Analysis of large-scale digital optical neural networks by Feynman diagrams

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ABSTRACT

A new method to study large scale neural networks is presented in this paper. The basis is the use of Feynman-like diagrams. These diagrams allow the analysis of collective and cooperative phenomena with a similar methodology to the employed in the Many Body Problem. The proposed method is applied to a very simple structure composed by an string of neurons with interaction among them. It is shown that a new behaviour appears at the end of the row. This behaviour is different to the initial dynamics of a single cell. When a feedback is present, as in the case of the hippocampus, this situation becomes more complex with a whole set of new frequencies, different from the proper frequencies of the individual neurons. Application to an optical neural network is reported.

Keywords: Neural networks, cooperative phenomena, hippocampus, complex dynamics.

1. INTRODUCTION

The highly interactive nature of brain dynamics is one of the most complex problems to be solved in the next future. This interest has been maintained at its higher level during the past years. The analysis of general design principles, cellular mechanisms, network modules, and functionally specialised multilevel architectures have proceeded closely, with each level of analysis clarifying the scientific understanding of the other levels. In any one of these models, mathematical analysis and computer simulations have played a very important role by demonstrating how complex emergent properties may arise from interactions among simpler network components.

From another point of view, it is interesting to point out the way different eras have considered the brain behaviour. This behaviour have had its roots in metaphors linked with the different interests the society, both scientific and technological, had at each time[1]. So, brain theories have gone from hydraulic and mechanical metaphors in the ancient times, to electronic and optical metaphors in the past years. This time is, perhaps, the time of the computational metaphor. At the core of this metaphor is the notion of formal rules for the manipulation of symbols as well as certain ideas about data structures for representing information. At the same time, if several working models were obtained from different physical areas, different mathematical models have had their role in the above mentioned metaphors. One of the most employed models to represent neurons as conductors of Electricity is the well known cable theory[2]. This model was adopted shortly after the formulation of Oliver Heaviside, in the late nineteenth century, of the transatlantic telephone transmission. Heaviside was the first to consider the effect of resistance leak through the insulation, equivalent to the membrane resistance in nerve. Cable theory was applied coherently to nerve fibres by Hodgkin and Rushton[3], who used extracellular electrodes to measure the spread of applied current along lobster axons. This theory is still very employed and, for instance, the phenomenological model of the squid giant axon due to Hodgkin and Huxley[4] is derived from it. Several other models have been reported at the literature and most of them correspond with equivalent theories applied in other field of Science and Technology.

Most of the above reported models deal with the physical status of a neuron when a perturbation is applied to some of its points. Variations with time and position are the main results. No indication is presented about other possible types of responses. Other important parameters, as new frequencies in the output signals, are no present in the work. Just when the final objective is to study non-linear behaviours, as for example those appearing in chaotic situations, those parameters are considered. These studies are centred, in almost every case, to the dynamics of simple elements, as single neurones, or autonomous systems as the hearth.

From the above situation it looks important to analyze the possibility to apply some other methods. The first point to be considered is the way to introduce the analysis of the frequency dependence with respect to the different values of the initial

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parameters. This fact is important from the point of view that most of the neurone responses are spikes with different properties depending on the input sensory signal. As it is well know, the difference between two inputs with different strength is perceived by the sensorial system as two different train of pulses with different repetition rate. Height is the same in both cases. It is because that we can say that a certain type of digital binary signal is the system response to the external action.

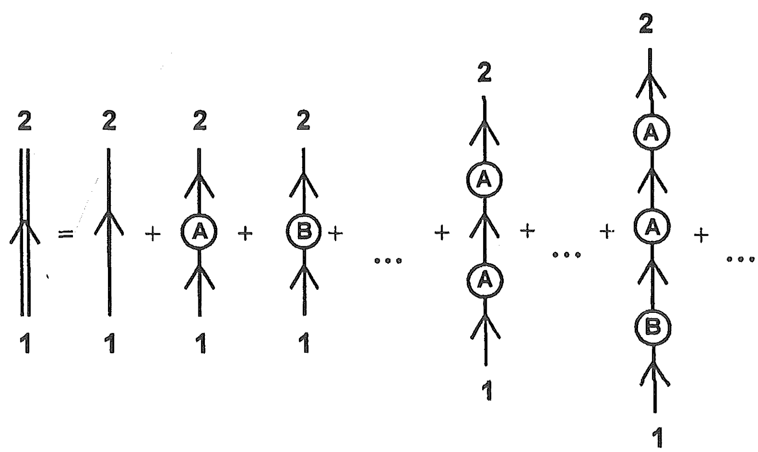
The second point to be considered in the proposed new model is the possibility to obtain a type of collective behaviour from the excitation of a single point in the system. This situation is similar to the multiple existing models in Physics. Solid State Theory, for example, is a very well known case where the properties of the whole system are very different from the individual properties of its constitutive units. Superconductivity is one of the most emblematic theories. Phase transitions and the Kondo problem are other cases where this situation have been solved[5]. These problems are now included in a general framework known as the Many Body Problem. Although different theories have been employed to explain the different phenomena one of the more powerful tools have been the use of Feynman diagrams[6]. These diagrams, introduced by R.P. Feynman in Quantum Electrodynamics, have been employed in many other fields ranging from liquid and gas states to High Energy Physics. Hence, there is the possibility to employ a similar method to study other complex systems as the neuronal ones.

From the above considerations, a new method to analyze collective behaviours in neuronal networks will be reported in this paper. Its basis was presented previously but new considerations have been taken into account[7]. It is not a real model in the sense that it does not follow closely the real behaviour of the living bodies. As a matter of fact, the exact characteristics of interactions among the different types of neurones has not been closely taken. Many simplifications have been adopted and, in certain cases, we have followed an heterodox approach. The adopted path has just the intention to show another way to handle these complex problems. We will give in Part 2 some indications about the way to operate with Feynman diagrams. Part 3 will offer a short view of the cable theory and some of the results that can be obtained from it. Part 4 will show the application of Feynman diagrams to a very straightforward situation where just one type of interaction among neurones is considered; the influence of feedbacking will be considered. Finally, part 5 will show the possibility to implement this type of network with a simple optical programmable logic cell, previously reported by us as a possible building block in Optical Computing.

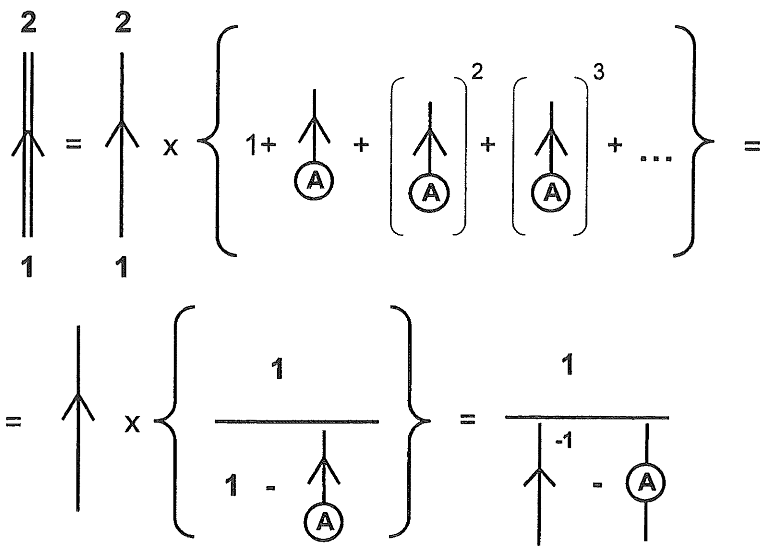
2. BIRD'S EYE VIEW OF FEYNMAN DIAGRAMS

Many body systems consisting of strongly interacting real particles can often be described as if they were composed of weakly interacting particles and collective interactions. The properties of these particles is by quantum field theoretical quantities known as Green's functions or propagators. Their importance is that they have a simple physical interpretation and they can be calculated in a way which is highly systematic and 'automatic', which appeals to one's physical intuition. The idea behind the propagator method is that the detailed description of a many-body system requires the position of each particle as a function of time. It turns out that in order to find the important physical properties of a system it is not necessary to know in detail behaviour of each particle but rather just the average behaviour of one or two typical particles. The quantities which describe this average behaviour are the one-particle propagator and the two-particle propagator respectively, and physical properties may be calculated directly from them. The one-particle propagator is the probability that the particle will be observed at a certain point at a particular time. This propagator yield directly the energies and momentum distribution of the particles and particle density and can be used to calculated some other physical parameters. This case is the situation for the 'classical' propagator. In quantum systems the 'quantum' propagator is the probability amplitude and not just the probability the involved concept.

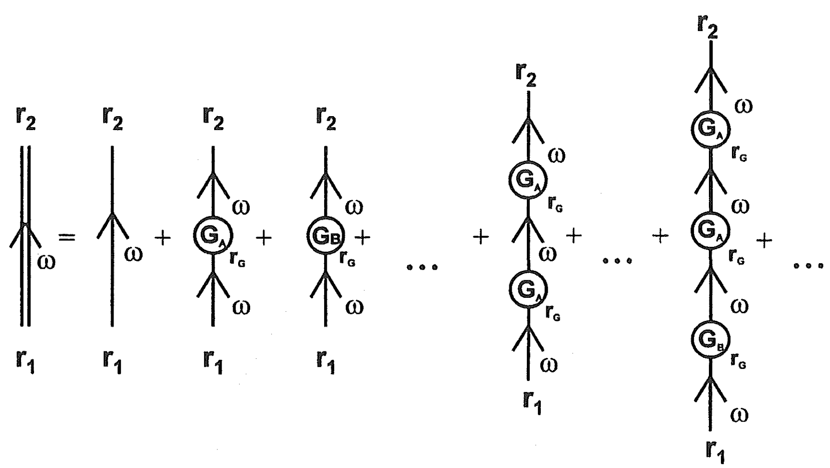
The situation for a particle going from status '1' to status '2' may be represented by the following equation



The meaning of this equation is that the probability of propagation from 1 to 2 is the sum of the probabilities for all the different ways it can propagate from 1 to 2 interacting with different situations. The first term indicates the probability to 'freely' propagate from 1 to 2, the second with an interaction A, the third with interaction B, and so on. In the case that one of these interactions is stronger than other interactions, this equations reduces to



An important point to be considered in this expression is that this propagator, going from a point x_1 at time t_1 to another point x_2 at time t_2, can be converted to the frequency space by its Fourier transform. In this case, previous equation may be written as



Similar procedure to the previous expression may be applied here. Further calculations with this type of diagrams will be shown later for the particular case we are dealing with.

3. CABLE EQUATION

The cable equation is a partial differential equation (PDE) that neurophysiologists usually express as

$$\lambda^2 \frac{\partial^2 V}{\partial x^2} - V - \tau \frac{\partial V}{\partial t} = 0$$

Here V represents the voltage difference across the membrane (interior minus exterior) as a derivation from its resting value; x represents distance along the axis of the membrane cylinder, and λ is the length constant of the core conductor. t represents time and τ is the membrane time constant of the passive membrane.

This PDE has many solutions; the problem is to construct a solution that satisfies not only the PDE but also the boundary conditions and an initial condition. There are two rather different basic solutions from which the more complicated solutions are constructed. The first one comes from the classical method of separation of variables. It can be expressed as

$$V(x,t) = (A\sin(\alpha x) + B\cos(\alpha x))e^{-(1+\alpha^2)t}$$

where A and B are arbitrary constants and α^2 is known as the separation constant.

The second one gives a different class of solutions. It can be constructed from the fundamental solution or the Green's function. This solution is known too as the instantaneous point source solution or the response function. It can be expressed as

$$V(x,t) = C_o\sqrt{\pi t}e^{-(t+x^2/4t)}$$

The singularity is located at x=0 when t=0. The value of C_o is $Q_o/(\lambda c_m)$ where λc_m represents the membrane capacitance of a λ length of cylinder.

A particular solution to this equation is represented in Fig. 1. V(x,t) appears as a function of both, x and t. According to initial conditions, perturbation is located at x=0 for t=0. In order to see a more clear representation of the variation with time, two different time instants appear at Figures 2 and 3. As it ca be seen, the variation with respect to time, at two particular points (in our case x=1 and x=4), corresponds to a typical diffusion equation for charge being transferred along the axis of the neuron.

The important point to be considered here is that no dynamical situation is obtained with this model. Because the initial perturbation is an static one, with no non-linear conditions present at the interior of the cylinder, the propagation is a classical one. There are no possibilities to obtain, for instance, a train of spikes or any other transient phenomena. It is because that a new model is necessary to handle this type of situations.

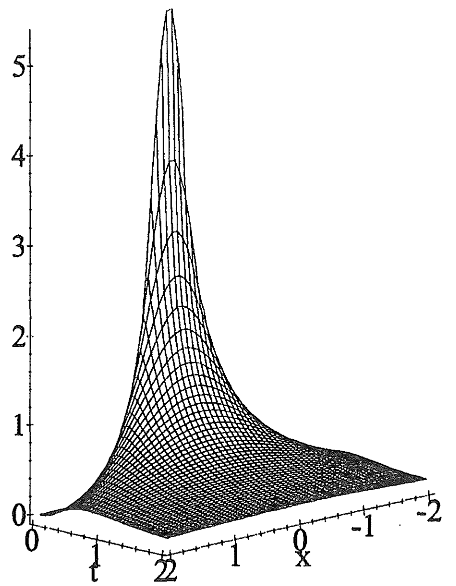


Figure 1.- Solution of the Cable equation for C_o=2

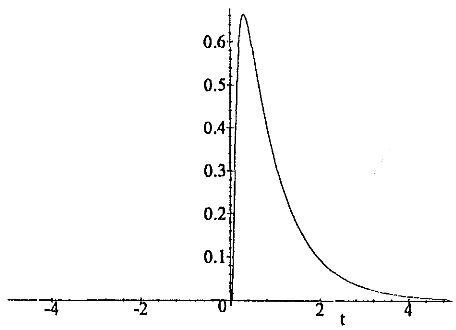


Figure 2.- Solution of the cable equation for C_o= 2 and x=1.

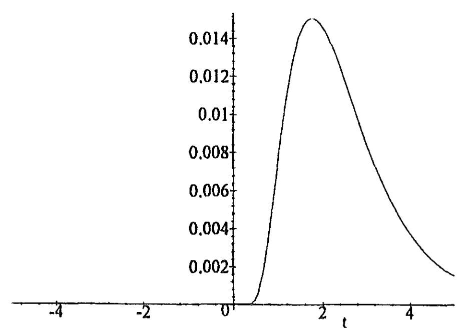


Figure 3.- Solution of the cable equation for C_o= 2 and x=4.

4. APPLICATION OF FEYNMAN DIAGRAMS TO A SIMPLE NEURAL NETWORK

A way to apply similar method to the above mentioned case is to postulate an equation for the neuronal behaviour different to the cable equation. The starting points to do that should be the possibility to introduce different external excitations to the neuron, the possibility to have interaction among different neurones and, finally, the possibility to obtain a dynamical output with some indication about the frequency content of it. According to these premises, the new equation should be

$$\left[\lambda^2 \frac{\partial^2}{\partial x^2} - 1 - S - \tau \frac{\partial}{\partial t}\right] V = 0$$

where S is the outward excitation coming either from an external action from outside the neural system or from another neuron inside the own system. It will be, in general, $S = \sum S_i$. We have kept τ as the membrane time constant of the passive membrane. But if we want to take into account the proper frequency of each neuron as well as the possibility to present oscillations of some kind this constant has to be modified. Under these conditions, the only possibility is that τ has to have an imaginary part. In order to simplify the solution of the equation, we can set equal to zero the real part of this time constant. This implies no dumping in the system. A posterior study should take into account this omission. Hence, the new equation should read

$$\left[\frac{\partial^2}{\partial x^2} - W + i \frac{\partial}{\partial t}\right] V = 0$$

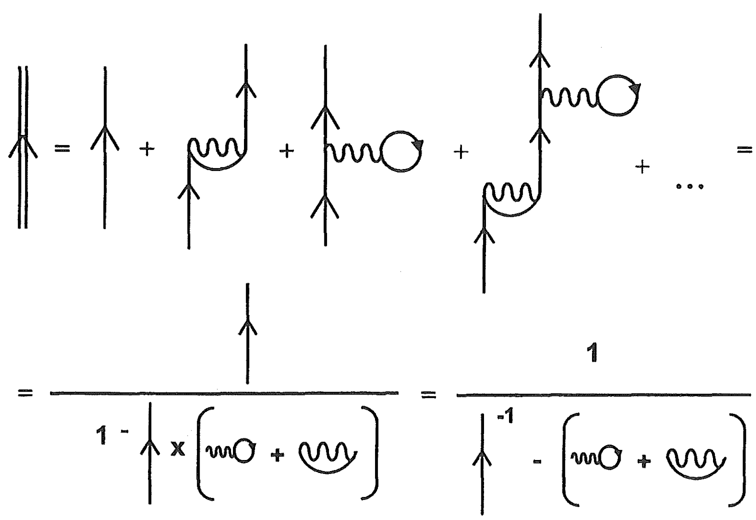
We have normalised the variables and made $W = 1 - S$. The associated Green's function will be given by

$$\left[\frac{\partial^2}{\partial x^2} - W + i \frac{\partial}{\partial t}\right] G(x, t - t') = \delta(t - t')$$

This equation may be solved in two steps. The first one should be to obtain the solution of the unperturbed state ($W = 0$) and afterwards to get the solution for the complete equation by conventional series expansion, In this case the final solution should be

$$G(t_2 - t_1) = G_o(t_2 - t_1) + \int_{-\infty}^{\infty} dt_M G_o(t_M - t_1) W G_0(t_2 - t_M) + \ldots\ldots$$

This expansion correspond to the propagator



Just two interactions have been introduced. Their physical-biological meaning will be reported elsewhere. G_0 correspond to the Green's function for the unperturbed state.

We can use for G_0 a *"displacement wave"*. This solution is not a fully real one but it is an easy way to obtain some preliminar results. The expression for G_o should read

$$G_o(t) = \left[\theta_t e^{-i\omega_o t} + \theta_{-t} e^{+i\omega t}\right]$$

with $\theta_t = \begin{cases} 1, & \text{for } t > 0 \\ 0, & \text{for } t < 0 \end{cases}$

It is convenient now to work with the Fourier transform of the above equation an so we have

$$G_o(t-t') = \frac{1}{\omega\ -\omega_o\ +i\delta} - \frac{1}{\omega+\omega_o+i\delta} = \frac{2\omega_o}{\omega^2-\omega_o^2}$$

where δ is a positive infinitesimal. In this ω-space, the solution for G is

$$G(\omega) = \frac{1}{(G_o)^{-1} - W(\omega)}$$

with W(ω) being the Fourier transform of the external perturbation. Hence, the final solution is

$$G = \frac{2\omega_o}{(\omega^2-\omega_o^2)-2\omega_o} = \frac{1}{1-\frac{2\omega_o W(\omega)}{\left(\omega^2-\omega_o^2\right)}}$$

A convenient perturbation is a pulse lasting a time T and with amplitude A. This perturbation has the form

$$W = \begin{cases} A & \text{for } |t| > T/2 \\ 0 & \text{for } |t| < T/2 \end{cases}$$

Its Fourier transform is

$$W(\omega) = AT\frac{\sin\frac{\omega T}{2}}{\frac{\omega T}{2}}$$

We can now study which type of situation will obtain when this type of perturbation is present at the system. Because the possible values for W to be obtained are in a very wide range we need to represent them in a logarithmic scale. At the same time, because we have employed a wave-like equation, we have to represent the value of G^2. The obtained result for the case $\omega_o = 3$ and A= 100 is shown in Fig. 3. We have represented W as a function of the pulse time duration T and the corresponding frequency components.

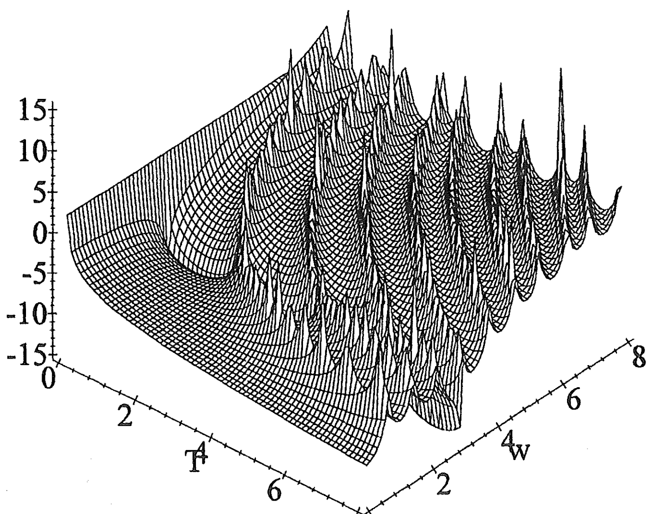


Figure 4.- Three dimensional space-frequency behaviour of the propagator at the end of a very large string of interacting neurones. Proper frequency of neuron, ω_o=3. Amplitude of interaction A=100.

Another representation is shown in Fig. 4. It corresponds to the contours or lines of equal height. As it can be seen, several new frequencies have appeared from the initial proper frequency of the neuron. At the same time, some regular structure appears. In order to get some more information about it, two lateral views are presented in Figs. 6-7. They show the profiles corresponding to views from the t-axis and from the ω-axis. It is clear from the last one that the content in frequencies goes to zero for values of ω larger than 20. Hence, no high frequency components are present, at least with a significant contribution. Another point that deserves some comments is the importance of the neuron proper frequency. At $\omega = 3$ the G value goes to almost zero (remember our representation is log G^2.

The above results indicate that if a large number of units are connected, the resulting activity offers a different behaviour than the obtained from a single neuron. This implies the beginning of a co-operative situation no present when other models are employed.

From this situation we have the possibility to analyze the effect of a possible feedback to the system. This situation is no difficult to find at higher levels of the brain. A place for that is the hippocampal circuitry. This part of the brain is a vital component in the experiential and cognitive life of human beings[8]. It is the location where significant cognitive representation of the internal and external world are formed, selected and embedded in the synaptic architecture of the brain. Bilateral ablation of the hippocampus puts an end to the embedding of new experiences. It is because that feedback constitutes here a fundamental function. As a matter of fact, there is a loop, referred in some places as the HML (hippocampal-mesocortical loop), that is seen as mediating standing reverberations that represent various candidate abstracted qualities of the external and internal worlds.

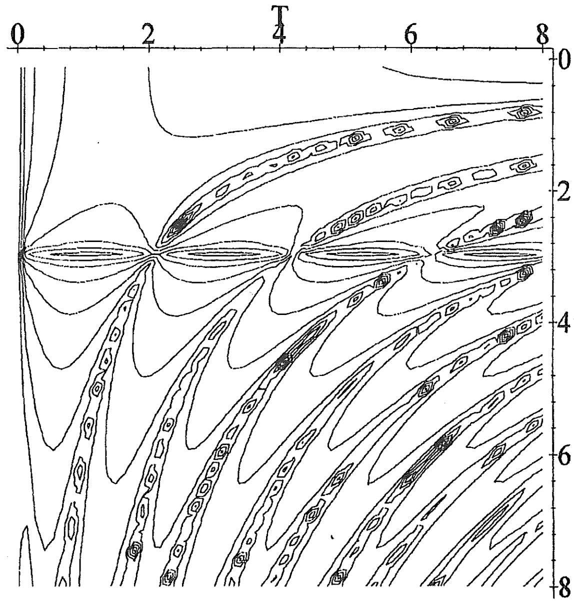


Figure 5.- Contours of Fig. 4

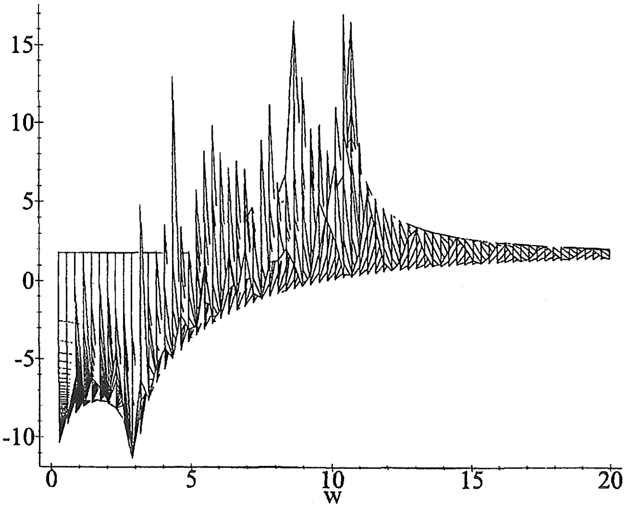


Figure 6.- Lateral view, from T=0, of Fig. 4.

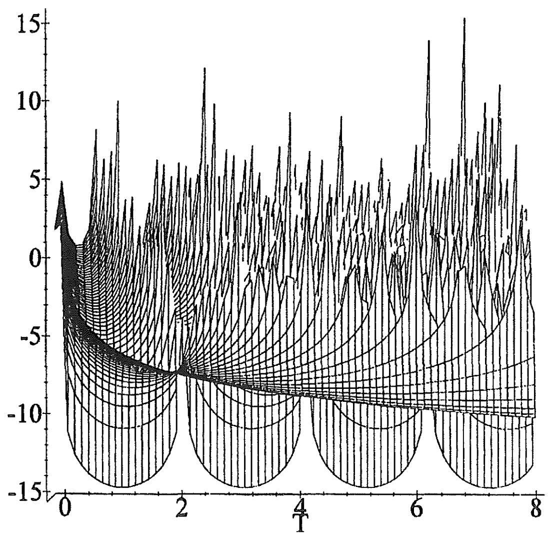


Figure 7.- Lateral view, from w=0, of Fig. 4.

In order to perform a simple analysis of how our model should apply to a configuration as the hippocampus, a feedback has been applied. The input signal to the system, that in the previously studied situation was a simple pulse, is changed now to the output signal from the previous analysis. The results are shown in Figs. 8 a-b. The main fact to be considered is that there is not a clear distinction for frequencies around $\omega = 3$. On the contrary, they appear regularly distributed on a range wider than before. If we repeat the feedback, new results are shown in Figs. 8 c-d. A new distinction appears now around $\omega = 3$. Figs 8 e-f and Figs. 9 a-d show iterations number 4 and 5. As it can be seen the distinction for $\omega = 3$ appears again at iteration 5. It disappear for higher iterations. The last run we have made, the tenth, Figs. 9 e-f, shows finally that no frequencies appear for $\omega < 3$. This indicates a substantial new behaviour, no present at the initial time. Frequencies extent up to around $\omega = 20$.

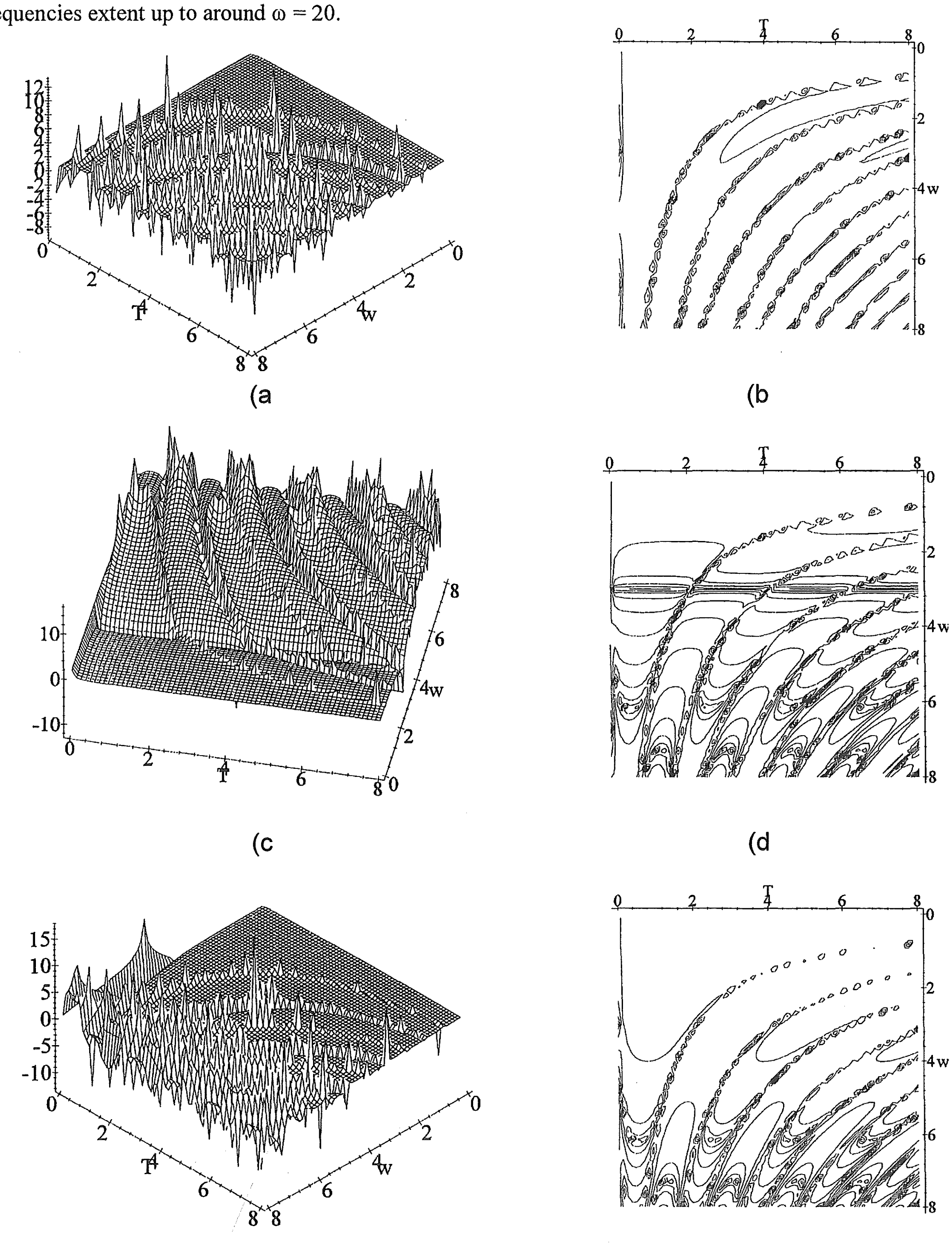


Figure 8.- Three dimensional space-frequency behaviour and contour of the propagator at the end of a very large string of interacting neurones when feedback is added. a)-b) first iteration. c)-d) second iteration. e)-f)Third iteration. Proper frequency of neuron, ω_o=3. Amplitude of interaction A=100.

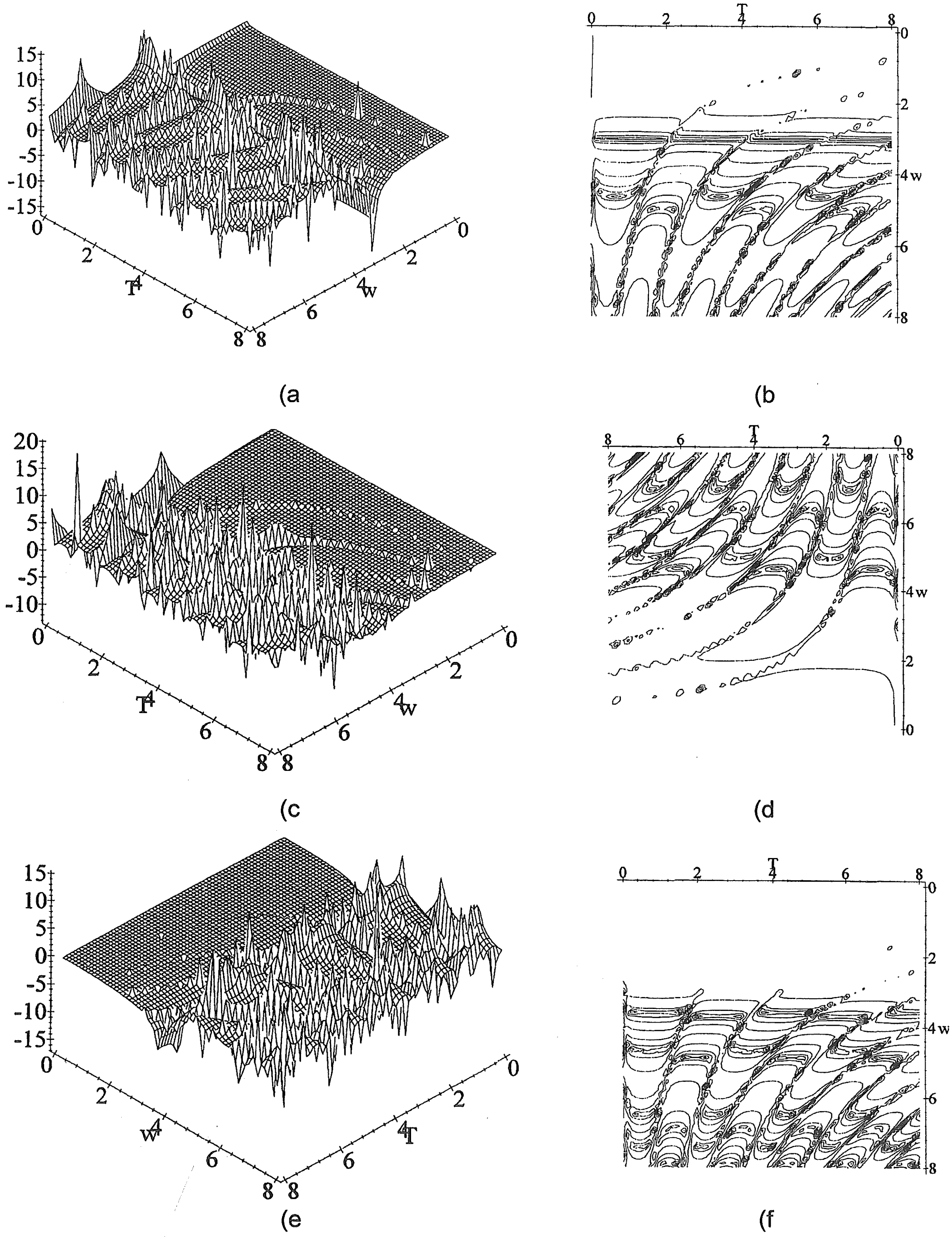


Figure 9.- Three dimensional space-frequency behaviour and contour of the propagator at the end of a very large string of interacting neurones when feedback is added. a)-b) fourth iteration. c)-d) fifth iteration. e)-f) tenth iteration. Proper frequency of neuron, ω_o=3. Amplitude of interaction A=100.

5. STUDY OF AN OPTICAL NEURAL NETWORK WITH FEEDBACK

In order to get a close analysis of above reported concepts, a simple optical neural network has been implemented. This structure was previously employed by us as a basic building block to emulate some sensorial process at the retina and the visual cortex[9-11]. The main scheme can be seen at [9]. It is composed by two nonlinear elements: an optical logic etalon and a SEED-like device. The cell has four inputs, two of them for control, and two outputs. When this block works as a logic cell is able to process the two digital input signals and to give at the output a set of fourteen logic functions of that input signals. The type of function performed depends on the input control signals. If this cell is feedbacked, connecting one of the output signals to one of the control gates, the behaviour if the system depends on the time constants of the involved elements. In some cases, a periodic output is obtained depending the period length on the feedback delay time. A nonperiodic, and under some conditions chaotic, output is obtained when there is a particular relation between internal and external delay times. When the internal delay of the cell gets close to zero, a nonperiodic situation is obtained and a very different situation appears in the system. This behaviour, as it has been shown[12], has many similarities to the above reported situation. Hence, the study of these kind of structures may bring a way to study these type of phenomena.

6. CONCLUSIONS

The study of cooperative phenomena in biological systems and living beings is of particular interest when these complex behaviours intend to explain process related to the cognitives sciences. Topics as cognition, language acquisition, the process of learning, the knowledge representation and, in general, any situation when some new concept, idea o representation is obtained, deserves the introduction of new tools. The creation activity is of particular interest from any point of view. To get any knowledge about how this activity may be generated is of paramount importance. If from an experimental point of view, technology is able to catch in the brain particular regions where these activities are generated, even more important is to develop a theory or, at a lower level, a model able to explain this phenomenon.

The model presented in this paper is just a proposal to this approach. If the lines pointed out here are going to help the theory, some better knowledge is needed about the type of interaction existing among neurons. This aspect requires a better understanding of the information transmission from cell to cell. And this understanding has to go from the biochemical to the biophysical aspects of the cells synapses. Moreover, this knowledge has to get information about the real activities in living cells, networks and systems. With this information, the value of the interaction potentials presented in this paper would have a value closer to the reality that our present simulation. But the theoretical model reported here can help in this way because we think can be easily extended to higher levels of complexity.

ACKNOWLEDGEMENTS

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REFERENCES

1. *Computational Neuroscience*, Ed.: Eric L. Schwartz. The MIT Press. Cambridge, Mass. 1990.
2. Koch, Christof and Idan Segev, *Methods in Neuronal Modelling. From Synapses to Networks*. The MIT Press. Cambridge, Mass. 1989.
3. Hodgkin, A.L. and Rushton, W.A.H., "The electrical constants of a crustacean nerve fibre" Proc. R. Soc. Lond. B, **133**, 444-479, 1946.
4. Hodgkin, A. L. and Huxley, A.F., "A quantitative description of membrane current and its application yo conduction and excitation in nerve". J. Physiol., **117**, 500-544. 1952.
5. March, N.H., W.H. Young and S. Sampanthar, "*The Many-Body Problem in Quantum Mechanics*. Cambridge University Press. Cambridge. 1967.
6. Mattuck, R.D., *A Guide to Feynman Diagrams in the Many-Body Problem* (Second Edition). Dover. New York. 1976.
7. J.A. Martín-Pereda and A. González-Marcos, "Use of Feynman diagrams in large-scale neural networks with non-linear behavior". **SPIE**, 2036, 162-173, 1993.
8. MacGregor, Ronald J., *Theoretical Mechanics of Biological Neural Networks*, Academic Press. Boston. 1993.
9. A. González-Marcos & J.A. Martín-Pereda, "Digital Chaotic Output from an Optically-Processing Element". Optical Engineering, 35, pp. 525-535, 1996.
10. J.A. Martín-Pereda & A. González-Marcos: "Some Connections between Neurophysiology and Optical Computing based on the Theory of Complexity". In "*Fluctuation Phenomena: Disorder and Nonlinarity*". Eds.: J. Bishop & L. Vázquez. pp. 107-113. World Scientific Press. Singapore. 1995.
11. J.A. Martín-Pereda and A. González-Marcos, "Image processing based on a model of the mammalian retina". In *Applications of Photonics Technology 3. Closing the Gap between Theory, Development, and Application.* Edited by G.A.Lampropoulos and R.A. Lessard. SPIE Volume 3491, 1185-1190, 1998.

12. A. González-Marcos and J.A. Martín-Pereda, "Chaos synchronization in Optically Programmable Logic Cells". In *Applications of Photonics Technology 3. Closing the Gap between Theory, Development, and Application*. Edited by G.A.Lampropoulos and R.A. Lessard. SPIE Volume 3491, 340-345. 1998.