sim f_\kappa(T_e, E)$ when $E - \epsilon_F \gg k_B T_e$. The low values of κ (solid lines) exhibit the long energy tails compared with the fast decay of the Maxwell-Boltzmann distribution (dotted line). This departure from the equilibrium energy spectrum of electrons is responsible for the increments observed in the thermionic current [9].

Finally, following the traditional Richardson-Dushmann model we calculate the emitted electron thermionic current. Equation (1) with $\kappa > 3/2$ might model the energy spectrum of a non equilibrium electron gas within the metal bulk. The thermionic current density $J_\kappa(T_e)$ from a hot emissive metallic surface along the perpendicular direction pointed by $\boldsymbol{u}_x$ is,

$$J_\kappa(T_e) = e \int_{v_{ox}}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} v_x \ \ f_\kappa(T_e, v) \ d\boldsymbol{v}. \qquad (3)$$

Here $v_x > 0$, and the energy $E_o = m_e v_{ox}^2/2$ would represent the *classical* work function, assuming that all electrons are originally at rest [16,18]. Hence,

$$J_\kappa(T_e) = \ e \ C_\kappa(T_e) \int_{v_{ox}}^{+\infty} v_x \, dv_x \int_{-\infty}^{+\infty} dv_y \times \int_{-\infty}^{+\infty} \left(1 \ + \ \frac{m_e\,(v_x^2 + v_y^2 + v_z^2) - 2\,\epsilon_F}{2\,k_B\,T_\kappa}\right)^{-(\kappa+1)} dv_z$$

and integrating along the Z-axis,

$$J_\kappa(T_e) = e\, C_\kappa(T_e) \sqrt{\frac{2\pi k_B\,T_\kappa}{m_e}}\, \frac{\Gamma(\kappa + \frac{1}{2})}{\Gamma(\kappa+1)} \int_{v_{ox}}^{+\infty} v_x\, dv_x \times \int_{-\infty}^{+\infty} dv_y \left(1 \ + \ \frac{m_e(v_x^2 + v_y^2) - 2\,\epsilon_F}{2\,k_B\,T_\kappa}\right)^{-(\kappa+1/2)}.$$

Finally we obtain,

$$J_\kappa(T_e) = \ e \ C_\kappa(T_e)\, \frac{\pi\ k_B^2\,T_e^2}{m_e^2} \times \frac{(2\kappa - 3 + 2\,\gamma_1)^2}{2\kappa\ (\kappa-1)} \left(1 \ + \ \frac{W_f}{k_B\,T_\kappa}\right)^{-\kappa+1}. \qquad (4)$$

Here, the term $W_f = E_o - \epsilon_F$ is the usual work function. In the limit for large κ, since $C_{\kappa\to\infty}(T_e) \to 2m^3/h^3$, $J_\kappa(T_e)$ reduces to the Richardson-Dushmann current density

$$J_{rd}(T_e) = e\ \frac{4\pi\ m_e\ k_B^2\ T_e^2}{h^3}\ \exp\left(-\frac{W_f}{k_B T_e}\right).$$

3 Results and conclusions

The emission of appreciable thermionic electron currents from metals needs temperatures of over 2000 K, as well as additional practical requisites such as low work function surface coatings. Under the usual experimental conditions, most of the electron emitting surfaces are frequently far from thermal equilibrium. In the experiments with pulsed laser heating the excited electron population temperature T_e and the lattice temperatures T differ for times shorter than few picoseconds. For longer times it is currently assumed that the energy transfer from the heated metal to the electrons is efficient enough to equal the electronic and the metal temperatures, as in the Richardson-Dushmann model.

Consequently, the energy spectrum of the electron population that causes the thermionic emission would depart from the equilibrium distribution as the theoretical and experimental results indicate [2,6,7,9–11].

In order to derive an analytical expression for the non equilibrium electron thermionic current from hot metals, we introduced the ad hoc modified Kappa energy distribution function of equation (1). We suggest that $f_\kappa(T_e, E)$ corresponds to the low temperature approximation of the more general κ-FD distribution of equation (2) which is quite similar to the generalized distributions introduced in references [15,17]. The equilibrium Fermi-Dirac distribution is recovered from this κ-FD distribution in the limit for large values of κ.

The approximated distribution $f_\kappa(T_e, E)$ is valid for $\kappa > 3/2$ and develops long high energy tails for small κ shown in Figure 3. The characteristic temperature T_κ that measures the electron average kinetic energy $\langle m_e\, v^2/2\rangle$ also accounts for the Fermi energy ϵ_F of metal electrons.

The comparison between the equilibrium Fermi-Dirac statistics with the κ-FD deformed distribution function is represented in Figure 1. For low values of $\kappa > 3/2$ the strong departure from thermal equilibrium for low values of κ is characterized by smoothed distribution compared with the FD statistics. This effect is similar to raising the electron temperature in the classical Fermi-Dirac distribution but, as shown in Figure 2, the κ-FD statistics is less sensitive to the temperature changes. The moderate

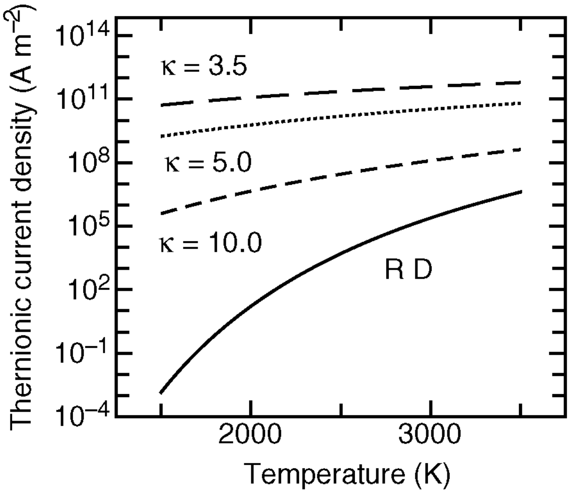

Fig. 4. The thermionic current predicted by equation (4) for tungsten (W_f = 4.5 eV) with different κ compared with the classical Richardson-Dushmann current density (RD).

departures from thermal equilibrium reported in the experiments in reference [9] would be explained by values of $\kappa \geq 20$.

Figure 4 shows the thermionic electron currents $J_\kappa(T_e)$ for tungsten calculated with the expression (4) derived using $f_\kappa(T_e, E)$. The electron high energy tails for low κ introduce dramatic increments in the thermionic electron current predicted by equation (4). This effect is more pronounced for low metal temperatures and reduces as κ increments. Moderate values of $\kappa \sim$ 20–50 only produce small deviations from the RD current that could explain the small departures observed in different experiments.

The calculations of Figure 4 suggest that materials with non equilibrium electron population when heated would be more efficient thermionic emitters. The thermionic currents for $\kappa < 5$ in Figure 4 are quite similar within the temperature range of 2000–3000 K. Therefore those higher currents would be obtained using lower working temperatures for such materials.

The modified Kappa function of equation (1) could be regarded as an effective distribution for the non equilibrium electron energy spectrum. Equation (3) provides an analytical expression for the electron emitted thermionic current that would be directly compared with the experimental results and theoretical calculations. The total thermionic current $I_\kappa(T_e)$ might be later obtained by further integration, considering the geometry of the emitting metal surfaces. This analytical approach could be used to explore testable effects in the emitted electron current to investigate the quantum kappa distribution.

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