CURRENT- VOLTAGE RESPONSE OF A SPHERICAL PLASMA CONTACTOR

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A theoretical model for a contactor, collecting electrons from an ambient, unmagnetized plasma and emitting a current I_i is discussed. The relation between I_i and the potential bias of the contactor is found to be crucial for the formation of a quasineutral core around the anode and, consequently, for the current colleted. Approximate analytical laws and charts for the current-voltage response are provided.

1. Introduction

The use of hollow cathodes as plasma contactors appears as a promising solution for current collection in electrodynamic tethers but both technology and theory still present gross uncertainties; on the one hand, several difficult phenomena are simultaneously present and, on the other hand, no real tests have yet been performed. Meanwhile, theoretical analysis of simplified models can be illuminating. Recently, we completed work on the steady-state response of a spherical anode immersed in an unmagnetized, quiescent plasma [1]. Compared with models found in the literature, we offer (i) an accurate kinetic treatment of the ambient plasma, taken from passive probe theories [2,3], (ii) a consistent use of asymptotic tools to find the electric potential profile in the different regions, (iii) the computation of the dimensionless C-V response, and (iv) the identification of, at least, two modes of operation. Section 2 briefly summarizes all the model hypotheses and equations, and the way we solved them. Section 3 discusses the contactor response in terms of dimensionless parameters. Some final remarks are made in Section 4.

2. The plasma-contactor model

Figure 1 sketches the two kinds of potential response we find for the plasma contactor. The contactor is a sphere of radius R, biased to a positive potential V_p

relative to the undisturbed ambient plasma. It emits ions (which are accelerated outwards) and electrons (which remain confined around the anode), the flow of this emitted plasma being characterized by the ion curent I_i. The undisturbed density and temperature of the ambient plasma are N_∞ and T_∞, respectively. The current-collection problem consists basically in determining the ambient-electron current I_e to the anode as a function of V_p, I_i, R and the thermodynamic state of the species; alternatively, we assume I_e given and determine R. The solution we worked out corresponds to a steady state without magnetic effects; we also assumed that the thermal energy of all species is much less than eV_p, and that collisional and ionization processes are negligible. The problem then reduces to solving Poisson's equation,

$$\frac{\epsilon_0}{r^2}\frac{d}{dr}\left(r^2\frac{dV}{dr}\right) = e(N_e - N_{ia} + N_{ec} - N_i), \tag{1}$$

where r is the radial distance to the center of the anode, N_e (N_{ia}) is the density of ambient electrons (ions), and N_{ec}, N_i are the corresponding densities of emitted species; ions are positive and singly-charged.

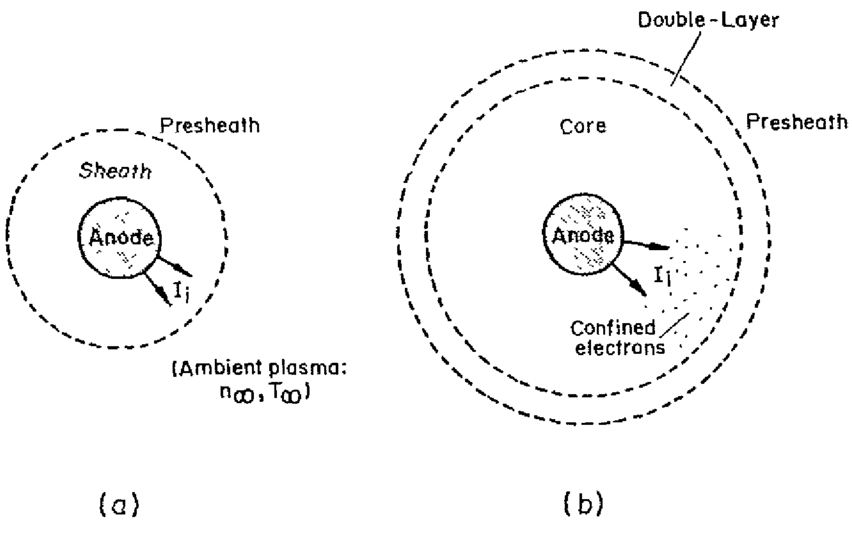

Figure 1.- Potential profile regions for the (a) no-core and (b) core modes. The plasma is quasineutral at presheath and core.

Kinetic theory was used to study the ambient plasma. For $eV_p \gg T_\infty$, the density of the repelled species (ions) is accurately given by the Boltzmann equilibrium law, [3].

$$N_{ia} \simeq N_\infty \exp\left(-\frac{eV}{T_\infty}\right). \tag{2}$$

Far from the contactor the attracted species (electrons) is assumed to be monoenergetic with a uniform distribution of angular momentum J; a relation between the undisturbed energy E_∞ and the electron temperature T_∞ must be provided. A comparison of this model with the exact maxwellian formulation, for a passive probe [4], has shown very good agreement when R is much larger than the ambient Debye lenght λ_D, the case of interest here; note, furthermore, that a maxwellian population would require numerical solution while an asymptotic analysis is possible for the monoenergetic model. Calling J_B the particular angular momentum, such that electrons with $J < J_B$ are absorbed by the anode while those with $J \geq J_B$ are turned back without reaching it, one has [2]

$$N_e = (N_\infty/2)\left(\sqrt{1+eV/E_\infty} \pm \sqrt{1+eV/E_\infty - b_B^2/r^2}\right), \tag{3}$$

with $b_B^2 \equiv J_B^2/2m_eE_\infty$ given by

$$I_e = \pi b_B^2 e N_\infty \sqrt{2E_\infty/m_e}. \tag{4}$$

For the emitted species we made an analysis similar to the ambient species. Although the exact distribution of confined electrons is difficult to compute, a plausible assumption, supported by experimental data, is that at least for a steady state, it also corresponds to a Boltzmann equilibrium

$$N_{ec} = N_0 \exp\left[e(V-V_p)/T_0\right], \tag{5}$$

where the constant N_0 is later determined by imposing that the total plasma is quasineutral around the contactor; T_0 is the (known) electron temperature, small compared with eV_p. For the emitted ions and assuming $E_i \ll eV_p$, where E_i is their kinetic energy when leaving the contactor, we may neglect the angular momentum distribution (as in the ABR model for collection of zero-temperature ions by a sphere [5]), because transverse velocities decrease like $1/r$ as the ions move outwards; the expansion is then radial and we have

$$N_i = \frac{I_i}{4\pi r^2 e}\sqrt{\frac{m_i/2}{e(V_p-V)+E_i}}. \tag{6}$$

Equations (1)-(6) constitute a complete set. To integrate it the easiest way is to start from $r = +\infty$ towards $r = R$, considering R, instead of I_e, as the unknown parameter. Far away from the anode there is a region, the presheath, where quasineutrality holds and the ambient species are dominant. Eq. (1) then becomes $N_e \simeq N_{ia}$, yielding

$$2r/b_B = \exp(eV/T_\infty)[(1 + eV/E_\infty)^{1/2} \exp(eV/T_\infty) - 1]^{-1/2} \tag{7}$$

for the potential profile $V(r)$. This solution asymptotically matches with a non-neutral sheath around the point, $r = r_1$, where $dV/dr \to -\infty$ [3]. Note that r_1 depends on the ratio E_∞/T_∞; a choice $E_\infty/T_\infty = 3/2$ will be later justified. We then obtain $eV(r_1)/T_\infty \simeq 0.75$ and $r_1/b_B \simeq 0.84$; eliminating now b_B in (4) yields a relation between I_e and r_1,

$$I_e \simeq 1.54 \times 4\pi r_1^2 \times eN_\infty \sqrt{T_\infty/2\pi m_e}. \tag{8}$$

For the sheath equations the following dimensionless variables and parameter are defined:

$$F \equiv (32e\epsilon_0^2\pi^2/m_e)^{1/3} V/I_e^{2/3}, \quad \tau \equiv r_1/r, \quad \mu \equiv I_i m_i^{1/2}/I_e m_e^{1/2}. \tag{9}$$

From a consistent use of asymptotic arguments, Eq.(1) may be simplified to

$$\tau^2 \frac{d^2F}{d\tau^2} \simeq \frac{1}{\sqrt{F}} - \frac{\mu}{\sqrt{F_p - F}} \tag{10}$$

with boundary conditions

$$\tau = 1^+ : \qquad F = 0, \qquad dF/d\tau = 0. \tag{11}$$

These equations express that inside the sheath (i) only the densities N_i and N_e of the accelerated species are important, (ii) the electric field is much higher than outside it, and (iii) the presheath potential and the thermal energies of the particles may be neglected. Integrating (10) from the sheath outer boundary with conditions (11) shows that $dF/d\tau$ starts increasing, reaches a maximum when $F = F_p/(1 + \mu^2)$ and decreases afterwards. Two kinds of solutions are found depending on the values of μ and F_P:

– **Core mode.**- For $\mu > \mu_s(F_p)$, where $\mu_s(F_p)$ is later determined, $dF/d\tau$ becomes zero at certain $\tau = \tau_2$ and $F = F_2$, with F_2 smaller than $F_p \equiv F(V_p)$; in dimensional form this gives

$$r_2 = r_1/\tau_2(F_p, \mu), \qquad V_2 = V_p F_2(F_p, \mu)/F_p. \tag{12}$$

Asymptotic analysis again shows that the confined electron density N_{ec} becomes of dominant order in Eq.(1) when $dF/d\tau$ tends to zero; around $\tau = \tau_2$ there is then a transition from Eq.(10) to the quasineutral equation

$$N_{ec} - N_i + N_e \simeq 0. \tag{13}$$

As the details of this transition are relevant only locally, we simply take $r = r_2$ as the inner boundary of the non-neutral region, which has now all the characteristics of a double layer. In the region $r < r_2$, a quasineutral core is formed; the potential $V(r)$ is obtained from (13), and setting $V = V_2$ and $V = V_p$, the constant N_0 in (5) and the probe radius R,

$$\frac{r_2^2}{R^2} = \exp\left(e\frac{V_p - V_2}{T_0}\right) \times \left[1 + e\frac{V_p - V_2}{E_i}\right]^{-1/2} \frac{\mu - \sqrt{(eV_p + E_i)/eV_2 - 1}}{\mu - \sqrt{E_i/eV_p}} \tag{14}$$

are finally obtained; when $V_p - V_2 \ll V_p$ we have $N_e \ll N_{ec}$ in (13) and the last factor in (14) is approximately one. We must note that Eq.(13) gives a univalued solution for $V(r)$ only if the Bohm-like condition $E_i \geq T_0/2$ is satisfied, i.e. ions cannot leave the anode subsonically; the double layer inner boundary $r = r_2$ is then, contrary to point 1, a regular supersonic point. From solutions (12) and (14) it is now clear that *the core mode disappears when $V_2 \to V_p$, that is when $dF/d\tau$ becomes zero just at* $F = F_p$; the limiting relation $\mu = \mu_s(F_p)$ comes out from $F_p = F_2(\mu, F_p)$.

– **No-core mode**.- When $\mu \leq \mu_s(F_p)$, Eqs.(10) and (11) give $dF/d\tau > 0$ at $F = F_p$. Thereafter, the non-neutral region is no longer a double layer and no quasineutral core is formed. The emitted electron population is now confined to a thin boundary layer around the contactor where it locally modifies the potential profile. Its effect on the global current-voltage response is however insignificant, the contactor operating now exactly as a pure ion-emitter. Calling $\tau = \tau_p$ the point where $F = F_p$ in the integration of (10) and (11), the probe radius R is

$$R = r_1/\tau_p(\mu, F_p). \tag{15}$$

Notice that the limiting case $\mu = \mu_s(F_p)$ corresponds to an ion-emitter operating at space-charge-limited conditions [6].

3. Current-voltage diagrams

Equations (8), (9) and (15) yield the dimensionless C-V characteristic for the no-core mode (or for an ion emitter) in the form $j_e(j_i, \tilde{\chi}_p)$ where

$$j_{e,i} = (I_{e,i}/\pi R^2 e N_\infty)(m_{e,i}/2E_\infty)^{1/2}, \qquad \tilde{\chi}_p = (eV_p/E_\infty)(\lambda_D/R)^{4/3},$$

or, measuring R in meters, V_p in volts, $I_{e,\imath}$ in amperes, N_∞ in $10^{11}\,\mathrm{m}^{-3}$ and T_∞ in $0.1\,\mathrm{eV}$,

$$
\begin{aligned}
j_{e,\imath} &\simeq 86.6 \times (m_{e,\imath}/m_e)^{1/2} I_{e,\imath}/R^2 N_\infty T_\infty^{1/2}, \\
\tilde{\chi}_p &\simeq 3.2 \times 10^{-2} \times V_p N_\infty^{2/3} T_\infty^{1/3}/R^{4/3}.
\end{aligned} \tag{16}
$$

In the following we shall use a convenient approximation of $j_e(j_\imath, \tilde{\chi}_p)$,

$$
j_e \simeq 2.1\tilde{\chi}_p^{6/7} + 0.25 j_\imath (j_e/\tilde{\chi}_p)^{1/2}, \tag{17}
$$

valid for $\tilde{\chi}_p \gg 1$, corresponding to the thick sheath limit, $r_1/R \gg 1$ [1]. The transition to the core mode is obtained by adding $\mu = \mu_s(F_p)$ to Eqs.(8), (9) and (15), showing that, for given $\tilde{\chi}_p$, there is an emission current value, $j_\imath = j_{\imath s}(\tilde{\chi}_p)$, such that if $j_\imath \leq j_{\imath s}$ the contactor does not develop a core; the following aproximate expressions,

$$
j_{\imath s} \simeq 2.8 j_{es} (\ln j_{es})^{1/2}, \qquad \tilde{\chi}_p \simeq 0.48 j_{es}^{2/3} \ln j_{es}, \tag{18}
$$

of $j_{\imath s}(\tilde{\chi}_p)$ and $j_{es}(\tilde{\chi}_p) \equiv j_e(j_{\imath s}, \tilde{\chi}_p)$ are valid for $\tilde{\chi} \gg 1$. For zero-emission, the sheath is negatively biased by the attracted electrons. With $j_\imath$ increasing the total charge decreases, vanishing just as the sheath becomes a double layer; the increase of j_e over this entire process is only moderate. Since we have $j_\imath \propto \tilde{\chi}_p^{3/2}$, roughly, the higher $\tilde{\chi}_p$, the larger the ion current required to electrically balance the sheath.

In the core mode we have $j_\imath > j_{\imath s}(\tilde{\chi}_p)$ and Eqs.(8), (9), (12) and (14) give the dimensionless C-V response in the form $j_e(j_\imath, \tilde{\chi}_p, E_\infty R^{4/3}/T_0 \lambda_D^{4/3}, E_\imath/T_0)$. Figure 2 shows j_e and $j_\imath$ versus $\tilde{\chi}_p$ for i) the zero-emission limit, ii) the transition to the core mode, and iii) several core-mode cases, each one corresponding to a fixed core potential drop ratio, $(V_p - V_2)/V_p$. Points B to F show different values of $j_\imath$ and $\tilde{\chi}_p$ that lead to the same collected current: $j_e = 2 \times 10^3$. Two important conclusions turn out: i) the minimum value of $j_\imath (\sim 3.9 \times 10^3)$ corresponds to *relatively low values* of $\tilde{\chi}_p$; and ii) in the core-mode, solutions are weakly dependent on $\tilde{\chi}_p$. Here again j_e and $j_\imath$ must balance the electric charge at the double-layer: this roughly yields $j_\imath \sim j_e (\ln j_e)^{1/2}$ for a thick layer, and $j_\imath \sim j_e$ for a thin layer. In points B to F the core *plus* double layer thickness is constant: $r_1/R \simeq 0.84 j_e^{1/2} \simeq 37.6$, according to Eq.(8), but the thickness ratio at the double layer decreases with $\tilde{\chi}_p$: we have $r_1/r_2 = r_1/R$ at B and $r_1/r_2 \simeq 1.6$ at E; for $\tilde{\chi}_p$ smaller (point F), $j_\imath$ and r_1/r_2 increase due to a decrease of the last factor in (14). Moreover, for j_e given, orbital-motion-limit (OML) effects set a lower-bound to $\tilde{\chi}_p$: $1 + \tilde{\chi}_p (R/\lambda_D)^{4/3} \geq j_e$; for instance, to collect

$j_e = 2 \times 10^3$ with $R/\lambda_D = 25$ requires that $\tilde{\chi}_p > 27.3$. Plasma contactors and ion emitters operating in an OML regime are discussed in [1].

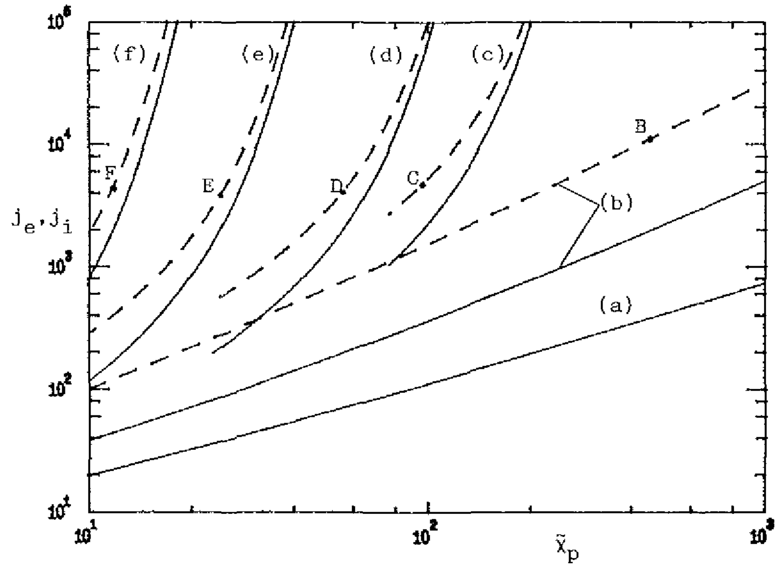

Figure 2.- Dimensionless C-V curves: solid (dashed) lines represent the collected (emitted) current versus the potential at the contactor for several no-core and core cases. The (a)-line corresponds to the zero-emission limit, $j_i = 0$, and the (b)-lines to the transition from the no-core to the core mode, $j_i = j_{is}(\tilde{\chi}_p)$ and $V_2/V_p = 1$. Each pair of lines representing a core-mode case has a constant value of the potential drop at the core: $1 - V_2/V_p$ is equal to .03 in (c), .06 in (d), .16 in (e), and .36 in (f). Points B to F represent five solutions of the C-V equation $2 \times 10^3 = j_e(j_i, \tilde{\chi}_p, 2, 0.5)$. In all curves we took $E_\infty R^{4/3}/T_0\lambda_D^{4/3} = 2$ and $E_i/T_0 = 0.5$. Dimensionless variables $\tilde{\chi}_p, j_e$, and j_i are defined in Eq.(16).

4. Final remarks

Regarding the kinetic treatment of the ambient electrons, a remarkable point is that Eqs.(3)-(4) are equivalent to the following fluid-like equations:

$$\dot{r}_e = -\frac{I_e}{4\pi r^2 N_e}, \tag{19}$$

$$\frac{m_e}{2}\left(\dot{r}_e^2 + \frac{J_B^2}{2m_e^2 r^2}\right) + E_\infty \left(\frac{N_e}{N_\infty}\right)^2 - eV = E_\infty. \tag{20}$$

Therefore, under the monoenergetic model, the electrons behave as a fluid with radial and centripetal motion; $\dot{r}_e$ being the radial velocity and $J_B/\sqrt{2}$ the angular momentum; note that for Eq.(20) exactly represent the momentum equation of a polytropic fluid with specific heat ratio equal to 3 (as in the case of electron-plasma waves in a collisionless plasma) we must set $E_\infty = 3T_\infty/2$. This fluid equivalence is relevant in two different directions. Firstly, compared with the kinetic equations, it might lead to a much simpler (still consistent) analysis of the stability of the steady-state solution. And secondly, it shows that ad hoc radial fluid models do not recover an essential feature of the particle motion: the reflection of particles with high angular momentum. This has a double consequence: i) radial models overestimate the current collected (for instance, the widely used, isothermal, radial model, gives in Eq.(8) a factor of 2.5 instead of 1.54, i.e. about a 60% overestimate on I_e); and ii) radial models cannot properly recover the OML regime [1].

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