

# CRITERIA FOR MATHEMATICAL MODEL SELECTION FOR SATELLITE VIBRO-ACOUSTIC ANALYSIS DEPENDING ON FREQUENCY RANGE

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## ABSTRACT

Satellites and space equipment are exposed to diffuse acoustic fields during the launch process. The use of adequate techniques to model the response to the acoustic loads is a fundamental task during the design and verification phases. Considering the modal density of each element is necessary to identify the correct methodology. In this report selection criteria are presented in order to choose the correct modelling technique depending on the frequency ranges. A model satellite's response to acoustic loads is presented, determining the modal densities of each component in different frequency ranges. The paper proposes to select the mathematical method in each modal density range and the differences in the response estimation due to the different used techniques. In addition, the methodologies to analyse the intermediate range of the system are discussed. The results are compared with experimental testing data obtained in an experimental modal test.

## 1. INTRODUCTION

The structural design of satellites and spacecraft is highly affected by acoustic loads, which in addition to shock loads are the main design loads in spacecraft structures. The noise generated during ignition of rocket engines manifests itself to launch vehicle, sensitive spacecraft or satellite and launch pad in the form of airborne acoustics and structure-borne vibration. Therefore, a successful space mission requires thorough consideration of vibration interaction of vibro-acoustic effects.

The primary source of structural vibrations and internal loads during launch is due to these acoustic loads. The vibration levels can be of sufficient magnitude to cause damage such as structural fatigue and destruction of the payload inside the fairing. Due to the vibro-acoustic environment during the launch of a space vehicle extends over a broad range of frequencies (10-10000 Hz), it is necessary to develop a correct methodology to calculate the pressure distribution on the fluid domain and the

displacement field on the structural domain. For this reason, several formulations are considered and an analysis at different frequencies is carried out. In each frequency band, the nature of the fluid (air) and the structure behaviour is fundamental when selecting the solving procedure for accurate results.

To cover the entire frequency range of interest, the analysis must take into account the large number of acoustic and structural modes contributing to the dynamic response. Modal analysis procedures can be extended to predict the interior acoustic environment by identifying the structural modes of the surfaces and acoustical modes of the interior space. It should be possible to compute the structural and acoustical modes of complex dynamic systems over a broad range of frequencies using Finite Element Method (FEM). Modal synthesis will enable efficient analysis of systems in which a limited number of modes are being excited. In practice, in vibro-acoustic problems, a lot of resonant modes may be excited, and the finite element method can become computationally unfeasible.

To make a prediction of the exterior and interior acoustic fields, the Boundary Element Method (BEM) must be combined with a FEM model of the structure. The main advantage of this procedure is that only the boundary of a component needs to be discretized and, in general, this leads to reduce the number of degrees of freedom that are needed by the finite element method. On the other hand, the matrices tend to be fully populated rather than banded as in the FEM and the computational effort to ensemble the equations can be significant.

The number of degrees of freedom required by FE and BE methods can become impracticable at high frequencies. In such cases carrying out a detailed prediction of the structure response is not possible. Statistical Energy Analysis (SEA) provides prediction procedures that are appropriate for high frequencies [1]. By using a statistical description of the system and by using vibrational energy to formulate the dynamic equations, these procedures provide great simplifications to the analysis. The method involves relatively few degrees of freedom, and it is possible to perform parameter studies with little computational effort. Furthermore, SEA allows a response prediction

to be made at frequencies for which no other analysis method is available. However, in using this statistical procedure, it is not possible to predict the acoustic environment at individual points and single frequencies. The statistical prediction gives averages over spatial locations and bands of frequency.

From aforementioned it can be inferred that it is not possible to use a single prediction response method for complex structures over the entire range of frequencies. The modelling technique used to model each element depends on several parameters, i.e the number of modes per frequency band (or the modal density), the modal overlap, the coupling between subsystems, etc. In general, the applicability of SEA formulation is valid for high mode count and modal overlap and for weakly coupled subsystems [1]. In particular, this paper has focused on the study of the modal density influence on the modelling technique.

In general, this problem is solved through the resolution of a FEM model for low modal density, a SEA model for high modal density and a unique hybrid model for intermediate modal density [2]. In this report the need to realize several hybrid models to predict the vibro-acoustic response is discussed. To perform this classification the elements of the structure must be classified according to its modal density in each frequency band.

This paper is organized as follows. In section 2 the criterion to select the number of necessary models to study the structure response to acoustic loads is presented. In section 3, this procedure is studied in a typical satellite structure. In section 4, the satellite modal behaviour is compared with experimental results obtained in a modal test. Finally, the advantages of the multi-hybrid procedure over the usual single-hybrid procedure are given in section 5.

## 2. MODAL DENSITY PROCEDURE

To study the correct modelling technique to predict the structure response to acoustic loads the first step is to calculate the modal density of each structural element. Thereby, the complete system has to be divided in different sub-structures or subsystems. For FE models, the concept of subsystem is not necessary, and it is appropriate to group elements to obtain the main response of significant regions.

However, for SEA modelling the system has to be divided in different subsystems, depending on its functional behaviour. To study the main parameters affecting the system behaviour and to obtain information about energy distribution it can be necessary to split the physical subsystem in several functional subsystems. The wave behaviour has to be conserved and, therefore, the functional subsystems dimensions must be corrected to reproduce the behaviour of the original physical subsystem.

Once the sub-structures have been identified, its modal densities are studied. This parameter will determine the employed modelling methodology.

In order to evaluate the applicability of the SEA

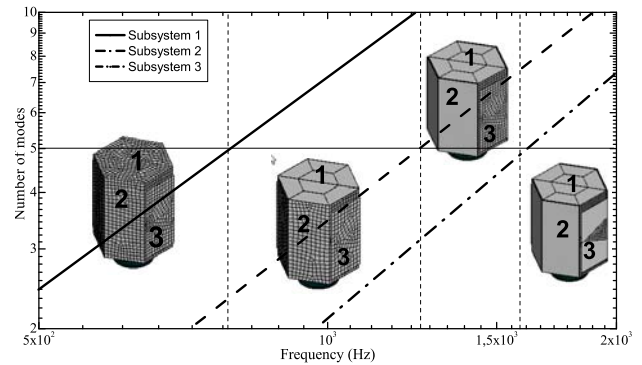


Figure 1. Application of the modal density criterion for a typical satellite structure.

formulation for each subsystem, the number of modes in each frequency band has to be studied. The critical modal count is defined as the number of modes beyond which the SEA methodology may be used. This value has been set to 5 modes per band, and it is the frontier between FEM and SEA technique. At each frequency band, subsystems with a number of modes greater than the critical modal count should be modelled with SEA while subsystems with a lower number of modes than the critical mode count are modelled with FEM.

Fig.1 shows an application example of the modal density procedure in a typical satellite structure. Modes in each frequency band versus frequency of the subsystems 1, 2 and 3 are studied. As it can be observed, several different models have to be considered in function of the frequency bands. Instead of the traditional unique hybrid model for intermediate modal density, in this example (for frequencies below 3000 Hz) three hybrid models should be studied.

From frequencies above 800 Hz, the FE model has to be replaced for a hybrid model, with the element 1 modelled through SEA. This hybrid model is valid for frequencies up to 1250 Hz. For frequencies above 2500 Hz two hybrid models have to be considered: one hybrid model for subsystems 1 and 2 modelled with SEA and another one with the three mentioned subsystems modelled through SEA.

## 3. APPLICATION OF THE MODAL DENSITY CRITERIA IN A TYPICAL SATELLITE STRUCTURE.

### 3.1. Description of the satellite structure

The proposed mock up reproduces the typical satellite structure: lower platform, upper platform, lateral faces, external solar arrays and adapter cone to the launcher.

The lower and upper platforms of the satellite are aluminium plates of 1 mm thickness for the upper and 10 mm for the base plate. The satellite body is simulated by six methacrylate lateral panels of 6 mm thickness placed

Table 1. Number of structural (or acoustic) modes in each frequency band for the elements of the system.

1/3 Octave band		Number of modes							
Center frequency (Hz)	Upper Platform	Acoustic cavity	Face	Upper/Lower solar array	Upper/Lower face	Medium face	Lower Platform	Medium solar array	Cone
630	3	0	2	2	0	0	2	0	0
800	6	3	2	2	2	1	0	1	0
1000	7	5	4	3	2	1	1	1	0
1250	8	6	5	3	3	1	1	1	0
1600	9	11	5	5	3	1	1	2	4
2000	10	22	10	6	4	0	4	2	0
2500	19	44	8	7	4	3	0	3	0
3150	14	66	14	6	5	2	2	3	0
4000	25	114	14	9	8	5	3	3	2
5000	28	235	24	11	9	6	5	4	3
6300	40	376	29	14	13	7	5	7	10
8000	47	498	42	18	20	7	10	8	5
10000	60	687	83	19	25	8	10	9	10

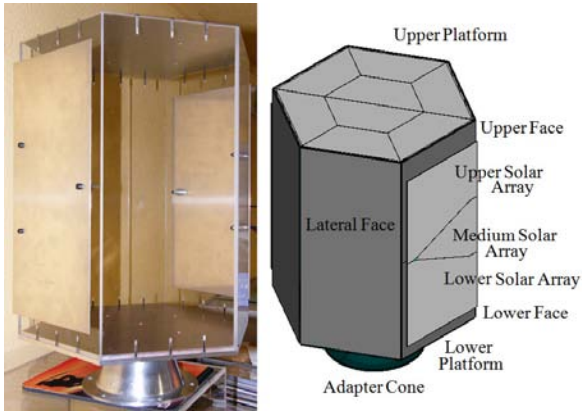


Figure 2. A picture of the satellite's mock-up (on the left) and a scheme of the subsystems used in the numerical model.

around the hexagon. The adapter cone is made of 4 mm thickness aluminium. The solar arrays are simulated using plates in aluminium of 1 mm and are attached to their corresponding lateral face through three points. In addition, the internal fluid is considered as an acoustic cavity. The entire mock up structure can be seen in Fig.2.

### 3.2. Numerical models description

The numerical proposed model reproduces the geometry of the satellite, for which the methacrylate and aluminium panels have been tested in order to calculate its mechanical properties, i.e. tensile modulus, density, etc. The upper and the lower platforms have been modelled dividing them into several parts: eight four-sided polygons in each sub-structure. In order to conserved the wave behaviour, the dimensions of each upper and lower part must be corrected to reproduce the behaviour of the original non divided subsystem. The modal response of

both structures is equal, but it is expected to obtain more information in the divided structure.

The junctions between the different elements of the satellite are as follows: line junctions between lateral faces, line junctions between lateral faces and the upper and lower platforms, point junctions between solar arrays and its corresponding lateral faces and line junctions between the adapter cone and the lower platform.

Additionally, some non real junctions have to be defined: line junctions between parts of the sub-divided panels of the upper and lower satellite platforms, and line junctions between the solar panels functional subsystems.

To predict the vibro-acoustic response of the satellite structure to acoustic loads, numerical models were developed in different frequency ranges: low, intermediate and high frequency ranges. The frequency ranges of interest are described in section 3.3.

To analyse the response of the system in the whole frequency ranges two modelling technique families were considered: deterministic formulations (Finite Element and Boundary Element Methods) and stochastic energetic formulations (Statistical Energy Analysis). For low frequencies, external surrounding air was modelled through BEM and the acoustic loads were a set of acoustic planes waves of a constant pressure of 1 Pa. For high frequencies, external air was modelled through semi-infinite fluids and the acoustic load was modelled as a Diffuse Acoustic Field of 1 Pa all over the frequency analysis range [3], [4].

### 3.3. Models classification.

Once the different subsystems have been chosen, it is necessary to specify the frequency bands for the analysis. In order to cover the entire range of frequencies during launch, the analysis has been made between 10-10000 Hz in standard third octave bands.

The number of modes per frequency band have been calculated by means of a FEM model extended to high frequencies, in order to consider the boundary conditions in the low frequency range.

This is presented in Tab.1. Subsystems with number of modes greater than the Critical Value should be modelled with SEA while subsystems with a lower number of modes than the Critical Value are modelled with FEM.

Thereby, to analyse the response of the system in the whole frequency range is necessary to model each satellite component using different techniques depending on its modal density. Taking this procedure into account and with the information of Tab. 1, ten different models have been used to predict the system response in the 10-10000 Hz range.

- Low modal density:

Model 1) FEM for structure, FEM for internal fluid and BEM for external fluid (0-300 Hz).

Model 2) FEM for structure, FEM for internal fluid and SEA for external fluid (300-800 Hz).

Model 3) FEM for structure, SEA for internal fluid and SEA for external fluid.

- Intermediate modal densities. Hybrid models:

Hybrid 1) From model 3, with SEA for the upper platform (800-1250 Hz).

Hybrid 2) From hybrid 1, with SEA for lateral faces (1250-1600 Hz).

Hybrid 3) From hybrid 2, with SEA for the upper and lower solar array plates (1600-3150 Hz).

Hybrid 4) From hybrid 3, with SEA for the upper and lower lateral faces (3150-4000 Hz).

Hybrid 5) From hybrid 4, with SEA for the medium lateral faces (4000-5000 Hz).

Hybrid 6) From hybrid 5, with SEA for the lower platform (5000-6300 Hz).

- High modal density:

Model 4) SEA for structural components and SEA for both cavity and external air (6300-10000 Hz).

As it can be observed in Tab. 1, the upper platform reaches the modes Critical value before the acoustic cavity, and the upper platform should be modelled with SEA in the frequency band of 800 Hz, while in this frequency range, the acoustic cavity should be modelled with FEM. For this reason, Model 3 should not have been considered. In the employed commercial software (VA-One) the area junction between FE acoustic cavity and SEA plate is not possible, and both elements have to be modelled with SEA simultaneously. To present the model results, Model 3 has been included to carry out a formal study of the system.

### 3.4. Results

The subsystem response to acoustic loads is shown in Figs. 3, 4, 5 and 6. In section 3.3 has been referred that

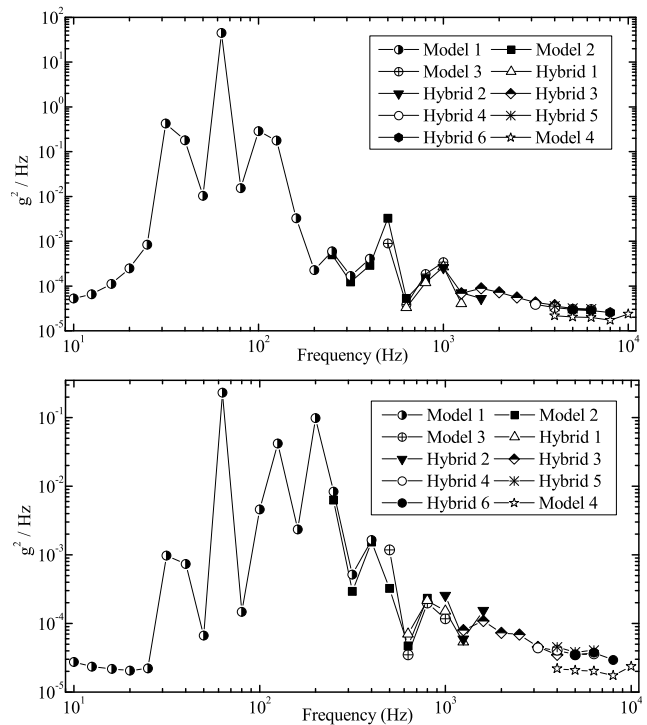


Figure 3. Upper solar array plate response (upper figure) and medium solar array plate response (lower figure) to a diffuse acoustic field of a constant pressure level (1 Pa) using the FEM-Hybrid VA-One model.

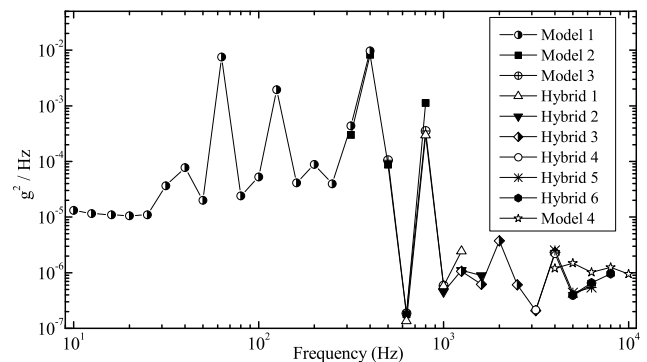


Figure 4. Adapter cone response to a diffuse acoustic field.

the acoustic cavity should be modelled through SEA for frequencies above 1000 Hz. However, this is not possible because of the characteristics of the commercial software employed.

As can be shown in Figs. 3, 4 and 5 for solar array panels, adapter cone and lateral faces, the Model 3 (FEM for structure and SEA for internal and external fluids) is quite close to the Model 2 (FEM for structure and acoustic cavity and SEA for external fluids). Hybrid model 1

response (marked with hollow triangles in those figures) is also close to the Model 3 response.

However, in Fig. 6 it can be observed a clear difference in the subsystems response for frequencies below 800 Hz. In this case, Model 3 should be excluded and the Hybrid model 1 is directly studied.

Differences between the response for BE fluids and SEA

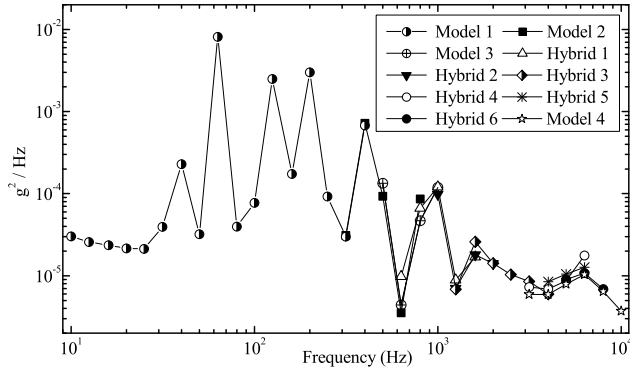


Figure 5. Lateral face response to a diffuse acoustic field.

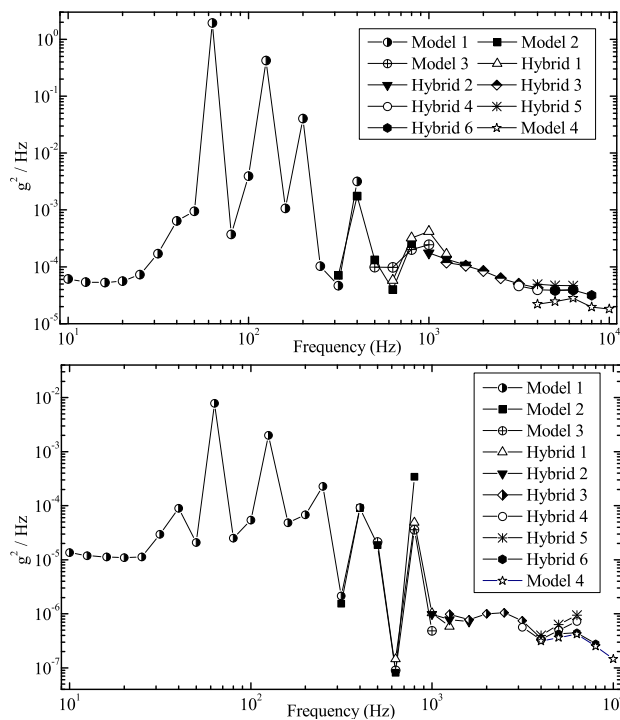


Figure 6. Upper and lower platforms response (respectively) to a diffuse acoustic field.

fluids have been found for several frequency ranges. The response of Models 1 and 2 have been studied for 1/24 octave bandwidth. The obtained results can be observed in Fig.7.

As can be shown in Fig. 7, for frequencies between 200 or 300 Hz and 700 Hz, the responses are quite equals, and SEA for external fluids will be used in this frequency range.

The results evidence that the criteria selected to model each sub-structure have been adequate, because a smooth continuity between the different models is shown, what allows to consider the whole response as a single one.

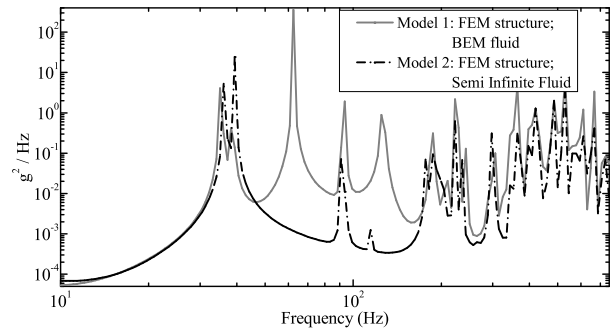


Figure 7. Differences between Models 1 and 2 to study the application range of semi infinite fluids.

#### 4. MODAL TEST

A modal test is performed to compare modal and frequency behaviour with experimental results. These experimental results are compared with the satellite numerical model. Given the uncoupled eigenvalues

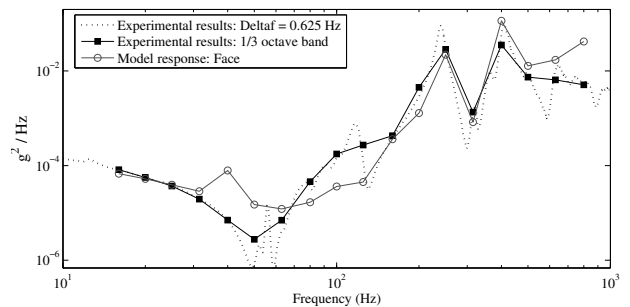


Figure 8. Correlation between experimental and numerical response on accelerometer placed in a lateral face in Narrow band and Standard Third Octaves.

analysis of the vibro-acoustic software (VA-One), the response of the system to a random point load applied on a generic point is considered. The model's natural frequencies are determined through the peaks on the response. It must be taken into account that this method depends on the analysis bandwidth and is not possible to distinguish several modes in the same band. Although a narrow band spectrum is considered, and it is enough resolution for the frequency range of study (10-1000 Hz). Instrumentation consist, basically, in 16 accelerometers distributed on the panels of the specimen: three in the upper platform, eight in the lateral faces, three in a solar array, one in the lower platform and one in the adapter cone. The mechanical excitation of the panels

is made with a shaker (Bruel and Kjaer, model 4809) and a force hammer (KISTLER, model 9724A2000). For data collection a 20 channel spectrum analyzer (OROS, model OR36) was used. In Figs. 8 and 9 can be observed a correlation between the experimental results and the numerical results obtained with Model 1 (FEM/BEM), for a lateral face and the upper platform. This figure shows a reasonable agreement between experimental and model data, and it evidences that both modal form and natural frequencies are quite similar.

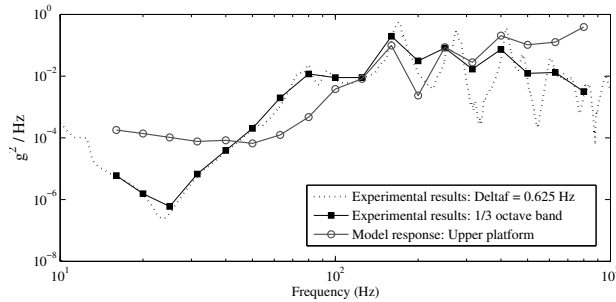


Figure 9. Correlation between experimental and numerical response on accelerometer placed in the upper platform in Narrow band and Standard Third Octaves.

## 5. ADVANTAGES OF THE MULTI-HYBRID PROCEDURE VERSUS THE USUAL SINGLE-HYBRID METHODOLOGY

The scope of this report is to analyse the vibro-acoustic response of a structure if the modelling technique depends on the frequency ranges (i.e. depending on the modal density of the structure subsystems) in order to exploit in future works the much lower computationally cost of the hybrid methodologies. The modal density criteria employed in this report has been compared with the usual methodology [2]: a unique hybrid model for the intermediate modal density range. This hybrid model has external and internal air modelled through SEA (acoustic cavity for internal air and semi infinite fluid for external air) and structural subsystems modelled through FEM except the upper platform, which is modelled with SEA (because is the satellite element with the highest modal density). If Fig.5 is compared with Fig. 10 some differences must be taken into account. While employing the multi-hybrid approach a continuity response exist, in Fig. 10 this continuity is only observed for low modal density, i.e. for frequencies below 800 Hz. For the intermediate modal density range is not possible to guarantee that continuity in the response. For this reason, several hybrid models has to be taken into account to realize a complete studio of the structure response over a broad range of frequencies. In vibro-acoustic problems, a lot of resonant modes may be excited and the computationally cost is high if the employed method is FEM or BEM. The predictions obtained using these procedures are generally quite

sensitive to small changes in design and geometry, requiring the analyses to be repeated several times as modifications are made.

Using hybrid methodologies in different frequency

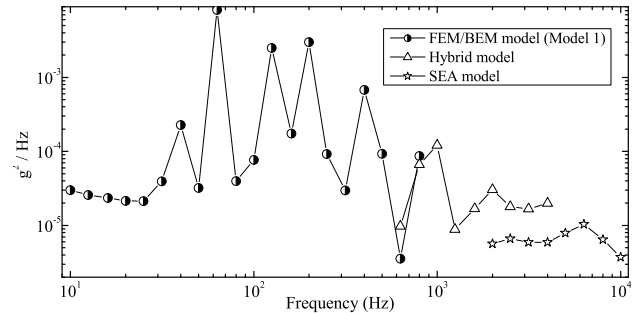


Figure 10. Subsystem response to a diffuse acoustic field of 1 Pa using the usual single-hybrid procedure.

ranges, i.e modelling some subsystems through SEA, computation time can be heavily reduced. Therefore, using the modal density criterion it is possible to obtain a better estimation of the vibro-acoustic response by reducing the computation time.

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## REFERENCES

- [1] A. Le Bot and V. Cotoni (2010). Validity diagrams of statistical energy analysis. *Journal of Sound and Vibration* **329**, 221-235.
- [2] V. Cotoni *et al.* (2007). Numerical and experimental validation of a hybrid FE-SEA method. *Journal of Acoustical Society of America* **122**, 259-270.
- [3] P. J. Shorter and R. S. Langley (2004). On the reciprocity relationship between direct field radiation and diffuse reverberant loading. *Journal of Acoustical Society of America* **117**, 85-95.
- [4] P. W. Smith (1962). Response and radiation of structural modes excited by sound, *Journal of Acoustical Society of America* **34** (5), 640-647.
- [5] A. Yarza *et al.* (2010). Acoustic response of large size space antenna reflectors in the middle frequency range. Proceedings of the Internoise Conference, Lisbon.