

Summary. We develop a remote sensing imaging system for the stereoscopic measurement of ocean waves via variational methods. This technique is non-invasive and enables a better understanding of the space-time dynamics of ocean waves over an area rather than at selected point locations of traditional monitoring methods (buoys, etc.).

The wave surface is obtained as the minimizer of an energy functional that combines image observations and space-time smoothness priors. Two energy functionals are considered (“elevation” and “disparity”), yielding similar results. Multigrid methods are used to numerically solve the partial differential equations (PDEs) derived from the optimality conditions of the functional. Experiments were carried out to measure and analyze ocean waves from real data collected at two offshore platforms in the Black Sea (Crimean Peninsula) and the Northern Adriatic Sea (Venice coast, Italy).

1. Variational Stereo Methods

1.1 Ocean surface represented as an **elevation map** $S(u, v) = (u, v, Z(u, v))$

Energy functional to be minimized:

$$E(S, f) = E_{\text{data}}(S, f) + \alpha E_{\text{geom}}(S) + \beta E_{\text{rad}}(f), \quad \alpha, \beta > 0.$$

- **Data fidelity:** measure **photo-consistency** of the video for a candidate surface.

$$E_{\text{data}} = \sum_{i=1}^{N_c} E_i \quad \text{with} \quad E_i(Z, f) = \int_T \int_{\Omega_i} \frac{1}{2} (I_i(\mathbf{x}_i) - f(\mathbf{x}_i))^2 d\mathbf{x}_i dt,$$

- **Regularizers:** enforce **spatial** and **temporal** smoothness of the solution.

$$\left\{ \begin{aligned} E_{\text{geom}}(Z) &= \int_T \int_U \frac{1}{2} \|\nabla Z\|^2 d\mathbf{u} dt, \\ E_{\text{rad}}(f) &= \int_T \int_U \frac{1}{2} \|\nabla f\|^2 d\mathbf{u} dt, \end{aligned} \right.$$

Minimization approach:

- Obtain Euler-Lagrange eqs (Necessary optimality) → set gradient descent PDEs
- Discretize and solve PDEs using 3-D **multigrid** methods.

Non-linear term (data-fidelity):

$$g(Z, f) = \nabla f \cdot \sum_{i=1}^{N_c} |M^i| \tilde{Z}_i^{-3} (I_i - f) (u - C_i^1, v - C_i^2),$$

Focal length
Distance to surface
Optical ray and surface normal

Radiance derivative
Photometric error

Advantages of variational methods:

- ✓ Enforce continuity of the wave surface in space and time (no holes).
- ✓ Improve robustness: less sensitive to image matching problems.
- ✓ Provide dense surface reconstructions.
- ✓ Allow controllability/priors on the unknowns.
- ✓ Can incorporate global properties of wave heights (statistics, etc.).
- ✓ Imply less post-processing than traditional methods.

1.2 Another variational stereo method:

- Ocean surface is represented as a **disparity map** λ (depth from the cameras).
- Energy functional with data fidelity and smoothness terms:

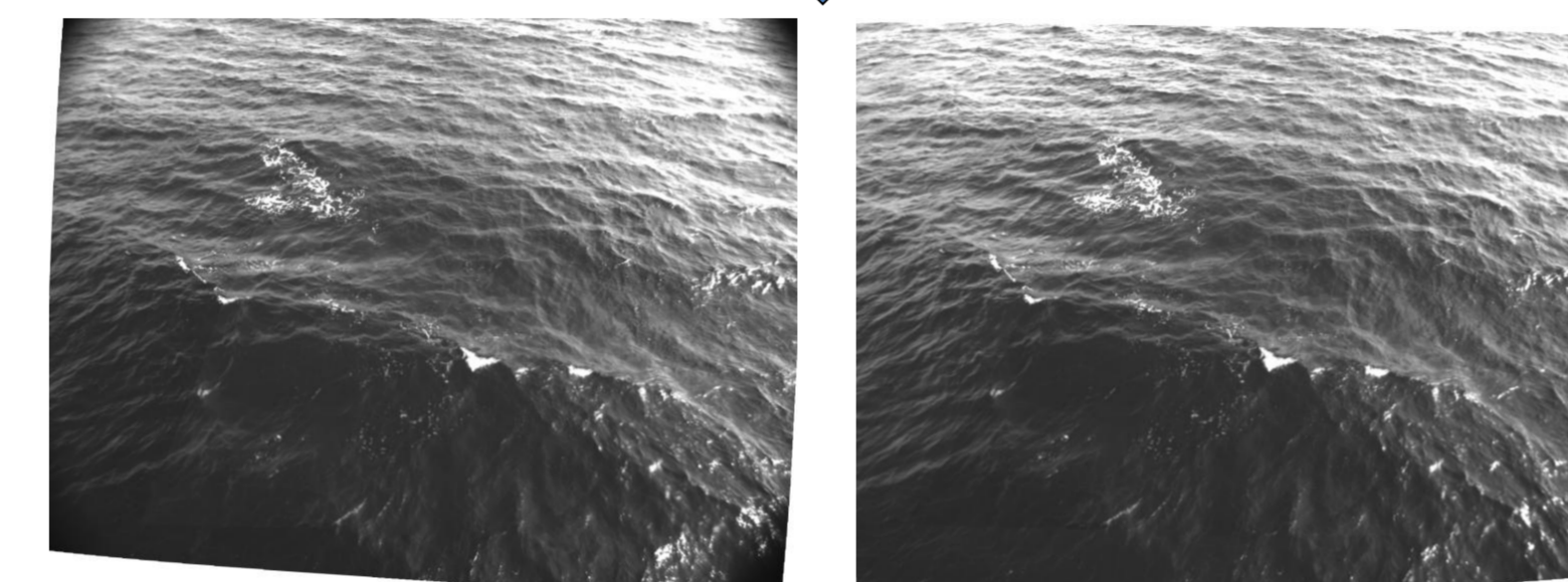
$$E'_{\text{data}}(\lambda) = \int_T \int_{\Omega} \frac{1}{2} (I_1(\mathbf{x}_1) - I_2(\mathbf{x}_2))^2 d\mathbf{x}_1 dt,$$

$$E_{\text{smooth}}(\lambda) = \int_T \int_{\Omega} \frac{1}{2} \|\nabla \lambda\|^2 d\mathbf{x}_1 dt,$$

- Same optimization approach: gradient descent PDE → multigrid methods.
- Then, back-project matched image points and fit a surface through 3D points.

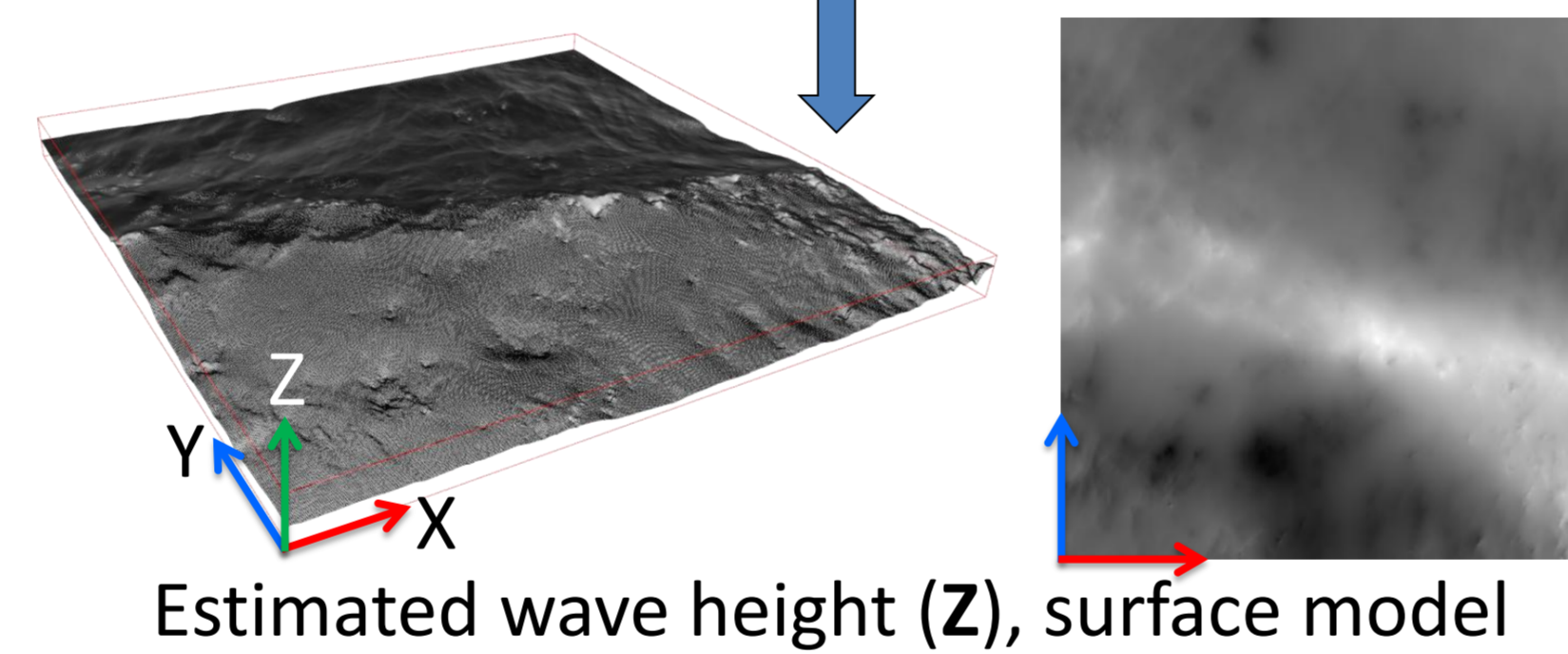


Vision **acquisition** system at offshore platforms: Adriatic and Black Seas



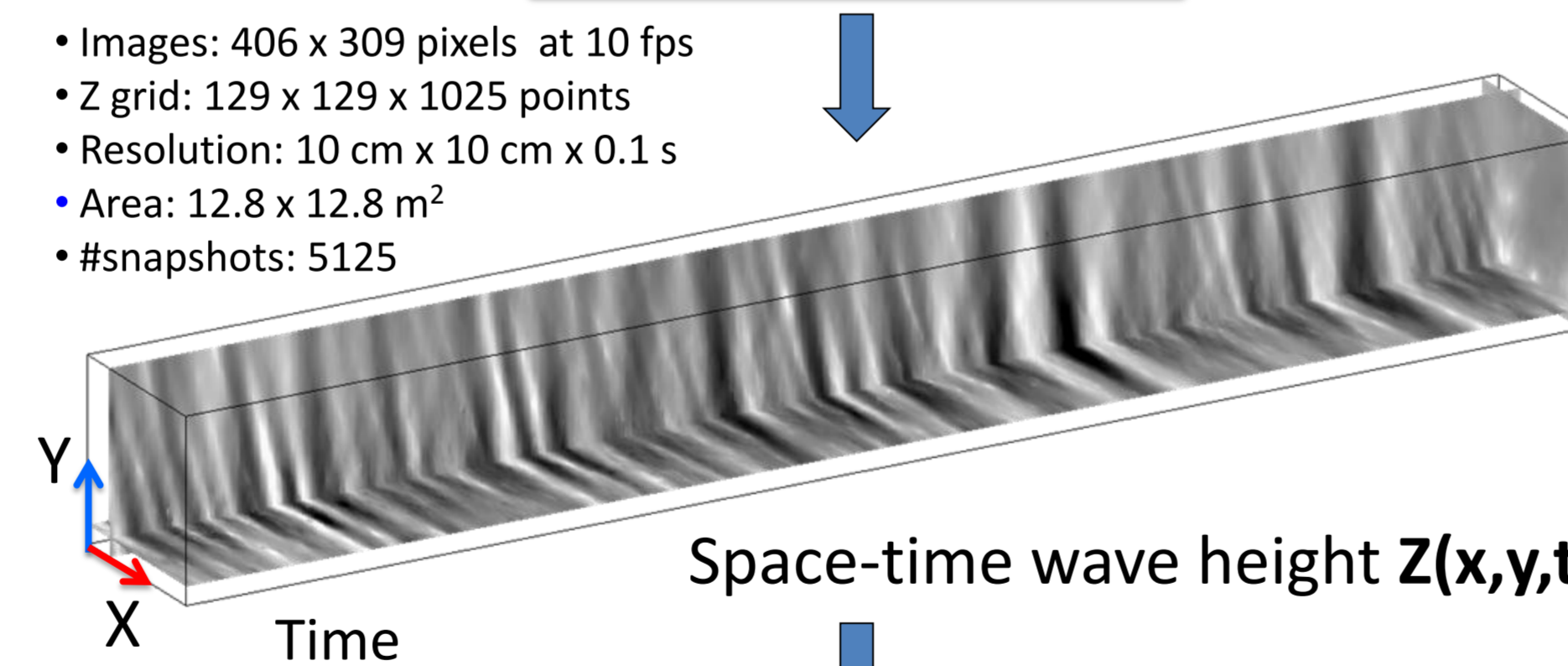
Multi-view videos

Variational stereo, Multigrid methods



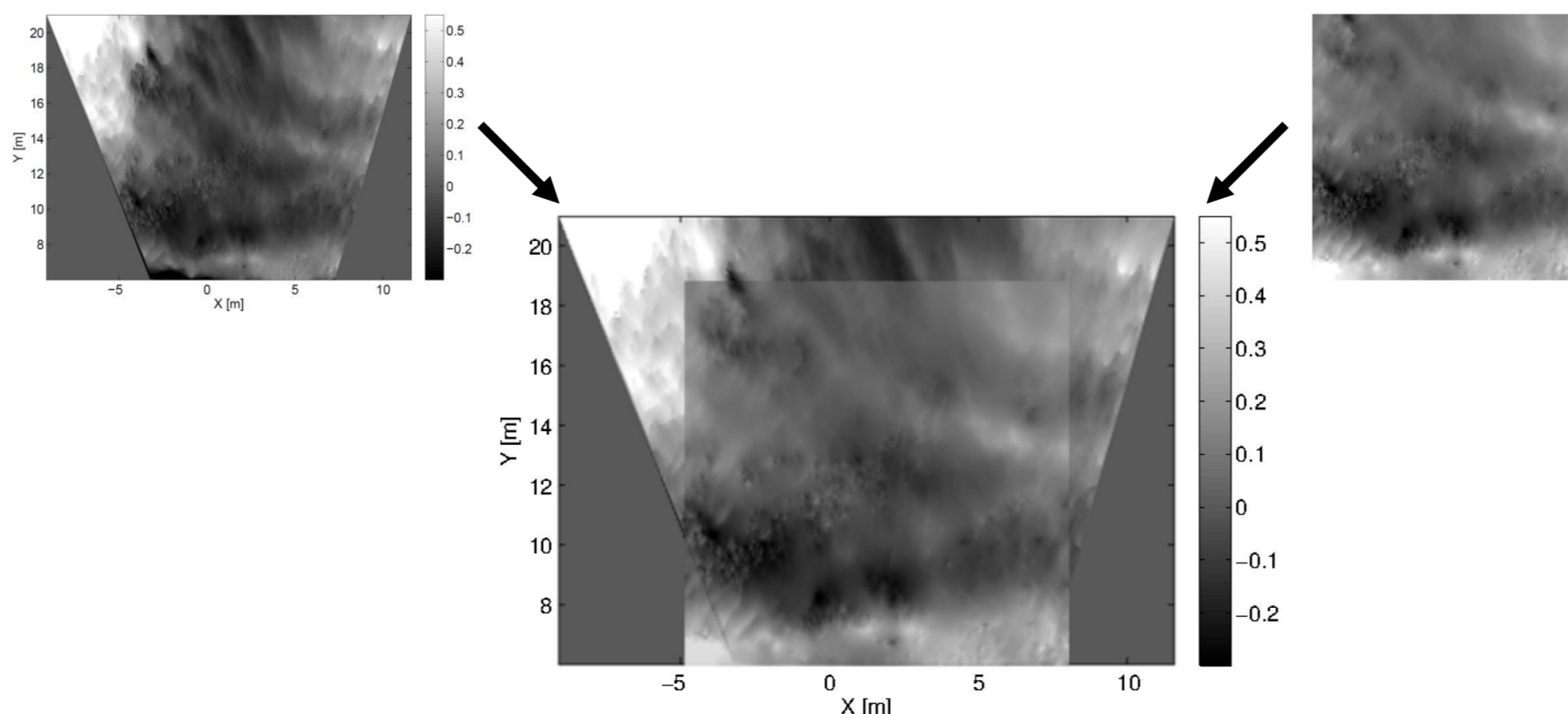
Estimated wave height (Z), surface model

Temporal processing

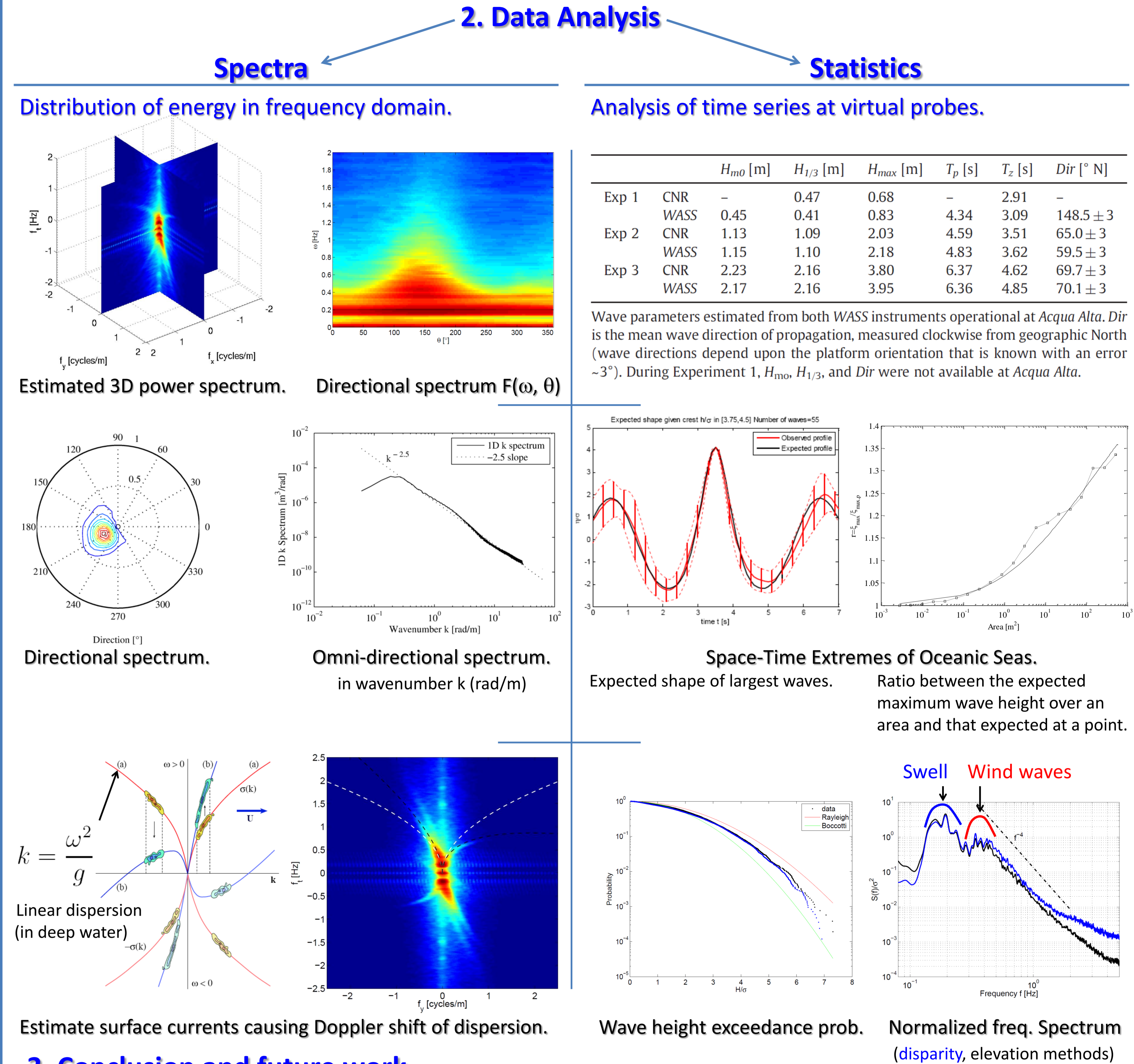


Data Analysis

Disparity method vs. Elevation method



- Images: 406 x 309 pixels at 10 fps
- Z grid: 129 x 129 x 1025 points
- Resolution: 10 cm x 10 cm x 0.1 s
- Area: 12.8 x 12.8 m²
- #snapshots: 5125



3. Conclusion and future work

Stereo wave measurement methods:

- have more advantages than classical wave measurements (area vs. point measurements).
 - provide dense wave height field estimations and allow to enforce continuity in space & time.
 - provide reliable statistics and predictions of ocean waves due to the rich information content of video data.
- Current and future research: improve processing power, scalability and computational efficiency. Improve robustness (camera calibration refinement).

References

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