ROBUST ROLLING STOCK UNDER UNCERTAIN DEMAND IN RAPID TRANSIT NETWORKS

Luis Cadarso
Universidad Politécnica de Madrid
Plaza del Cardenal Cisneros,3 - 28040 Madrid, Spain
luis.cadarso@upm.es
Ángel Marín
Universidad Politécnica de Madrid
Plaza del Cardenal Cisneros,3 - 28040 Madrid, Spain
angel.marin@upm.es
Luis Torres
Universidade Federal Fluminense
Rua Passos da Pátria 156, Niterói, RJ, Brasil
tepletg@vm.uff.br

Abstract

This paper focuses on the railway rolling stock circulation problem in rapid transit networks where the known demand and train schedule must be met by a given fleet. In rapid transit networks the frequencies are high and distances are relatively short. Although the distances are not very large, service times are high due to the large number of intermediate stops required to allow proper passenger flow. The previous circumstances and the reduced capacity of the depot stations and that the rolling stock is shared between the different lines, force the introduction of empty trains and a careful control on shunting operation.
In practice the future demand is generally unknown and the decisions must be based on uncertain forecast. We have developed a stochastic rolling stock formulation of the problem. The computational experiments were developed using a commercial line of the Madrid suburban rail network operated by RENFE (The main Spanish operator of suburban trains of passengers). Comparing the results obtained by deterministic scenarios and stochastic approach some useful conclusions may be obtained.

Keywords. Suburban railways, stochastic, rolling stock.

1. Introduction

In the RS problem for a daily planning period, the data and the decisions must be considered in a space-time network. The known demand and scheduling are met by a given fleet. The RS model takes decisions about the aggregation and disaggregation of the different RS in the depot stations. The problem can be stated as follows in the context of the metropolitan rapid transit networks: Given the train services' departure and arrival times as well as the expected numbers of passengers at each arc and period, find the optimal assignment of the RS to the train services considering composition changes.

For passenger demand, we use the expected number of passengers for each service given by RENFE. The expectation is taken from historical data from year 2008 corresponding to autumn season. The RS model (RSM) considers the passenger capacity in the trains with certain flexibility, as it attempts to provide a comfortable service for passengers which is also as compatible as possible with an efficient RS.

In the RS literature it has been very common to assume that demand is known. Typically this will not be the case, the actual demand is in general unknown at the time at which decisions are made. This means that the decisions on RS cannot be based on actual demand. The only information that is available is the partial knowledge of demand given through its distribution. It is known that the expected-value obtained replacing random demand may not produce very good solutions to the RS problem. In this paper we formulate the RS problem as a two-stage stochastic program with integer first stage and continuous second stage, hence, explicitly taking uncertainty into account in the decision process.

The robustness in rapid transit networks is introduced considering the empty movements and shunting operations, these operations may be sometimes difficult to operate and they can easily malfunction causing a localized incidence that could propagate through the entire network due to cascading effects. This is the case of composition changes. They will be penalized to try to avoid them selectively due to the high probability of malfunction. Alternatively, we can introduce robustness by avoiding empty train movements at rush hours. Similarly, at rush hours, the number of passengers per time period arriving at stations' platforms may be huge, so, in order to avoid lack of capacity due to train services' delays, critical train services are introduced, which are forced to have a bigger capacity.

This paper is organized as follows. A literature overview is given in Section 2. We describe the deterministic RS problem and model in Section 3. In Section 4 the stochastic RS model is defined. Section 5 contains the computational experience based on a test network and a realistic case provided by RENFE. Finally, we present our conclusions in Section 6.

2. State of Art

Alfieri, et al. (2006) determines the RS circulation for multiple RS types on a single line and on a single day. They use the concept of a transition graph. The problem is an integer multi-commodity flow problem, where a feasible path in the transition graph is to be found at the same time for each train. The objective is to minimize the number of units or the carriage-kilometers such the given passenger demand is satisfied. The above model is extended by Fioole, et al. (2006), with the possibility of combining and splitting of trains, as it happens in several locations in the Dutch timetable. They use an extended set of variables to get locally an improved description of the convex hull of the integer solutions. This method appears to improve the lower bounds substantially. Robustness is considered by counting the number of composition changes. Maróti (2006) focuses on planning problems that arise at NS. He identifies tactical, operational and short-term rolling stock planning problems and develops operations research models for describing them. Then, he analyses the considered models, investigates their computational complexity and proposes solution methods.

The allocation of RS units to the French TGV trains is studied by Ben-Khedher, et al. (1998). The RS circulation must be adjusted to the latest demand known from the seat reservation system. Therefore, this problem has a strong re-scheduling component. The objective is to maximize the expected profit for the company. A RS circulation problem related to the circulation of ICE train units in the German network was described by Mellouli and Suhl (2007). In this case, the required capacities of the trains are known a priori. Carriages and locomotives have first to be combined into train units of certain pre-specified groups, and these train units have to be routed then through the network in an optimal way. The problem is modeled as an integer multi-commodity flow problem on a multiple-layered network. Cadarso and Marín (2011) define a model to study a suburban rapid transit RS with convoys formed by three cars of the same type. The trains may be composed of one or two convoys in a dense network to attend an asymmetric demand and scheduling.

Problems with uncertain coefficients can be solved by stochastic programming techniques. Solutions approaches to stochastic programming have been studied in Rockafeller and Wets (1991). A difficulty in these approaches is to strike a proper balance between the terms of the objective function. One must achieve a tradeoff between the mean and variance of the solution, and deviations from feasibility under all scenarios.

Books on stochastic programming are Birge and Louveaux (1997), Prékopa (1995) and Kall (2005), the last of which is the one that is most easily accessible. The first two give more technical insight and are more comprehensive. A research survey on various aspects of stochastic programming is given in Ruszczynski and Shapiro (2003).

3. Deterministic Rolling Stock Problem in Rapid Transit Networks

In this section, the RS problem is described. First, the rapid transit network is introduced. Next, we describe the train services. Then, we introduce the passenger demand.

3.1 The Rapid Transit Network

In the network we can distinguish two main different types of stations represented by $s \in S$. The first type is characterized by the fact that train services only attend passenger demand. The second type is called depot station. In these, shunting operations can also be performed, that is, there exists a depot attached to the passenger station where trains are driven to be parked or to perform shunting operations.

The existing infrastructure linking different stations are represented by arcs, $a \in A$. Between two stations two different arcs exist, the first one of the senses of the movement and the second one in the opposite. Therefore, every arc a is defined by its departure and arrival station and by its length (e.g., in kilometers).

The planning time is discretized into time periods, $t \in T$. Due to the high train frequencies, the duration of a time period is set to one minute. The previous physical network is replicated as many times as time periods exist in the planning period (e.g., 20 hours).

3.2 Train Services

Once the space-time network is defined, train services within the network are known. Each train service is represented by $l \in L$. Train services are defined as commercial trains operating in the network to attend passenger demand. They are characterized by: their departure depot station; their arrival depot station; every arc they come through, defined by $a \in Al \subset A$; and, their departure time.

Each RS material (type of train) $m \in M$ must be assigned to a train service l. Since a train service starting in one line must stay in the same line until it finishes the material assigned must be equal for the entire train service, that is, different material cannot be mixed to form other/new train services.

As mentioned before, different cars and locomotives types exist, and naturally, a carriage cannot move without a locomotive. In order to enable train movement carriages are attached to locomotives of the same material. When locomotives are attached to both sides of the carriage a convoy is obtained, which can move in both directions. The composition $c \in C$ assigned to a train service will be given by the number of convoys forming the train service, being always from the same material. A train type and composition is given by $m,c \in M, C$.

3.3 Passenger Demand

The passenger demand for this problem is treated as a passenger flow $g_{a,l}$ through each arc a belonging to each train service l. In the deterministic RS the passenger flow is obtained from historical data under normal conditions, that is, assuming that the train services matched the designed timetable. In this case, the passenger flow $g_{a,l}$ is known.

Under the above hypothesis, the model will treat the passengers under a centralized point of view, that is, only the operator criteria are optimized. However, since the proposed problem is a suburban rapid transit network problem, it is obvious that every passenger will have the choice of choosing any other available company or transportation mode. Thus, the operator has to factor in the passenger behavior in order to avoid losing passengers to other transportation companies.

For each convoy formed by one material type m, the passenger capacity is known. There is a fixed seating passenger capacity, and then, the standing passenger capacity. For standing passenger capacity multiple possibilities arises. The aim is to obtain for every train service an adequate passenger capacity. This may be obtained with different configurations for standing passengers. It is considered that when the seating capacity is full and there are less than 3 pax/m^2 standing, there is a comfortable capacity. If this capacity is exceeded, the passengers above this level are only modestly penalized, and they are deemed passengers in excess. However, if standing passengers are below 4 pax/m^2 standing they receive a medium penalty. This is due to the fact that the operator would like to obtain 3.5 pax/m^2. Passengers exceeding 4 pax/m^2 standing are highly penalized because this situation is deemed uncomfortable.

3.4 Robustness

As mentioned before, robustness is introduced through composition changes and empty movements.

When a composition change is performed, multiple failures can occur, forcing the train to be parked for a long time and causing an incident. Containment of cascading effects is easier if the incident occurs during off-peak hours when more RS material is available at the depot station. In our model, this is treated by harshly penalizing composition changes during rush hours (see coefficient $\hat{\upsilon}_{s,t}$, which depends on the station s and the time period t).

Similarly, empty movements during rush hours complicate network network operation. Therefore, empty movements during rush hours are also heavily penalizaed. This idea is represented by the $\theta_{s,s',t}$ coefficient, which penalizes empty movements between depot stations s, s' within departure time period t.

Another aspect that could be interpreted as robustness is the critical train. A small delay at departure time may change the actual passenger flow. A train service is considered a critical train if it comes through stations that have a large number of passengers in a given time period arriving at the platform during rush hours. This is reflected in our model through a greater penalty per passenger in excess for these train services, according to the operator's wishes.

Finally, the system is made more robust by assigning only one material type per line (i.e., for every train service operating the same line, the material must be equal). This constraint allows for all material on one line to be swapped between different train services at depot stations serving that line.

3.5 The Deterministic Rolling Stock Model

In the RS model (RSM) the type and composition to form the train services are determined. The convoys are usually formed by three cars of the same type. Convoys from different types are not compatible and they cannot be mixed in the same train.

In the RS model, the relationships between the data and variables are considered within a directed space-time graph, G(S,A), where S is the stations' set and A is the arc' set. Each arc a is defined by (s,t,s',t'), where s and s' are the origin and destination nodes, t is the departure time, and t' is the arrival time. This is, $t'=t+t_a$, where t_a is the arc train time to move from s to s'. It is assumed that this time is known and fixed for each arc. This means that in the RSM where an arc is denoted by a, this may be understood as a=(s,s',t).

The RSM arises as an extension of the model proposed in Cadarso and Marín (2011). In the model presented in this paper, special attention is given to shunting in depot stations and robustness aspects. The RSM mathematical formulation follows:

- Sets:

- L(l): train services set. Each train service is characterized by an origin, a destination and a departure time.
- T(t): time periods set.
- S(s): stations set.
- A(a): arcs set.
- M(m): convoy types set.
- C(c): convoy number set. A convoy is a carriage composition. It can move in both directions. However, a carriage needs a locomotive to move. The index of this set is the number of convoys composing the train.
- Al(a,l): = 1, if arc a is used by the train service l; = 0, otherwise.
- Sl(s,l): = 1, if station s has the minimum platform length in service l; = 0, otherwise.
- Sc(s): depot stations set.

- Parameters:

- $c_{m,c}$: operating cost per rolled kilometer of convoy type m using c convoys.
- ic_m : convoy of type m investment cost.
- $pen_{a,l}^{3\text{-}4}$: penalty per passenger in excess between 3 and 4 pax/m^2 in arc a and train service l.
- $pen_{a,l}^{4\text{-}10}$: penalty per passenger in excess between 4 and 10 pax/m^2 in arc a and train service l.
- $\theta_{s,s',t}$: penalty for empty movement between deposit stations s,s' with departure time period t.
- $\ddot{\upsilon}_{s,t}$: cost for composition change in station s and time period t.
- $g_{a,l}$: expected passenger flow in arc a used by train service l.
- q_m^3: passenger capacity (seating+standing) for 3 pax/m^2 configuration in convoys of type m.
- q_m^4: passenger capacity (seating+standing) for 4 pax/m^2 configuration in convoys of type m.
- q_m^{10}: passenger capacity (seating+standing) for 10 pax/m^2 configuration in convoys of type m. This is a large number in order to avoid infeasible solutions.
- o_c: ordinal of c.
- Km_l: kilometers rolled by train service l.

- Variables:

- $x_{l,m,c}$: = 1, if train service l uses convoy type and composition (m,c); = 0, otherwise.
- yn_m: integer variable, number of convoy type m to buy.

- $\pi_{a,l}^{3\text{-}4}$: positive variable, number of passengers in excess between 3 and 4 pax/m^2 that use the train service l at arc a.
- $\pi_{a,l}^{4\text{-}10}$: positive variable, number of passengers in excess between 4 and 10 pax/m^2 that use the train set l at arc a.
- $cc_{s,t}^{m,c}$: = 1, if a composition change is performed at station s at period t from type and composition (m,c); = 0, otherwise.
- $em_{s,s't}^{m,c}$: =1, if empty movement from s to s' is started at station s at period t, with type and composition (m,c); = 0, otherwise.

The RSM for rapid transit networks can be formulated as multicommodity flow model presented in the next paragraphs.

3.6 Objective Function

$$\min \; z = \sum_{l\in L}\sum_{m\in M}\sum_{c\in C} c_{m,c}.km_l.x_{l,m,c} + \sum_{s,s'\in Sc}\sum_{t\in T}\sum_{m\in M}\sum_{c\in C} \theta_{s,s',t}.c_{m,c}.km_{s,s'}\, sem_{,s',t}^{m,c} +$$
$$\sum_{s\in Sc}\sum_{t\in T}\sum_{m\in M}\sum_{c\in C} \ddot{\upsilon}_{s,t}.cc_{s,t}^{m,c} +$$
$$\sum_{m\in M} ic_m.yn_m + \sum_{l\in L}\sum_{a\in Al} pen_{a,l}^{3\text{-}4}.\pi_{a,l}^{3\text{-}4} + \sum_{l\in L}\sum_{a\in Al} pen_{a,l}^{4\text{-}10}.\pi_{a,l}^{4\text{-}10}$$

In the objective function, different costs are minimized. First, commercial train services' operating costs (TSOC) are minimized. In the second term, empty movements' operating costs (EMOC) are also minimized. They are equal to those related with commercial service trains. However, the coefficient $\theta_{s,s't}$ increases these costs for some empty movements in order to introduce robustness in the system. Another shunting cost (CCC) is also minimized in the third term, related to composition changes. Through coefficient $\ddot{\upsilon}_{s,t}$, the composition change cost depending on station and time period is introduced. Minimizing the composition changes, robustness is introduced because they usually malfunction. A special cost (NTC) is introduced in the fourth term to take into account the possibility of buying new material. For computational purposes, this is equivalent to an infinity cost in order to avoid infeasibilities in the model. Finally, costs related to passengers in excess are introduced. A first cost (EDP1) appears for standing passengers between 3 pax/m^2 and 4 pax/m^2. Then, another cost (EDP2) for standing passengers between 4 pax/m^2 and 10 pax/m^2. Both terms contribute to minimize the number of passenger in excess.
Decision variables are subject to the demand and other constraints

3.7 Demand Constraints

$$\sum_{m\in M}\sum_{c\in C} o_c.q_m^{3}.x_{l,m,c} \geq g_{a,l} - \pi_{a,l}^{3\text{-}4} - \pi_{a,l}^{4\text{-}10}, \qquad \forall\, l\; \varepsilon\; L,\; a\; \varepsilon\; Al,$$
$$\pi_{a,l}^{3\text{-}4} \leq \sum_{m\in M}\sum_{c\in C} o_c.(q_m^{4} - q_m^{3}).\, x_{l,m,c}, \qquad \forall\, l\; \varepsilon\; L,\; a\; \varepsilon\; Al,$$
$$\pi_{a,l}^{4\text{-}10} \leq \sum_{m\in M}\sum_{c\in C} o_c.(q_m^{10} - q_m^{4}).\, x_{l,m,c}, \qquad \forall\, l\; \varepsilon\; L,\; a\; \varepsilon\; Al$$

The first constraint ensures that the assigned capacity to each train service is enough to satisfy demand requirements for passengers in 3 pax/m^2. If the capacity is not enough, passengers in

excess are calculated by the constraint. These passengers are limited in number by the second and third constraints, one for each group of passengers.
Other constraints are the train service, the material and the shunting constraints. These constraints may be study in the reference described by Cadarso and Marín (2011).

4. Stochastic Rolling Stock

So far, we only considered the deterministic Rolling Stock, where the assumption that the demand is known will not be justified since uncertainty is almost always an inherent feature of the railway operations involving the assessment of future demand. The demand is highly uncertain, so expected demand values may not reflect properly the final demand realization. Hence, we explicitly consider several potential market scenarios in making decisions. This future scenario based concepts lead naturally to a two-stage stochastic approach. The scenarios must reflect the different distribution of the demand (disaggregated by arcs and periods) for each day of the week.
In this section we explicitly take this uncertainty into account by formulating the RS problem as a two-stage stochastic program with integer first stage and continuous second stage.
The probability space is often modeled as a finite set of scenarios. Then, in principle, a linear optimization problem with stochastic input data, chance constraints and some stochastic objective function is a linear program itself. But the size of the original program is multiplied by the number of scenarios.
In the stochastic model, the time is modeled discretely by means of stages, corresponding to the available information. Each step decisions can be made after observing the realizations of the random parameters. If all uncertainty is resolved at the same moment, this is captured by a recourse model with two stages: present and future, and such models are known as two-stage stochastic programming models. The objective is minimizing the total expected cost.
We consider, first stochastic RSM (SRSM) as a classical two stage stochastic programming with fixed recourses, second the SRSM is formulated.

4.1 The Stochastic RS model as a two Stage Stochastic Mixed-Integer Programming

The fact that demand is not known with certainty is incorporated in the RS problem by allowing demand in each arc to depend on the outcome of some random variable. The number of RS to assign on the network must be decided in advance to the point in time at which they are actually installed and operated. The routing of passengers is naturally postponed until the actual realization of the demand is observed.
Hence decisions can be split in two: first stage decisions x based solely on the demand information given by its distribution. The first stage decision variable x is defined by the variables x, em, yt, yn, cc of the RSM and a second stage decisions variable y defined by the variables $\pi_{a,l}^{3\text{-}4,w}$ and $\pi_{a,l}^{4\text{-}5,w}$ of the RSM determined after arc demand g(w) has been observed.
The classical two stage stochastic linear program with fixed recourse is the problem of finding:

$$\min\ z = c^T x + E_{w\varepsilon\Omega}[Q(x,w)]$$

subject to:

$$\begin{aligned} Ax &= b, \\ W(w)\, y(w) &= g(w) - T(w)\, x, \\ x \,\varepsilon\, X &= \{0,1\}^n, \\ y(w) &\geq 0. \end{aligned}$$

Corresponding to x are the first stage vectors and matrices c, b, and A, where c are the RSM objective function coefficients of the variable x.

In the second stage, a number of random events $w \varepsilon \Omega$ may realize. For a given realization w, the second stage problem data q(w), g(w), W(w) and T(w) become known, where q are the RSM objective function coefficients (pen^{1-4}, pen^{4-10}) of the variable y. Then, the second stage decision y(w) must be made.
The objective function of the stochastic RS model contains a deterministic term $c^T x$ and the expectation of the second stage objective Q(x,w) taken over all realizations of the random event w. This second stage is the more difficult one because, for each w, the value y_w is the solution of a linear program. To stress this fact, a deterministic equivalent program can be used. For a given realization w, let

$$Q(x,w) = \min_y\{q(w)^T y \ ; W(w)\, y = g(w) - T(w)\, x; \ y \geq 0\}$$

be the second stage value function.
We assume that the random vector ξ has finite support. Let e=1,...,E index its possible realizations and let p_e be their probabilities. Under this assumption, we now write the deterministic equivalent program, in the extensive form:

$$\min \ z = c^T x + \sum_{e \in E} p_e q_e^T y_e$$

subject to:

$$\begin{aligned} Ax &= b, \\ W_e\, y_e &= g_e - T_e\, x, \quad \forall e \varepsilon E \\ x \varepsilon X &= \{0,1\}^n, \\ y_e &\geq 0, \qquad \forall e \varepsilon E. \end{aligned}$$

4.2 Stochastic RS model

In this way, the deterministic equivalent problem to the Stochastic RSM (SRSM) would be similar to that introduced by demand constraints of the RS model, but now considering the different possible realizations. Thus, some RS parameters and variables will depend on the scenario index $e \varepsilon E$. The SRSM arises as follows:

- Sets:

  E(e): finite scenarios' set.

- Parameters:

  $g_{a,j}^e$: expected passenger flow in arc a using the train service l at scenario e.
  p_e: probability of scenario e.
  $pen_{a,l}^{3-4,e}$: penalty pe r passenger in excess between 3 and 4 pax/m^2 in arc a and train service l at scenario e.
  $pen_{a,l}^{4-10,e}$: penalty per passenger in excess between 4 and 10 pax/m^2 in arc a and train service l at scenario e.

- Variables:

  $\pi_{a,l}^{3-4,e}$: positive variable, number of passengers in excess between 3 and 4 of scenario e, pax/m^2 that use the train service l at arc a.
  $\pi_{a,l}^{4-10,e}$: positive variable, number of passengers in excess between 4 and 10 of scenario e, pax/m^2 that use the train service l at arc a.

4.2.1 Objective Function

$$\min z = \sum_{l\in L}\sum_{m\in M}\sum_{c\in C} c_{m,c} \cdot km_l \cdot x_{l,m,c} + \sum_{s,s'\in Sc}\sum_{t\in T}\sum_{m\in M}\sum_{c\in C} \theta_{s,s',t} \cdot c_{m,c} \cdot km_{s,s'} \cdot sem_{,s't}^{m,c} +$$
$$\sum_{s\in Sc}\sum_{t\in T}\sum_{m\in M}\sum_{c\in C} \ddot{\upsilon}_{s,t} \cdot cc_{s,t}^{m,c} + \sum_{m\in M} ic_m \cdot yn_m +$$
$$\sum_{c\in C}\sum_{l\in L}\sum_{a\in Al} p_e \cdot pen_{a,l}^{3\text{-}4,e} \cdot \pi_{a,l}^{3\text{-}4,e} + \sum_{c\in C}\sum_{l\in L}\sum_{a\in Al} p_e \cdot pen_{a,l}^{4\text{-}10,e} \cdot \pi_{a,l}^{4\text{-}10,e}$$

subject to:

4.2.2 Demand Constraints

$$g_{a,l}^{e} - \pi_{a,l}^{3\text{-}4,e} - \pi_{a,l}^{4\text{-}10,e} \le \sum_{m\in M}\sum_{c\in C} o_c \cdot q_m^{3} \cdot x_{l,m,c}, \qquad \forall\, l \in L, a \in Al, e \in E$$
$$\pi_{a,l}^{3\text{-}4,e} \le \sum_{m\in M}\sum_{c\in C} o_c \cdot (q_m^{4} - q_m^{3}) \cdot x_{l,m,c}, \qquad \forall\, l \in L, a \in Al, e \in E$$
$$\pi_{a,l}^{4\text{-}10,e} \le \sum_{m\in M}\sum_{c\in C} o_c \cdot (q_m^{10} - q_m^{4}) \cdot x_{l,m,c}, \qquad \forall\, l \in L, a \in Al, e \in E$$

4.2.3 Other Constraints

The rest of constraints are the same, plus the new variable domain:

$$\pi_{a,l}^{3\text{-}4,e} \in R^{+}, \qquad \forall\, l \in L, a \in Al, e \in E,$$
$$\pi_{a,l}^{4\text{-}10,e} \in R^{+}, \qquad \forall\, l \in L, a \in Al, e \in E$$

5 Computational Experiences

The network used in the tests is the line C5, a realistic case drawn from RENFE's regional network in Madrid, also known as "Cercanías Madrid". We have chosen this line because it is the one with the highest frequency number in the network. RENFE performs the same planning during some days, for example one week: they perform identical commercial services, empty movements, shunting operations, etc. However, demand varies slightly from one day to the next one; in order to obtain an optimal planning for different demand scenarios, the previous SRSM is applied. Then, the planning is changed for the weekend. The presented study case has common depot stations. Our runs have been performed on a Personal Computer with an Intel Core2 Quad Q9950 CPU at 2.83 GHz and 8 GB of RAM, running under Windows Vista 64Bit, and our programs have been implemented in GAMS/Cplex 11.1.

5.1 Study Case: Line C5

Line C5 has more than 320 train services scheduled for a single day with frequencies in the order of 3 minutes at rush hours, equivalent to the rotation time in this line. The line is composed of 22 stations and 4 depot stations. There is one material type available and the train services can go in simple (one convoy) or double (two convoys) composition.

The convoys for the material in Line C5 have determined characteristics, which we can see in Table 1. The train capacity is divided into seating and standing passengers. Seats are fixed but for standing passengers a density value is defined. In every convoy, for example, for a density of 3 pax/m^2 we would have the 240 fixed seats plus 261 standing passengers, that is, 501 passengers by convoy. But, for a density of 4 pax/m^2, we would have the same 240 seats but a total of 588 passengers capacity.

For a daily planning time from 5:00 am. to 1:00 am. divided into one minute periods, we have 1140 time periods. The rotation time is of 3 minutes for every depot station.
We generate a number of demands, in these instances, with three demand scenarios, with the same probability of 1/3, such that one of the scenario is the expected of normal passenger flow, denoted by nd. The first line of table 2 corresponds to (nd,1.20nd,0.80nd) case, the second lines corresponds to (nd,1.30nd,0.70nd) case. The next lines 3 and 4 of table 2 correspond to (nd,1.40nd,0.60nd) and (nd,1.50nd,0.50nd) respectively.
For the stochastic case, the number of single equations is 112537, the number of single variables is 250032, the number of discrete variables is 70526 and the number of non zero elements is 890201. For the deterministic case, the number of single equations is 89317, of the single variables is 234552, of the discrete variables is 70526 and finally the number of the non zero elements is 797321. It can be seen that as the number of scenarios increasing, the stochastic problem increases the number of variables and the number of constraints.
We present three sets of experiments. The first set of experiments, table 2, is on problems with stochastic demand processes. The second one, table 3, includes problems with deterministic demand processes. The penalties for these experiments are $pen_{a,l}^{3\text{-}4,e} = 1$ and $pen_{a,l}^{4\text{-}10,e} = 5$. The fourth table compares the stochastic demand instance with the above penalties and the $pen_{a,l}^{3\text{-}4,e} = 3$ and $pen_{a,l}^{4\text{-}10,e} = 6$ penalties. In all these sets of experiments, the running times, given in seconds, are not shown because of the low computational times.

Table 1: Convoy Capacity and Length

Material Type	Seats	Standing	Density (Pax/m^2)	Length (m)
		261	3	
m1	240	348	4	80
		870	10	

Table 2: Computational Results for the Stochastic Case

# C	# CC	TSOC	EMOC	EDP1	EDP2	z
67	22	85548.72	1320.24	4258.06	2742.63	94309.65
69	20	89142.96	1124.64	6081.80	6575.42	103324.82
72	20	93411.12	1350.32	8091.93	12257.19	115510.56
72	18	95283.12	1425.20	11576.07	23941.50	132585.89

The table 2 does not include the NTC cost, because of the respective variable yn_m is zero, it means it is not necessary to buy convoy type m. Also, it is not included the CC cost for the relatively low value and almost constant.
It can be seen from this table 2, that the value of z is greatly influenced by the commercial train services' operating costs (TSOC).
It is noted from line 4, table 2, that we are using all the convoys available # C = 72, this is because of the high demand.
Table 3 reports the stochastic case taking the second instance of table 2, that is, the demand scenarios are (nd,1.30nd,0.70nd). The deterministic cases are defined as Det1, Det2, Det3 related to each demand scenario, and Mean, line 5, represents the mean taking the three scenarios with the respective probability. From this table, it is observed that the values related to the Mean case are very similar to the Det1, which represents the expected passenger flow. When the demand is low, that is 0.70nd for the Det3 case, we only need 51 convoys and the cost (EDP2) for standing passengers between 4 pax/m^2 and 10 pax/m^2 is practically null.

Table 3: Computational Results for Stochastic and Deterministic Cases

Cases	# C	# CC	TSOC	EMOC	EDP1	EDP2	z
Stoch	69	20	89142.96	1124.64	6081.80	6575.42	103324.82
Det1	64	20	80099.76	1265.04	3079.00	475.00	85318.80
Det2	72	22	93439.92	1198.72	12552.60	16050.50	123681.74
Det3	51	6	65895.60	415.04	1786.10	7.00	68223.74
Mean	64	20	80099.76	1265.04	3098.57	248.71	85112.08

Now, when we are using the fourth table which compares different penalties for the (nd,1.30nd,0.70nd) case, practically all the values of the corresponding line of $pen_{a,l}^{3\text{-}4,e} = 3$ and $pen_{a,l}^{4\text{-}10,e} = 6$ penalties are higher than the original penalties of our problem.

Table 4: Computational Results for Stochastic Case with different Penalties

Penalties	# C	# CC	TSOC	EMOC	EDP1	EDP2	z
1, 5	69	20	89142.96	1124.64	6081.80	6575.42	103324.82
3, 6	72	22	93126.00	1198.72	13509.84	6396.59	114671.15

6. Conclusions

We have considered the robust rolling stock with uncertain demand in Rapid Transit Networks. We have investigated the stochastic rolling stock formulation and presented the computational results for several examples, with three scenario demand, using a realistic problem. The decision maker now has several options to decide what alternatives he has to make for solving the respective model. Certainly, as the number of scenarios increases, the decision maker has a better information of the related problem, but the number of variables and the number of constraints increases enormously, and the stochastic problem will be huge and may be almost imposible to solve it. If this is the case, some decomposition techniques could be used in order to reduce in some form the dimension of the problem. Finally, the computational approach allows us to explore many situations that will be considered for future research.

7. Acknowledgments

This research was supported by project grant TRA2008-06782-C02-01 of the "Ministerio de Ciencia e Innovación, Spain". This work was done while the third author was visiting the UPM and he is grateful to Professor Marín for his hospitality.

References

Alfieri, A., Groot, R., Kroon, L.G. and Schrijver, A., (2006), A. Efficient circulation of railway rolling stock. Transportation Science, 40, 378-391.

Ben-Khedher, N., Kintanar, J., Queille, C. and Stripling, W. (1998), Schedule Optimization at SNCF: From Conception to Day of Departure. Interfaces, 28, 6-23.

Birge, J.R. and Louveaux, F., (1997) Introduction to Stochastic programming, Springer Series in operations Research.

Cadarso, L., Marín, A., (2011). Robust Rolling Stock in Rapid Transit Networks. Computers and Operations Research, 32, 1131-1142.

Fioole, P.G., Kroon, L.G., Maróti, G. and Schrijver, A., (2006), A rolling stock circulation model for combining and splitting of passenger trains. European Journal of Operational Research, 174(2):1281-1297.

Kall, P. and Mayer, J., (2005). Stochastic linear programming: Models, Theory and Computation, Kluwer Academic Publishers, New York.

Maróti, G., (2006), Operations Research Models for Railway Rolling Stock Planning, PhD thesis, Eindhoven, Technische Universiteit Eindhoven.

Mellouli, T. and Suhl, L. (20007). Rotation planning of locomotive and carriage groups with shared capacities. In F. Geraets, L.G. Kroon, A. Schbel, D. Wagner, and C. Zaroliagis, editors, Algorithmic Methods for Railway Optimization. Springer, Berlin, Germany.

Prékopa, A., (1995), Stochastic Programming, Kluwer Academic Publishers, Dordrecht.

Rockafeller, R.T. and Wets, R.J.B., (1991), Scenarios and policy in optimization under uncertainty, Mathematics of Operations Research,16, 119-147.

Ruszczynski, A. and Shapiro, A., (2003), Stochastic Programming. Handbook in Operations Research and Management Science, volume 10. Elsevier.