# SPHERICAL COLLECTORS VERSUS BARE TETHERS FOR DRAG, THRUST, AND POWER GENERATION

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**Abstract**

Performances of ED-tethers using either spherical collectors or bare tethers for drag, thrust, or power generation, are compared. The standard Parker-Murphy model of current to a full sphere, with neither space-charge nor plasma-motion effects considered, but modified to best fit TSS1R results, is used (the Lam, Al'pert/Gurevich space-charge limited model will be used elsewhere). In the analysis, the spherical collector is assumed to collect current well beyond its random-current value (thick-sheath). Both average current in the bare-tether and current to the sphere are normalized with the short-circuit current in the absence of applied power, allowing a comparison of performances for all three applications in terms of characteristic dimensionless numbers. The sphere is always substantially outperformed by the bare-tether if ohmic effects are weak, though its performance improves as such effects increase.

## I. Introduction

A comparison of the performances of an electrodynamic (ED) tether using the tether itself, left bare, for electron (anodic) collection, and an ED tether using a large sphere as anodic device, as used in the TSS1 and TSS1R tether missions, involves a proper evaluation of the current-collection capabilities. This requires carrying the respective current laws to some common formulation, which is discussed in the next section. Independently, gauging performance is different for drag (or deorbit), power-generation, and thrust applications, and is carried out separately in Secs. III, IV, and V, respectively.

In this work we shall assume that current to the sphere follows a modified Parker-Murphy law, as claimed describing TSS1R results [1] (the well known space-charge-limited current law, also claimed as fitting those results, [2] will be considered elsewhere). That modified PM law is

$$I_{sph} = J_{th} \times 4\pi R_{sph}^2 \times a\, f_{PM}(\psi), \qquad \psi = \frac{e\Phi_{sph}}{kT}\left(\frac{l_e}{R_{sph}}\right)^2, \tag{1a, b}$$

where $R_{sph}$ and $\Phi_{sph}$ are the sphere radius and bias, and $T$ is plasma temperature. Also, $a \approx 2.5$ is a fitting factor coming from TSS1R results, and $J_{th}$, $f_{PM}$ and $l_e$ are the thermal or random current density, the Parker-Murphy law function, and the electron thermal gyroradius, respectively,

$$J_{th} = eN_\infty \sqrt{\frac{kT}{2\pi m_e}}, \qquad f_{PM} = \frac{1+\sqrt{8\psi}}{2}, \qquad l_e = \frac{\sqrt{m_e kT}}{eB}, \tag{2a-c}$$

with $N_\infty$ being plasma density, and $B$ a magnetic field. We shall assume that the sphere is an efficient collector in the sense that it is not so large that it just collects the thermal current. This requires $\psi$ not to be small in (2b), the factor ½ describing magnetic guiding of electrons.

We shall also assume that the bare-tether current follows the OML (orbital-motion-limited) collection law for a cylinder, whether a tape or a round wire. For an uniform bias $\Phi_{cyl}$, this law is

$$I_{cyl}(OML) \approx J_{th} \times pL \times \sqrt{\frac{4}{\pi}\frac{e\Phi_{cyl}}{kT}} \equiv eN_\infty \frac{pL}{\pi} \times \sqrt{\frac{2e\Phi_{cyl}}{m_e}}, \tag{3}$$

where $p$ and $L$ are cross-section perimeter and length of the cylinder. In the case of a bare tether, the current $\bar{I}_{bt}$ involves some average of bias over the collecting length, which may be different for drag, thrust, and power generation applications.

## II. Normalized Currents

The double appearance of the radius $R_{sph}$ in Eq.(1a) makes comparing performances difficult. It thus proves convenient to use Eq.(1b) itself to rearrange (1a) so that $R_{sph}$ only appears once (through its $\psi$-dependence),

$$I_{sph} = J_{th} \times 4\pi l_e^2 \times a \frac{f_{PM}(\psi)}{\psi} \times \frac{e\Phi_{sph}}{kT} . \tag{4}$$

We note that the ratio $f_{PM}(\psi)/\psi$ in Eq. (4) decreases with $\psi$, keeping about unity over a wide $\psi > 1$ interval; it ranges from 1.91 at $\psi = 1$ to 0.38 at $\psi = 16$. TSS1R results reach as high as $\psi \sim 10$. [1, 2]

It is also convenient to normalize currents with the short-circuit value, $\sigma_c E_m A$, which is a bound to current in case of drag or power-generation applications; here $\sigma_c$, $A$ and $E_m$ are tether conductivity and cross-section area, and motional field, respectively. As we shall see, ohmic limitations are fundamental in the comparison between bare tethers and TSS1R-like spheres. We first introduce dimensionless values

$$i_{sph} \equiv \frac{I_{sph}}{\sigma_c E_m A}, \qquad \frac{\bar{I}_{bt}}{\sigma_c E_m A} \equiv \bar{i}_{bt} , \tag{5a, b}$$

and next introduce a characteristic length $L^*$ that gauges bare-tether collection impedance against the ohmic impedance, [3]

$$\frac{4}{3} e N_\infty \frac{pL^*}{\pi} \sqrt{\frac{2eE_m L^*}{m_e}} \equiv \sigma_c E_m A . \tag{6}$$

For the relevant aluminum case one has

$$L^* = 1.47\,km \times \left( \frac{E_m}{150 V / km} \right)^{1/3} \left( \frac{h}{0.1\,mm} \times \frac{3 \times 10^{11}\, m^{-3}}{N_\infty} \right)^{2/3} , \tag{7}$$

where $h$ is thickness for a thin tape and radius for a round wire.

Results for $\bar{i}_{bt}$ versus $L/L^*$ (and additional dimensionless numbers appropriate to the application considered) have been determined in the past and will be recalled in Secs. III-V. As regards current to a sphere, we directly obtain

$$i_{sph} = i^* \times \frac{\Phi_{sph}}{E_m L^*}, \qquad i^* \equiv \frac{3\pi^{3/2} a}{2} \times \frac{f_{PM}(\psi)}{\psi} \frac{l_e}{p} \sqrt{\frac{eE_m l_e}{kT}} \sqrt{\frac{l_e}{L^*}} . \tag{8a, b}$$

For typical values

$$l_e \approx 2.5 cm, \qquad E_m \approx 150 V / km, \qquad kT = 0.15\, eV ,$$

and, say, $L^* = 1.5$ km in case of a thin tape, we have

$$i^* \approx 0.003 \frac{f(\psi)}{\psi} \times \frac{2 l_e}{p} , \tag{9}$$

$i^*$ thus turning out to be very small, typically about $10^{-2}$ - $10^{-3}$. Clearly, $L^*$ would be always much larger for a round wire.

**III. Deorbit missions**

For the sphere, neglecting the small voltage drop at a hollow cathode that would establish electrical contact at the cathodic end, the total induced voltage would here equal the sphere bias plus the ohmic voltage drop,

$$E_m L = \Phi_{sph} + Z_t I_{sph} \tag{10}$$

where $Z_t = L / \sigma_c A$ is the tether resistance. One then finds $\Phi_{sph} = E_m L (1 - i_{sph})$, Eq. (8a) now becoming

$$i_{sph} = \frac{i^* \times L / L^*}{1 + i^* \times L / L^*} . \tag{11}$$

As regards the bare tether, $\bar{i}_{bt}$ is a function of just the ratio $L/L^*$, which the analysis in Ref. 4 easily shows to be defined as

$$\bar{i}_{bt} = 1 - \frac{L^*}{L} \times \phi_A, \qquad \int_0^{\phi_A} \frac{d\phi}{\sqrt{1 + \phi^{3/2} - \phi_A^{3/2}}} = \frac{L}{L^*}, \qquad for \quad L/L^* < 4, \tag{12a}$$

with $\bar{i}_{bt} \approx 0.3 \times (L/L^*)^{3/2}$ for $L/L^*$ small; and

$$\bar{i}_{bt} = 1 - \frac{L^*}{L}, \qquad for \quad L/L^* > 4. \tag{12b}$$

To gauge performances we consider the ratio between the mass of the system dedicated to producing thrust, and the total impulse of the deorbiting mission, written as the product of drag-force $F$ times the mission duration $\tau$. That ratio, which should be as small as possible, is the inverse of the velocity of exhaust gases ($g_0 \times$ *specific impulse*) in the case of chemical propulsion, where system mass is basically propellant mass. In the case of electrical propulsion one must also allow for the mass of a power supply. An ED-tether needs no power supply for deorbiting, and consumes very little expellant (mass) at the hollow cathode. Dedicated mass is then basically tether mass times some factor $\alpha_t \sim 2.5$ that accounts for tether-related hardware (*deployer* / *end ballast*).

Writing $F$ as *average current* times $LB$, we have

$$\frac{Dedicated\ mass}{Mission\ impulse} \approx \frac{\alpha_t\ \rho AL}{F\tau} = \frac{\alpha_t\ \rho U_{orb}}{\sigma_c E_m^2\ \bar{i}\ \tau}, \tag{13}$$

where $\bar{i}$ is given by Eqs.(11) and (12a, b) for sphere and bare-tether, respectively. Since all other factors in (13) may have values common to both systems, performance is measured by the dimensionless average current, which should be as large as possible.

Both $i_{sph}$ and $\bar{i}_{bt}$ approach unity at large $L/L^*$, with ohmic effects limiting current collection in either case. This occurs for a bare tether at moderate $L/L^*$ values, whereas for the sphere it occurs at much larger values. These are easier to reach with long, thin tapes at daytime. For $L = 20$ km, $N_\infty = 10^{12}$ m$^{-3}$, tape *thickness* $h = 0.1$ mm and *width* = 25 mm, and $\psi \sim 3$ (corresponding to a 4m sphere diameter), we find $i_{sph} \sim 0.12$ (note that $i^* \times L/L^*$ is independent of $E_m$). With $L^{*3/2} \propto N_\infty$, current to the sphere would drop by a factor of 10 at night, effectively switching off the drag $F$. For the sphere we would then have a normalized current

$$\bar{i}\,(sphere) = f_d \times i_d \equiv f_d \times \frac{i_d^* \times L/L_d^*}{1 + i_d^* \times L/L_d^*} \tag{14}$$

where $f_d$ is the *noneclipse* or *daytime* fraction, and $i_d^*$, $L_d^*$ are $i^*$, $L^*$ computed at day density. For a given (deorbiting) mission impulse, the bare-tape system appears then to be 10-20 times lighter than the system using a spherical anode.

## IV. Power generation

For the sphere we would have

$$E_m L = \Phi_{sph} + (Z_t + Z_l)\, I_{sph}\,. \tag{15}$$

As with all generators, the impedance ratio $Z_l / Z_t$ determines the efficiency $\eta_g$ in taking energy (from the orbital motion) into useful energy at an electrical load of impedance $Z_l$ in the tether circuit. Using

$$\eta_g = \frac{Z_l I_{sph}^2}{E_m L I_{sph}} = \frac{Z_l}{Z_t}\, i_{sph} \tag{16}$$

instead of $Z_l / Z_t$ as free parameter in (15) leads to $\Phi_{sph} = E_m L\ (1 - i_{sph} - \eta_g)$, Eq. (8a) now becoming

$$i_{sph} = (1-\eta_g) \times \frac{i^* \times L/L^*}{1 + i^* \times L/L^*}. \tag{17a}$$

As regards the bare tether, $\bar{i}_{bt}$ is here a function of the ratio $L/L^*$ and $Z_L/Z_t$, or alternatively $\eta_g$. From the analysis in Ref. 3 and assuming a large $L/L^*$ ratio one finds

$$\bar{i}_{bt} \approx 1 - \eta_g - \frac{L^*}{L} \times F(\eta_g), \tag{17b}$$

$$(1-\eta_g) \times F(\eta_g) \equiv \frac{4}{3}\eta_g^2 \int_0^1 \frac{d\eta}{(\eta^2 - \eta_g^2)^{1/3}} - (2\eta_g - 1)\times(1-\eta_g^2)^{2/3}.$$

At an ED-tether, power is generated at the expense of orbital energy; a tether can thus serve as a primary power source only for very short times. It turns out, however, that for longer than 1 - 2 weeks, with solar power not available, power generation by a combination *ED-tether / rocket*, with the tether providing electrical power and the chemical rocket providing thrust to compensate the magnetic drag on the tether, proves more efficient as regards fuel consumption than the alternative power source, which would be a fuel cell for direct generation. Note that the magnetic power $\dot{m}_{rk} v_{rk} U_{orb}$ (magnetic drag $F$ being assumed equal to rocket thrust $\dot{m}_{rk} v_{rk}$ to keep the orbit stationary) is greater than the rocket output power $\dot{m}_{rk} v_{rk}^2/2$. Here $\dot{m}_{rk}$ and $v_{rk}$ are propellant mass-flow-rate and velocity at the rocket exhaust; for LOX-$LH_2$ (*specific impulse* ~ 460 s), we have $v_{rk}$ ~ 4.5 Km/s < $2U_{orb}$ ~ 15 km/s. The decrease in rocket energy of motion due to the fuel-mass loss makes for the excess over the rocket output.

Performance is gauged by the ratio between the mass of the system dedicated to producing power, and the total energy generated, written as the product of mission duration $\tau$ times the electrical power generated, $W_g = \eta_g \times FU_{orb}$. System mass is basically made of rocket propellant mass and tether related mass,

$$\frac{Dedicated\ mass}{Energy\ generated} \approx \frac{\dot{m}_{rk}\tau + \alpha_t \rho AL}{\eta_g F U_{orb}\tau}. \tag{18}$$

Using $\dot{m}_{rk} v_{rk} = \bar{I}LB$ we find

$$\frac{Dedicated\ mass}{Energy\ generated} = \frac{1}{\eta_g U_{orb} v_{rk}} + \frac{\alpha_t \rho}{\eta_g \sigma_c E_m^2 \bar{i}\,\tau} \propto \frac{1}{\eta_g}\left[1 + \frac{\tilde{\tau}/\tau}{\bar{i}(\eta_g)}\right], \tag{18'}$$

$$\tilde{\tau} \equiv \alpha_t \rho U_{orb} v_{rk} / \sigma_c E_m^2 \sim 0.5\ weeks \tag{19}$$

for aluminum and $E_m$ ~ 150 V/km.

At the large $L/L^*$ values favoring the spherical collector we just set $\bar{i}_{bt} \approx 1 - \eta_g$ in (16b) for the bare teher. For the sphere, with drag again effectively switched off at night, we set $\bar{i}$ (*sphere*) $\approx (1 - \eta_g)\ f_d\, i_d$, with $i_d$ as given by Eq. (14). For a given mission duration $\tau$, the ratio in (18') has a minimum at

$$\frac{2\eta_g - 1}{(1-\eta_g)^2} = \frac{\tau}{\tilde{\tau}} \quad \text{for the bare tether,} \tag{20a}$$

$$= \frac{f_d\, i_d \times \tau}{\tilde{\tau}} \quad \text{for the sphere.} \tag{20b}$$

One finds $\eta_g \approx 0.5$ at short times; $\eta_g \approx 0.59$ at $\tau = \tilde{\tau}$ (or $\tilde{\tau}/f_d\, i_d$); $\eta_g \approx 1 - \sqrt{\tilde{\tau}/\tau}$ at long times. For the case considered in Sec.III, having $1/f_d\, i_d$ about 15, the system with the spherical collector is heavier than the bare-tether system by a factor of 10 for mission duration $\tau \approx \tilde{\tau}$, and a factor of 4.5 for $\tau \approx \tilde{\tau}/f_d\, i_d$, or about two months. Both systems would have similar masses for durations much longer than two months, where system mass is basically propellant mass for the rocket.

## V. Thrusting missions

For the sphere we would now have

$$W_s = \mathcal{E}_s \times I_{sph} = (E_m L + \Phi_{sph} + I_{sph} Z_t) \times I_{sph}, \tag{21a}$$

where $W_s$ and $\mathcal{E}_s$ are the supply power and voltage, with $I_{sph}$ and $\Phi_{sph}$ related by Eq. (8a).

In thrusting, a bare tether is more efficient if some upper segment is insulated. [3] The analysis in Ref.5 shows that at low $L^*/L$, the lower segment left bare should be a small fraction of the full tether legnth. [5] Then the required power supply is just

$$W_s = \mathcal{E}_s \times \bar{I}_{bt} = (E_m L + \bar{I}_{bt} Z_t) \times \bar{I}_{bt}, \tag{21b}$$

corresponding to negligible impedance in bare-tether collection.

To gauge performances we consider the ratio between the mass of the system dedicated to producing thrust, and the total impulse of the deorbiting mission,

$$\frac{Dedicated\ \ mass}{Mission\ \ impulse} = \frac{\beta W_s + \alpha_t \rho A L}{F\,\tau} \tag{22}$$

where $\beta$ is the inverse specific power of the supply. Typically, $\beta$ is a few tens of kg/kW, where a dedicated solar-power is required, as in the case of a 'space-tug'; and less than 10 kg/kW, where a solar-power system can be taken for free, as in the case of reboost of the International Space Station.

For the bare tether one then finds

$$\frac{Dedicated\ \ mass}{Mission\ \ impulse} = \frac{\beta U_{orb}}{\tau}\left(1 + \bar{i}_{bt} + \frac{1}{\tilde{E}_m^{\,2}\,\bar{i}_{bt}}\right). \tag{23a}$$

The expression in the parenthesis is minimum for $W_s$ such that $\bar{i}_{bt} = 1/\tilde{E}_m$, yielding a minimum *mass-to-impulse* ratio

$$\frac{\beta U_{orb}}{\tau}\left(1 + \frac{2}{\tilde{E}_m}\right).$$

Here, $\tilde{E}_m$ is a normalized value of the motional field, $E_m / \sqrt{\alpha_t \rho / \beta \sigma_c}$, which is typically near unity.

For the tether with a sphere, taking $\Phi_{sph}$ from Eq. (8a), one finds

$$\frac{Dedicated\ \ mass}{Mission\ \ impulse} \approx \frac{\beta U_{orb}}{f_d\,\tau}\left[1 + \frac{i(sph)}{i_d} + \frac{1}{\tilde{E}_m^{\,2}\, i(sph)}\right]. \tag{23b}$$

A minimum now occurs at *i(sph)* = $\sqrt{i_d} / \tilde{E}_m$ yielding a *mass-to-impulse* ratio

$$\frac{\beta U_{orb}}{f_d\,\tau}\left(1 + \frac{2}{\tilde{E}_m \sqrt{i_d}}\right).$$

For a given mission impulse in the case considered in Sec.III, the system with the spherical collector would be heavier than the bare-tether system by a factor of about 4.

## VI. Conclusions

We have shown that, in the absence of ohmic effects, bare tethers substantially outperform tethers using a sphere as passive anodic collector. For wires with not unreasonably thin cross sections, ohmic effects are typically small. On the other hand, ohmic effects on (long) thin tapes, may be strong. The greater current-collecting capability of a bare tape makes it more vulnerable to ohmic limitations. As a result, spherical-collector performances do approach performances by bare tether. Systems with tapes 20 km long and as thin as 0.1 mm, left bare, are lighter than insulated-tape systems with collecting spheres, by factors of about 10-15 in deorbiting; about 6-8 in power generation; and about 4 in thrusting.

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